Electromagnetics:
Antenna Patterns and Antenna Parameters
Thin Linear Antennas
Antenna Arrays
(11-3, 11-4, 11-5)

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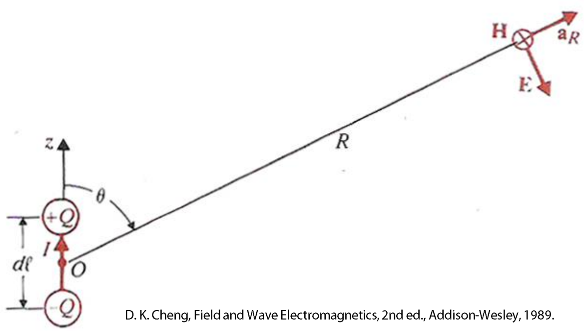
Radiation Pattern or Antenna Pattern

Relative far-zone field strength at a fixed distance from an antenna:

→ *E-plane pattern: Normalized field strength vs θ for a constant ϕ*

→ *H-plane pattern: Normalized field strength vs ϕ for $\theta = \pi/2$*

Herzian dipole:



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Far fields: $\beta R = 2\pi R / \lambda >> 1$

$$E_\theta = j \frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0 \beta \sin\theta$$

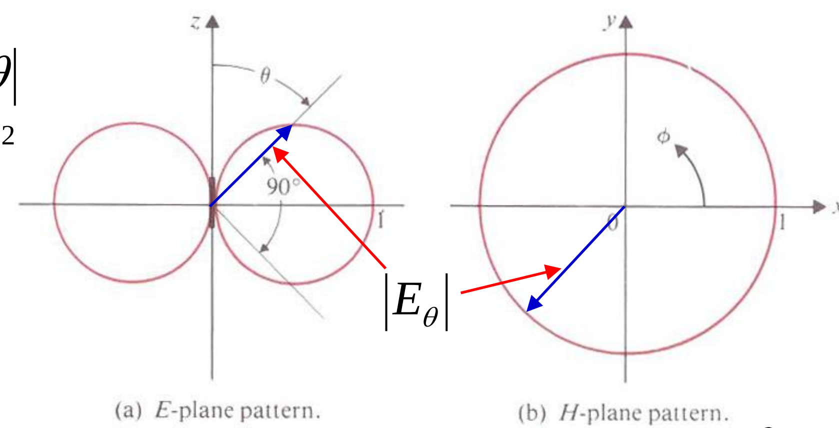
$$H_\phi = j \frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta \sin\theta$$

E-plane pattern: $\rightarrow$ *Normalized* $|E_\theta| = |\sin\theta|$

$$\rightarrow |E_\theta| = \left|\frac{x}{\sqrt{x^2+z^2}}\right| \rightarrow \left(x \pm \frac{1}{2}\right)^2 + z^2 = \left(\frac{1}{2}\right)^2$$

H-plane pattern:

$$\rightarrow \textit{Normalized } |E_\theta| = |\sin\theta|_{\theta=\pi/2} = 1$$

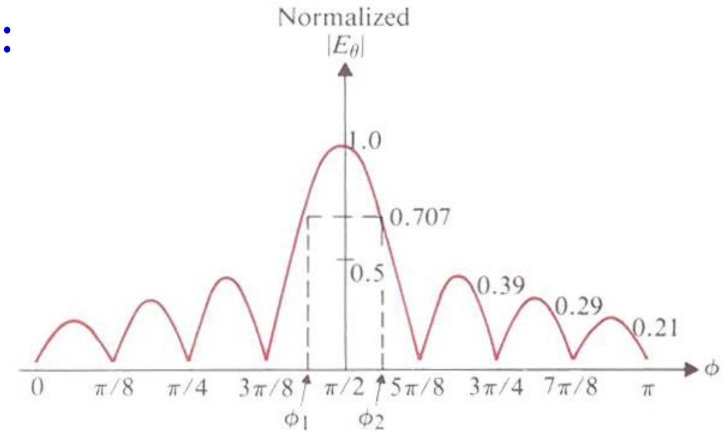


(a) *E*-plane pattern. (b) *H*-plane pattern.

