Optical solitons

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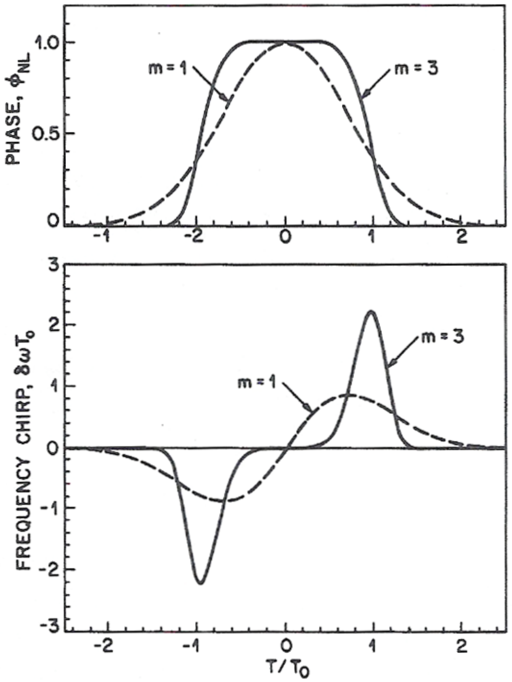
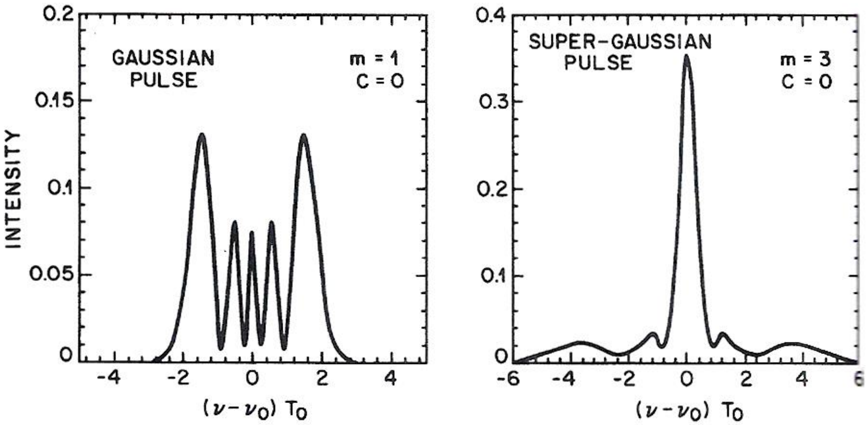
Self-phase modulation (SPM)

For Gaussian and super-Gaussian pulses:





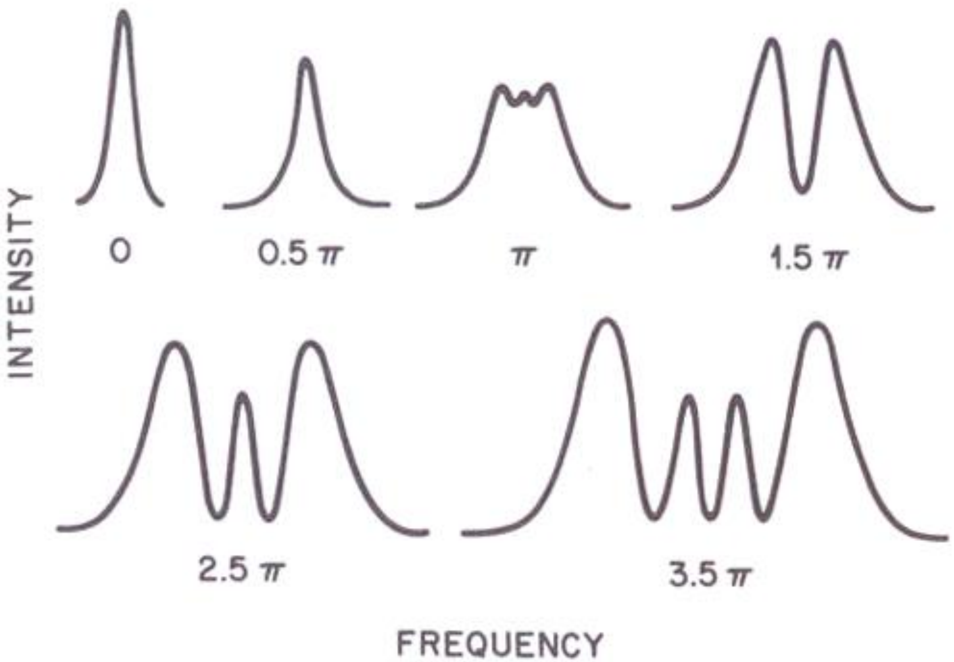
$$\phi_{NL,\max} = 4.5\pi$$

Frequency chirp incurred

Source: Nonlinear Fiber Optics, G. P. Agrawal

Self-phase modulation (SPM)

