

Electro-Optics:

Electromagnetic Propagation in Anisotropic Media (4)

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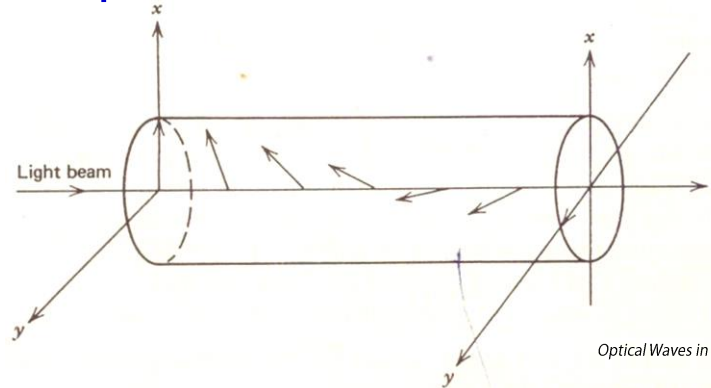
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Optical Activity (1)

Rotation of the plane of polarization:



Eigenwaves:

$$\hat{\mathbf{R}} = \exp\left[i\omega\left(t - \frac{zn_r}{c}\right)\right], \quad \hat{\mathbf{L}} = \exp\left[i\omega\left(t - \frac{zn_l}{c}\right)\right]$$

Linearly polarized beam (x-pol.) at $z = 0$:

$$\rightarrow \frac{D_0}{\sqrt{2}} e^{i\omega t} (\hat{\mathbf{R}} e^{-i\omega zn_r/c} + \hat{\mathbf{L}} e^{-i\omega zn_l/c})$$

$$\rightarrow D_0 \hat{\mathbf{p}} \exp\left\{i\omega\left[t - \frac{z(n_r + n_l)}{2c}\right]\right\}$$

$$\rightarrow \hat{\mathbf{p}} = \hat{\mathbf{x}} \cos\left[\frac{\omega(n_l - n_r)}{2c} z\right] + \hat{\mathbf{y}} \sin\left[\frac{\omega(n_l - n_r)}{2c} z\right]$$

→ Right-handed if $n_r < n_l$

Optical Activity (2)

Specific rotatory power:

$$\rightarrow \rho = \frac{\pi}{\lambda} (n_l - n_r)$$

Induced dipole moment:

$$\rightarrow p = \alpha E - \beta \dot{H}$$

Material equation:

$$\rightarrow \mathbf{D} = \varepsilon \mathbf{E} + i\varepsilon_0 \mathbf{G} \times \mathbf{E} \quad \leftarrow \mathbf{G} // \mathbf{k} \quad \rightarrow \mathbf{G} = G\mathbf{s}$$

$$\rightarrow \mathbf{D} = (\varepsilon + i\varepsilon_0 [G]) \mathbf{E} \quad \leftarrow [G]_{23} = -[G]_{32} = -G_x$$

$$\rightarrow \varepsilon' = \varepsilon + i\varepsilon_0 [G] \quad [G]_{31} = -[G]_{13} = -G_y$$

← Hermitian

$$[G]_{12} = -[G]_{21} = -G_z$$

Fresnel equation:

← Reciprocal

$$\rightarrow \frac{s_x^2}{n^2 - \varepsilon_x / \varepsilon_0} + \frac{s_y^2}{n^2 - \varepsilon_y / \varepsilon_0} + \frac{s_z^2}{n^2 - \varepsilon_z / \varepsilon_0} - \frac{1}{n^2} = G^2 \frac{s_x^2 n_x^2 + s_y^2 n_y^2 + s_z^2 n_z^2}{n^2 (n^2 - n_x^2)(n^2 - n_y^2)(n^2 - n_z^2)}$$

Optical Activity (3)

Fresnel equation:

$$\rightarrow \frac{s_x^2}{n^2 - \varepsilon_x / \varepsilon_0} + \frac{s_y^2}{n^2 - \varepsilon_y / \varepsilon_0} + \frac{s_z^2}{n^2 - \varepsilon_z / \varepsilon_0} - \frac{1}{n^2} = G^2 \frac{s_x^2 n_x^2 + s_y^2 n_y^2 + s_z^2 n_z^2}{n^2 (n^2 - n_x^2)(n^2 - n_y^2)(n^2 - n_z^2)}$$

$$\rightarrow (n^2 - n_1^2)(n^2 - n_2^2) = G^2$$

For propagation along the optic axes:

$$\rightarrow n_1 = n_2 = \bar{n}$$

$$\rightarrow n^2 = \bar{n}^2 \pm G \quad \rightarrow n \cong \bar{n} \pm \frac{G}{2\bar{n}} \quad \rightarrow \rho = \frac{\pi G}{\lambda \bar{n}}$$

