

# Electromagnetics:

## Transients on Transmission Lines

### (9-5)

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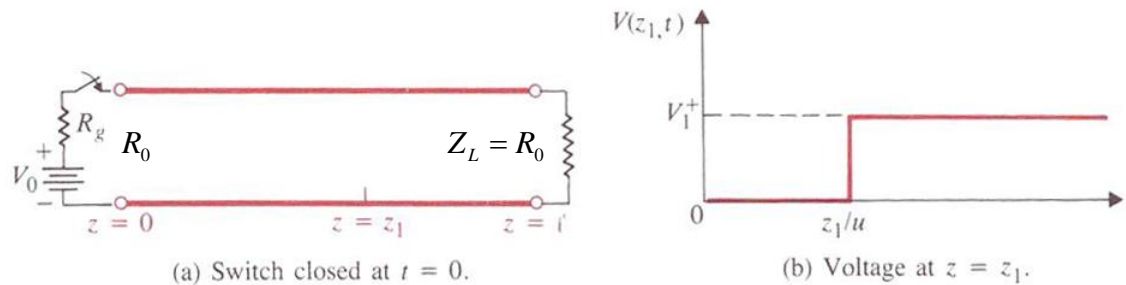
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# Transients on Transmission Lines (1)

Consider a dc source with  $R_g$ :



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$V_1^+ = \frac{R_0 V_0}{R_0 + R_g}, \quad I_1^+ = \frac{V_1^+}{R_0} = \frac{V_0}{R_0 + R_g}, \quad u = 1/\sqrt{LC}$$

**No back-reflection!**  $\leftarrow Z_L = R_0$

# Transients on Transmission Lines (2)

Consider:  $R_L \neq R_0$  &  $R_g \neq R_0$

$$V_1^+ = \frac{R_0 V_0}{R_0 + R_g}, \quad V_1^- = \Gamma_L V_1^+ \quad \leftarrow \Gamma_L = \frac{R_L - R_0}{R_L + R_0}$$

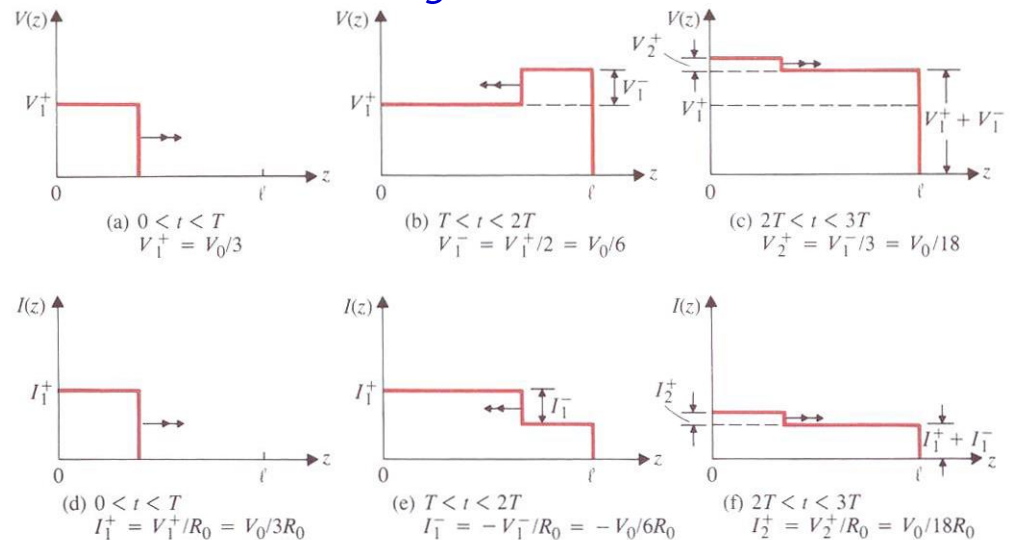
$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+ \quad \leftarrow \Gamma_g = \frac{R_g - R_0}{R_g + R_0}$$



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

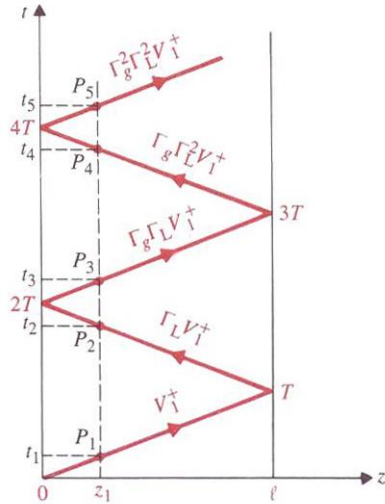
$$\begin{aligned} \rightarrow V_L &= V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \dots \\ &= V_1^+ (1 + \Gamma_L + \Gamma_g \Gamma_L + \Gamma_g \Gamma_L^2 + \Gamma_g^2 \Gamma_L^2 + \Gamma_g^2 \Gamma_L^3 + \dots) \\ &= V_1^+ \left[ (1 + \Gamma_g \Gamma_L + \Gamma_g^2 \Gamma_L^2 + \dots) + \Gamma_L (1 + \Gamma_g \Gamma_L + \Gamma_g^2 \Gamma_L^2 + \dots) \right] \\ &= V_1^+ \left( \frac{1 + \Gamma_L}{1 - \Gamma_g \Gamma_L} \right) \\ \rightarrow I_L &= \frac{V_1^+}{R_0} \left( \frac{1 - \Gamma_L}{1 - \Gamma_g \Gamma_L} \right) \end{aligned}$$