For Gaussian pulses:



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Nonlinear pulse propagation

NLSE:

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2}A = i\gamma|A|^2 A \qquad \textit{for} \quad \exp(-i\omega t)$$

$$\rightarrow \quad \frac{\partial A}{\partial z} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \frac{\alpha}{2}A = i\gamma|A|^2 A,$$

where $T = t - z/v_g = t - \beta_1 z$

Modulation instability

If fibre losses are ignored:

$$i\frac{\partial A}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \gamma |A|^2 A$$

Steady-state solution:

$$\bar{A} = \sqrt{P_0}\,\exp(i\phi_{NL}),$$

$$\text{where } \phi_{NL} = \gamma P_0 z.$$

With a small perturbation:

$$A = (\sqrt{P_0} + a)\exp(i\phi_{NL})$$

$$\rightarrow \quad i\frac{\partial a}{\partial z} = \frac{\beta_2}{2}\frac{\partial^2 a}{\partial T^2} - \gamma P_0 (a + a^*)$$

With a perturbation in the amplitude, i.e. $A=(\sqrt{P_0}+a)\exp(i\phi_{NL})$

$$i\frac{\partial A}{\partial z}=i\frac{\partial}{\partial z}[(\sqrt{P_0}+a)\exp(i\phi_{NL})]$$

$$=i\frac{\partial a}{\partial z}\exp(i\phi_{NL})+i(\sqrt{P_0}+a)\cdot i\frac{\partial \phi_{NL}}{\partial z}\exp(i\phi_{NL})$$

$$=i\frac{\partial a}{\partial z}\exp(i\phi_{NL})-(\sqrt{P_0}+a)\gamma P_0\exp(i\phi_{NL})$$

$$-\gamma|A|^2A=-\gamma(\sqrt{P_0}+a)\exp(i\phi_{NL})\cdot(\sqrt{P_0}+a^*)\exp(-i\phi_{NL})\cdot(\sqrt{P_0}+a)\exp(i\phi_{NL})$$

$$=-\gamma[P_0(\sqrt{P_0}+a)+P_0(a+a^*)+\sqrt{P_0}(a^2+2a^*a)+a^*a^2]\exp(i\phi_{NL})\ \ (\text{as } a\to O^2)$$

$$\approx-\gamma P_0(\sqrt{P_0}+a)\exp(i\phi_{NL})-\gamma P_0(a+a^*)$$

In result (w/o loss of generality):

$$i\frac{\partial a}{\partial z}=\frac{\beta_2}{2}\frac{\partial^2 a}{\partial T^2}-\gamma P_0(a+a^*)$$

Modulation instability

Consider its solution in the form:

$$a(z,T) = a_1 \exp[i(Kz - \Omega T)] + a_2 \exp([-i(Kz - \Omega T)]$$

Solution with the dispersion relation:

$$K = \pm \frac{1}{2} |\beta_2 \Omega| [\Omega^2 + \mathrm{sgn}(\beta_2)\Omega_c^2]^{1/2},$$

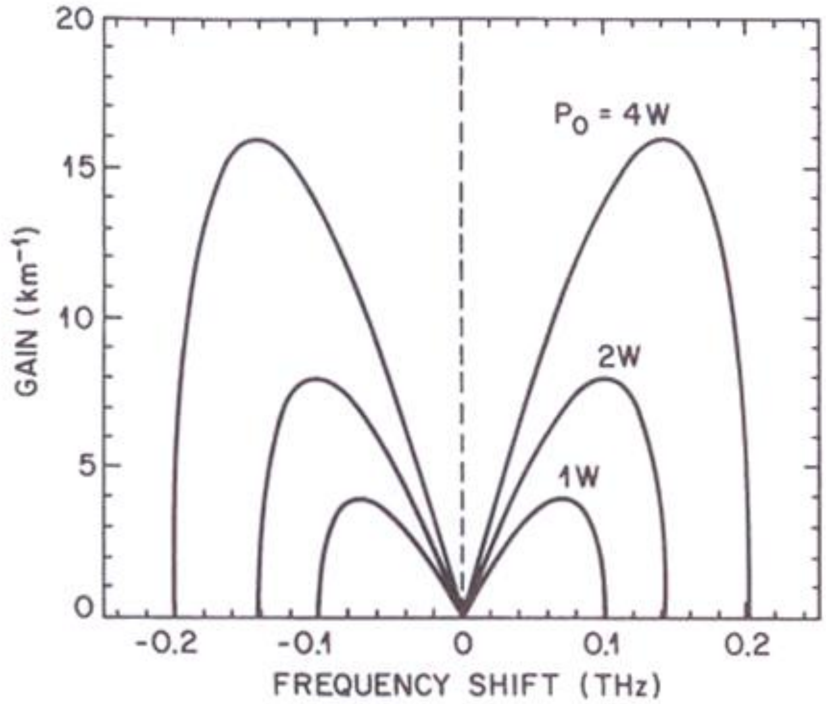
where $\mathrm{sgn}(\beta_2) = \pm 1$ depending on the sign of β_2,

