Electro-Optics:

Electromagnetic Propagation in Anisotropic Media (3)

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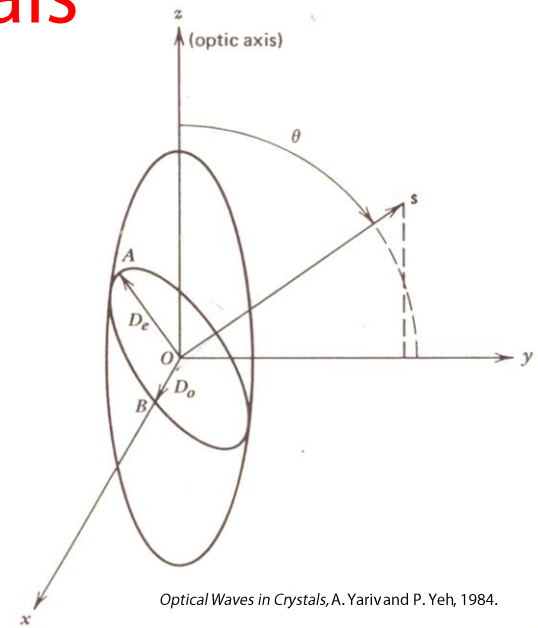
Light Propagation in Uniaxial Crystals

Index ellipsoid:

$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}=1$$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Polarizations for the displacement vector:

$$\mathbf{d}_o=\frac{\mathbf{k}\times\mathbf{c}}{|\mathbf{k}\times\mathbf{c}|} \quad \leftarrow \text{Ordinary polarization}$$

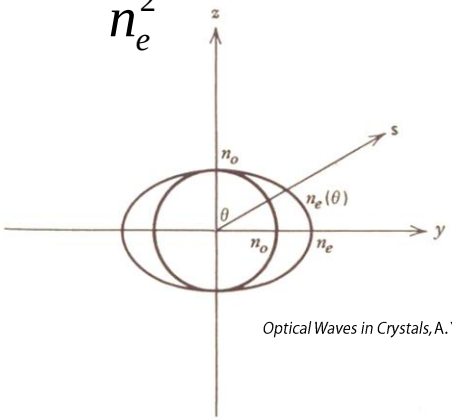
$$\mathbf{d}_e=\frac{\mathbf{d}_o\times\mathbf{k}}{|\mathbf{d}_o\times\mathbf{k}|} \quad \leftarrow \text{Extraordinary polarization}$$

For $k_x = 0$:

$$\cancel{\frac{x^2}{n_o^2}}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}=1 \quad \rightarrow \quad \frac{1}{n_e^2(\theta)}=\frac{\cos^2\theta}{n_o^2}+\frac{\sin^2\theta}{n_e^2}$$

Extraordinary electric field:

$$\rightarrow \begin{pmatrix} 0 \\ \dfrac{\sin\theta}{n_e^2(\theta)-n_o^2} \\ \dfrac{\cos\theta}{n_e^2(\theta)-n_e^2} \end{pmatrix}$$



