Electromagnetics:
Introduction to Antennas and Radiating Systems
Radiation Fields of Elemental Dipoles
(11-1, 11-2)

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Antennas

For example:

Antennas: Structures designed for radiating or collecting electromagnetic energy !

Some Equations To Recall (Not Too Difficult)

Vector magnetic potential:

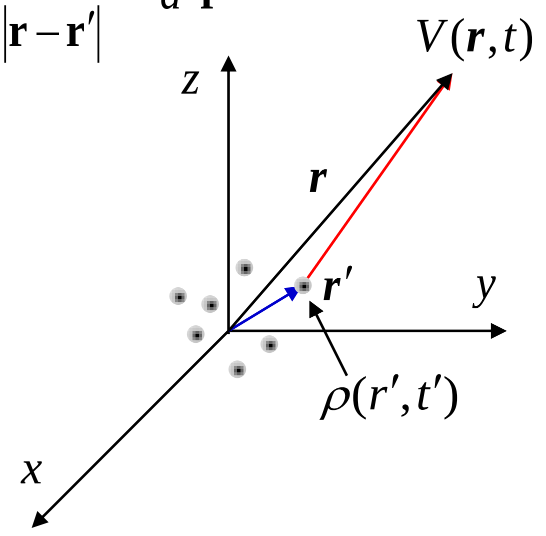
$$\nabla \cdot \mathbf{B} = 0 \quad \rightarrow \mathbf{B} = \nabla \times \mathbf{A} \quad \rightarrow \mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A}$$

$$\rightarrow \nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A}) \quad \rightarrow \nabla \times (\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}) = 0$$

$$\rightarrow \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V \quad \rightarrow \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \quad \rightarrow \mathbf{E} = -\nabla V - j\omega \mathbf{A}$$

Retarded potentials:

$$\rightarrow \mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}') e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}' \quad \rightarrow V(\mathbf{r}) = \frac{1}{4\pi\varepsilon} \int \frac{\rho(\mathbf{r}') e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}'$$



Principle of conservation of charge:

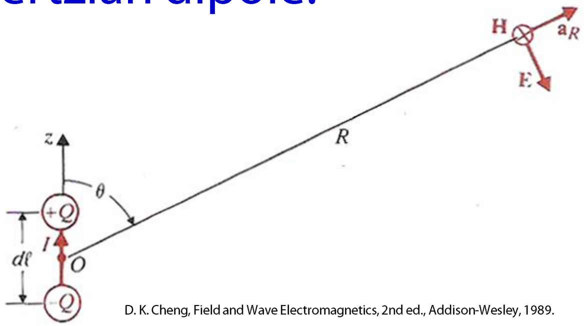
$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad \rightarrow \nabla \cdot \mathbf{J} = -j\omega\rho$$

For source-free regions:

$$\mathbf{E} = \frac{1}{j\omega\varepsilon} \nabla \times \mathbf{H}$$

Elemental Electric Dipole (1)

Hertzian dipole:



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$$i(t) = I\cos\omega t = \mathrm{Re}[Ie^{j\omega t}]$$

$$\rightarrow i(t) = \pm\frac{dq(t)}{dt} \quad \leftarrow q(t) = \mathrm{Re}[Qe^{j\omega t}]$$

$$\rightarrow I = \pm j\omega Q$$

$$\rightarrow Q = \pm\frac{I}{j\omega}$$

Electric dipole moment: $\rightarrow \mathbf{p} = \mathbf{a}_z Q dl$

Magnetic vector potential:

$$\rightarrow \mathbf{A} = \mathbf{a}_z \frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right) \qquad \leftarrow \beta = k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

With spherical coordinates:

$$\rightarrow A_R = A_z\cos\theta = \frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\cos\theta$$

$$\rightarrow A_\theta = -A_z\sin\theta = -\frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\sin\theta$$

$$\rightarrow A_\phi = 0$$

Elemental Electric Dipole (2)

$$A_R = A_z \cos\theta = \frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\cos\theta$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu_0 I dl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\sin\theta$$

$$A_\phi = 0$$

$$\rightarrow \mathbf{H} = \frac{1}{\mu_0}\nabla \times \mathbf{A}$$

$$= \mathbf{a}_\phi \frac{1}{\mu_0 R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial \theta}\right]$$

$$= -\mathbf{a}_\phi \frac{I dl}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]e^{-j\beta R}$$

$$\rightarrow \mathbf{E} = \frac{1}{j\omega\varepsilon_0}\nabla \times \mathbf{H}$$

$$= \frac{1}{j\omega\varepsilon_0}\left[\mathbf{a}_R \frac{1}{R\sin\theta}\frac{\partial}{\partial\theta}(H_\phi \sin\theta) - \mathbf{a}_\theta \frac{1}{R}\frac{\partial}{\partial R}(RH_\phi)\right]$$

$$\rightarrow E_R = -\frac{I dl}{4\pi}\eta_0\beta^2 2\cos\theta\left[\frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}$$

$$\rightarrow E_\theta = -\frac{I dl}{4\pi}\eta_0\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}$$

$$\rightarrow E_\phi = 0$$

Elemental Electric Dipole (3)

$$H_\phi = -\frac{Idl}{4\pi}\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]e^{-j\beta R}$$

$$E_R = -\frac{Idl}{4\pi}\eta_0\beta^2 2\cos\theta\left[\frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}$$

$$E_\theta = -\frac{Idl}{4\pi}\eta_0\beta^2 \sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3}\right]e^{-j\beta R}$$

Wavelength dependent!