Parameter G :

$$G = g_{11}s_x^2 + g_{22}s_y^2 + g_{33}s_z^2 + 2g_{12}s_x s_y + 2g_{23}s_y s_z + 2g_{31}s_x s_z$$

$$\rightarrow G = g_{ij}s_i s_j, \quad i, j = x, y, z$$

Wave equation:

$$n^2 \mathbf{s} \times \left(\mathbf{s} \times \frac{\varepsilon_0}{\varepsilon'} \right) \mathbf{D} + \mathbf{D} = 0 \quad \leftarrow \frac{1}{\varepsilon'} = \frac{1}{\varepsilon} - \varepsilon_0 \frac{1}{\varepsilon} [G] \frac{1}{\varepsilon}$$

Optical Activity (4)

Wave equation:

$$\rightarrow [s][s]\{\eta - i\eta[G]\eta\}\mathbf{D} = -\frac{1}{n^2}\mathbf{D}$$

Eigenmodes in the absence of optical activity ($G = 0$):

$$\rightarrow [s][s]\{\eta + \frac{1}{n_{1,2}}\}\mathbf{D}_{1,2} = 0 \quad \leftarrow \mathbf{D}_i \cdot \mathbf{D}_j = \delta_{ij}$$

In the coordinates system formed by \mathbf{D}_1 , \mathbf{D}_2 , & s :

$$\rightarrow \begin{pmatrix} \frac{1}{n_1^2} & \frac{iG}{n_1^2 n_2^2} \\ -\frac{iG}{n_1^2 n_2^2} & \frac{1}{n_2^2} \end{pmatrix} \mathbf{D} = \frac{1}{n^2} \mathbf{D} \rightarrow \begin{pmatrix} \frac{1}{n_1^2} & -\frac{1}{n^2} \\ \frac{1}{n_2^2} & -\frac{1}{n^2} \end{pmatrix} \begin{pmatrix} \frac{1}{n_1^2} & -\frac{1}{n^2} \\ \frac{1}{n_2^2} & -\frac{1}{n^2} \end{pmatrix} = \left(\frac{G}{n_1^2 n_2^2} \right)^2$$

$$\rightarrow \frac{1}{n^2} = \frac{1}{2} \left(\frac{1}{n_1^2} + \frac{1}{n_2^2} \right) \pm \sqrt{\frac{1}{4} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)^2 + \left(\frac{G}{n_1^2 n_2^2} \right)^2}$$

Jones vectors:

$$\rightarrow \mathbf{J}_{\pm} = \begin{pmatrix} \frac{1}{2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \pm \sqrt{\frac{1}{4} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)^2 + \left(\frac{G}{n_1^2 n_2^2} \right)^2} \\ -\frac{iG}{n_1^2 n_2^2} \end{pmatrix}$$

Optical Activity (5)

Ellipticity:

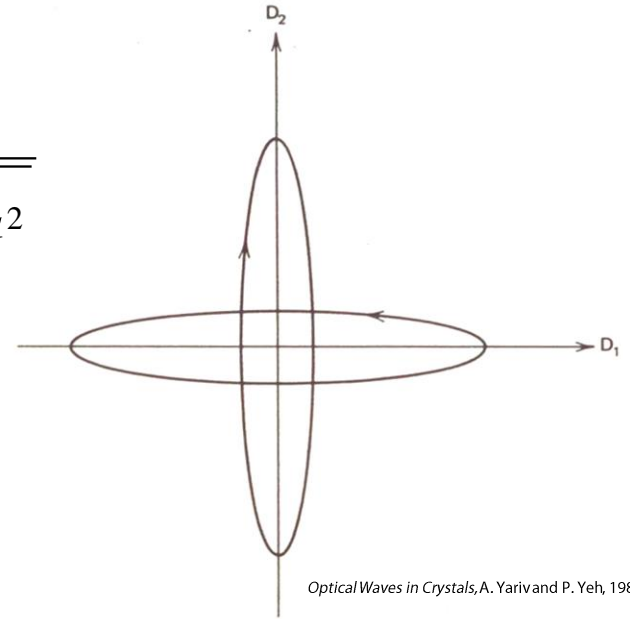
$$e = \frac{-G}{\frac{1}{2}(n_2^2 - n_1^2) \pm \sqrt{\frac{1}{4}(n_2^2 - n_1^2)^2 + G^2}}$$

Isotropic medium:

$$\rightarrow \frac{1}{n^2} = \frac{1}{\bar{n}^2} \pm \frac{G}{\bar{n}^4} = \frac{1}{\bar{n}^2} \left(1 \pm \frac{G}{\bar{n}^2} \right)$$

Jones vectors:

$$\rightarrow \mathbf{J}_{\pm} = \begin{pmatrix} \pm 1 \\ -i \end{pmatrix}$$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Faraday Rotation

Faraday effect:

$$\mathbf{G} = \gamma \mathbf{B}_{ext} \quad \leftarrow \text{Magnetogyration coefficient}$$

Verdet constant:

$$\rightarrow \rho = V B_{ext}$$

Material relation:

$$\rightarrow \mathbf{D} = \epsilon \mathbf{E} + i \epsilon_0 \gamma \mathbf{B}_{ext} \times \mathbf{E}$$

\leftarrow Nonreciprocal

\rightarrow Application to optical isolators