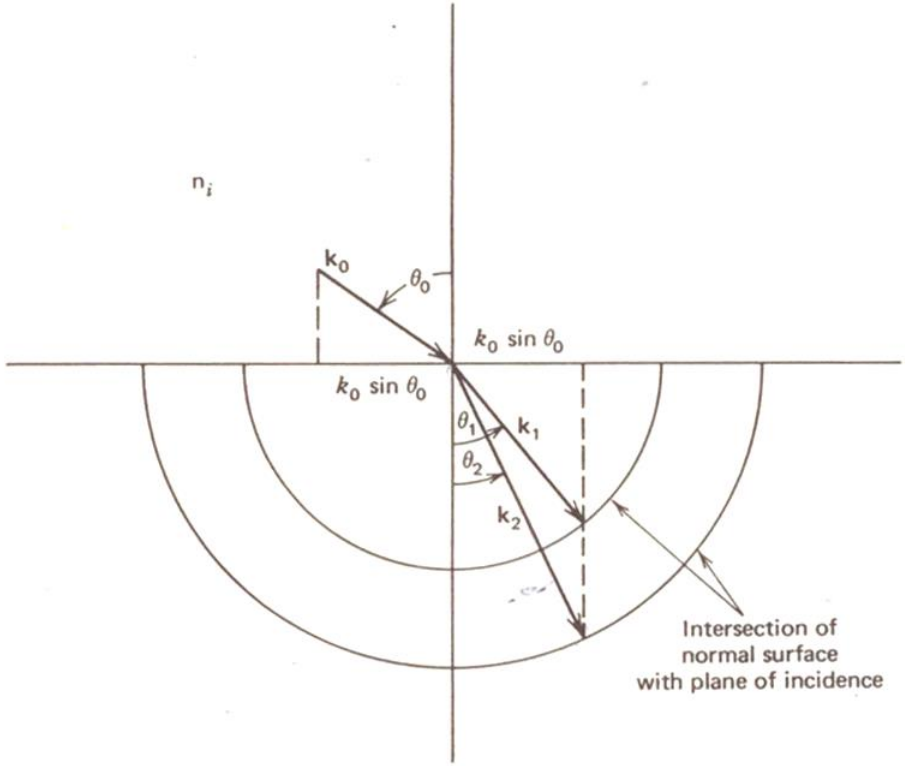
Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Double Refraction at a Boundary

Boundary condition:

$$k_0 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2$$

Snell's law: $\rightarrow \; n_i \sin\theta_0 = n_o \sin\theta_1 = n_e(\theta) \sin\theta_2$

