Electromagnetics:

Wave Characteristics on Finite Transmission Lines

(9-4)

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Lines with Resistive Termination (1)

Recall:

$$V(z') = \frac{I_L}{2}\left[(Z_L + Z_0)e^{\gamma z'} + (Z_L - Z_0)e^{-\gamma z'}\right]$$

$$I(z') = \frac{I_L}{2Z_0}\left[(Z_L + Z_0)e^{\gamma z'} - (Z_L - Z_0)e^{-\gamma z'}\right]$$

$$\rightarrow V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{\gamma z'}\left[1 + \frac{Z_L - Z_0}{Z_L + Z_0}e^{-2\gamma z'}\right] \quad \leftarrow \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}$$

$$= \frac{I_L}{2}(Z_L + Z_0)e^{\gamma z'}\left[1 + \Gamma e^{-2\gamma z'}\right]$$

Voltage reflection coefficient

$$\rightarrow I(z') = \frac{I_L}{2Z_0}(Z_L + Z_0)e^{\gamma z'}\left[1 - \Gamma e^{-2\gamma z'}\right]$$

For a lossless transmission line:

$$V(z') = \frac{I_L}{2}(Z_L + R_0)e^{j\beta z'}\left[1 + \Gamma e^{-j2\beta z'}\right] = \frac{I_L}{2}(Z_L + R_0)e^{j\beta z'}\left[1 + |\Gamma| e^{j(\theta_\Gamma - 2\beta z')}\right]$$

$$I(z') = \frac{I_L}{2Z_0}(Z_L + R_0)e^{j\beta z'}\left[1 - \Gamma e^{-j2\beta z'}\right] = \frac{I_L}{2Z_0}(Z_L + R_0)e^{j\beta z'}\left[1 - |\Gamma| e^{j(\theta_\Gamma - 2\beta z')}\right]$$

Lines with Resistive Termination (2)

Also Recall:

$$V(z') = I_L(Z_L \cosh \gamma z' + Z_0 \sinh \gamma z')$$

$$I(z') = \frac{I_L}{Z_0}(Z_L \sinh \gamma z' + Z_0 \cosh \gamma z')$$

For a lossless transmission line:

$$\rightarrow V(z') = V_L \cos \beta z' + jI_L R_0 \sin \beta z'$$

$$\rightarrow I(z') = I_L \cos \beta z' + j\frac{V_L}{R_0} \sin \beta z'$$

For purely resistive termination:

$$\rightarrow Z_L = R_L$$

$$\rightarrow V_L = I_L R_L$$

$$|V(z')| = V_L \sqrt{\cos^2 \beta z' + (R_0 / R_L)^2 \sin^2 \beta z'}$$

$$|I(z')| = I_L \sqrt{\cos^2 \beta z' + (R_L / R_0)^2 \sin^2 \beta z'}$$

$\leftarrow$ *Standing waves*

Lines with Resistive Termination (3)

Recall for a lossless transmission line:

$$V(z') = \frac{I_L}{2}(Z_L + R_0)e^{j\beta z'}\left[1 + |\Gamma| e^{j(\theta_\Gamma - 2\beta z')}\right]$$

$$I(z') = \frac{I_L}{2Z_0}(Z_L + R_0)e^{j\beta z'}\left[1 - |\Gamma| e^{j(\theta_\Gamma - 2\beta z')}\right]$$

$$\leftarrow \Gamma = \frac{Z_L - R_0}{Z_L + R_0} = |\Gamma| e^{j\theta_\Gamma}$$

Standing-wave ratio (SWR): $S = \left|\frac{V_{\max}}{V_{\min}}\right| = \left|\frac{I_{\max}}{I_{\min}}\right| = \frac{1+|\Gamma|}{1-|\Gamma|} \quad \rightarrow |\Gamma| = \frac{S-1}{S+1}$

$\Gamma = 0 \quad \rightarrow S = 1 \quad \leftarrow Z_L = Z_0$ → *Matched load*

$\Gamma = -1 \quad \rightarrow S = \infty \quad \leftarrow Z_L = 0$ → *Short circuit*

$\Gamma = +1 \quad \rightarrow S = \infty \quad \leftarrow Z_L = \infty$ → *Open circuit*

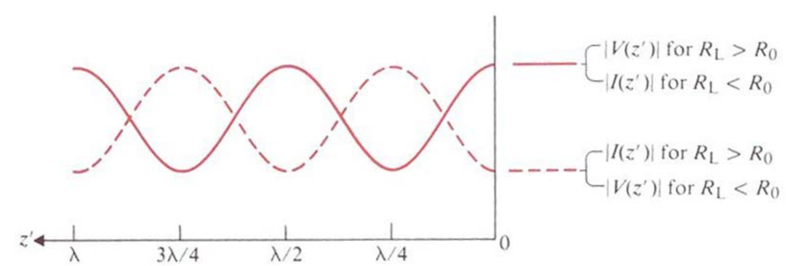
$\theta_\Gamma - 2\beta z'_M = -2n\pi \quad \rightarrow V_{\max}$ & $I_{\min}$