$$\Omega_c^2 = \frac{4\gamma P_0}{|\beta_2|} = \frac{4}{|\beta_2| L_{NL}}.$$

The sign of β_2 (GVD) determines the stability against the perturbation.

Modulation instability

Gain spectra: $g(\Omega) = 2\,\mathrm{Im}(K) = \left|\beta_2 \Omega\right| (\Omega_c^2 - \Omega^2)^{1/2}$



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$$\beta_2 = -20\,ps^2/km, \;\; \gamma = 2W^{-1}/km$$

Fibre solitons

Recall NLSE:

$$i\frac{\partial A}{\partial z}=\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2}-\gamma|A|^2 A$$

Introduce three dimensionless variables:

$$U=\frac{A}{\sqrt{P_0}},\ \ \xi=\frac{z}{L_D},\ \ \tau=\frac{T}{T_0}$$

$$\text{where}\ \ L_D=\frac{T_0^2}{|\beta_2|}.$$

NLSE:

$$i\frac{\partial U}{\partial \xi}=\text{sgn}(\beta_2)\frac{1}{2}\frac{\partial^2 U}{\partial \tau^2}-N^2|U|U,$$

$$\text{where}\ \ N^2=\frac{L_D}{L_{NL}}=\frac{\gamma P_0 T_0^2}{|\beta_2|}.$$

$$\rightarrow\qquad i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}-|u|u=0,$$

$$\text{where}\ \ u=NU=\sqrt{\gamma L_D}\,A\ \text{and}\ \text{sgn}(\beta_2)=-1.$$

Fundamental solitons

For N = 1:

$$i\frac{\partial u}{\partial \xi}+\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2}-|u|^2 u=0$$

Shape-preserving solution:

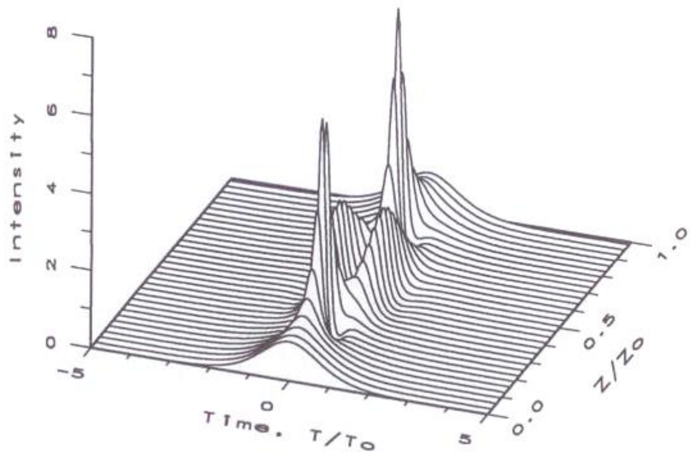
$$u(\xi,\tau)=\mathrm{sech}(\tau)\exp(i\xi/2)$$

How to solve?

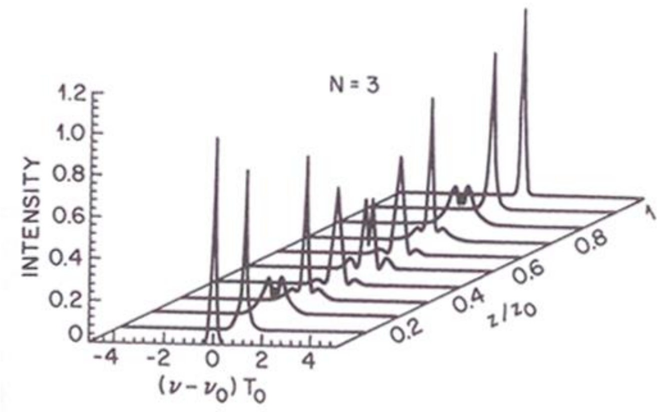
Higher-order solitons

e.g. N = 3:

