Four-wave mixing I

Dr Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953
Email: yoonchan@snu.ac.kr

Tensor notation for the third-order susceptibility

Constitutive relations for E-field:

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_o\mathbf{E} + \mathbf{P},$$

$$\mathbf{P} = \varepsilon_o\chi\mathbf{E} = \varepsilon_o\left(\chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{EE} + \underline{\chi^{(3)}}\mathbf{EEE} + \cdots\right)$$

Tensor notation for $\chi^{(3)}$:

$$\chi^{(3)} \to \chi^{(3)}_{ijkl}(\omega_4 = \omega_1 + \omega_2 + \omega_3)$$

$$P_{i,NL} \leftarrow \sum_{j,k,l}\chi^{(3)}_{ijkl}E_jE_kE_l$$

Note that we normally omit summation notation.

Tensor nature of the third-order susceptibility

How many elements for $\chi_{ijkl}^{(3)}$?   *81 elements*

For isotropic media:

$$\chi_{1111}^{(3)} = \chi_{2222}^{(3)} = \chi_{3333}^{(3)},$$

$$\chi_{1122}^{(3)} = \chi_{1133}^{(3)} = \chi_{2211}^{(3)} = \chi_{2233}^{(3)} = \chi_{3311}^{(3)} = \chi_{3322}^{(3)},$$

$$\chi_{1212}^{(3)} = \chi_{1313}^{(3)} = \chi_{2323}^{(3)} = \chi_{2121}^{(3)} = \chi_{3131}^{(3)} = \chi_{3232}^{(3)},$$

$$\chi_{1221}^{(3)} = \chi_{1331}^{(3)} = \chi_{2112}^{(3)} = \chi_{2332}^{(3)} = \chi_{3113}^{(3)} = \chi_{3223}^{(3)}.$$ *→ 21 nonzero elements*

In addition:

$$\chi_{1111}^{(3)} = \chi_{1122}^{(3)} + \chi_{1212}^{(3)} + \chi_{1221}^{(3)}$$ *→ Why?*

→ In the compact form: $\chi_{ijkl}^{(3)} = \chi_{1122}^{(3)}\delta_{ij}\delta_{kl} + \chi_{1212}^{(3)}\delta_{ik}\delta_{jl} + \chi_{1221}^{(3)}\delta_{il}\delta_{jk}$

Cross-phase modulation (XPM)

Suppose that there are two monochromatic waves:

$$\mathbf{E}(r,t) = \frac{1}{2}\hat{x}[E_1\exp(-i\omega_1 t) + E_2\exp(-i\omega_2 t)] + c.c.$$

Induced nonlinear polarisation:

$$\mathbf{P}_{NL}(r,t) = \varepsilon_0\chi^{(3)}\mathbf{EEE} = \frac{1}{2}\hat{x}\{P_{NL}(\omega_1)\exp(-i\omega_1 t) + P_{NL}(\omega_1)\exp(-i\omega_2 t)$$

$$+ P_{NL}(2\omega_1 - \omega_2)\exp[-i(2\omega_1 - \omega_2)t]$$

$$+ P_{NL}(2\omega_2 - \omega_1)\exp[-i(2\omega_2 - \omega_1)t]\} + c.c$$

$$+ \cdots$$

where

$$P_{NL}(\omega_1) = \chi_{eff}(|E_1|^2 + 2|E_2|^2)E_1,$$

$$P_{NL}(\omega_2) = \chi_{eff}(|E_2|^2 + 2|E_1|^2)E_2,$$

$$\chi_{eff} = \frac{3\varepsilon_0\chi^{(3)}_{xxxx}}{4}.$$

Nonlinear pulse propagation with XPM

NLSE:

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2}A = i\gamma|A|^2 A \qquad \textit{for} \quad \exp(-i\omega t)$$

Coupled NLSEs:

$$\rightarrow \quad \frac{\partial A_1}{\partial z} + \beta_{11}\frac{\partial A_1}{\partial t} + i\frac{\beta_{21}}{2}\frac{\partial^2 A_1}{\partial t^2} + \frac{\alpha_1}{2}A_1 = i\gamma_1(|A_1|^2 + 2|A_2|^2)A_1,$$

$$\frac{\partial A_2}{\partial z} + \beta_{12}\frac{\partial A_2}{\partial t} + i\frac{\beta_{22}}{2}\frac{\partial^2 A_2}{\partial t^2} + \frac{\alpha_2}{2}A_2 = i\gamma_2(|A_2|^2 + 2|A_1|^2)A_2.$$

Modulation instability?

Optical solitons?