Second harmonic generation

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Nonlinear polarisation

Constitutive equations

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_o\mathbf{E} + \mathbf{P},$$

$$\mathbf{P} = \varepsilon_o\chi\mathbf{E}$$

Origin of nonlinear response

Related to anharmonic motion of bound electrons under the influence of an applied field.

$$\mathbf{P} = \varepsilon_o\chi\mathbf{E} = \varepsilon_o\left(\chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{E}\mathbf{E} + \chi^{(3)}\mathbf{E}\mathbf{E}\mathbf{E} + \cdots\right)$$

Note: $\chi^{(2)}$ is non-zero only for media that lack an inversion symmetry (centrosymmetry).

Nonlinear perturbation

- **Coupled-mode theory with nonlinear perturbation terms**

With second-order nonlinearity

$$\nabla\cdot(\boldsymbol{E}'_{\omega}\times\boldsymbol{H}^{*}_{\omega,p}+\boldsymbol{E}^{*}_{\omega,p}\times\boldsymbol{H}'_{\omega})=-i\omega\boldsymbol{E}^{*}_{\omega,p}\cdot\Delta\boldsymbol{P}'_{\omega},\quad (p=1,2,\ldots),$$

$$\Delta\boldsymbol{P}'_{\omega_1}=\Delta\varepsilon_{ij}(\omega_1)E'_{\omega_1,j}+2d_{ijk}(-\omega_1,\omega_2,-\omega_1)E'_{\omega_2,j}E'^{*}_{\omega_1,k}$$
$$+\left\{3\chi_{ijkl}(-\omega_1,\omega_1,-\omega_1,\omega_1)E'_{\omega_1,j}E'^{*}_{\omega_1,k}+6\chi_{ijkl}(-\omega_1,\omega_2,-\omega_2,\omega_1)E'_{\omega_2,j}E'^{*}_{\omega_2,k}\right\}E'_{\omega_1,l},$$

$$\Delta\boldsymbol{P}'_{\omega_2}=\Delta\varepsilon_{ij}(\omega_2)E'_{\omega_2,j}+d_{ijk}(-\omega_2,\omega_1,\omega_1)E'_{\omega_1,k}E'_{\omega_1,k}$$
$$+\left\{3\chi_{ijkl}(-\omega_2,\omega_2,-\omega_2,\omega_2)E'_{\omega_2,j}E'^{*}_{\omega_2,k}+6\chi_{ijkl}(-\omega_2,\omega_1,-\omega_1,\omega_2)E'_{\omega_1,j}E'^{*}_{\omega_1,k}\right\}E'_{\omega_2,l}.'$$

Without second-order nonlinearity

$$\nabla\cdot(E'\times H^{*}_{p}+E^{*}_{p}\times H')=-i\omega E^{*}_{p}\cdot(\Delta\varepsilon_{L}+\Delta\varepsilon_{NL})E',\quad (p=1,2,3,\ldots)$$

$$\Delta\varepsilon_{NL(q)}=\varepsilon_{o}\frac{3}{4}\chi^{(3)}(r,\phi,z)\cdot\sum_{s}\alpha_{(q,s)}\left|E_{s}(t,z)\right|^{2}\left|\hat{e}_{s}\right|^{2},\ \alpha_{(q,s)}=\begin{cases}1 & (q=s),\\ 2 & (q\neq s),\end{cases}$$

Second harmonic generation

- **With plane waves**

$$E^{\omega_j}(t,z) = E_j(z)\exp[i(\omega_j t - k_j z)] \quad (j = 1,\ 2,\ \omega_2 = 2\omega_1)$$

- **Second-order nonlinear coupled-wave equations**

$$\frac{dE_1}{dz} = -i\frac{\omega_1}{\varepsilon_o n_1 c} d_1 E_2 E_1^* \exp(-i\Delta kz),$$

$$\frac{dE_2}{dz} = -i\frac{\omega_2}{2\varepsilon_o n_2 c} d_2 E_1^2 \exp(i\Delta kz),$$

