

Electro-Optics:

Electromagnetic Propagation in Anisotropic Media (3)

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Light Propagation in Uniaxial Crystals

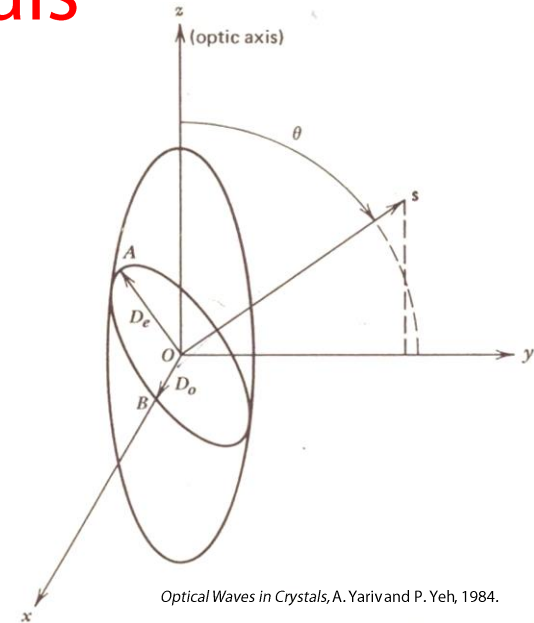
Index ellipsoid:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$$

Polarizations for the displacement vector:

$$\mathbf{d}_o = \frac{\mathbf{k} \times \mathbf{c}}{|\mathbf{k} \times \mathbf{c}|} \quad \leftarrow \text{Ordinary polarization}$$

$$\mathbf{d}_e = \frac{\mathbf{d}_o \times \mathbf{k}}{|\mathbf{d}_o \times \mathbf{k}|} \quad \leftarrow \text{Extraordinary polarization}$$



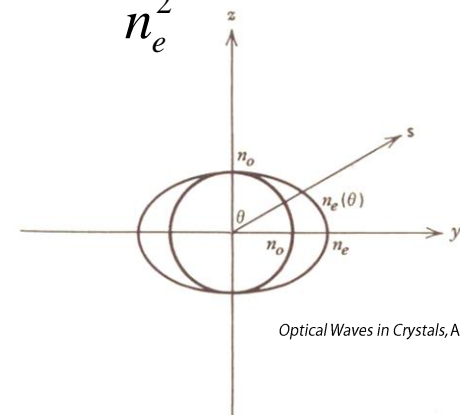
Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

For $k_x = 0$:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1 \quad \rightarrow \quad \frac{1}{n_e^2(\theta)} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2}$$

Extraordinary electric field:

$$\rightarrow \begin{pmatrix} 0 \\ \sin \theta \\ \frac{n_e^2(\theta) - n_o^2}{\cos \theta} \\ \frac{n_e^2(\theta) - n_e^2}{\cos \theta} \end{pmatrix}$$



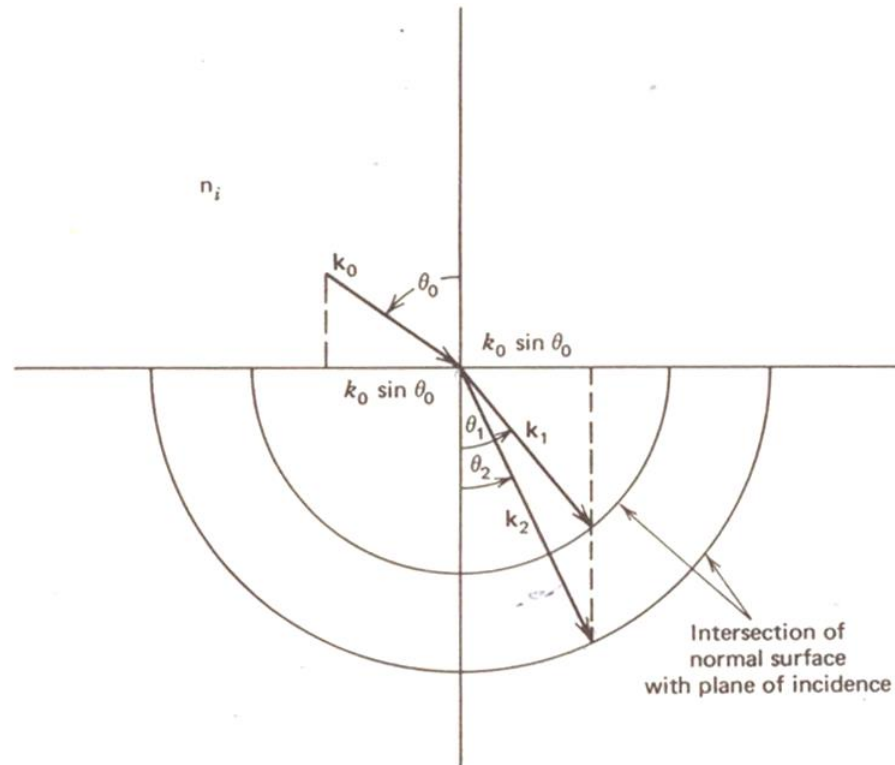
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Double Refraction at a Boundary

