

Electromagnetics:

Wave Characteristics on Finite Transmission Lines

(9-4)

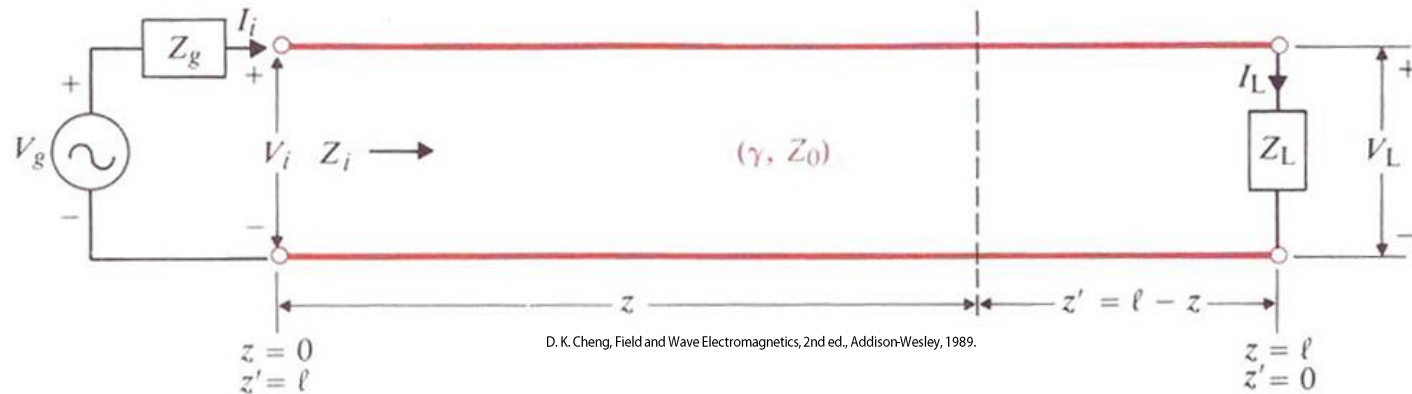
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Finite Transmission Lines (1)



General solution:

$$V(z) = V^+(z) + V^-(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

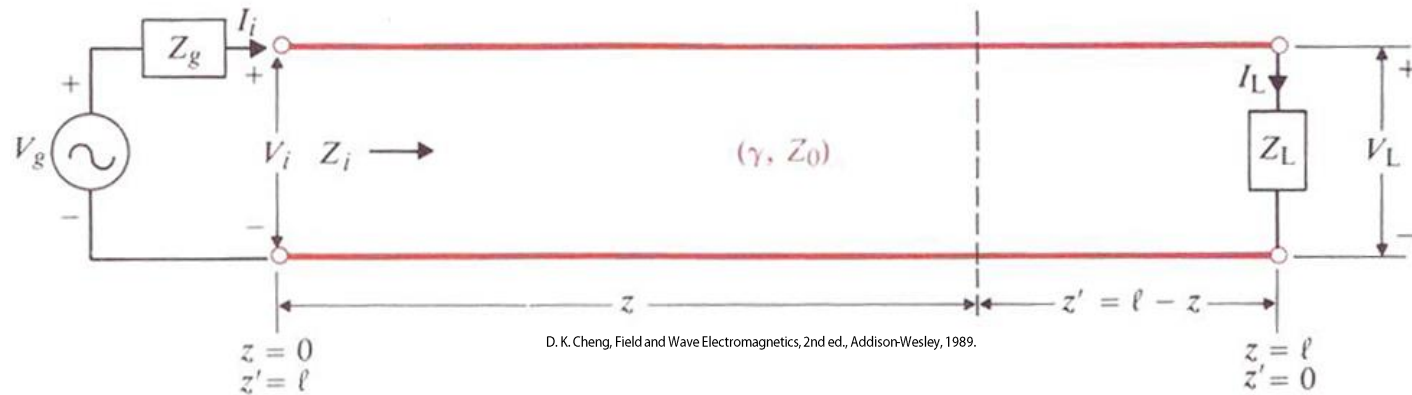
$$I(z) = I^+(z) + I^-(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} \quad \leftarrow \text{Characteristic impedance}$$