where $\Delta k = k_2 - 2k_1$

- **Phase matching condition:** $\Delta k = k_2 - 2k_1 = 0$

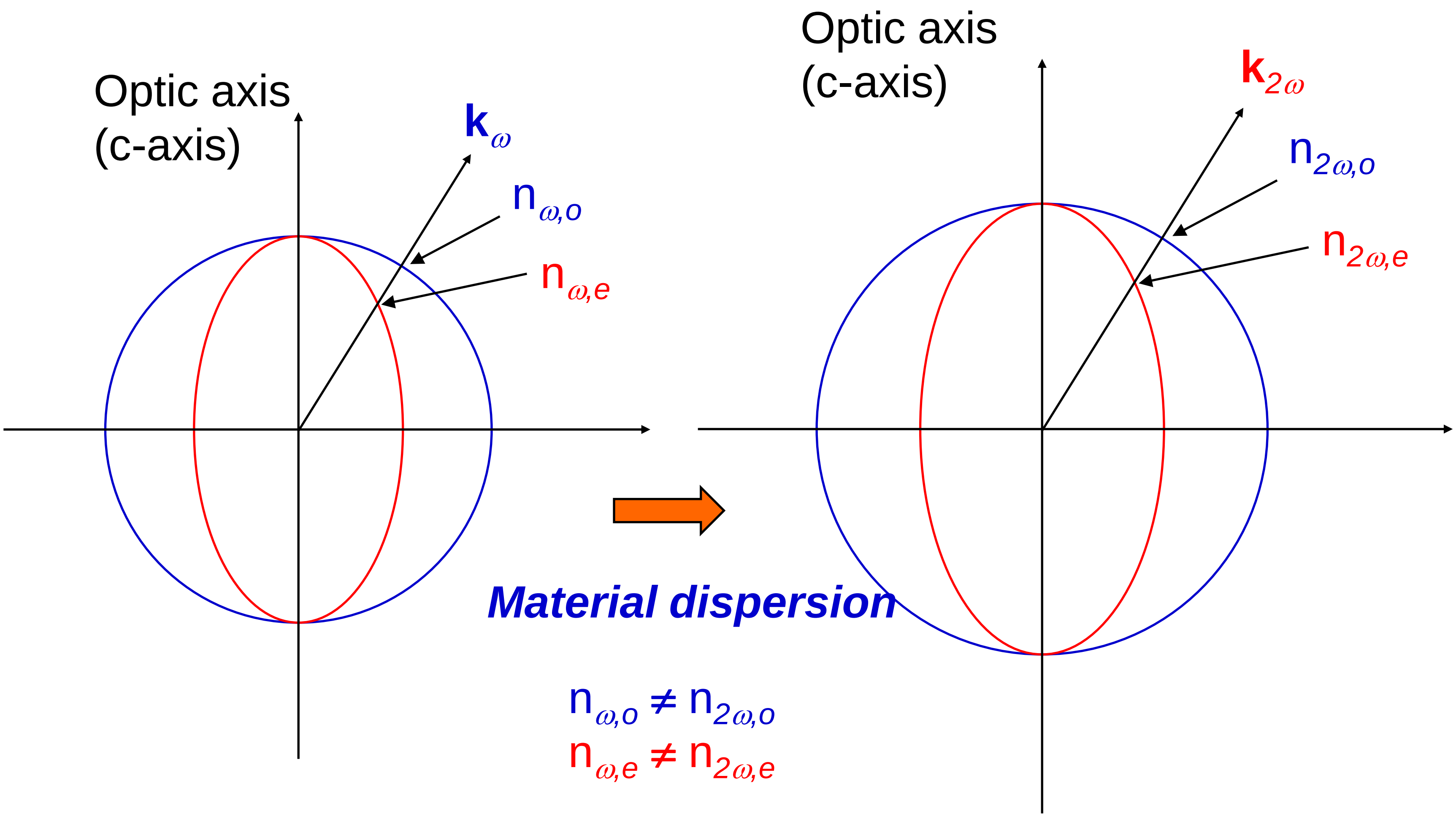
Birefringence: Type I or Type II

Modal dispersion: Higher-order modes, waveguide modes

Quasi-phase matching: Modulation of nonlinearity

Phase matching in birefringent media

eg. Negative uniaxial crystal



$n_{\omega,o} \neq n_{2\omega,o}$

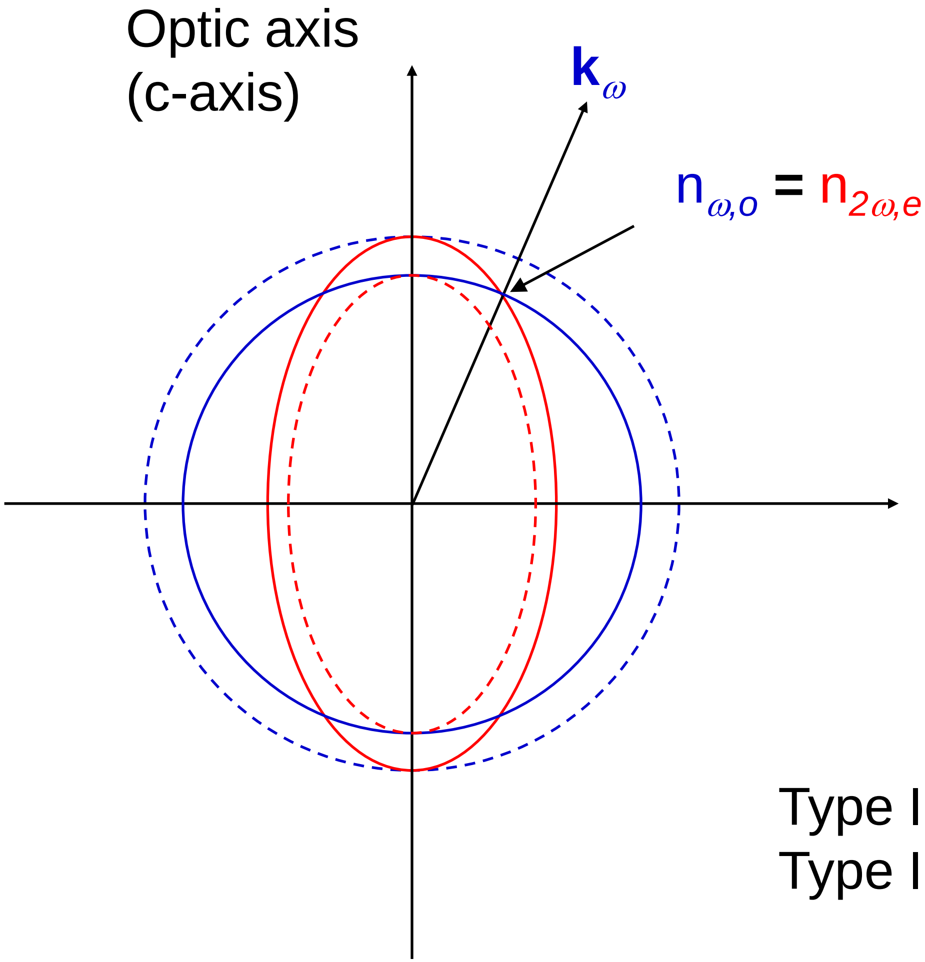
