

Electromagnetics:

Introduction to Transmission Lines

TEM Wave along Parallel-Plate Transmission Line

(9-1, 9-2)

Yoonchan Jeong

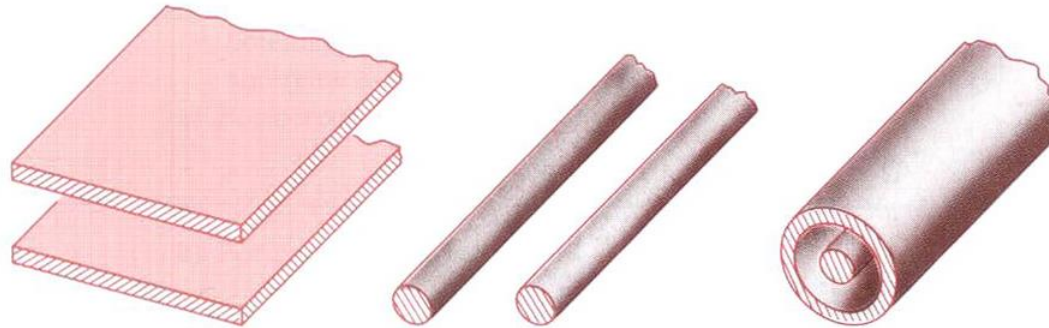
School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: yunchan@snu.ac.kr

Why Transmission Lines?

Common types of transmission lines:



(a) Parallel-plate transmission line.

(b) Two-wire transmission line. (c) Coaxial transmission line.

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Why proper transmission lines?

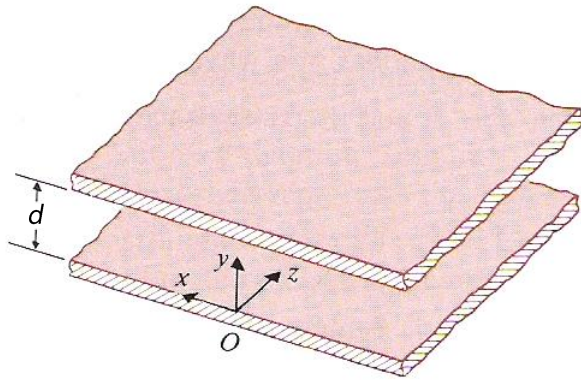
Otherwise, we cannot avoid extreme power loss by radiation & ohmic heat dissipation!

Good conductors: $\rightarrow \eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{\mu}{\epsilon}} (1 - j \frac{\sigma_c}{\omega \epsilon})^{-1/2} \approx \frac{1 + j}{\sqrt{2}} \sqrt{\frac{\omega \mu}{\sigma_c}}$

Low loss dielectrics: $\rightarrow \eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{\mu}{\epsilon}} (1 - j \frac{\sigma}{\omega \epsilon})^{-1/2} \approx \sqrt{\frac{\mu}{\epsilon}} (1 + j \frac{\sigma}{2\omega \epsilon})$

Parallel-Plate Waveguides

Just recall what we've already learned:



← An infinite parallel-plate waveguide

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Conditions given:

- Two perfectly conducting plates
- Dielectric medium with ϵ, μ
- Waves propagating in the z -direction
- Infinitely wide in the x -direction: No variation in the x -direction

TM Waves Between Parallel Plates (1)

Recall:

$$\gamma = \alpha + j\beta \quad \rightarrow \text{Propagation constant}$$

$$\mathbf{E}(x, y, z; t) = \text{Re}[\mathbf{E}^0(x, y)e^{(j\omega t - \gamma z)}]$$

$$H_z = 0 \quad \rightarrow \nabla_{xy}^2 E_z^0 + (\gamma^2 + k^2)E_z^0 = 0 \quad \rightarrow \nabla_{xy}^2 E_z^0 + h^2 E_z^0 = 0$$

$$\leftarrow h^2 = \gamma^2 + k^2$$

Characteristic equation:

$$\rightarrow \frac{\partial^2 E_z^0}{\partial y^2} + h^2 E_z^0 = 0 \quad (\because \frac{\partial}{\partial x} \rightarrow 0)$$

Recall:
BCs for a D-PEC Interface

$$E_{1t} = 0, \quad E_{2t} = 0$$

$$\mathbf{a}_{n2} \times \mathbf{H}_1 = \mathbf{J}_s, \quad H_{2t} = 0$$

$$\mathbf{a}_{2n} \cdot \mathbf{D}_1 = \rho_s, \quad D_{2n} = 0$$

$$B_{1n} = 0, \quad B_{2n} = 0$$

Boundary conditions:

$$E_z^0 = 0 \quad \leftarrow y = 0 \text{ \& } y = b$$

Solution:

$$E_z^0 = A_n \sin hy$$

$$\leftarrow h = \frac{n\pi}{b}, \quad n = 0, 1, 2, \dots$$

TM Waves Between Parallel Plates (2)