## Transient voltage and current waves



# Reflection Diagrams

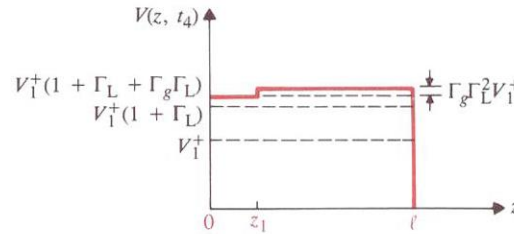
## Voltage reflection diagram:



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

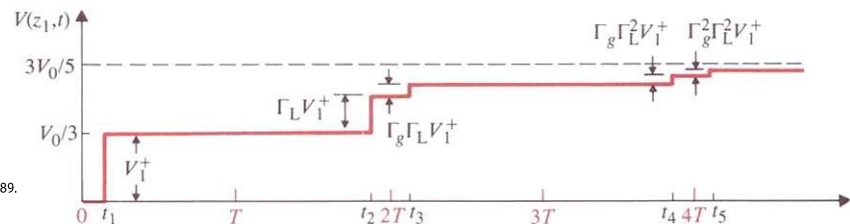
Time Range	Voltage	Voltage Discontinuity
$0 \leq t < t_1$ ( $t_1 = z_1/u$ )	0	0
$t_1 \leq t < t_2$ ( $t_2 = 2T - t_1$ )	$V_1^+$	$V_1^+$ at $t_1$
$t_2 \leq t < t_3$ ( $t_3 = 2T + t_1$ )	$V_1^+(1 + \Gamma_L)$	$\Gamma_L V_1^+$ at $t_2$
$t_3 \leq t < t_4$ ( $t_4 = 4T - t_1$ )	$V_1^+(1 + \Gamma_L + \Gamma_g \Gamma_L)$	$\Gamma_g \Gamma_L V_1^+$ at $t_3$
$t_4 \leq t < t_5$ ( $t_5 = 4T + t_1$ )	$V_1^+(1 + \Gamma_L + \Gamma_g \Gamma_L + \Gamma_g \Gamma_L^2)$	$\Gamma_g \Gamma_L^2 V_1^+$ at $t_4$
$\vdots$	$\vdots$	$\vdots$

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.



(a)  $V(z, t_4)$  versus  $z$ ;

$$\Gamma_L = \frac{1}{2}, \Gamma_g = \frac{1}{3}, V_1^+ = V_0/3.$$



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

(b)  $V(z_1, t)$  versus  $t$ ;  $V(z_1, \infty) = 3V_0/5$ .

# Pulse Excitation

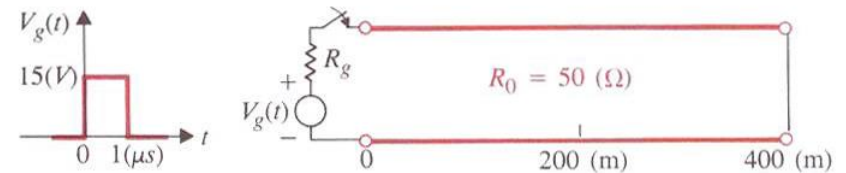
A step function:

$$v_g(t) = V_0 U(t) \quad \leftarrow U(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

*"Heaviside" step function*

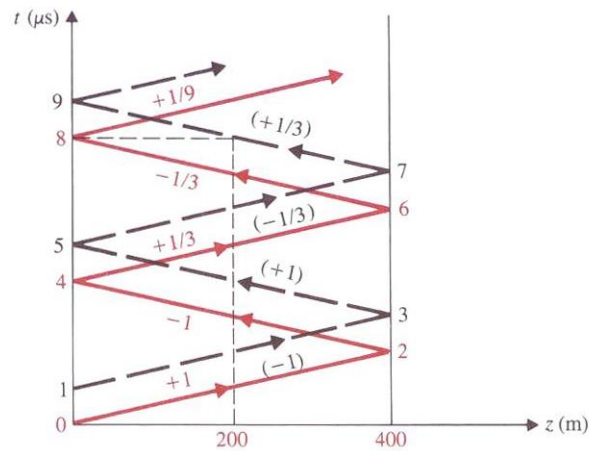
Pulse excitation:

$$v_g(t) = V_0 [U(t) - U(t - T_0)]$$

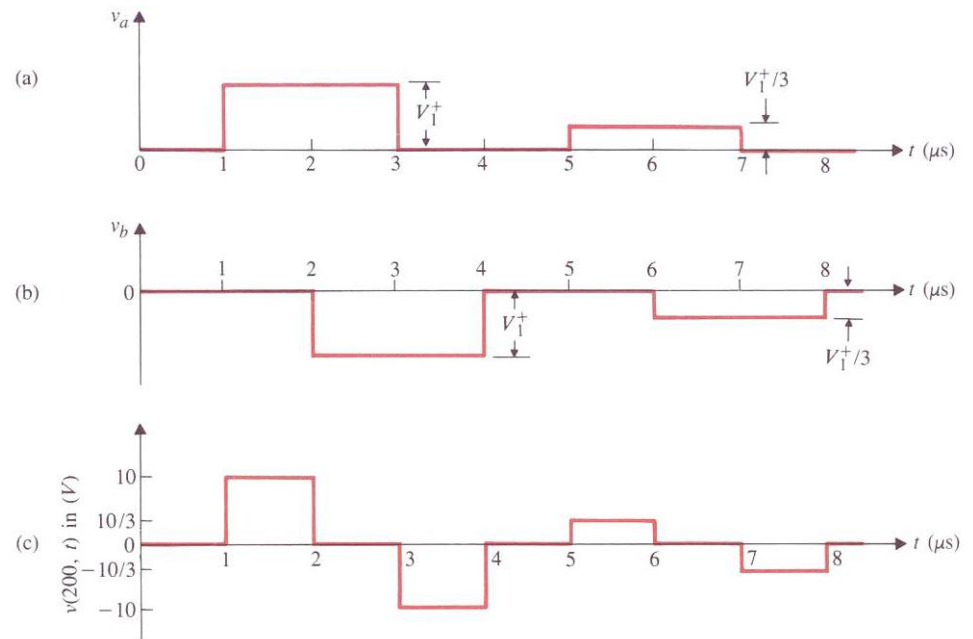


*Superposition of two step functions!*

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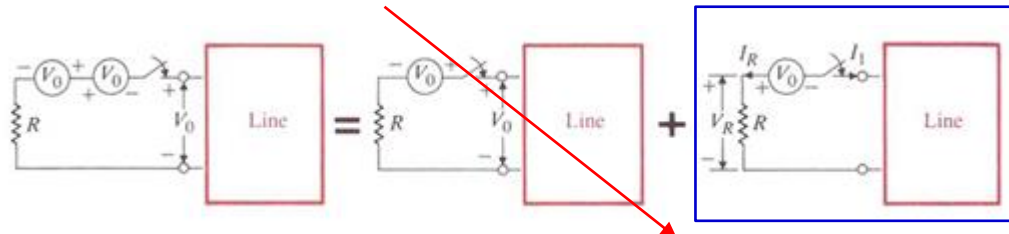
# Initially Charged Line

Consider:

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

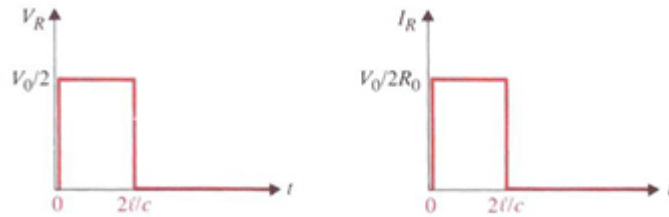


Superposition of two circuits:



→ Of our interest

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$V_1^+ = -\frac{R_0}{R + R_0} V_0 = -\frac{V_0}{2} \quad \leftarrow R = R_0$$

$$V_1^- = V_1^+ = -\frac{V_0}{2} \quad \leftarrow \text{Open-circuited termination}$$

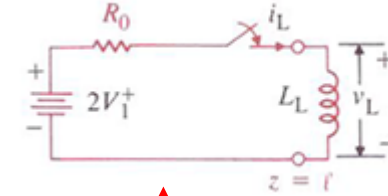
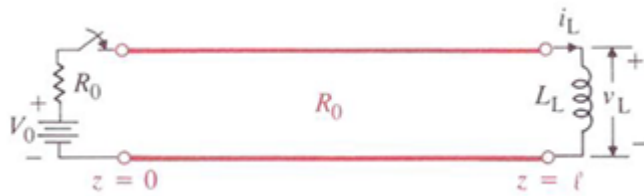
$$I_R = -I_1$$

$$I_1 = I_1^+ = \frac{V_1^+}{R_0} = -\frac{V_0}{2R_0} \quad \text{for } 0 \leq t < 2l/c$$

$$I_1^- = -I_1^+ = \frac{V_0}{2R_0}$$

# Line with Reactive Load (1)

For a lossless line with an inductive termination:



← Equivalent circuit  
( $t \geq T$ )

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$V_1^+ = \frac{V_0}{2}$$

$$v_L(t) = V_1^+ + V_1^-(t) \quad \text{at } z = l \text{ \& } t \geq T$$

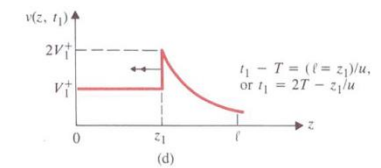
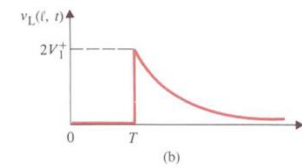
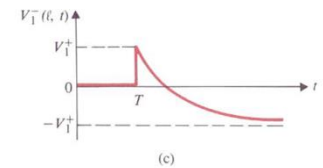
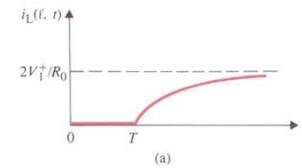
$$i_L(t) = \frac{1}{R_0} [V_1^+ - V_1^-(t)] \quad \left\{ \begin{array}{l} v_L(t) = 2V_1^+ - R_0 i_L(t) \end{array} \right.$$

$$v_L(t) = L_L \frac{di_L(t)}{dt} \quad \left\{ \begin{array}{l} L_L \frac{di_L(t)}{dt} + R_0 i_L(t) = 2V_1^+, \quad t \geq T \end{array} \right.$$

**Solution:**

$$i_L(t) = \frac{2V_1^+}{R_0} [1 - e^{-(t-T)R_0/L_L}], \quad t \geq T$$

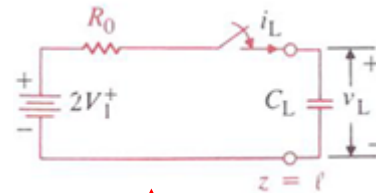
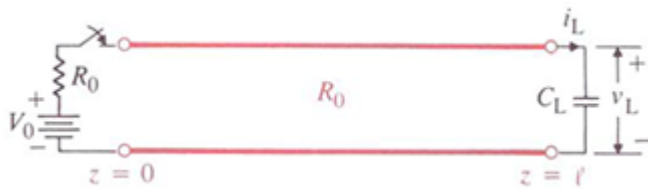
$$v_L(t) = L_L \frac{di_L(t)}{dt} = 2V_1^+ e^{-(t-T)R_0/L_L}, \quad t \geq T$$



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

# Line with Reactive Load (2)

For a lossless line with a capacitive termination:



← Equivalent circuit  
( $t \geq T$ )

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$V_1^+ = \frac{V_0}{2}$$

$$v_L(t) = V_1^+ + V_1^-(t) \quad \text{at } z = l \text{ \& } t \geq T$$

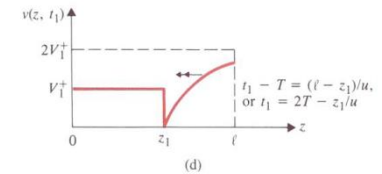
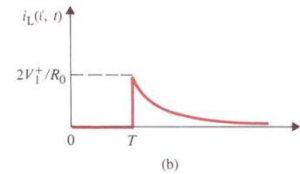
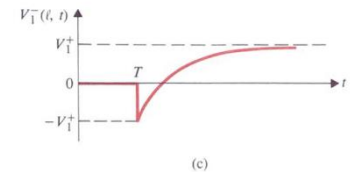
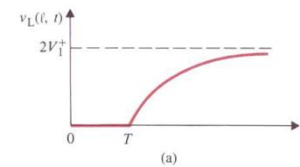
$$i_L(t) = \frac{1}{R_0} [V_1^+ - V_1^-(t)] \quad \checkmark \quad v_L(t) = 2V_1^+ - R_0 i_L(t)$$

$$i_L(t) = C_L \frac{dv_L(t)}{dt} \quad \checkmark \quad C_L \frac{dv_L(t)}{dt} + \frac{1}{R_0} v_L(t) = \frac{2}{R_0} V_1^+, \quad t \geq T$$

**Solution:**

$$v_L(t) = 2V_1^+ \left[ 1 - e^{-(t-T)/R_0 C_L} \right], \quad t \geq T$$

$$i_L(t) = C_L \frac{dv_L(t)}{dt} = \frac{2V_1^+}{R_0} e^{-(t-T)/R_0 C_L}, \quad t \geq T$$



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