Load impedance:

$$\left(\frac{V}{I} \right)_{z=l} = \frac{V_L}{I_L} = Z_L$$

Finite Transmission Lines (2)



$$\begin{aligned} \rightarrow V_L &= V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} & \rightarrow V_0^+ &= \frac{1}{2} (V_L + I_L Z_0) e^{\gamma l} \\ \rightarrow I_L &= \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{\gamma l} & \rightarrow V_0^- &= \frac{1}{2} (V_L - I_L Z_0) e^{-\gamma l} \end{aligned}$$

$$\begin{aligned} \rightarrow V(z) &= \frac{I_L}{2} \left[(Z_L + Z_0) e^{\gamma(l-z)} + (Z_L - Z_0) e^{-\gamma(l-z)} \right] \\ \rightarrow I(z) &= \frac{I_L}{2Z_0} \left[(Z_L + Z_0) e^{\gamma(l-z)} - (Z_L - Z_0) e^{-\gamma(l-z)} \right] \end{aligned}$$

$$\begin{aligned} l-z \rightarrow z' & \\ \rightarrow V(z') &= \frac{I_L}{2} \left[(Z_L + Z_0) e^{\gamma z'} + (Z_L - Z_0) e^{-\gamma z'} \right] \\ \rightarrow I(z') &= \frac{I_L}{2Z_0} \left[(Z_L + Z_0) e^{\gamma z'} - (Z_L - Z_0) e^{-\gamma z'} \right] \end{aligned}$$

Finite Transmission Lines (3)

Recall: $V(z') = \frac{I_L}{2} [(Z_L + Z_0)e^{\gamma z'} + (Z_L - Z_0)e^{-\gamma z'}]$ $\leftarrow \frac{e^{\gamma z'} + e^{-\gamma z'}}{2} = \cosh \gamma z'$

$I(z') = \frac{I_L}{2Z_0} [(Z_L + Z_0)e^{\gamma z'} - (Z_L - Z_0)e^{-\gamma z'}]$ $\leftarrow \frac{e^{\gamma z'} - e^{-\gamma z'}}{2} = \sinh \gamma z'$

$\rightarrow V(z') = I_L (Z_L \cosh \gamma z' + Z_0 \sinh \gamma z')$

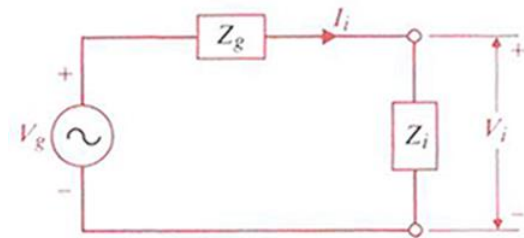
$\rightarrow I(z') = \frac{I_L}{Z_0} (Z_L \sinh \gamma z' + Z_0 \cosh \gamma z')$

Impedance:

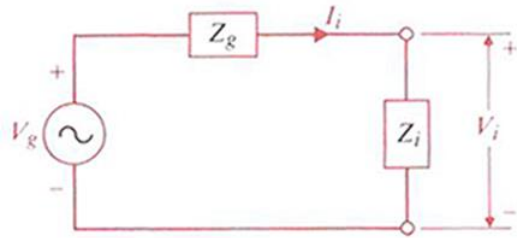
$$Z(z') = \frac{V(z')}{I(z')} = Z_0 \frac{Z_L \cosh \gamma z' + Z_0 \sinh \gamma z'}{Z_L \sinh \gamma z' + Z_0 \cosh \gamma z'} = Z_0 \frac{Z_L + Z_0 \tanh \gamma z'}{Z_0 + Z_L \tanh \gamma z'}$$

Input impedance: $\leftarrow z = 0$ or $z' = l$

$$Z_i = (Z)_{\substack{z=0 \\ z'=l}} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$



Finite Transmission Lines (4)



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$\rightarrow V_i = \frac{Z_i}{Z_g + Z_i} V_g$$

$$\rightarrow I_i = \frac{V_g}{Z_g + Z_i}$$

Time-averaged power delivered to the input terminals:

$$\rightarrow (P_{av})_i = \frac{1}{2} \text{Re}[V_i I_i^*]_{z=0, z'=l}$$

Time-averaged power delivered to the load:

$$\rightarrow (P_{av})_L = \frac{1}{2} \text{Re}[V_L I_L^*]_{z=l, z'=0} = \frac{1}{2} \left| \frac{V_L}{Z_L} \right|^2 R_L = \frac{1}{2} |I_L|^2 R_L$$