Solution:

$$E_z^0 = A_n \sin hy$$

$$\rightarrow H_x^0 = \frac{j\omega\epsilon}{h} A_n \cos hy$$

$$\rightarrow E_y^0 = -\frac{\gamma}{h} A_n \cos hy$$

Recall:

$$H_x^0 = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z^0}{\partial x} - j\omega\epsilon \frac{\partial E_z^0}{\partial y} \right)$$

$$H_y^0 = -\frac{1}{h^2} \left(\gamma \frac{\partial H_z^0}{\partial y} + j\omega\epsilon \frac{\partial E_z^0}{\partial x} \right)$$

$$E_x^0 = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z^0}{\partial x} + j\omega\mu \frac{\partial H_z^0}{\partial y} \right)$$

$$E_y^0 = -\frac{1}{h^2} \left(\gamma \frac{\partial E_z^0}{\partial y} - j\omega\mu \frac{\partial H_z^0}{\partial x} \right)$$

$$\rightarrow \gamma = \sqrt{\left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu\epsilon}$$

$$\rightarrow f_c = \frac{n}{2b\sqrt{\mu\epsilon}}$$

If $n = 0$:

$$\rightarrow E_z^0 = 0 \rightarrow E_y^0 \neq 0 \text{ \& } H_x^0 \neq 0$$

$$\rightarrow \gamma = ik$$

TEM mode:

$$\mathbf{E} = \mathbf{a}_y E_y = \mathbf{a}_y E_0 e^{-\gamma z}$$

$$\mathbf{H} = \mathbf{a}_x H_x = -\mathbf{a}_x \frac{E_0}{\eta} e^{-\gamma z}$$

$TM_0 \rightarrow$ TEM mode

$$\leftarrow \eta = \sqrt{\frac{\mu}{\epsilon}}$$

Transmission Line Parameters (1)

Recall:

$$\mathbf{E} = \mathbf{a}_y E_y = \mathbf{a}_y E_0 e^{-\gamma z}$$

$$\mathbf{H} = \mathbf{a}_x H_x = -\mathbf{a}_x \frac{E_0}{\eta} e^{-\gamma z}$$

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H} \quad \rightarrow \quad \frac{dE_y}{dz} = j\omega\mu H_x$$

$$\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E} \quad \rightarrow \quad \frac{dH_x}{dz} = j\omega\varepsilon E_y$$

$$\frac{dE_y}{dz} = j\omega\mu H_x \quad \rightarrow \quad \frac{d}{dz} \int_0^d E_y dy = j\omega\mu \int_0^d H_x dy \quad \leftarrow \quad \int \mathbf{E} \cdot d\mathbf{l} = -\int (\nabla V + \frac{\partial \mathbf{A}}{\partial t}) \cdot d\mathbf{l}$$

Why?

$$\rightarrow -\frac{dV(z)}{dz} = j\omega\mu J_{su}(z)d = j\omega \left(\mu \frac{d}{w} \right) [J_{su}(z)w]$$

$$= j\omega LI(z) \quad \leftarrow \quad L = \mu \frac{d}{w} \quad \leftarrow \text{Inductance per unit length}$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y \quad \rightarrow \quad \frac{d}{dz} \int_0^w H_x dx = j\omega\varepsilon \int_0^w E_y dx$$

$$\rightarrow -\frac{dI(z)}{dz} = -j\omega\varepsilon E_y(z)w = j\omega \left(\varepsilon \frac{w}{d} \right) [-E_y(z)d]$$

$$= j\omega CV(z) \quad \leftarrow \quad C = \varepsilon \frac{w}{d} \quad \leftarrow \text{Capacitance per unit length}$$

Transmission Line Parameters (2)

$$\begin{aligned} -\frac{dV(z)}{dz} &= j\omega LI(z) & \rightarrow \frac{d^2V(z)}{dz^2} &= -\omega^2 LCV(z) & \rightarrow V(z) &= V_0 e^{-j\beta z} \\ -\frac{dI(z)}{dz} &= j\omega CV(z) & \rightarrow \frac{d^2I(z)}{dz^2} &= -\omega^2 LCI(z) & \rightarrow I(z) &= I_0 e^{-j\beta z} \end{aligned}$$

$$\leftarrow \beta = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon}$$

\rightarrow Phase constant

Characteristic impedance:

$$Z_0 = \frac{V(z)}{I(z)} = \frac{V_0}{I_0} = \sqrt{\frac{L}{C}} = \frac{d}{w} \sqrt{\frac{\mu}{\varepsilon}} = \frac{d}{w} \eta$$

Velocity of propagation:

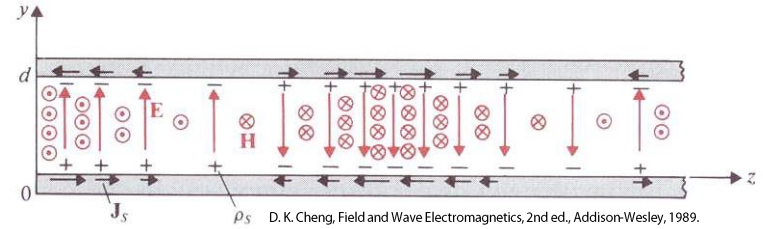
$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\varepsilon}} \quad \rightarrow \text{Phase velocity of a TEM plane wave}$$