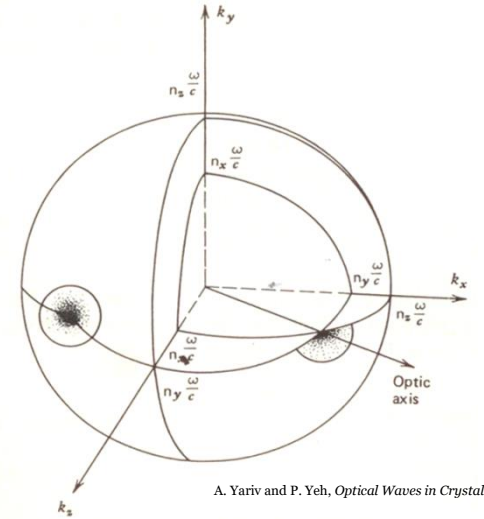


Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

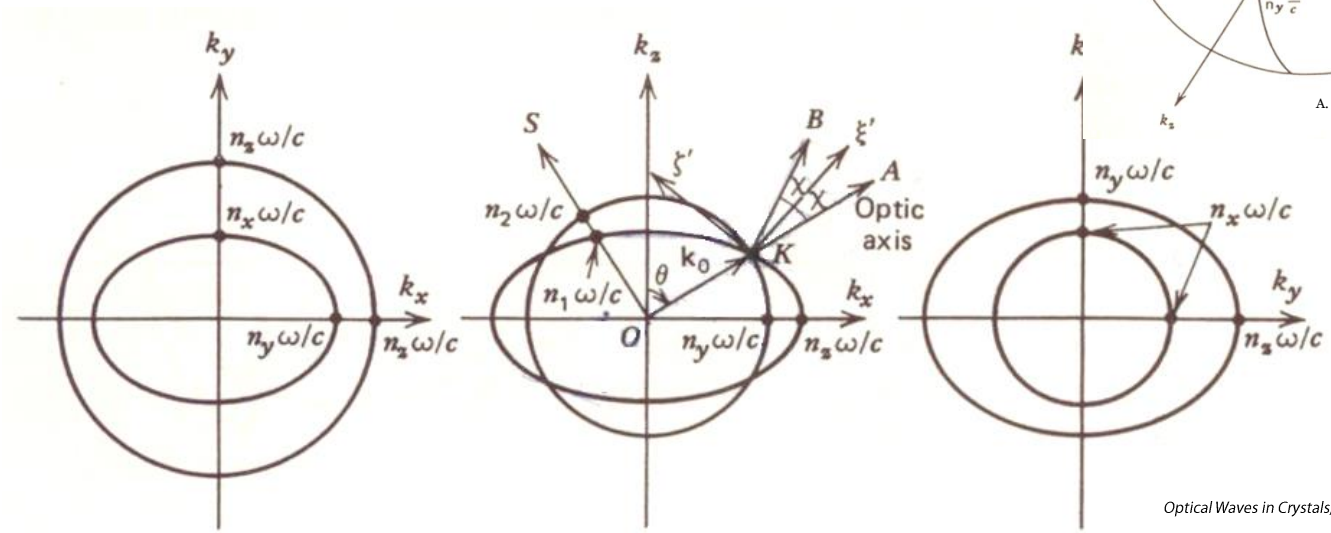
Light Propagation in Biaxial Crystals

Biaxial media: $\rightarrow n_x < n_y < n_z$

Normal surface:



A. Yariv and P. Yeh, *Optical Waves in Crystals*, 1984.



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

k_x-k_y plane:

$$\rightarrow \frac{k_x^2 + k_y^2}{n_z^2} = \left(\frac{\omega}{c}\right)^2$$

$$\rightarrow \frac{k_x^2}{n_y^2} + \frac{k_y^2}{n_x^2} = \left(\frac{\omega}{c}\right)^2$$

k_z-k_x plane:

$$\rightarrow \frac{k_z^2 + k_x^2}{n_y^2} = \left(\frac{\omega}{c}\right)^2$$

$$\rightarrow \frac{k_z^2}{n_x^2} + \frac{k_x^2}{n_z^2} = \left(\frac{\omega}{c}\right)^2$$

k_y-k_z plane:

$$\rightarrow \frac{k_y^2 + k_z^2}{n_x^2} = \left(\frac{\omega}{c}\right)^2$$

$$\rightarrow \frac{k_y^2}{n_z^2} + \frac{k_z^2}{n_y^2} = \left(\frac{\omega}{c}\right)^2$$

Conical Refraction (1)

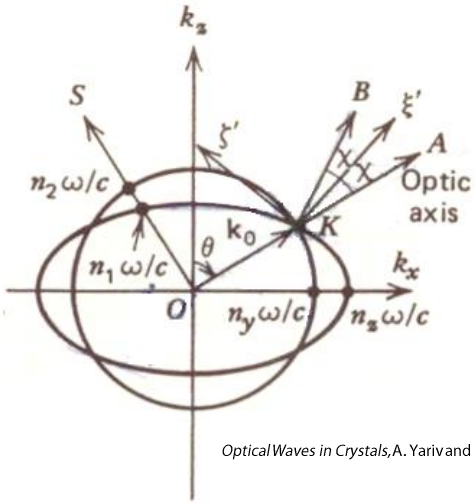
For light propagation in the direction of the optic axis:

$$\mathbf{k}_0 = \hat{\mathbf{x}}k_{x0} + \hat{\mathbf{y}}k_{y0} + \hat{\mathbf{z}}k_{z0}$$

$$\rightarrow \; k_{x0} = n_y \frac{\omega}{c}\sin\theta$$

$$\rightarrow \; k_{y0} = 0 \qquad \rightarrow \; \tan\theta = \frac{n_z}{n_x}\left(\frac{n_y^2 - n_x^2}{n_z^2 - n_y^2}\right)^{1/2}$$

$$\rightarrow \; k_{z0} = n_y \frac{\omega}{c}\cos\theta$$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Taylor series expansion:

$$\rightarrow \; k_x = k_{x0} + \xi$$

$$\rightarrow \; k_y = k_{y0} + \eta \qquad \rightarrow \; k_x^2 YZ + k_y^2 ZX + k_z^2 XY - XYZ = 0$$

$$\rightarrow \; k_z = k_{z0} + \varsigma$$

$$\rightarrow \; 4(k_{x0}\xi + k_{z0}\zeta)(n_x^2 k_{x0}\xi + n_z^2 k_{z0}\varsigma) + \eta^2(n_y^2 - n_x^2)(n_y^2 - n_z^2)\left(\frac{\omega}{c}\right)^2 = 0$$

Rotation of the coordinates:

$$\rightarrow \; \alpha\xi'^2 + \beta\varsigma'^2 + \eta^2(n_y^2 - n_x^2)(n_y^2 - n_z^2) = 0$$

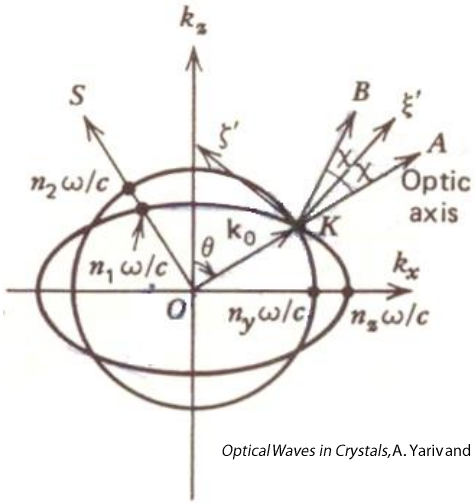
Conical Refraction (2)

Rotation of the coordinates:

$$\rightarrow \alpha\xi'^2 + \beta\varsigma'^2 + \eta^2(n_y^2 - n_x^2)(n_y^2 - n_z^2) = 0$$

$$\rightarrow \begin{pmatrix} \xi' \\ \varsigma' \end{pmatrix} = \begin{pmatrix} \cos(\frac{\pi}{2} + \chi - \theta) & \sin(\frac{\pi}{2} + \chi - \theta) \\ -\sin(\frac{\pi}{2} + \chi - \theta) & \cos(\frac{\pi}{2} + \chi - \theta) \end{pmatrix} \begin{pmatrix} \xi \\ \varsigma \end{pmatrix}$$

$$= \begin{pmatrix} -\sin(\chi - \theta) & \cos(\chi - \theta) \\ -\cos(\chi - \theta) & -\sin(\chi - \theta) \end{pmatrix} \begin{pmatrix} \xi \\ \varsigma \end{pmatrix}$$



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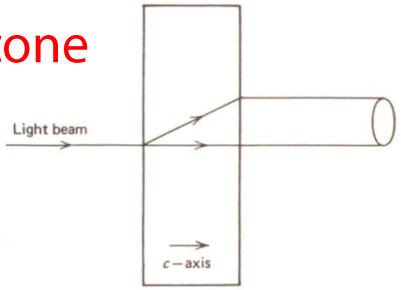
$$\rightarrow \varsigma'^2 + \frac{1}{1 - \tan^2\chi}\eta^2 = \xi'^2 \cot^2\chi$$ ← Eq. of an elliptical cone

$$\rightarrow \tan^2 2\chi = \frac{(n_z^2 - n_y^2)(n_y^2 - n_x^2)}{n_x^2 n_z^2}$$



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Energy flow:

$$\leftarrow \begin{pmatrix} \varsigma' & \frac{1}{1 - \tan^2\chi}\eta & -\xi'\cot^2\chi \end{pmatrix} \cdot \begin{pmatrix} \varsigma' & \eta & \xi' \end{pmatrix} = 0$$

$$\rightarrow \varsigma'^2 + (1 - \tan^2\chi)\eta^2 = \xi'^2 \tan^2\chi$$