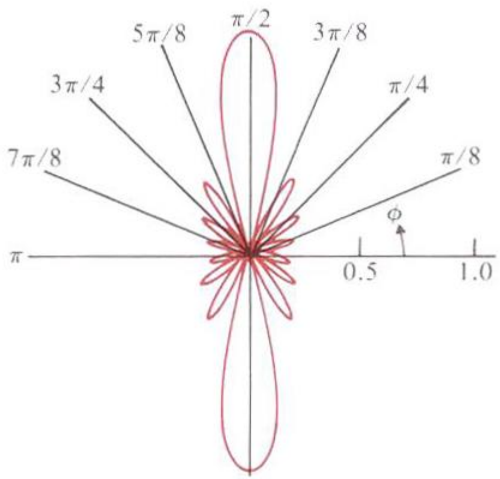
D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Antenna Patterns

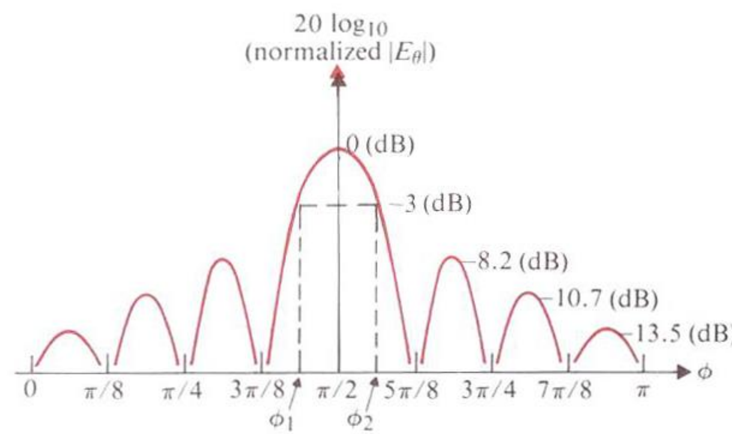
Typical H-plane radiation patterns:



(b) Radiation pattern
in rectangular coordinates.



(a) A typical radiation pattern
in polar coordinates.



(c) Radiation pattern in rectangular
coordinates plotted in dB scale.

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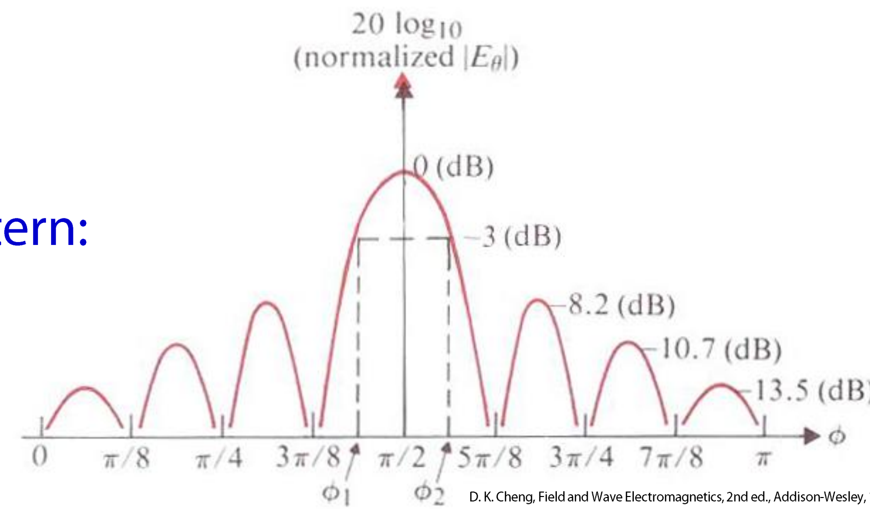
Antenna Parameters (1)

A typical H-plane radiation pattern:



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

(c) Radiation pattern in rectangular coordinates plotted in dB scale.

1. Bandwidth (Width of the main beam):

→ *Angular width between the half-power or –3-dB points (occasionally, –10-dB points)*

2. Sidelobe levels: → *Typically, –40-dB required for radar applications*

3. Directivity: → *Radiation intensity (time-averaged power per unit solid angle):* $U = R^2 P_{av}$

Time-average Poynting vector

→ *Total time-average power radiated:* $P_r = \oint \mathbf{P}_{av} \cdot d\mathbf{s} = \oint U d\Omega$

→ *Directive gain:* $G_D(\theta,\phi) = \dfrac{U(\theta,\phi)}{U_{av}} = \dfrac{U(\theta,\phi)}{\oint U d\Omega / 4\pi} = \dfrac{U(\theta,\phi)}{P_r / 4\pi}$

