Electro-Optics:

Propagation of Laser Beams (2)

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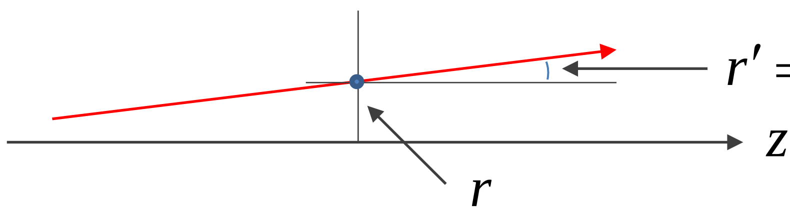
Paraxial Rays and Ray Matrix

Ray: A stream of light normal to the optical wavefront

Paraxial ray: Angular deviation from the reference longitudinal axis small enough $\rightarrow \sin\theta \approx \theta, \ \tan\theta \approx \theta$

Ray distance from the axis and its slope:



$$r' = \frac{dr}{dz} \quad \rightarrow \begin{bmatrix} r \\ r' \end{bmatrix}$$

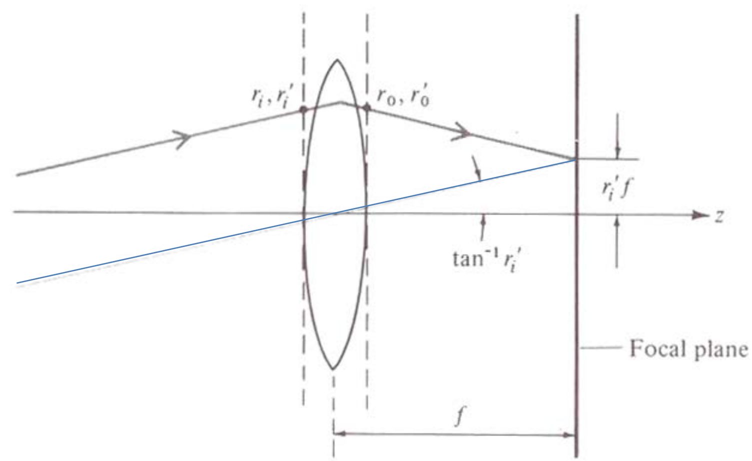
Deflection of a ray by a thin lens:

$$r_{out} = r_{in}$$

$$r'_{out} = r'_{in} - \frac{r_{in}}{f}$$

$$\rightarrow \begin{bmatrix} r_{out} \\ r'_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} r_{in} \\ r'_{in} \end{bmatrix}$$

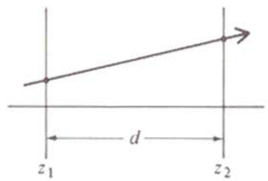
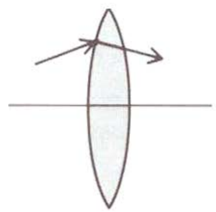
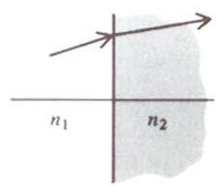
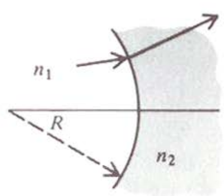
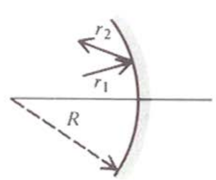
$\rightarrow$ Ray matrix



A. Yariv, *Optical Electronics, 4th ed. Saunders, 1991.*

Ray Matrices

Ray matrices for some common optical elements and media:

(1) Straight Section: Length d		$\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$
(2) Thin Lens: Focal length f ($f > 0$, converging; $f < 0$, diverging)		$\begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$
(3) Dielectric Interface: Refractive indices n_1, n_2		$\begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$
(4) Spherical Dielectric Interface: Radius R		$\begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_1}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$
(5) Spherical Mirror: Radius of curvature R		$\begin{bmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{bmatrix}$

← (1):
$$r_2 = r_1 + r_1' d$$
$$r_2' = r_1'$$

← (3):
$$r_2 = r_1$$
$$n_2 r_2' = n_1 r_1'$$

(4), (5): H.W.

A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.

Rays in Lenslike Media (1)

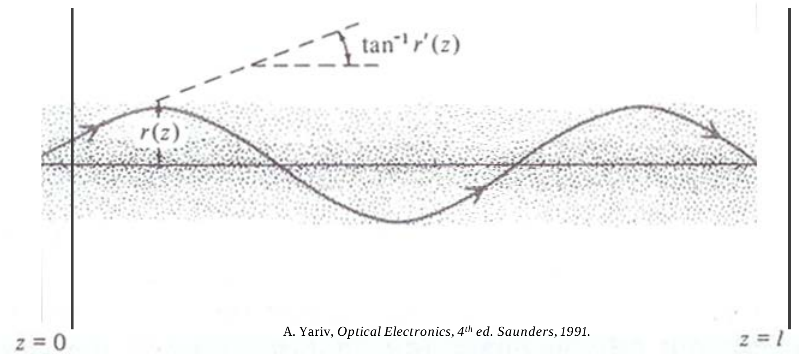
Ideal thin lens:

$$r' = -\frac{r}{f} \quad \rightarrow \; E_R(x,y) = E_L(x,y)\exp\left(+ik\frac{x^2+y^2}{2f}\right)$$

← Phase delay

Lenslike medium:

$$n(x,y) = n_0\left[1 - \frac{k_2}{2k}(x^2+y^2)\right]$$



A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.

