Electro-Optics:

Electromagnetic Propagation in Anisotropic Media (4)

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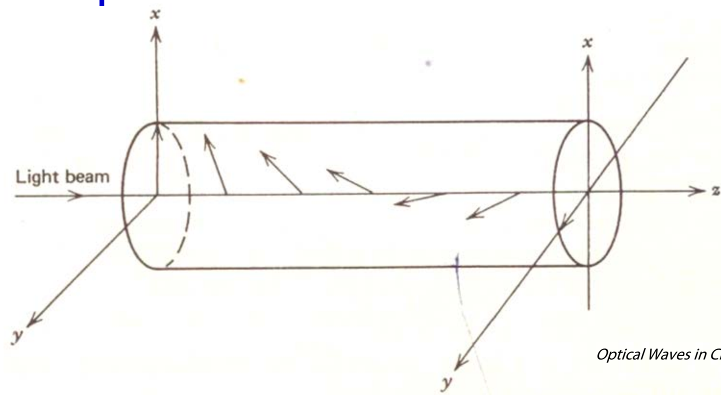
Optical Activity (1)

Rotation of the plane of polarization:



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Recall:

$$\hat{\mathbf{R}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - i\hat{\mathbf{y}})$$

$$\hat{\mathbf{L}} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})$$

Eigenwaves:

$$\hat{\mathbf{R}} \exp\left[i\omega\left(t - \frac{zn_r}{c} \right)\right], \quad \hat{\mathbf{L}} \exp\left[i\omega\left(t - \frac{zn_l}{c} \right)\right]$$

Linearly polarized beam (x-pol.) at $z = 0$:

$$\rightarrow \frac{D_0}{\sqrt{2}} e^{i\omega t} (\hat{\mathbf{R}} e^{-i\omega z n_r / c} + \hat{\mathbf{L}} e^{-i\omega z n_l / c})$$

$$\rightarrow D_0 \hat{\mathbf{p}} \exp\left\{ i\omega\left[t - \frac{z(n_r + n_l)}{2c} \right]\right\}$$

$$\rightarrow \hat{\mathbf{p}} = \hat{\mathbf{x}} \cos\left[\frac{\omega(n_l - n_r)}{2c} z \right] + \hat{\mathbf{y}} \sin\left[\frac{\omega(n_l - n_r)}{2c} z \right]$$

$\rightarrow$ Right-handed if $n_r < n_l$

Optical Activity (2)

Specific rotatory power:

$$\rightarrow \quad \rho = \frac{\pi}{\lambda}(n_l - n_r)$$

Induced dipole moment:

$$\rightarrow \quad p = \alpha E - \beta \dot{H}$$

Material equation:

$$\rightarrow \quad \mathbf{D} = \varepsilon \mathbf{E} + i\varepsilon_0 \mathbf{G} \times \mathbf{E} \quad \leftarrow \quad \mathbf{G} \,//\, \mathbf{k} \quad \rightarrow \quad \mathbf{G} = G\mathbf{s}$$

$$\rightarrow \quad \mathbf{D} = (\varepsilon + i\varepsilon_0 [G])\mathbf{E} \qquad \leftarrow \quad [G]_{23} = -[G]_{32} = -G_x$$

$$\rightarrow \quad \varepsilon' = \varepsilon + i\varepsilon_0 [G] \qquad [G]_{31} = -[G]_{13} = -G_y$$

$\leftarrow$ Hermitian $\qquad [G]_{12} = -[G]_{21} = -G_z$

$\leftarrow$ Reciprocal

Fresnel equation:

$$\rightarrow \quad \frac{s_x^2}{n^2 - \varepsilon_x/\varepsilon_0} + \frac{s_y^2}{n^2 - \varepsilon_y/\varepsilon_0} + \frac{s_z^2}{n^2 - \varepsilon_z/\varepsilon_0} - \frac{1}{n^2} = G^2 \frac{s_x^2 n_x^2 + s_y^2 n_y^2 + s_z^2 n_z^2}{n^2(n^2 - n_x^2)(n^2 - n_y^2)(n^2 - n_z^2)}$$

Optical Activity (3)

Fresnel equation:

$$\rightarrow \frac{s_x^2}{n^2-\varepsilon_x/\varepsilon_0}+\frac{s_y^2}{n^2-\varepsilon_y/\varepsilon_0}+\frac{s_z^2}{n^2-\varepsilon_z/\varepsilon_0}-\frac{1}{n^2}=G^2\frac{s_x^2n_x^2+s_y^2n_y^2+s_z^2n_z^2}{n^2(n^2-n_x^2)(n^2-n_y^2)(n^2-n_z^2)}$$

$$\rightarrow (n^2-n_1^2)(n^2-n_2^2)=G^2$$

For propagation along the optic axes:

$$\rightarrow n_1=n_2=\overline{n}$$

$$\rightarrow n^2=\overline{n}^2\pm G \quad \rightarrow n\cong\overline{n}\pm\frac{G}{2\overline{n}} \quad \rightarrow \rho=\frac{\pi G}{\lambda\overline{n}}$$