→ *Directivity:* $D = \dfrac{U_{max}}{U_{av}} = \dfrac{U_{\max}}{P_r / 4\pi} = \dfrac{4\pi |E_{max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E(\theta,\phi)|^2 \sin\theta d\theta d\phi}$

Antenna Parameters (2)

4. Power gain:

$$G_P = \frac{U_{\max}}{P_i / 4\pi} \quad \leftarrow \underbrace{P_i}_{\text{Total input power}} = \overbrace{P_r}^{\text{Radiated power}} + \underbrace{P_l}_{\text{Power loss (Ohmic, structural, etc.)}}$$

5. Radiation efficiency:

$$\eta_r = \frac{P_r}{P_i} = \frac{G_p}{D} \quad \leftarrow \boxed{D = \frac{U_{max}}{U_{av}} = \frac{U_{\max}}{P_r / 4\pi}}$$

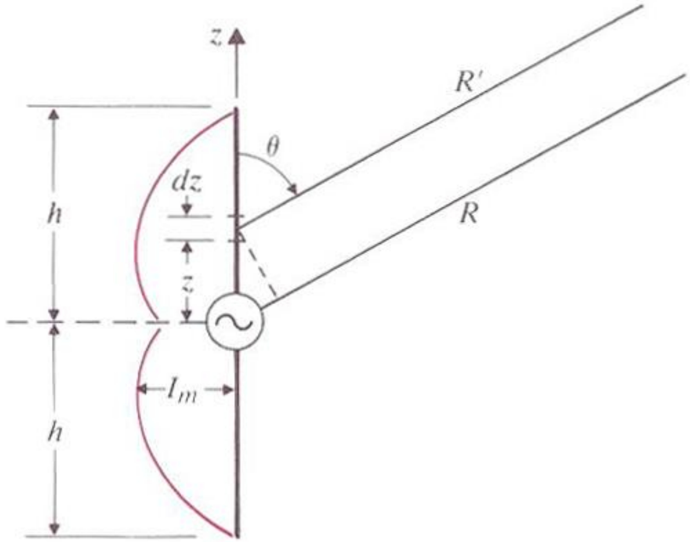
6. Radiation resistance:

$$P_r = \frac{1}{2} I^2 R_r$$

7. Loss resistance:

$$P_l = \frac{1}{2} I^2 R_l$$

Thin Linear Antennas (1)



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Current distribution:

Why?

$$\rightarrow I(z) = I_m \sin\beta(h-|z|)$$

Recall: Hertzian dipole $(\beta R = 2\pi R/\lambda >> 1)$

$$E_\theta = j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\eta_0\beta\sin\theta$$

$$H_\phi = j\frac{Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\beta\sin\theta$$

Elemental contribution

Far fields:

$$\rightarrow dE_\theta = \eta_0 dH_\phi = j\frac{Idz}{4\pi}\left(\frac{e^{-j\beta R'}}{R'}\right)\eta_0\beta\sin\theta \quad \leftarrow R' = \left(R^2+z^2-2Rz\cos\theta\right)^{1/2} \cong R - z\cos\theta$$

$$\rightarrow E_\theta = \eta_0 H_\phi \cong j\frac{I_m\eta_0\beta\sin\theta}{4\pi R}e^{-j\beta R}\int_{-h}^{h}\sin\beta(h-|z|)e^{j\beta z\cos\theta}dz$$

Why?

$$= j\frac{I_m\eta_0\beta\sin\theta}{4\pi R}e^{-j\beta R}\int_{-h}^{h}\sin\beta(h-|z|)[\cos(\beta z\cos\theta) + \cancel{j\sin(\beta z\cos\theta)}]dz$$

$$= j\frac{I_m\eta_0\beta\sin\theta}{2\pi R}e^{-j\beta R}\int_{0}^{h}\sin\beta(h-z)\cos(\beta z\cos\theta)dz$$

Thin Linear Antennas (2)

Far fields:

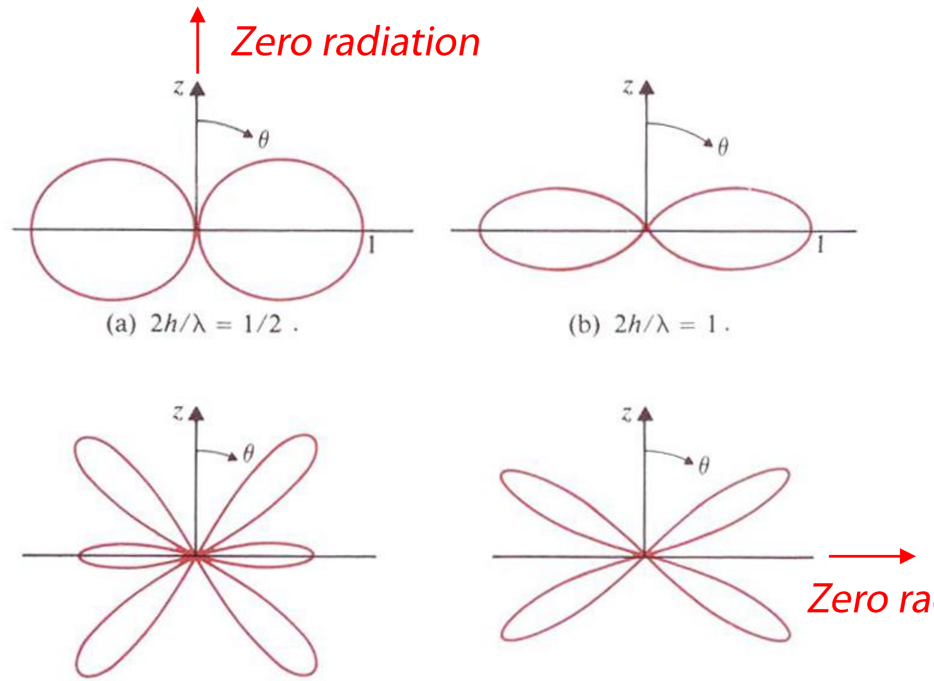
$$\rightarrow E_\theta = \eta_0 H_\phi \cong j \frac{I_m \eta_0 \beta \sin\theta}{2\pi R} e^{-j\beta R} \int_0^h \sin\beta(h-z)\cos(\beta z\cos\theta)dz$$

$$= j\frac{60 I_m}{R} e^{-j\beta R} F(\theta) \quad \leftarrow F(\theta) = \frac{\cos(\beta h\cos\theta) - \cos\beta h}{\sin\theta}$$

← Pattern function

Radiation pattern:

$$\rightarrow \beta h = \frac{2\pi}{\lambda} h$$



(a) $2h/\lambda = 1/2$. (b) $2h/\lambda = 1$.

(c) $2h/\lambda = 3/2$. (d) $2h/\lambda = 2$.

Why?

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Half-Wave Dipole (1)

E-plane Pattern function:

$$F(\theta) = \frac{\cos(\beta h \cos\theta) - \cos\beta h}{\sin\theta}$$

For a half-wave dipole: $2h = \lambda/2 \;\rightarrow\; \beta h = 2\pi h/\lambda = \pi/2$

Far-zone fields: $$\rightarrow E_\theta = \frac{j60 I_m}{R} e^{-j\beta R} \left\{ \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \right\}$$

$$\rightarrow H_\phi = \frac{jI_m}{2\pi R} e^{-j\beta R} \left\{ \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \right\}$$

Time-average Poynting vector:

$$P_{av} = \frac{1}{2} E_\theta H_\phi^* = \frac{15 I_m^2}{\pi R^2} \left\{ \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \right\}^2$$

Total power radiated:

$$P_r = \int_0^{2\pi} \int_0^{\pi} P_{av} R^2 \sin\theta \, d\theta \, d\phi$$

$$= 30 I_m^2 \int_0^{\pi} \frac{\cos^2[(\pi/2)\cos\theta]}{\sin\theta} d\theta \;\; \cong 36.54 I_m^2$$

Half-Wave Dipole (2)

Radiation resistance:

$$R_r = \frac{2P_r}{I_m^2} = 73.1\ (\Omega) \leftarrow P_r = \frac{1}{2} I^2 R_r$$

Directivity:

$$U_{\max} = R^2 P_{av}(\theta = \pi/2) = \frac{15}{\pi} I_m^2 \qquad \leftarrow P_{av} = \frac{15 I_m^2}{\pi R^2} \left\{ \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \right\}^2$$

$$\rightarrow D = \frac{U_{\max}}{P_r / 4\pi} = \frac{60}{36.54} = 1.64 \qquad \leftarrow P_r \cong 36.54 I_m^2$$