Near fields: $\rightarrow \beta R = 2\pi R/\lambda << 1$

$$H_\phi = \frac{Idl}{4\pi R^2}\sin\theta$$

$$E_R = \frac{p}{4\pi\varepsilon_0 R^3} 2\cos\theta$$

$$E_\theta = \frac{p}{4\pi\varepsilon_0 R^3}\sin\theta$$

$\leftarrow p = Qdl = \frac{Idl}{j\omega}$

→ Quasi-static fields

Far fields: $\rightarrow \beta R = 2\pi R/\lambda >> 1$

$$H_\phi = j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta$$

$$E_R = 0$$

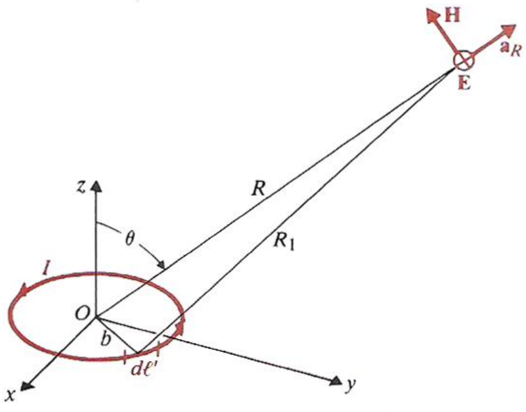
$$E_\theta = j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0\beta\sin\theta$$

$\rightarrow E_\theta / H_\phi = \eta_0$

$\rightarrow \mathbf{E} \perp \mathbf{H}$

→ In space quadrature *→ In time phase*

Elemental Magnetic Dipole (1)



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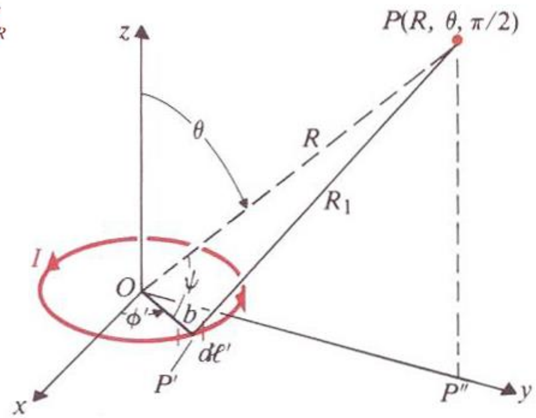

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Magnetic vector potential:

$$\mathbf{A} = \frac{\mu_0 I}{4\pi}\oint \frac{e^{-j\beta R_1}}{R_1} d\mathbf{l}'$$

$$\leftarrow e^{-j\beta R_1} = e^{-j\beta R} e^{-j\beta(R_1 - R)}$$

$$\cong e^{-j\beta R}[1 - j\beta(R_1 - R)]$$

(See pp. 239-240)

$$\to \mathbf{A} = \frac{\mu_0 I}{4\pi} e^{-j\beta R}\left[(1 + j\beta R)\oint \frac{d\mathbf{l}'}{R_1} - j\beta \oint d\mathbf{l}'\right]$$

$$\leftarrow d\mathbf{l}' = (-\mathbf{a}_x \sin\phi' + \mathbf{a}_y \cos\phi') b\, d\phi'$$

$$\to \mathbf{a}_\phi$$

$$\to \mathbf{A} = \mathbf{a}_\phi \frac{\mu_0 I}{4\pi} e^{-j\beta R}(1 + j\beta R)\int_0^{2\pi} \frac{b \sin\phi'}{R_1} d\phi'$$

$$= \mathbf{a}_\phi \frac{\mu_0 m}{4\pi R^2}(1 + j\beta R) e^{-j\beta R} \sin\theta$$

Magnetic dipole moment:

$$\to \mathbf{m} = \mathbf{a}_z I \pi b^2$$

$$\frac{1}{R_1} = \left(R^2 + b^2 - 2bR\cos\psi\right)^{-1/2}$$

$$\cong \frac{1}{R}\left(1 + \frac{b}{R}\cos\psi\right) \cong \frac{1}{R}\left(1 + \frac{b}{R}\sin\phi' \sin\theta\right)$$