Wave equation:

$$\nabla^2\Psi - \frac{n^2}{c^2}\frac{\partial^2\Psi}{\partial t^2} = 0$$

$$\rightarrow \; \Psi = A(\mathbf{r})e^{i[\omega t - \phi(\mathbf{r})]}$$

$$\rightarrow \; \nabla^2 A - 2i\nabla A\cdot\nabla\phi - A|\nabla\phi|^2 - iA\nabla^2\phi + \frac{n^2\omega^2}{c}A = 0$$

Real part: $\rightarrow \; \nabla^2 A - A|\nabla\phi|^2 + (\frac{2\pi}{\lambda}n)^2 A = 0$

← Slowly varying amplitude

Eikonal equation: $|\nabla\phi|^2 = (\frac{2\pi}{\lambda}n)^2$

Rays in Lenslike Media (2)

Deflection of a ray normal to an ideal thin lens:

$$|\nabla\phi|^2 = (\frac{2\pi}{\lambda}n)^2 \quad \rightarrow \nabla\phi$$: Normal to the wavefront

$$\rightarrow \nabla\phi = \frac{2\pi}{\lambda} n \frac{d\mathbf{r}}{ds}$$

Unit vector in the direction of $\nabla\phi$

$$\rightarrow \phi(\mathbf{r}) = \frac{2\pi}{\lambda}\int n\,ds$$

Ray trajectory:

$$\rightarrow \frac{d}{ds}(\nabla\phi) = \frac{d\mathbf{r}}{ds}\cdot\nabla(\nabla\phi) = \left(\frac{2\pi}{\lambda}\right)^{-1}\frac{1}{n}\nabla\phi\cdot\nabla(\nabla\phi)$$

$$= \left(\frac{2\pi}{\lambda}\right)^{-1}\frac{1}{2n}\nabla(\nabla\phi\cdot\nabla\phi) = \left(\frac{2\pi}{\lambda}\right)^{-1}\frac{1}{2n}\nabla\left[\left(\frac{2\pi}{\lambda}n\right)^2\right]$$

$$= \left(\frac{2\pi}{\lambda}\right)\nabla n$$

$$\rightarrow \frac{d}{ds}(n\frac{d\mathbf{r}}{ds}) = \nabla n$$

Rays in Lenslike Media (3)

For paraxial rays: $\frac{d}{ds} \to \frac{d}{dz}$

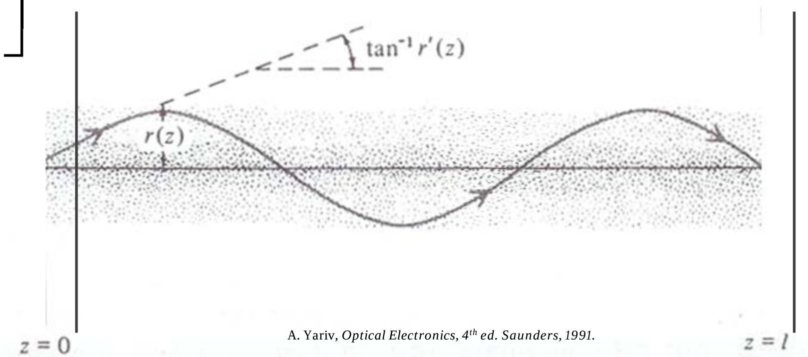
$$\frac{d}{ds}\left(n\frac{d\mathbf{r}}{ds}\right) = \nabla n \;\to\; \frac{d}{dz}\left(n\frac{d\mathbf{r}}{dz}\right) = \nabla n$$

Lenslike medium:

$$n(x,y) = n_0\left[1 - \frac{k_2}{2k}(x^2+y^2)\right]$$

$$\to\; n\frac{d^2r}{dz^2} = -2n_0\frac{k_2}{2k}r$$

$$\to\; \frac{d^2r}{dz^2} + \left(\frac{k_2}{k}\right)r = 0$$



A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.