Half-power bandwidth:

$$E_\theta = \frac{j60 I_m}{R} e^{-j\beta R} \left\{ \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \right\}$$

$$\rightarrow \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} = \frac{1}{\sqrt{2}} \qquad \rightarrow \theta_{FWHM} \approx 78^\circ$$

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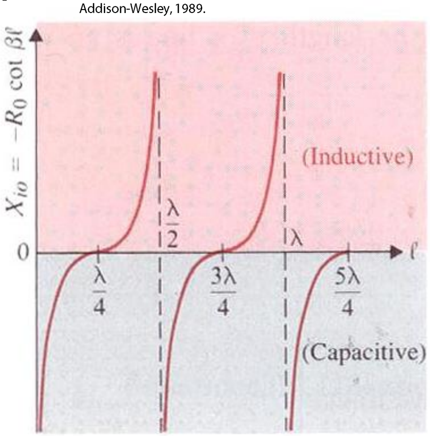


What about the total impedance?

$$Z_r = R_r + jX_r$$

For the impedance matching,
the dipole length should be slightly shorter than $\lambda/4$!

Quarter-Wave Monopole

Recall: Image method $\leftarrow$ Radiation limited into the upper half-space!

Total power radiated:

For HW DP

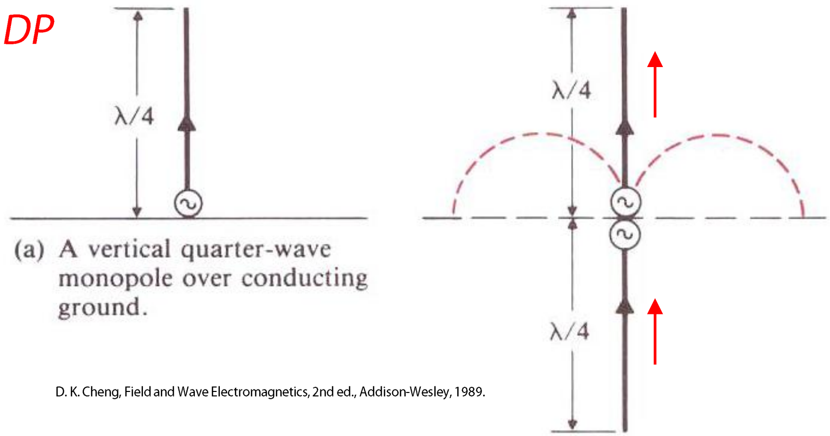
$$\rightarrow P_r = \frac{1}{2} \times (36.54 I_m^2)$$

Radiation resistance:

$$\rightarrow R_r = \frac{2P_r}{I_m^2} = 36.54 \ (\Omega)$$

Directivity:

$$\rightarrow D = \frac{U_{max}}{U_{av}} = \frac{U_{max}}{P_r / 2\pi} = 1.64$$



(a) A vertical quarter-wave monopole over conducting ground.

(b) Equivalent half-wave dipole radiating into upper half-space.