For a lossless line: $(P_{av})_i = (P_{av})_L$

When a finite transmission line matched: $\leftarrow Z_L = Z_0$

$$\rightarrow Z(z') = Z_0 \frac{Z_L + Z_0 \tanh \gamma z'}{Z_0 + Z_L \tanh \gamma z'} = Z_0$$

Recall:

$$V(z) = \frac{I_L}{2} \left[(Z_L + Z_0) e^{\gamma(l-z)} + (Z_L - Z_0) e^{-\gamma(l-z)} \right] = (I_L Z_0 e^{\gamma l}) e^{-\gamma z} = V_i e^{-\gamma z}$$

$$I(z) = \frac{I_L}{2Z_0} \left[(Z_L + Z_0) e^{\gamma(l-z)} - (Z_L - Z_0) e^{-\gamma(l-z)} \right] = (I_L e^{\gamma l}) e^{-\gamma z} = I_i e^{-\gamma z}$$

Transmission Lines as Circuit Elements (1)

Ordinary lump-circuit elements possible for high frequencies, e.g. UHF (300 MHz to 3 GHz)?

Recall: \rightarrow Transmission-line segments

Input impedance: $z = 0$ or $z' = l$

$$Z_i = (Z)_{\substack{z=0 \\ z'=l}} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

For lossless lines:

$$Z_i = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

1. Open-circuit termination: $Z_L = \infty$

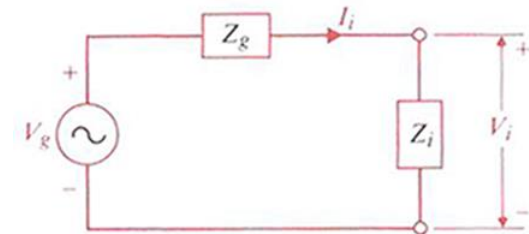
$$Z_{io} = -jR_0 \cot \beta l = jX_{io}$$

\rightarrow Purely reactive
(capacitive or inductive)

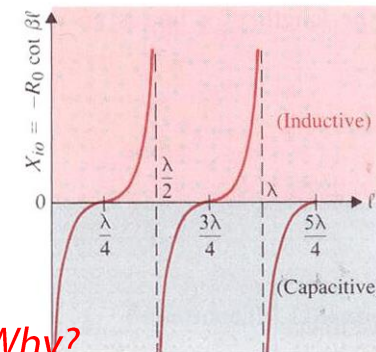
\leftarrow No ideal open-circuit termination: Why?

For a short line: $\beta l \ll 1$

$$Z_{io} = jX_{io} \cong -j \frac{R_0}{\beta l} = -j \frac{\sqrt{L/C}}{\omega l \sqrt{LC}} = -j \frac{1}{\omega C l}$$



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Transmission Lines as Circuit Elements (2)

For lossless lines:

$$Z_i = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

2. Short-circuit termination: $Z_L \rightarrow 0$

$$Z_{is} = jR_0 \tan \beta l = jX_{is}$$

→ Purely reactive (capacitive or inductive)

Shifted by an odd multiple of $\lambda/4$, compared to Z_{io} !

For a short line: $\beta l \ll 1$

$$Z_{is} = jX_{is} \cong jR_0 \beta l = j \sqrt{\frac{L}{C}} \omega \sqrt{LC} l = j\omega L l$$

3. Quarter-wave section: $l = \lambda/4$, $\beta l = \pi/2$

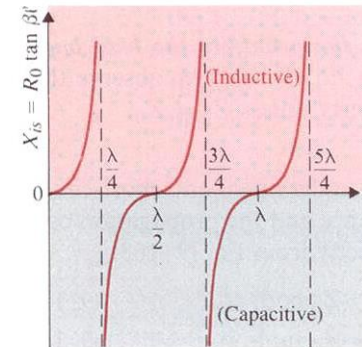
$$\beta l = \frac{2\pi}{\lambda} (2n-1) \frac{\lambda}{4} = (2n-1) \frac{\pi}{2} \rightarrow \tan \beta l = \pm \infty$$

$$\rightarrow Z_i = \frac{R_0^2}{Z_L}$$

4. Half-wave section: $l = \lambda/2$, $\beta l = \pi$

$$\beta l = \frac{2\pi}{\lambda} \left(\frac{n\lambda}{2} \right) = n\pi \rightarrow \tan \beta l = 0$$

$$\rightarrow Z_i = Z_L$$



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Transmission Lines as Circuit Elements (3)

