

Electro-Optics:

Propagation of Laser Beams (2)

Dr Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

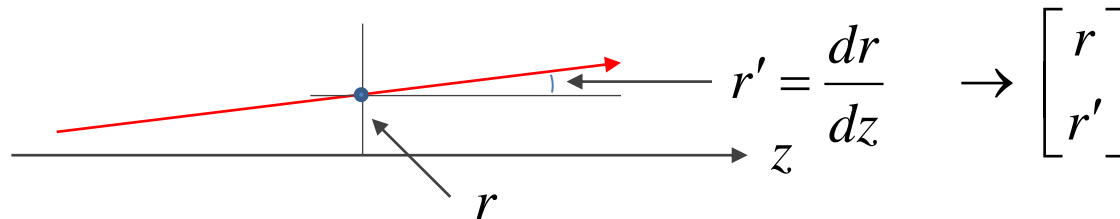
Email: yunchan@snu.ac.kr

Paraxial Rays and Ray Matrix

Ray: A stream of light normal to the optical wavefront

Paraxial ray: Angular deviation from the reference longitudinal axis small enough $\rightarrow \sin \theta \approx \theta, \tan \theta \approx \theta$

Ray distance from the axis and its slope:



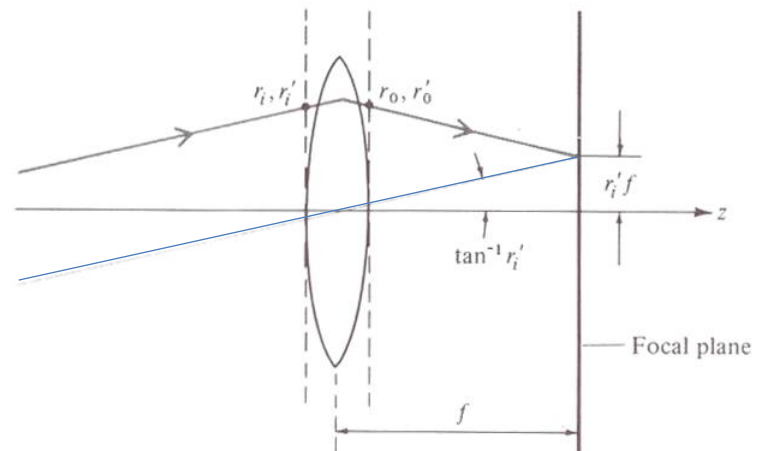
Deflection of a ray by a thin lens:

$$r_{out} = r_{in}$$

$$r'_{out} = r'_{in} - \frac{r_{in}}{f}$$

$$\rightarrow \begin{bmatrix} r_{out} \\ r'_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} r_{in} \\ r'_{in} \end{bmatrix}$$


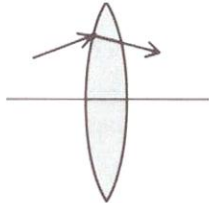
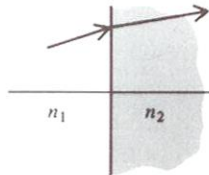
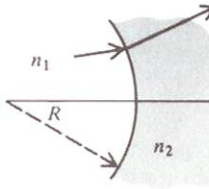
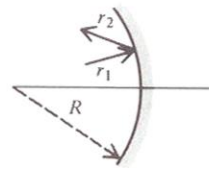
\rightarrow Ray matrix



A. Yariv, Optical Electronics, 4th ed. Saunders, 1991.

Ray Matrices

Ray matrices for some common optical elements and media:

<p>(1) Straight Section: Length d</p>		$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$	$\leftarrow \begin{aligned} r_2 &= r_1 + r_1' d \\ r_2' &= r_1' \end{aligned}$
<p>(2) Thin Lens: Focal length f ($f > 0$, converging; $f < 0$, diverging)</p>		$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$	
<p>(3) Dielectric Interface: Refractive indices n_1, n_2</p>		$\begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix}$	$\leftarrow \begin{aligned} r_2 &= r_1 \\ n_2 r_2' &= n_1 r_1' \end{aligned}$
<p>(4) Spherical Dielectric Interface: Radius R</p>		$\begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_1}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$	$\leftarrow \begin{aligned} r_2 &= r_1 \\ n_2 \theta_2' &= n_1 \theta_1' \rightarrow r_1' = \theta_{r_1} + \theta_1' \\ &\rightarrow r_2' = \theta_{r_1} + \theta_2' \end{aligned}$
<p>(5) Spherical Mirror: Radius of curvature R</p>		$\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}$	$\leftarrow \begin{aligned} r_2 &= r_1 \\ \theta_2' &= \theta_1' \rightarrow r_1' = \theta_{r_1} + \theta_1' \\ &\rightarrow r_2' = \theta_2' - \theta_{r_1} \end{aligned}$

Rays in Lenslike Media (1)

Ideal thin lens:

$$r' = -\frac{r}{f} \rightarrow E_R(x, y) = E_L(x, y) \exp\left(+ik \frac{x^2 + y^2}{2f}\right)$$

Lenslike medium:

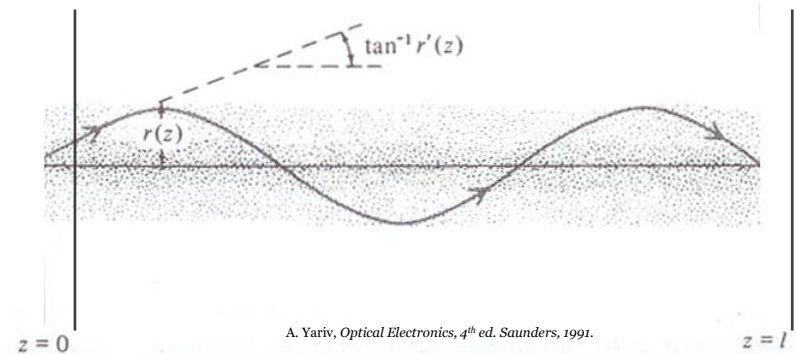
$$n(x, y) = n_0 \left[1 - \frac{k_2}{2k} (x^2 + y^2) \right]$$

← Phase delay

Wave equation:

$$\nabla^2 \Psi - \frac{n^2}{c^2} \frac{\partial^2 \Psi}{\partial t^2} = 0$$

$$\rightarrow \Psi = A(\mathbf{r}) e^{i[\omega t - \phi(\mathbf{r})]}$$



$$\rightarrow \nabla^2 A - 2i\nabla A \cdot \nabla \phi - A(\nabla \phi)^2 - iA\nabla^2 \phi + \frac{n^2 \omega^2}{c} A = 0$$

Real part: $\rightarrow \nabla^2 A - A(\nabla \phi)^2 + \left(\frac{2\pi}{\lambda} n\right)^2 A = 0$

← Slowly varying amplitude

Eikonal equation:

$$(\nabla \phi)^2 = \left(\frac{2\pi}{\lambda} n\right)^2$$

Rays in Lenslike Media (2)

Deflection of a ray normal to an ideal thin lens:

$$(\nabla\phi)^2 = \left(\frac{2\pi}{\lambda}n\right)^2 \quad \rightarrow \nabla\phi : \text{Normal to the wavefront}$$

$$\rightarrow \nabla\phi = \frac{2\pi}{\lambda}n \frac{d\mathbf{r}}{ds} \quad \rightarrow \text{Unit vector in the direction of } \nabla\phi$$

$$\rightarrow \phi(\mathbf{r}) = \frac{2\pi}{\lambda} \int n ds$$

Ray trajectory:

$$\nabla(\nabla\phi)^2 = \nabla\left(\frac{2\pi}{\lambda}n\right)^2$$

$$\rightarrow 2\nabla\phi \cdot \nabla(\nabla\phi) = 2\frac{2\pi}{\lambda}n \frac{2\pi}{\lambda}\nabla n$$

$$\rightarrow \frac{2\pi}{\lambda}n \frac{d\mathbf{r}}{ds} \cdot \nabla(\nabla\phi) = \left(\frac{2\pi}{\lambda}\right)^2 n \nabla n \quad \leftarrow \frac{d\mathbf{r}}{ds} \cdot \nabla = \frac{d}{ds}$$

$$\rightarrow \frac{d}{ds}(\nabla\phi) = \frac{2\pi}{\lambda}\nabla n$$

$$\rightarrow \frac{d}{ds}\left(n \frac{d\mathbf{r}}{ds}\right) = \nabla n$$

Rays in Lenslike Media (3)

For paraxial rays: $\frac{d}{ds} \rightarrow \frac{d}{dz}$

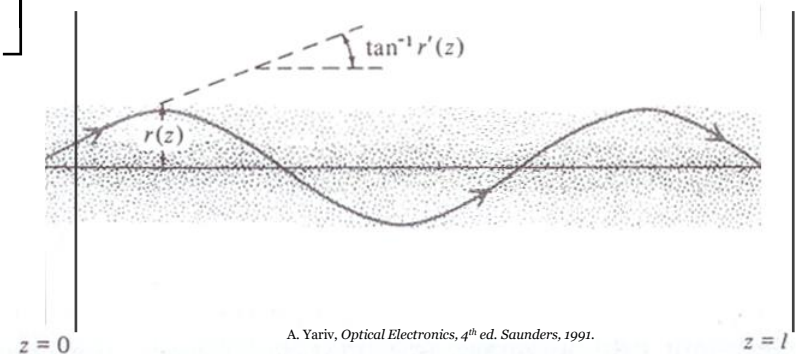
$$\frac{d}{ds} \left(n \frac{d\mathbf{r}}{ds} \right) = \nabla n \rightarrow \frac{d}{dz} \left(n \frac{d\mathbf{r}}{dz} \right) = \nabla n$$