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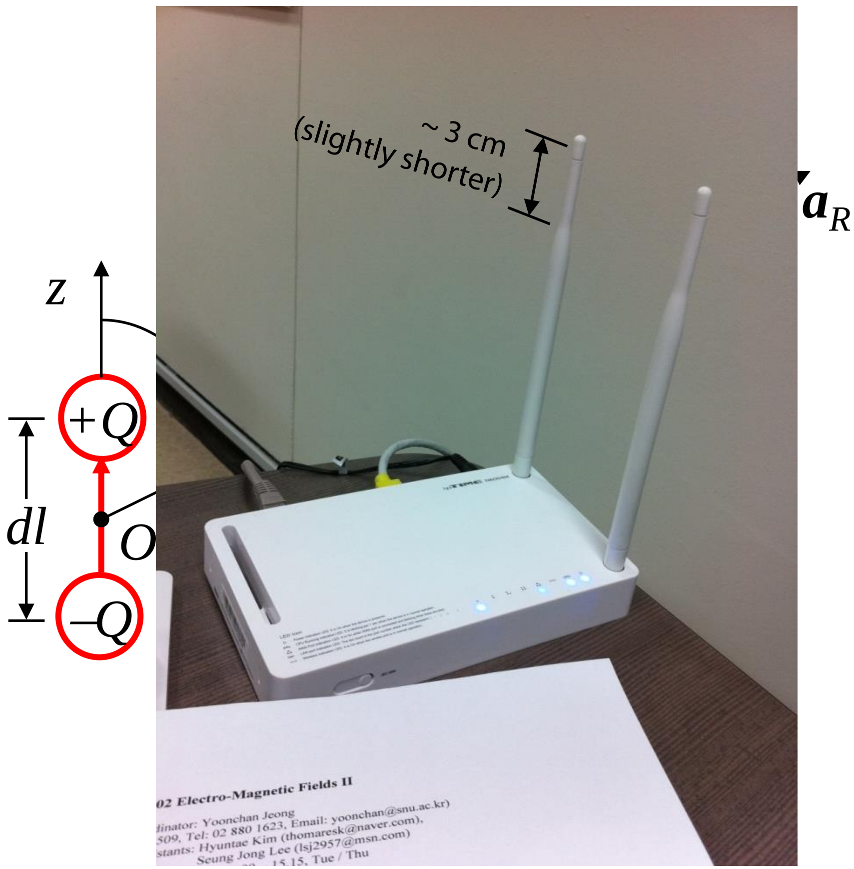
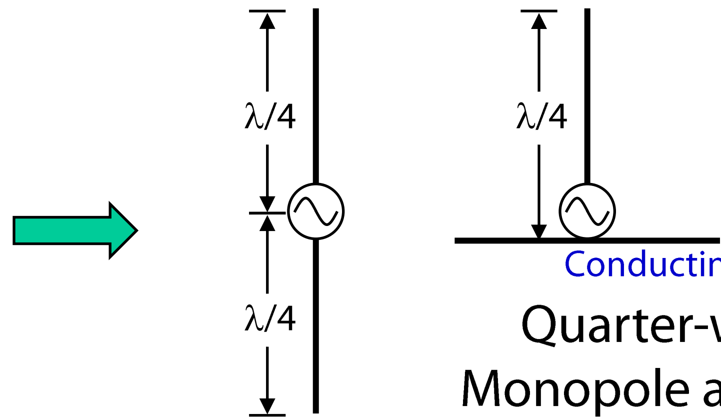
Recall: Electromagnetic Waves (RF/MW)





Quarter-wave
Monopole antenna

Half-wave
dipole antenna

$$\boxed{c = f\lambda}$$

Speed of EM wave — Frequency — Wavelength

WiFi router (f = 2.4 GHz)

$$\boxed{\lambda/4 = 3\times10^{8} / 2.4\times10^{9} / 4 \approx 0.031 \text{ m}}$$

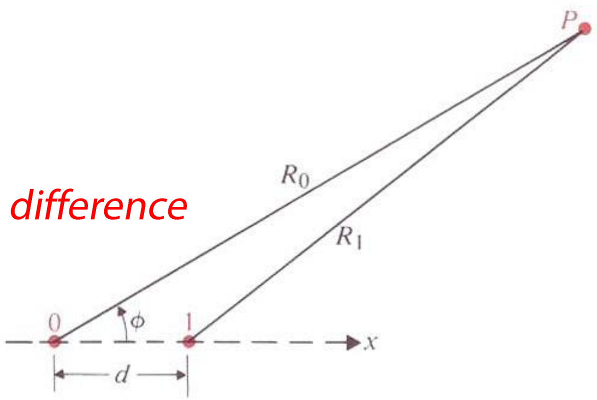
Antenna Arrays

Two-element arrays:

$$E_0 = E_m F(\theta,\phi)\frac{e^{-j\beta R_0}}{R_0}$$

Initial phase difference

$$E_1 = E_m F(\theta,\phi)\frac{e^{j\xi}e^{-j\beta R_1}}{R_1}$$



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Total electric field:

Why?

