

Electro-Optics:

Polarization of Light Waves (1)

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Polarization of Monochromatic Plane Waves

Electric field vector:

$$\mathbf{E}(z, t) = \text{Re}[\mathbf{A}e^{i(\omega t - kz)}] \quad \leftarrow \quad \mathbf{A} = \hat{\mathbf{x}}A_x e^{i\delta_x} + \hat{\mathbf{y}}A_y e^{i\delta_y}$$

$$\rightarrow E_x = A_x \cos(\omega t - kz + \delta_x)$$

$$\rightarrow E_y = A_y \cos(\omega t - kz + \delta_y)$$

Time-evolution locus:

$$\rightarrow \left(\frac{E_x}{A_x}\right)^2 + \left(\frac{E_y}{A_y}\right)^2 - 2\frac{\cos \delta}{A_x A_y} E_x E_y = \sin^2 \delta \quad \leftarrow \delta = \delta_y - \delta_x$$

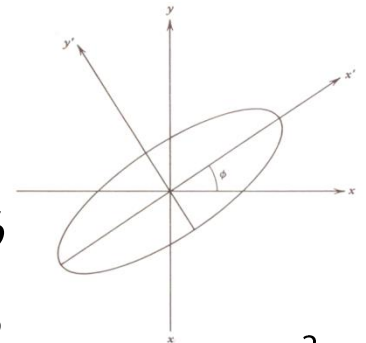
← Polarization ellipse

In the principal coordinates system: $\rightarrow \begin{pmatrix} E_{x'} \\ E_{y'} \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$

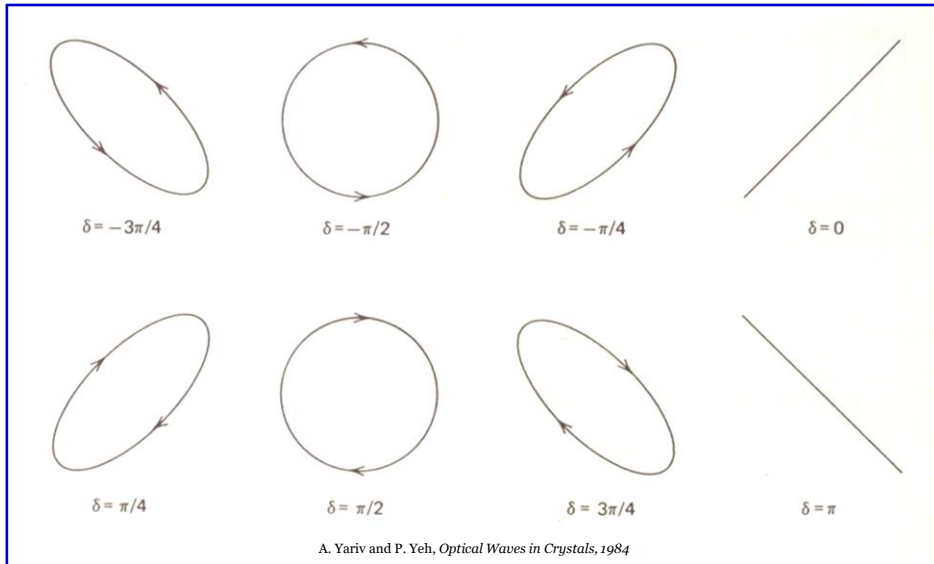
$$\rightarrow \left(\frac{E_{x'}}{a}\right)^2 + \left(\frac{E_{y'}}{b}\right)^2 = 1 \quad \rightarrow \tan 2\phi = \frac{2A_x A_y}{A_x^2 - A_y^2} \cos \delta$$

$$\leftarrow a^2 = A_x^2 \cos^2 \phi + A_y^2 \sin^2 \phi + 2A_x A_y \cos \delta \cos \phi \sin \phi$$

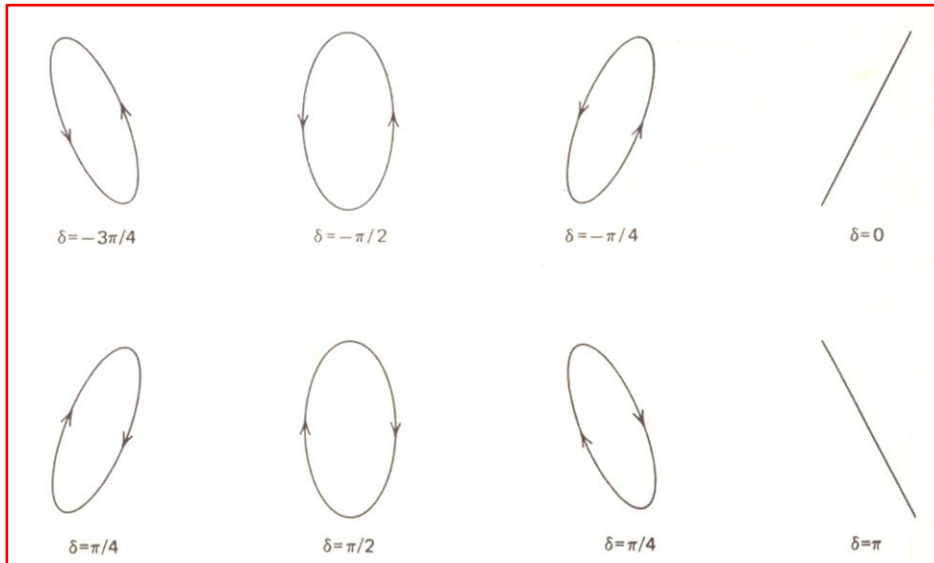
$$\leftarrow b^2 = A_x^2 \sin^2 \phi + A_y^2 \cos^2 \phi - 2A_x A_y \cos \delta \cos \phi \sin \phi$$



Polarization Ellipses



$$\leftarrow \begin{aligned} E_x &= \cos(\omega t - kz) \\ E_y &= \cos(\omega t - kz + \delta) \end{aligned}$$



$$\leftarrow \begin{aligned} E_x &= \frac{1}{2} \cos(\omega t - kz) \\ E_y &= \cos(\omega t - kz + \delta) \end{aligned}$$

Linear and Circular Polarizations

Linear polarization:

$$\delta = \delta_y - \delta_x = m\pi \quad (m = 0, 1)$$

$$\rightarrow \frac{E_y}{E_x} = (-1)^m \frac{A_y}{A_x}$$

Circular polarization:

$$\delta = \delta_y - \delta_x = \pm \frac{1}{2} \pi \quad \& \quad A_y = A_x$$

$$\rightarrow \delta = -\frac{1}{2} \pi \quad \leftarrow \text{Right-hand circularly polarized}$$

$$\rightarrow \delta = +\frac{1}{2} \pi \quad \leftarrow \text{Left-hand circularly polarized}$$

Elliptical polarization:

$$e = \pm \frac{b}{a} \quad \leftarrow \text{Ellipticity of a polarization ellipse}$$