Temporal evolution



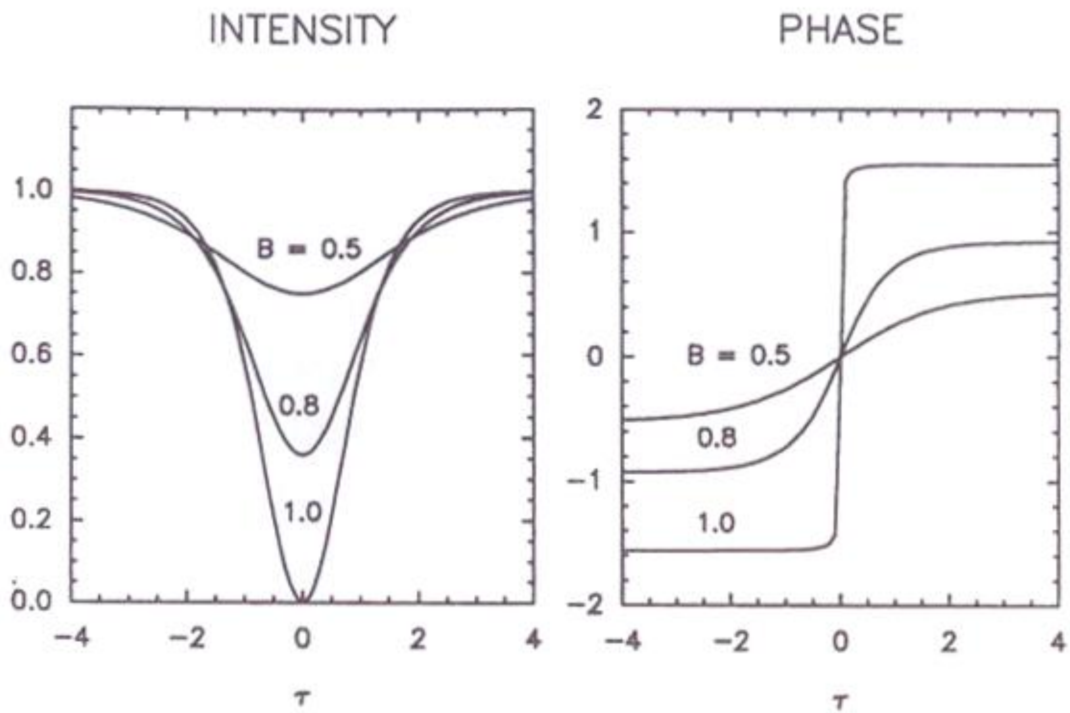
Spectral evolution



Source: Nonlinear Fiber Optics, G. P. Agrawal

Dark solitons

What if $\beta_2 > 0$?

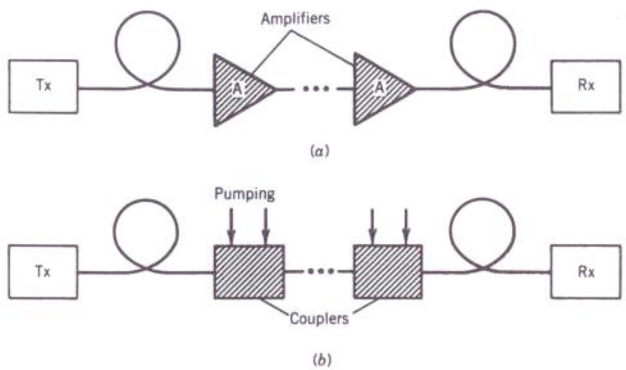


Source: Nonlinear Fiber Optics, G. P. Agrawal

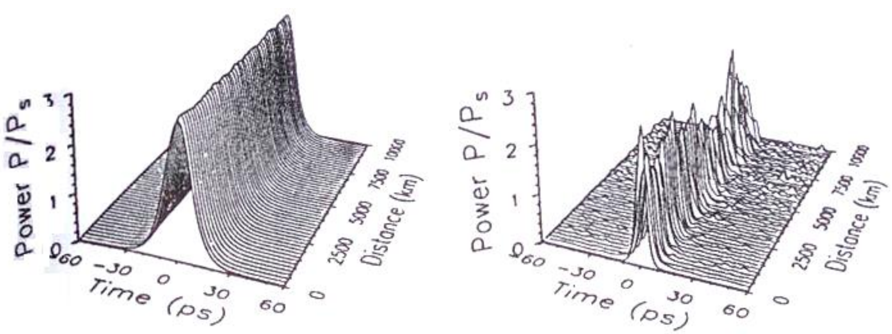
Solition transmission

Fibre losses?

Lumped or distributed amplification:



Experimental results:



Source: Nonlinear Fiber Optics, G. P. Agrawal

$L_A < L_D$ $L_A > L_D$