Lenslike medium:

$$n(x, y) = n_0 \left[1 - \frac{k_2}{2k} (x^2 + y^2) \right]$$

$$\rightarrow n \frac{d^2 r}{dz^2} = -2n_0 \frac{k_2}{2k} r$$

$$\rightarrow \frac{d^2 r}{dz^2} + \left(\frac{k_2}{k} \right) r = 0$$



Ray trajectory vector:

$$r(z) = \cos \left(\sqrt{\frac{k_2}{k}} z \right) r_0 + \sqrt{\frac{k}{k_2}} \sin \left(\sqrt{\frac{k_2}{k}} z \right) r'_0$$

$$r'(z) = -\sqrt{\frac{k_2}{k}} \sin \left(\sqrt{\frac{k_2}{k}} z \right) r_0 + \cos \left(\sqrt{\frac{k_2}{k}} z \right) r'_0$$

Fundamental Gaussian Beam in a LL Medium

For a lenslike medium: $\rightarrow k_2 \neq 0$

$$\left(\frac{1}{q}\right)^2 + \left(\frac{1}{q}\right)' + \frac{k_2}{k} = 0, \quad P' = -\frac{i}{q}$$

$$\rightarrow \frac{1}{q} = \frac{u'}{u} \rightarrow u'' + u\left(\frac{k_2}{k}\right) = 0$$

$$\rightarrow u(z) = a \sin \sqrt{\frac{k_2}{k}} z + b \cos \sqrt{\frac{k_2}{k}} z$$

$$\rightarrow u'(z) = a \sqrt{\frac{k_2}{k}} \cos \sqrt{\frac{k_2}{k}} z - b \sqrt{\frac{k_2}{k}} \sin \sqrt{\frac{k_2}{k}} z$$

Complex beam radius:

$$q(z) = \frac{u}{u'} = \frac{q_0 \cos \sqrt{\frac{k_2}{k}} z + \sqrt{k/k_2} \sin \sqrt{\frac{k_2}{k}} z}{-q_0 \sqrt{k_2/k} \sin \sqrt{\frac{k_2}{k}} z + \cos \sqrt{\frac{k_2}{k}} z}$$

\rightarrow Recall the ray matrix!

The ABCD Law

Ray matrix for a lenslike medium:

$$\frac{d^2 r}{dz^2} + \left(\frac{k_2}{k} \right) r = 0 \quad \rightarrow \text{Identical eq. to } u$$

$$\begin{bmatrix} r \\ r' \end{bmatrix}_2 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r \\ r' \end{bmatrix}_1 \quad \rightarrow \quad (r/r')_2 = \frac{A(r/r')_1 + B}{C(r/r')_1 + D}$$

ABCD law:

$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \quad \rightarrow \text{All the ray matrices applicable}$$

Beam propagation:

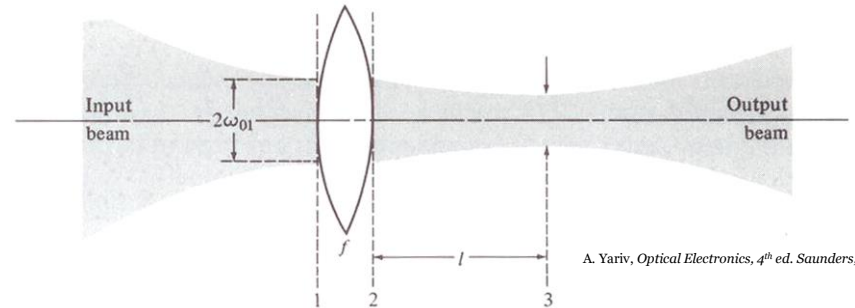
$$q_2 = \frac{A_1 q_1 + B_1}{C_1 q_1 + D_1} \quad \rightarrow \quad q_3 = \frac{A_2 q_2 + B_2}{C_2 q_2 + D_2} = \frac{A_T q_1 + B_T}{C_T q_1 + D_T}$$

$$\rightarrow \begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$$

Gaussian Beam Focusing

At plane 1:

$$\frac{1}{q_1} = \frac{1}{R_1} - i \frac{\lambda}{\pi \omega_1^2 n} = -i \frac{\lambda}{\pi \omega_1^2 n}$$



At plane 2:

$$q_2 = \frac{q_1}{-\frac{1}{f}q_1 + 1} = \frac{1}{-1/f - i(\lambda/\pi\omega_1^2 n)} = \frac{-a + ib}{a^2 + b^2}$$

At plane 3:

$$q_3 = q_2 + l = \frac{-a}{a^2 + b^2} + \frac{ib}{a^2 + b^2} + l$$

$$\rightarrow \frac{1}{q_3} = \frac{1}{R_3} - i \frac{\lambda}{\pi \omega_3^2 n} = \frac{\left(\frac{-a}{a^2 + b^2} + l\right) - i \frac{b}{a^2 + b^2}}{\left(\frac{-a}{a^2 + b^2} + l\right)^2 + \left(\frac{b}{a^2 + b^2}\right)^2} \quad \leftarrow R_3 = \infty$$

$$\rightarrow l = \frac{a}{a^2 + b^2} = \frac{f}{1 + (f/z_1)^2} \quad \rightarrow \frac{\omega_3}{\omega_1} = \frac{f/z_1}{\sqrt{1 + (f/z_1)^2}}$$