$n_{\omega,e} \neq n_{2\omega,e}$

For fundamental harmonic

For second harmonic

Phase matching in birefringent media

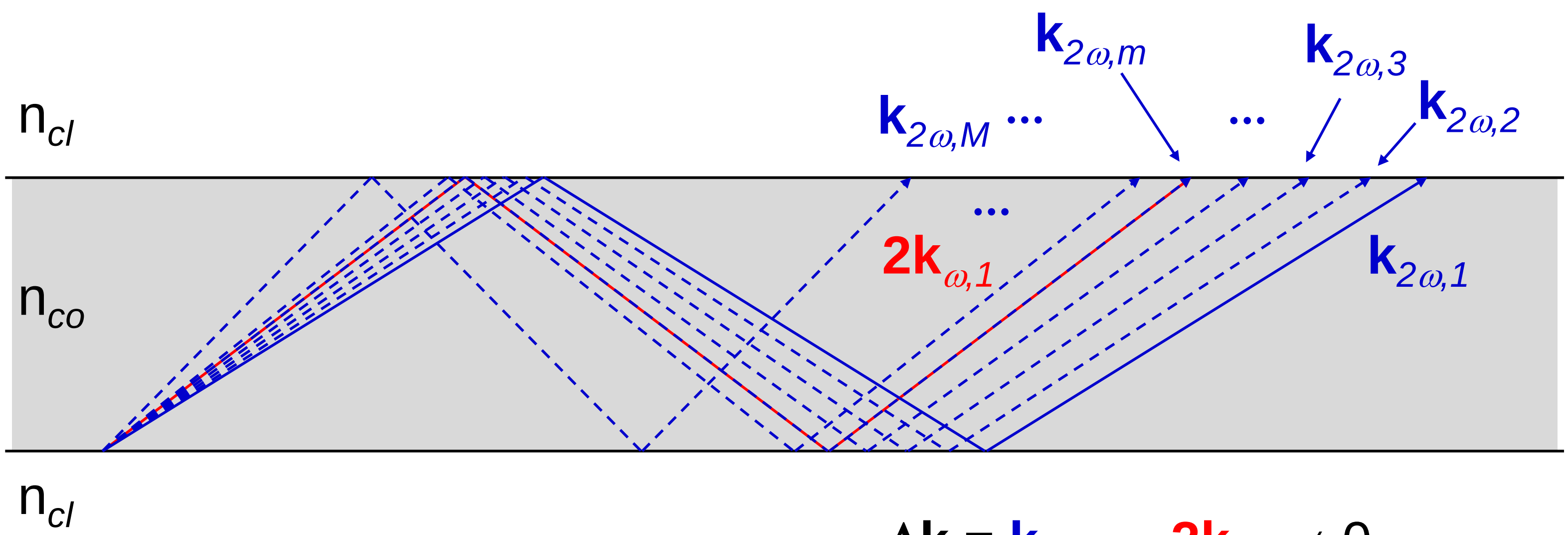
eg. Negative uniaxial crystal



Type I: ooe, eeo
Type II: oeo, oee

Phase matching in waveguides

eg. Dispersive waveguide



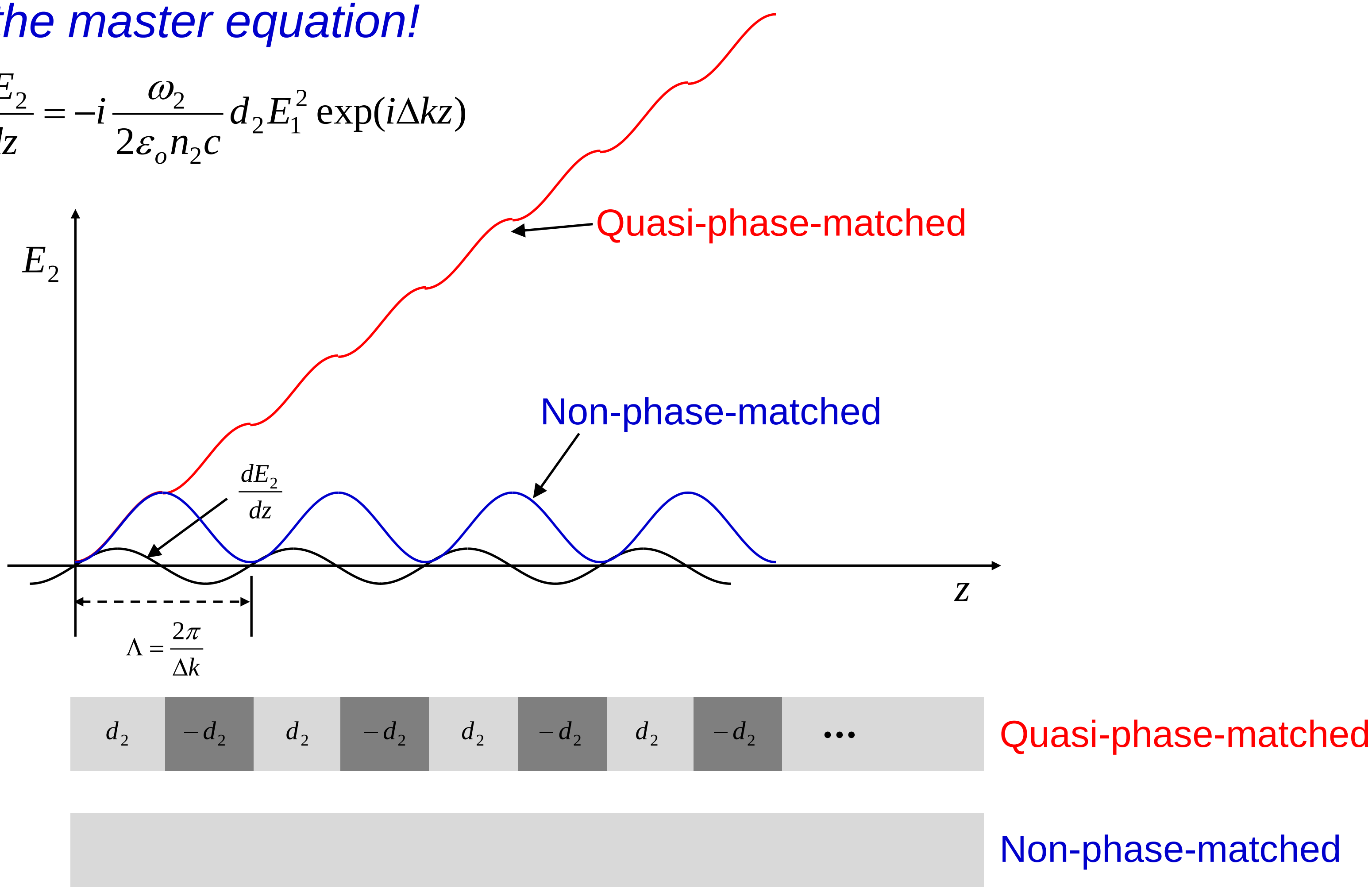
$\Delta\mathbf{k} = \mathbf{k}_{2\omega,1} - \mathbf{2k}_{\omega,1} \neq 0$

$\Delta\mathbf{k} = \mathbf{k}_{2\omega,m} - \mathbf{2k}_{\omega,n} = 0$, i.e. the phase-matching condition can be satisfied for some set of modes of *m* and *n*, *exploiting the modal dispersion in waveguides*.

Quasi-phase matching

Recall the master equation!

$$\frac{dE_2}{dz} = -i\frac{\omega_2}{2\varepsilon_o n_2 c} d_2 E_1^2 \exp(i\Delta k z)$$



Periodic spatial modulation of the nonlinear coefficient: e.g. PPLN (periodically-poled lithium niobate)