$\theta_\Gamma - 2\beta z'_m = -(2n+1)\pi \rightarrow V_{\min}$ & $I_{\max}$

For resistive terminations: $\rightarrow \Gamma = \frac{R_L - R_0}{R_L + R_0}$

$R_L > R_0 \quad \rightarrow V_{\max}$ *at the terminating resistance*

$R_L < R_0 \quad \rightarrow V_{\min}$ *at the terminating resistance*



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Lines with Resistive Termination (4)

For resistive terminations:

$$|V(z')| = V_L\sqrt{\cos^2\beta z' + (R_0/R_L)^2\sin^2\beta z'}$$

$$|I(z')| = I_L\sqrt{\cos^2\beta z' + (R_L/R_0)^2\sin^2\beta z'}$$

$$\rightarrow \Gamma = \frac{R_L - R_0}{R_L + R_0}$$

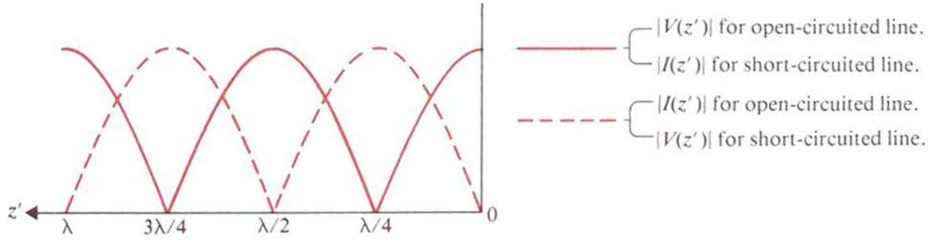
$$R_L = \infty \qquad \rightarrow |V(z')| = V_L|\cos\beta z'| \qquad \rightarrow \Gamma = 1 \ \& \ S \rightarrow \infty$$

$$\rightarrow |I(z')| = \frac{V_L}{R_0}|\sin\beta z'|$$

$$R_L = 0 \qquad \rightarrow |V(z')| = I_L R_0|\sin\beta z'| \qquad \rightarrow \Gamma = -1 \ \& \ S \rightarrow \infty$$

$$\rightarrow |I(z')| = I_L|\cos\beta z'|$$

→ All the minima go to zero.



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Lines with Arbitrary Termination

What if the terminating impedance is not a pure resistance?

$$\rightarrow Z_L = R_L + jX_L \leftarrow TL(l_m) \text{ terminated by } R_m$$

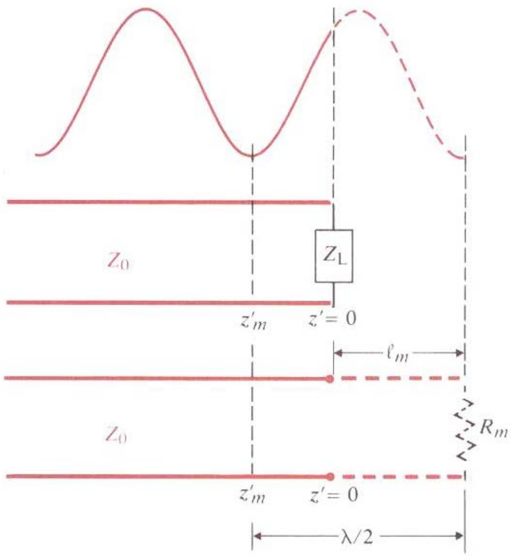
"Equivalent"

$$\leftarrow R_m < R_0$$

Recall:

$$Z_i = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$

$$\rightarrow R_i + jX_i = R_0 \frac{R_m + jR_0 \tan \beta l_m}{R_0 + jR_m \tan \beta l_m}$$



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Experimental procedure to determine Z_L :

1. Find $|\Gamma|$ from S: $|\Gamma| = \frac{S-1}{S+1}$
2. Find θ_Γ from z'_m: $\theta_\Gamma - 2\beta z'_m = -\pi$
3. Find Z_L: $Z_L = R_L + jX_L = R_0 \frac{1+|\Gamma|e^{j\theta_\Gamma}}{1-|\Gamma|e^{j\theta_\Gamma}}$

Recall:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = |\Gamma| e^{j\theta_\Gamma}$$

Also recall:

$$V(z') = V_L \cos \beta z' + jI_L R_0 \sin \beta z'$$
$$= V_L \cos \beta z' + jV_L \frac{R_0}{R_L} \sin \beta z'$$

$$z'_m + l_m = \lambda / 2$$
$$S = R_0 / R_m$$

$$\leftarrow R_L = R_m$$

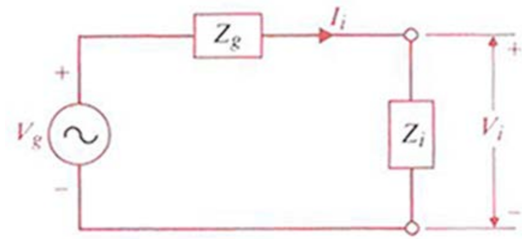
Transmission-Line Circuits (1)