Ray trajectory vector:

$$r(z) = \cos\left(\sqrt{\frac{k_2}{k}}z\right)r_0 + \sqrt{\frac{k}{k_2}}\sin\left(\sqrt{\frac{k_2}{k}}z\right)r_0'$$

$$r'(z) = -\sqrt{\frac{k_2}{k}}\sin\left(\sqrt{\frac{k_2}{k}}z\right)r_0 + \cos\left(\sqrt{\frac{k_2}{k}}z\right)r_0'$$

Fundamental Gaussian Beam in a LL Medium

For a lenslike medium: $\rightarrow\ k_2 \neq 0$

$$\left(\frac{1}{q}\right)^2 + \left(\frac{1}{q}\right)' + \frac{k_2}{k} = 0, \quad P' = -\frac{i}{q}$$

$$\rightarrow\ \frac{1}{q} = \frac{u'}{u} \quad \rightarrow\ u'' + u\left(\frac{k_2}{k}\right) = 0$$

$$\rightarrow u(z) = a \sin\sqrt{\frac{k_2}{k}}z + b\cos\sqrt{\frac{k_2}{k}}z$$

$$\rightarrow u'(z) = a\sqrt{\frac{k_2}{k}}\cos\sqrt{\frac{k_2}{k}}z - b\sqrt{\frac{k_2}{k}}\sin\sqrt{\frac{k_2}{k}}z$$

Complex beam radius:

$$q(z) = \frac{u}{u'} = \frac{q_0\cos\sqrt{\frac{k_2}{k}}z + \sqrt{k/k_2}\sin\sqrt{\frac{k_2}{k}}z}{-q_0\sqrt{k_2/k}\sin\sqrt{\frac{k_2}{k}}z + \cos\sqrt{\frac{k_2}{k}}z}$$

$\rightarrow$ Recall the ray matrix!

The ABCD Law

Ray matrix for a lenslike medium:

$$\rightarrow\ q(z) = \frac{u}{u'}$$

$$\frac{d^2 r}{dz^2} + \left(\frac{k_2}{k}\right) r = 0 \quad \rightarrow \text{Identical eq. to } u \rightarrow\ u'' + u\left(\frac{k_2}{k}\right) = 0$$

$$\begin{bmatrix} r \\ r' \end{bmatrix}_2 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} r \\ r' \end{bmatrix}_1 \quad \rightarrow\ (r/r')_2 = \frac{A(r/r')_1 + B}{C(r/r')_1 + D}$$

ABCD law:

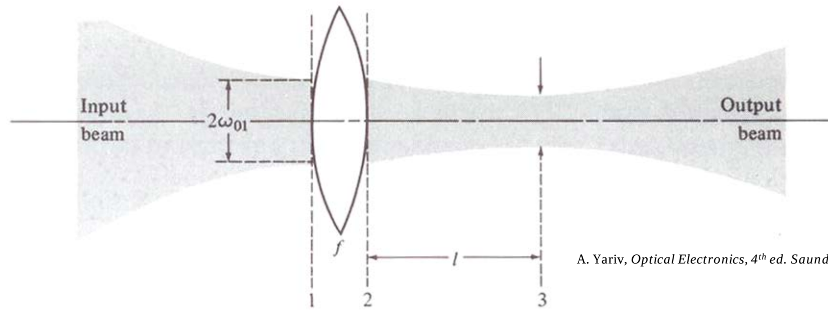
$$q_2 = \frac{Aq_1 + B}{Cq_1 + D} \quad \rightarrow \text{All the ray matrices applicable}$$

Beam propagation:

$$q_2 = \frac{A_1 q_1 + B_1}{C_1 q_1 + D_1} \ \rightarrow\ q_3 = \frac{A_2 q_2 + B_2}{C_2 q_2 + D_2} = \frac{A_T q_1 + B_T}{C_T q_1 + D_T}$$

$$\rightarrow \begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}$$

Gaussian Beam Focusing



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At plane 1:

$$\frac{1}{q_1} = \frac{1}{R_1} - i\frac{\lambda}{\pi\omega_1^2 n} = -i\frac{\lambda}{\pi\omega_1^2 n}$$

At plane 2:

$$q_2 = \frac{q_1}{-\frac{1}{f}q_1 + 1} = \frac{1}{-1/f - i(\lambda/\pi\omega_1^2 n)} = \frac{-a+ib}{a^2+b^2}$$

$$\leftarrow\ a \equiv \frac{1}{f},\quad b \equiv \frac{\lambda}{\pi\omega_1^2 n}$$

At plane 3:

$$q_3 = q_2 + l = \frac{-a}{a^2+b^2} + \frac{ib}{a^2+b^2} + l$$

$$\rightarrow \frac{1}{q_3} = \frac{1}{R_3} - i\frac{\lambda}{\pi\omega_3^2 n} = \frac{\left(\frac{-a}{a^2+b^2} + l\right) - i\frac{b}{a^2+b^2}}{\left(\frac{-a}{a^2+b^2} + l\right)^2 + \left(\frac{b}{a^2+b^2}\right)^2}$$

$\leftarrow R_3 = \infty$

$$\rightarrow l = \frac{a}{a^2+b^2} = \frac{f}{1+(f/z_1)^2} \quad \rightarrow \frac{\omega_3}{\omega_1} = \frac{f/z_1}{\sqrt{1+(f/z_1)^2}}$$