Electro-Optics:

Electromagnetic Fields (1)

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Topics To Learn

- Textbook:

  Optical Waves in Crystals (Propagation and Control of Laser Radiation), A. Yariv and P. Yeh, Wiley, 1983.

  - *Chap. 1. Electromagnetic Fields*
  - *Chap. 2. Propagation of Laser Beams*
  - *Chap. 3. Polarization of Light Waves*
  - *Chap. 4. Electromagnetic Propagation in Anisotropic Media*

    } Ex 1
  - *Chap. 5. Jones Calculus and its Application to Birefringent Optical Systems*
  - *Chap. 6. Electromagnetic Propagation in Periodic Media*

    } Ex 2
  - *Chap. 7. Electro-optics*
  - *Chap. 8. Electro-optic Devices*
  - *Chap. 11. Guided Waves*

    } Ex 3

Electromagnetic Fields and Waves

"The ideal laser emits coherent electromagnetic radiation which can be described by its electric and magnetic field vectors. The propagation of this radiation field is governed by Maxwell's equations"

Maxwell's equations & constitutive (material) equations:

Energy density

Energy flow

Poynting theorem

Conservation laws

Wave equations: Monochromatic plane waves

Phase velocity & group velocity

Maxwell's Equations

Findings of 19th century:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday's law

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

"Displacement current"

Ampère's law

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

Gauss's law

$$\mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

$$\mathbf{H} = \frac{1}{\mu}\mathbf{B} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

Constitutive (material) equations

The two divergence equations can be derived from the two curl equations! $\rightarrow$ *12 unknowns & 12 equations!*

So simple and elegant!

Electromagnetic Boundary Conditions

Continuity conditions:

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \;\rightarrow\; \oint_C \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

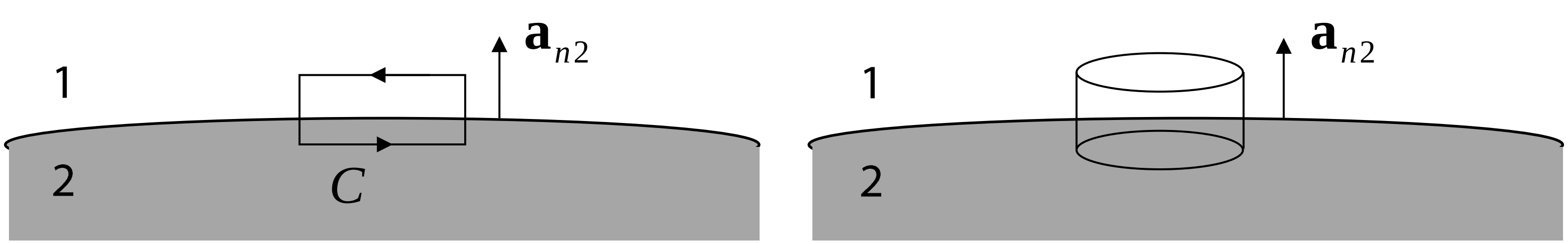
$$\rightarrow\; E_{1t} = E_{2t} \quad (\text{V/m})$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \;\rightarrow\; \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot d\mathbf{s}$$

$$\rightarrow\; \mathbf{a}_{n2} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s \quad (\text{A/m})$$

$$\nabla \cdot \mathbf{D} = \rho \;\rightarrow\; \mathbf{a}_{n2} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s \quad (\text{C/m}^2)$$

$$\nabla \cdot \mathbf{B} = 0 \;\rightarrow\; B_{1n} = B_{2n} \quad (\text{T})$$



Poynting Theorem and Conservation Laws

Ampère's law:

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \quad \rightarrow \; \mathbf{J} \cdot \mathbf{E} = \mathbf{E} \cdot (\nabla \times \mathbf{H}) - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t}$$

$$\leftarrow \; \nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H})$$

$$\rightarrow \; \mathbf{J} \cdot \mathbf{E} = -\nabla \cdot (\mathbf{E} \times \mathbf{H}) - \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} - \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t}$$

Energy density and Poynting vector:

$$U = \frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$$

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

Poynting theorem (Conservation of energy):

$$\frac{\partial U}{\partial t} + \nabla \cdot \mathbf{S} = -\mathbf{J} \cdot \mathbf{E}$$

Outward power flow ($\nabla \cdot \mathbf{S}$)

Internal heat dissipation ($-\mathbf{J} \cdot \mathbf{E}$)