Voltage generator V_g with an internal impedance Z_g:

$$V_i = V_g - I_i Z_g$$

$$V_i = \frac{I_L}{2}(Z_L + Z_0)e^{\gamma l}[1 + \Gamma e^{-2\gamma l}]$$

$$I_i = \frac{I_L}{2Z_0}(Z_L + Z_0)e^{\gamma l}[1 - \Gamma e^{-2\gamma l}]$$



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$$\rightarrow \frac{I_L}{2}(Z_L + Z_0)e^{\gamma l}[1 + \Gamma e^{-2\gamma l}] = V_g - \frac{I_L}{2Z_0}(Z_L + Z_0)e^{\gamma l}[1 - \Gamma e^{-2\gamma l}]Z_g$$

$$\rightarrow \frac{I_L}{2}(Z_L + Z_0)e^{\gamma l} = \frac{Z_0 V_g}{Z_0 + Z_g}\frac{1}{[1 - \Gamma_g \Gamma e^{-2\gamma l}]} \qquad \leftarrow \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$$

Voltage reflection coefficient at the generator end

$$\leftarrow \boxed{z' = l - z}$$

$$\rightarrow V(z') = \frac{I_L}{2}(Z_L + Z_0)e^{\gamma z'}[1 + \Gamma e^{-2\gamma z'}] = \frac{Z_0 V_g}{Z_0 + Z_g} e^{-\gamma z}\left(\frac{1 + \Gamma e^{-2\gamma z'}}{1 - \Gamma_g \Gamma e^{-2\gamma l}}\right)$$

$$\rightarrow I(z') = \frac{I_L}{2Z_0}(Z_L + Z_0)e^{\gamma z'}[1 - \Gamma e^{-2\gamma z'}] = \frac{V_g}{Z_0 + Z_g} e^{-\gamma z}\left(\frac{1 - \Gamma e^{-2\gamma z'}}{1 - \Gamma_g \Gamma e^{-2\gamma l}}\right)$$

Transmission-Line Circuits (2)

Recall: $$V(z') = \frac{Z_0 V_g}{Z_0 + Z_g} e^{-\gamma z} \left(\frac{1 + \Gamma e^{-2\gamma z'}}{1 - \Gamma_g \Gamma e^{-2\gamma l}} \right)$$

$$= \frac{Z_0 V_g}{Z_0 + Z_g} e^{-\gamma z} (1 + \Gamma e^{-2\gamma z'})(1 + \Gamma_g \Gamma e^{-2\gamma l} + \Gamma_g^2 \Gamma^2 e^{-4\gamma l} + \cdots)$$

$$= \frac{Z_0 V_g}{Z_0 + Z_g} [e^{-\gamma z} + (\Gamma e^{-\gamma l}) e^{-\gamma z'} + \Gamma_g (\Gamma e^{-2\gamma l}) e^{-\gamma z} + \cdots]$$

$$= V_1^+ + V_1^- + V_2^+ + V_2^- + \cdots$$

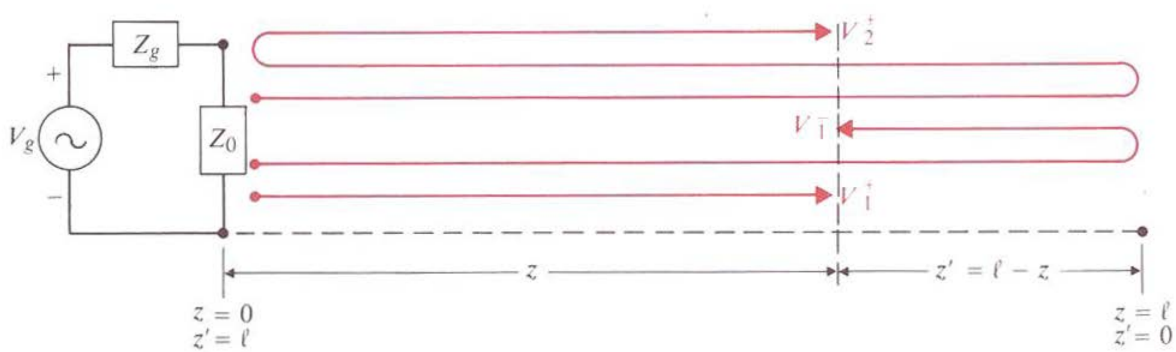
What if: $\Gamma = 0$ *or* $\Gamma_g = 0$?

$$\leftarrow V_1^+ = \frac{V_g Z_0}{Z_0 + Z_g} e^{-\gamma z} = V_M e^{-\gamma z}$$

$$\leftarrow V_1^- = \Gamma (V_M e^{-\gamma l}) e^{-\gamma z'}$$

$$\leftarrow V_2^+ = \Gamma_g (\Gamma V_M e^{-2\gamma l}) e^{-\gamma z}$$

⋮



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