Elemental Magnetic Dipole (2)

$$\rightarrow \mathbf{H} = \frac{1}{\mu_0}\nabla\times\mathbf{A}$$

$$\rightarrow \mathbf{E} = \frac{1}{j\omega\varepsilon_0}\nabla\times\mathbf{H}$$

$$E_\phi = \frac{j\omega\mu_0 m}{4\pi}\beta^2\sin\theta\left[\frac{1}{j\beta R}+\frac{1}{(j\beta R)^2}\right]e^{-j\beta R}$$

$$H_R = -\frac{j\omega\mu_0 m}{4\pi\eta_0}\beta^2 2\cos\theta\left[\frac{1}{(j\beta R)^2}+\frac{1}{(j\beta R)^3}\right]e^{-j\beta R}$$

$$H_\theta = -\frac{j\omega\mu_0 m}{4\pi\eta_0}\beta^2\sin\theta\left[\frac{1}{j\beta R}+\frac{1}{(j\beta R)^2}+\frac{1}{(j\beta R)^3}\right]e^{-j\beta R}$$

Far fields:

$$E_\phi = \frac{\omega\mu_0 m}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta$$

$$H_\theta = -\frac{\omega\mu_0 m}{4\pi\eta_0}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta$$

Recall: Principle of duality

$$\mathbf{E}_e = \eta_0\mathbf{H}_m$$

$$\mathbf{H}_e = -\frac{\mathbf{E}_m}{\eta_0}$$

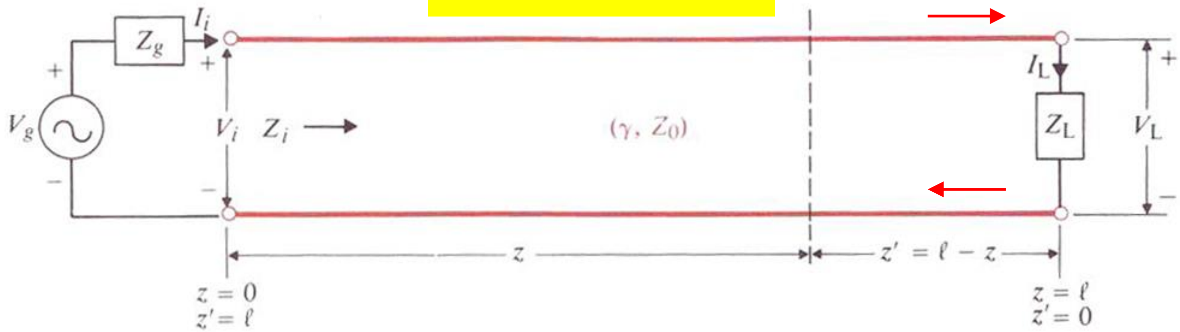
Recall: Hertzian dipole

$$E_\theta = j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0\beta\sin\theta$$

Note: $E_{\theta,p}$ & $E_{\phi,m}$ are in both space and time quadrature!

Antenna vs Transmission Line

Transmission line



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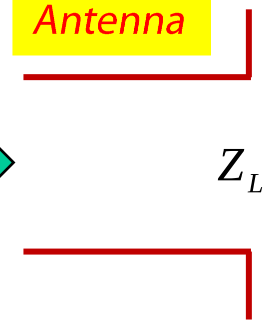
Antenna



$dl = \lambda / 4?$

$Z_L = R_L + jX_L?$

Input impedance (lossless): $z = 0$ or $z' = l$

$$Z_i = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

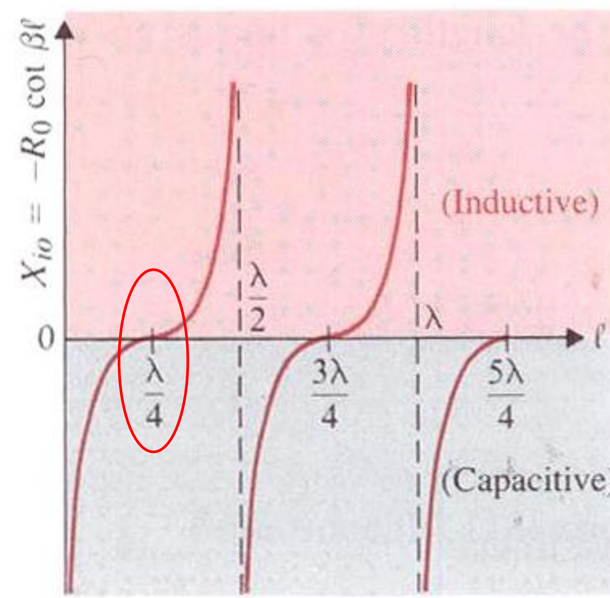
Open-circuit termination: $Z_L = \infty$

$$Z_{io} = -jR_0 \cot \beta l = jX_{io}$$

→ Purely reactive
(capacitive or inductive)

Recall:

$$\rightarrow C = \varepsilon \frac{w}{d} \quad \rightarrow L = \mu \frac{d}{w}$$



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