Lossy Parallel-Plate Transmission Lines

For lossy dielectrics ($\sigma \neq 0$):

$$C = \frac{Q}{V} = \frac{\oint_S \mathbf{D} \cdot d\mathbf{s}}{-\int_L \mathbf{E} \cdot d\mathbf{l}} = \frac{\oint_S \epsilon \mathbf{E} \cdot d\mathbf{s}}{-\int_L \mathbf{E} \cdot d\mathbf{l}}$$

$$R = \frac{1}{G} = \frac{V}{I} = \frac{-\int_L \mathbf{E} \cdot d\mathbf{l}}{\oint_S \mathbf{J} \cdot d\mathbf{s}} = \frac{-\int_L \mathbf{E} \cdot d\mathbf{l}}{\oint_S \sigma \mathbf{E} \cdot d\mathbf{s}}$$



$$\rightarrow G = \frac{\sigma}{\epsilon} C = \sigma \frac{w}{d}$$

\rightarrow Conductance per unit length

For the finite conductivity of the plates ($\sigma_c \neq \infty$):

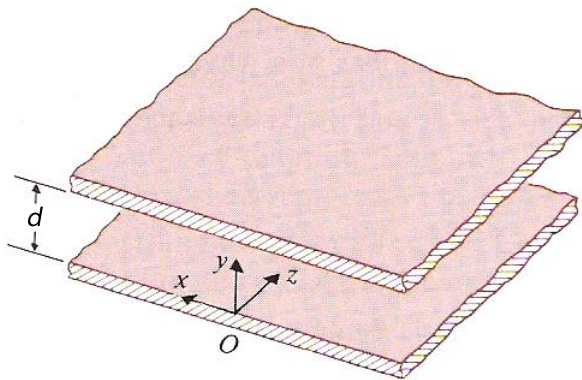
$$\mathbf{P}_{av} = \mathbf{a}_y p_\sigma = \frac{1}{2} \text{Re}(\mathbf{a}_z E_z \times \mathbf{a}_x H_x^*) \leftarrow \text{Average power per unit area in each plate}$$

$$Z_s = \frac{E_t}{J_s} = \frac{E_z}{H_x} = \eta_c = (1 + j) \sqrt{\frac{\pi f \mu_c}{\sigma_c}} = R_s + jX_s \quad \rightarrow \text{Surface impedance}$$

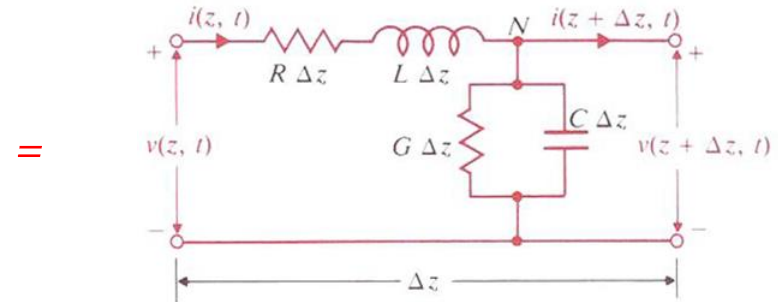
$$\rightarrow p_\sigma = \frac{1}{2} \text{Re}(|J_{su}|^2 Z_s) = \frac{1}{2} |J_{su}|^2 R_s \quad \text{Resistance per unit length}$$

Power dissipated per unit length: $\rightarrow P_\sigma = wp_\sigma = \frac{1}{2} I^2 \left(\frac{R_s}{w} \right) \rightarrow R = 2 \left(\frac{R_s}{w} \right) = \frac{2}{w} \sqrt{\frac{\pi f \mu_c}{\sigma_c}}$

Distributed Parameters



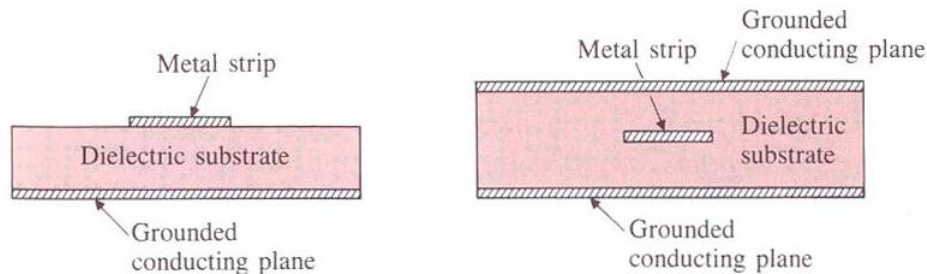
→ Equivalent circuit



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$R = \frac{2}{w} \sqrt{\frac{\pi f \mu_c}{\sigma_c}} \quad G = \sigma \frac{w}{d} \quad L = \mu \frac{d}{w} \quad C = \epsilon \frac{w}{d}$$

Microstrip lines:



Why triplate lines?
Characteristic impedance?

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.