Input impedance: $Z_i = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$

Open-circuited lines: $Z_L \rightarrow \infty \rightarrow Z_{io} = Z_0 \coth \gamma l$

Short-circuited lines: $Z_L = 0 \rightarrow Z_{is} = Z_0 \tanh \gamma l$

$$\rightarrow Z_0 = \sqrt{Z_{io} Z_{is}}$$

$$\rightarrow \gamma = \frac{1}{l} \tanh^{-1} \sqrt{\frac{Z_{is}}{Z_{io}}}$$

← Technically very useful!

For a lossy line with a short-circuit termination:

$$\rightarrow Z_{is} = Z_0 \tanh \gamma l = Z_0 \frac{\sinh(\alpha + j\beta)l}{\cosh(\alpha + j\beta)l} = Z_0 \frac{\sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l}{\cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l}$$

Series-resonant circuit condition: $l = n\lambda / 2 \rightarrow \beta l = n\pi \rightarrow \sin \beta l = 0$

$$\rightarrow Z_{is} = Z_0 \tanh \alpha l \cong Z_0(\alpha l) \quad \text{if } \alpha l \ll 1$$

Parallel-resonant circuit condition: $l = n\lambda / 4 \rightarrow \beta l = n\pi / 2 \rightarrow \cos \beta l = 0$ (n : odd int.)

$$\rightarrow Z_{is} = Z_0 \coth \alpha l \cong Z_0 / \alpha l \quad \text{if } \alpha l \ll 1$$

Transmission Lines as Circuit Elements (4)

$$l = n\lambda/4 \rightarrow \beta l = n\pi/2 \quad (n: \text{odd int.})$$

Frequency selective characteristics (Parallel-resonant circuit):

$$\beta l = \frac{2\pi f}{u_p} l = \frac{2\pi(f_0 + \delta f)}{u_p} l$$

Resonant freq.

$$= \frac{n\pi}{2} + \frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right), \quad n = \text{odd int.}$$

$$\rightarrow \cos \beta l = -\sin \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \right] \cong -\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right)$$

$$\rightarrow \sin \beta l = \cos \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \right] \cong 1$$

$$\alpha l \ll 1 \rightarrow Z_{is} = Z_0 \frac{\sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l}{\cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l}$$

$$Z_{is} \cong \frac{Z_0}{\alpha l + j \frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right)} \rightarrow |Z_{is}|^2 \cong \frac{|Z_0|^2}{(\alpha l)^2 + \left[\frac{n\pi}{2} \left(\frac{\delta f}{f_0} \right) \right]^2} \rightarrow \frac{|Z_{is}|^2}{|Z_{is}|_{\max}^2} \cong \frac{1}{1 + \left[\frac{n\pi}{2\alpha l} \left(\frac{\delta f}{f_0} \right) \right]^2}$$

Half-power frequencies: $\delta f = \pm \Delta f / 2$

$$\rightarrow \frac{n\pi}{2\alpha l} \left(\frac{\delta f}{f_0} \right) = \frac{n\pi}{2\alpha l} \left(\frac{\Delta f}{2f_0} \right) = \frac{\beta}{2\alpha} \left(\frac{\Delta f}{f_0} \right) = 1 \rightarrow Q = \frac{\omega W}{P_L} = \frac{\beta}{2\alpha} = \frac{f_0}{\Delta f}$$

For low-loss lines:

$$\rightarrow Q = \frac{\omega \sqrt{LC}}{R \sqrt{C/L} + G \sqrt{L/C}} = \frac{\omega L}{R + GL/C} = \frac{1}{R/\omega L + G/\omega C} \cong \frac{\omega L}{R}$$