Boundary condition:

$$k_0 \sin \theta_0 = k_1 \sin \theta_1 = k_2 \sin \theta_2$$

Snell's law: $\rightarrow n_i \sin \theta_0 = n_o \sin \theta_1 = n_e(\theta) \sin \theta_2$

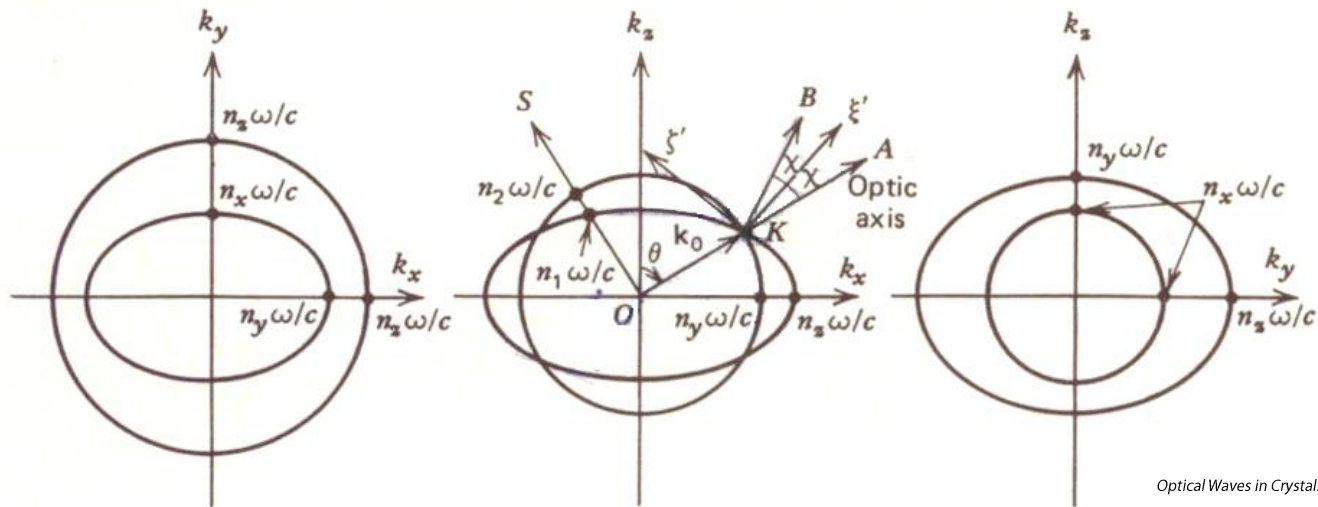


Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Light Propagation in Biaxial Crystals

Biaxial media: $\rightarrow n_x < n_y < n_z$

Normal surface:



k_x - k_y plane:

$$\rightarrow \frac{k_x^2 + k_y^2}{n_z^2} = \left(\frac{\omega}{c}\right)^2$$

$$\rightarrow \frac{k_x^2}{n_y^2} + \frac{k_y^2}{n_x^2} = \left(\frac{\omega}{c}\right)^2$$

k_z - k_x plane:

$$\rightarrow \frac{k_z^2 + k_x^2}{n_y^2} = \left(\frac{\omega}{c}\right)^2$$

$$\rightarrow \frac{k_z^2}{n_x^2} + \frac{k_x^2}{n_z^2} = \left(\frac{\omega}{c}\right)^2$$

k_y - k_z plane:

$$\rightarrow \frac{k_y^2 + k_z^2}{n_x^2} = \left(\frac{\omega}{c}\right)^2$$

$$\rightarrow \frac{k_y^2}{n_z^2} + \frac{k_z^2}{n_y^2} = \left(\frac{\omega}{c}\right)^2$$

Conical Refraction (1)

For light propagation in the direction of the optic axis:

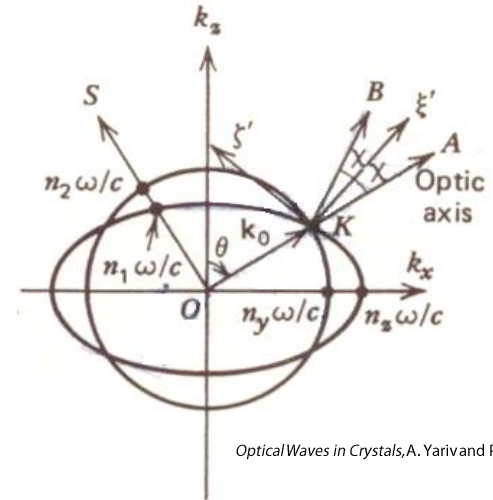
$$\mathbf{k}_0 = \hat{\mathbf{x}}k_{x0} + \hat{\mathbf{y}}k_{y0} + \hat{\mathbf{z}}k_{z0}$$

$$\rightarrow k_{x0} = n_y \frac{\omega}{c} \sin \theta$$

$$\rightarrow k_{y0} = 0$$

$$\rightarrow \tan \theta = \frac{n_z}{n_x} \left(\frac{n_y^2 - n_x^2}{n_z^2 - n_y^2} \right)^{1/2}$$

$$\rightarrow k_{z0} = n_y \frac{\omega}{c} \cos \theta$$



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Taylor series expansion:

$$\rightarrow k_x = k_{x0} + \xi$$

$$\rightarrow k_y = k_y + \eta \quad \rightarrow k_x^2 YZ + k_y^2 ZX + k_z^2 XY - XYZ = 0$$

$$\rightarrow k_z = k_{z0} + \zeta$$

$$\rightarrow 4(k_{x0}\xi + k_{z0}\zeta)(n_x^2 k_{x0}\xi + n_z^2 k_{z0}\zeta) + \eta^2 (n_y^2 - n_x^2)(n_y^2 - n_z^2) \left(\frac{\omega}{c} \right)^2 = 0$$

Rotation of the coordinates:

$$\rightarrow \alpha \xi'^2 + \beta \zeta'^2 + \eta^2 (n_y^2 - n_x^2)(n_y^2 - n_z^2) = 0$$

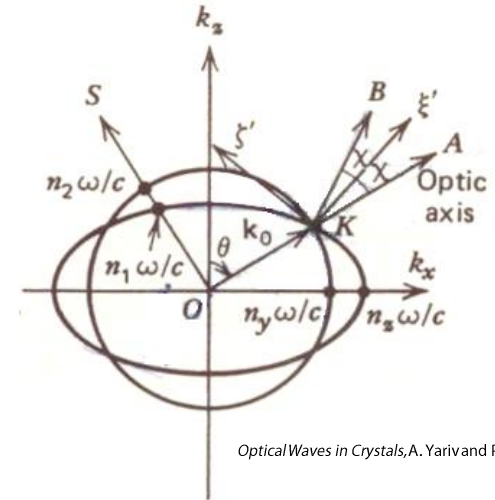
Conical Refraction (2)

Rotation of the coordinates:

$$\rightarrow \alpha \xi'^2 + \beta \zeta'^2 + \eta^2 (n_y^2 - n_x^2)(n_y^2 - n_z^2) = 0$$

$$\rightarrow \begin{pmatrix} \xi' \\ \zeta' \end{pmatrix} = \begin{pmatrix} \cos(\frac{\pi}{2} + \chi - \theta) & \sin(\frac{\pi}{2} + \chi - \theta) \\ -\sin(\frac{\pi}{2} + \chi - \theta) & \cos(\frac{\pi}{2} + \chi - \theta) \end{pmatrix} \begin{pmatrix} \xi \\ \zeta \end{pmatrix}$$

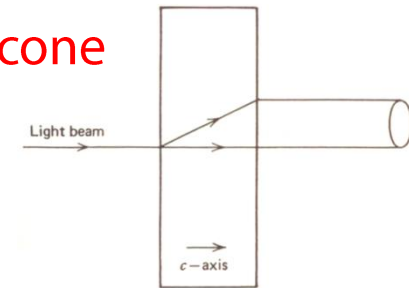
$$= \begin{pmatrix} -\sin(\chi - \theta) & \cos(\chi - \theta) \\ -\cos(\frac{\pi}{2} + \chi - \theta) & -\sin(\chi - \theta) \end{pmatrix} \begin{pmatrix} \xi \\ \zeta \end{pmatrix}$$



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$$\rightarrow \zeta'^2 + \frac{1}{1 - \tan^2 \chi} \eta^2 = \xi'^2 \cot^2 \chi \quad \leftarrow \text{Eq. of an elliptical cone}$$

$$\rightarrow \tan^2 2\chi = \frac{(n_z^2 - n_y^2)(n_y^2 - n_x^2)}{n_x^2 n_z^2}$$



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Energy flow:

$$\left(\zeta' \quad \frac{1}{1 - \tan^2 \chi} \eta \quad -\xi' \cot^2 \chi \right) \cdot \begin{pmatrix} \zeta' \\ \eta \\ \xi' \end{pmatrix} = 0$$

$$\rightarrow \zeta'^2 + (1 - \tan^2 \chi) \eta^2 = \xi'^2 \tan^2 \chi$$