Parameter G:

$$G=g_{11}s_x^2+g_{22}s_y^2+g_{33}s_z^2+2g_{12}s_xs_y+2g_{23}s_ys_z+2g_{31}s_xs_z$$

$$\rightarrow G=g_{ij}s_is_j,\quad i,j=x,y,z$$

Wave equation:

$$n^2\mathbf{s}\times(\mathbf{s}\times\frac{\varepsilon_0}{\varepsilon'}\mathbf{D})+\mathbf{D}=0 \qquad \leftarrow \frac{1}{\varepsilon'}=\frac{1}{\varepsilon}-i\varepsilon_0\frac{1}{\varepsilon}[G]\frac{1}{\varepsilon}$$

Optical Activity (4)

Wave equation:

$$\rightarrow [s][s]\{\eta - i\eta[G]\eta\}\mathbf{D} = -\frac{1}{n^2}\mathbf{D}$$

Eigenmodes in the absence of optical activity ($G = 0$):

$$\rightarrow [s][s]\{\eta + \frac{1}{n_{1,2}^2}\}\mathbf{D}_{1,2} = 0 \qquad \leftarrow \; \mathbf{D}_i \cdot \mathbf{D}_j = \delta_{ij}$$

In the coordinates system formed by $\mathbf{D}_1$, $\mathbf{D}_2$, & $\mathbf{s}$:

$$\rightarrow \begin{pmatrix} \frac{1}{n_1^2} & \frac{iG}{n_1^2 n_2^2} \\ -\frac{iG}{n_1^2 n_2^2} & \frac{1}{n_2^2} \end{pmatrix}\mathbf{D} = \frac{1}{n^2}\mathbf{D} \;\rightarrow \left(\frac{1}{n_1^2} - \frac{1}{n^2}\right)\left(\frac{1}{n_2^2} - \frac{1}{n^2}\right) = \left(\frac{G}{n_1^2 n_2^2}\right)^2$$

$$\rightarrow \frac{1}{n^2} = \frac{1}{2}\left(\frac{1}{n_1^2} + \frac{1}{n_2^2}\right) \pm \sqrt{\frac{1}{4}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)^2 + \left(\frac{G}{n_1^2 n_2^2}\right)^2}$$

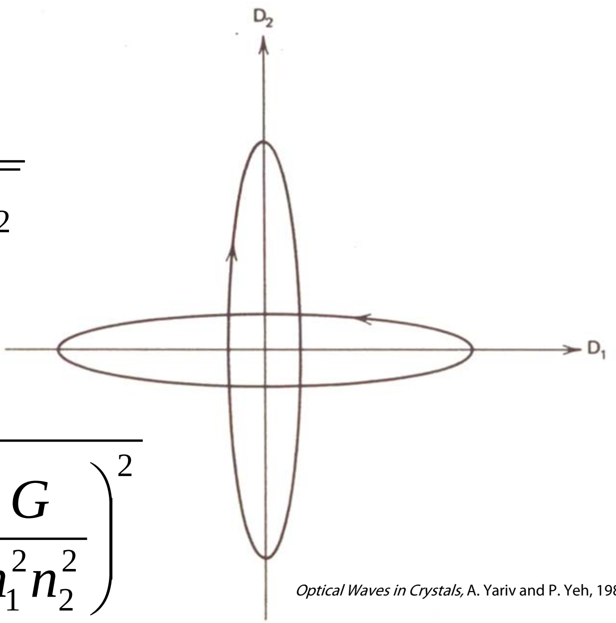
Jones vectors:

$$\rightarrow \mathbf{J}_{\pm} = \begin{pmatrix} \frac{1}{2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \pm \sqrt{\frac{1}{4}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)^2 + \left(\frac{G}{n_1^2 n_2^2}\right)^2} \\ -\frac{iG}{n_1^2 n_2^2} \end{pmatrix}$$

Optical Activity (5)

Ellipticity:

$$e=\frac{-G}{\frac{1}{2}\left(n_2^2-n_1^2\right)\pm\sqrt{\frac{1}{4}\left(n_2^2-n_1^2\right)^2+G^2}}$$



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Isotropic medium:

$$\rightarrow\frac{1}{n^2}=\frac{1}{2}\left(\frac{1}{n_1^2}+\frac{1}{n_2^2}\right)\pm\sqrt{\frac{1}{4}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)^2+\left(\frac{G}{n_1^2n_2^2}\right)^2}$$

$$\rightarrow\frac{1}{n^2}=\frac{1}{\bar{n}^2}\pm\frac{G}{\bar{n}^4}=\frac{1}{\bar{n}^2}\left(1\pm\frac{G}{\bar{n}^2}\right)$$

$$\rightarrow\mathbf{J}_\pm=\begin{pmatrix}\frac{1}{2}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)\pm\sqrt{\frac{1}{4}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)^2+\left(\frac{G}{n_1^2n_2^2}\right)^2}\\-\frac{iG}{n_1^2n_2^2}\end{pmatrix}$$

$$\rightarrow\mathbf{J}_\pm=\begin{pmatrix}\pm1\\-i\end{pmatrix}$$

Faraday Rotation

Faraday effect:

$$\mathbf{G} = \gamma \mathbf{B}_{ext} \quad \leftarrow \text{Magnetogyration coefficient}$$

Verdet constant:

$$\rightarrow \quad \rho = V B_{ext}$$

Material relation:

$$\rightarrow \quad \mathbf{D} = \varepsilon \mathbf{E} + i \varepsilon_0 \gamma \mathbf{B}_{ext} \times \mathbf{E}$$

← Nonreciprocal

→ Application to optical isolators