Electro-Optics:

Polarization of Light Waves (2)

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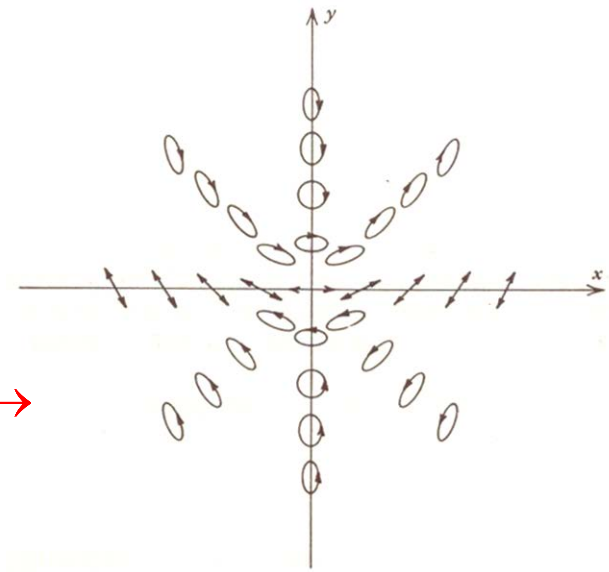
Complex-Number Representation (1)

Polarization state in the complex plane:

$$\chi = e^{i\delta} \tan\psi = \frac{A_y}{A_x} e^{i(\delta_y - \delta_x)}$$

Complex-number representation $\rightarrow$



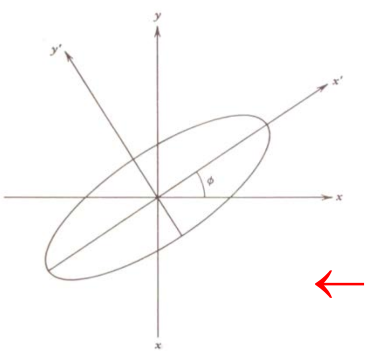
A. Yariv and P. Yeh, *Optical Waves in Crystals*, 1984

Inclination angle:

$$\tan 2\phi = \frac{2A_xA_y}{A_x^2 - A_y^2}\cos\delta = \frac{2\,\mathrm{Re}[\chi]}{1-|\chi|^2}$$

$$\leftarrow \mathrm{Re}[\chi] = \frac{A_y}{A_x}\cos\delta$$

$$\leftarrow 1-|\chi|^2 = 1-\frac{A_y^2}{A_x^2}$$



$\leftarrow$ Polarization ellipse

A. Yariv and P. Yeh, *Optical Waves in Crystals*, 1984

Complex-Number Representation (2)

Ellipticity angle:

$$\theta = \tan^{-1} e = \tan^{-1} \pm \frac{b}{a} \quad \leftarrow \pm\text{: Right- \& left-handed rotation}$$

$$\rightarrow \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{\pm 2\frac{b}{a}}{1+\frac{b^2}{a^2}} = \frac{\pm 2ab}{a^2+b^2}$$

$$\leftarrow \boxed{\begin{aligned} a^2 &= A_x^2\cos^2\phi + A_y^2\sin^2\phi + 2A_xA_y\cos\delta\cos\phi\sin\phi \\ b^2 &= A_x^2\sin^2\phi + A_y^2\cos^2\phi - 2A_xA_y\cos\delta\cos\phi\sin\phi \end{aligned}}$$

$$\rightarrow a^2+b^2 = A_x^2 + A_y^2$$

$$\rightarrow a^2b^2 = A_x^2A_y^2\sin^2\delta \quad \rightarrow ab = \mp A_xA_y\sin\delta$$

$$\rightarrow \sin 2\theta = \frac{\pm 2(\mp A_xA_y\sin\delta)}{A_x^2+A_y^2} = \frac{-2\frac{A_y}{A_x}\sin\delta}{1+\left(\frac{A_y}{A_x}\right)^2} = -\frac{2\,\mathrm{Im}[\chi]}{1+|\chi|^2}$$

Jones-Vector Representation (1)

Column vector of complex amplitudes:

$$\mathbf{J} = \begin{pmatrix} A_x e^{i\delta_x} \\ A_y e^{i\delta_y} \end{pmatrix}$$

Normalized Jones vector:

$$\rightarrow \ \mathbf{J}^* \cdot \mathbf{J} = 1$$

Linearly polarized light:

$$\begin{pmatrix} \cos\psi \\ \sin\psi \end{pmatrix} \perp \begin{pmatrix} -\sin\psi \\ \cos\psi \end{pmatrix}$$

$$\rightarrow \ \hat{\mathbf{x}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \hat{\mathbf{y}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Circularly polarized light:

$$\hat{\mathbf{R}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}, \quad \hat{\mathbf{L}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\rightarrow \ \hat{\mathbf{R}}^* \cdot \hat{\mathbf{L}} = 0$$

Jones-Vector Representation (2)

Superposition of polarizations:

$$\hat{\mathbf{R}} = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{x}} - i\hat{\mathbf{y}}\right)$$

$$\hat{\mathbf{L}} = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{x}} + i\hat{\mathbf{y}}\right)$$

$$\hat{\mathbf{x}} = \frac{1}{\sqrt{2}}\left(\hat{\mathbf{R}} + \hat{\mathbf{L}}\right)$$

$$\hat{\mathbf{y}} = \frac{i}{\sqrt{2}}\left(\hat{\mathbf{R}} - \hat{\mathbf{L}}\right)$$

General elliptical polarization:

$$\chi = e^{i\delta}\tan\psi = \frac{A_y}{A_x} e^{i(\delta_y - \delta_x)}$$

$$\rightarrow \mathbf{J} = \begin{pmatrix} \cos\psi \\ e^{i\delta}\sin\psi \end{pmatrix}$$