$$E = E_0 + E_1 = E_m F(\theta,\phi)\left(\frac{e^{-j\beta R_0}}{R_0} + \frac{e^{j\xi}e^{-j\beta R_1}}{R_1}\right) \quad \leftarrow R_1 \cong R_0 - d\,\underline{\sin\theta}\cos\phi$$

$$\rightarrow E = E_m \frac{F(\theta,\phi)}{R_0} e^{-j\beta R_0}(1 + e^{j\beta d\sin\theta\cos\phi}e^{j\xi}) = E_m \frac{F(\theta,\phi)}{R_0} e^{-j\beta R_0}(1+e^{j\psi})$$

$$= E_m \frac{F(\theta,\phi)}{R_0} e^{-j\beta R_0} e^{j\psi/2}\left(2\cos\frac{\psi}{2}\right) \quad \leftarrow \psi = \beta d\sin\theta\cos\phi + \xi$$

$$\rightarrow |E| = \frac{2E_m}{R_0}|F(\theta,\phi)|\left|\cos\frac{\psi}{2}\right|$$

Array factor

Element factor

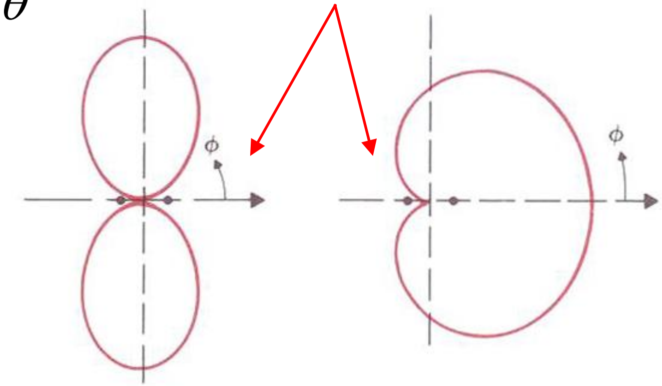
Multi-Element Parallel Dipole Arrays

For two parallel half-wave dipoles:

$$|E| = \frac{2E_m}{R_0}|F(\theta,\phi)|\left|\cos\frac{\psi}{2}\right| \quad \leftarrow F(\theta) = \frac{\cos(\beta h\cos\theta) - \cos\beta h}{\sin\theta}$$

$$= \frac{2E_m}{R_0}\left|\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right|\left|\cos\frac{\psi}{2}\right|$$

Why zero?



(a) $d = \lambda/2$, $\xi = 0$. (b) $d = \lambda/4$, $\xi = -\pi/2$.

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$\rightarrow$ Normalized array factor:

$$|A(\psi)| = \left|\cos\frac{\psi}{2}\right| = \left|\cos\frac{1}{2}(\beta d\cos\phi + \xi)\right|$$

$$\leftarrow \psi = \beta d\sin\theta\cos\phi + \xi \quad \leftarrow \theta = \pi/2$$

Arrays of equal magnitude and a uniform progressive phase shift:

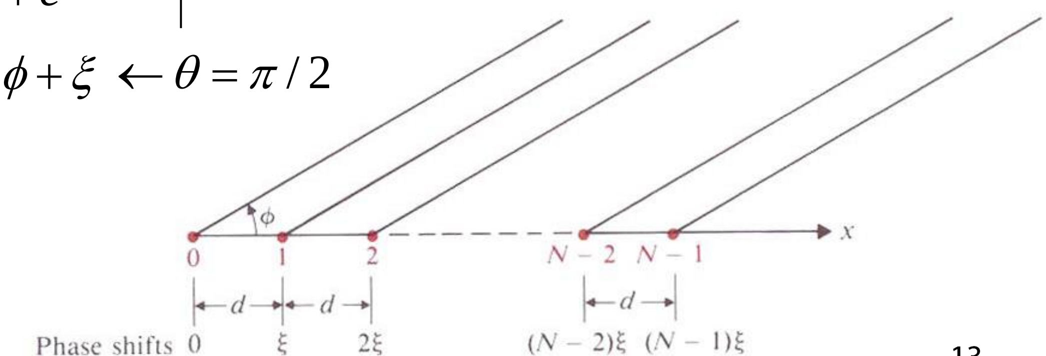
$$|A(\psi)| = \frac{1}{N}\left|1 + e^{j\psi} + e^{j2\psi} + \cdots + e^{j(N-1)\psi}\right|$$

$$\leftarrow \psi = \beta d\cos\phi + \xi \leftarrow \theta = \pi/2$$

$$\rightarrow |A(\psi)| = \frac{1}{N}\left|\frac{1-e^{jN\psi}}{1-e^{j\psi}}\right|$$

$$= \frac{1}{N}\left|\frac{\sin(N\psi/2)}{\sin(\psi/2)}\right|$$

$\rightarrow$ Broadside or endfire?



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