Electro-Optics:

Jones Calculus and its Application to Birefringent Optical Systems (1)

Dr Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953
Email: yoonchan@snu.ac.kr

Jones Matrix Formulation (1)

Jones vector:

$$\mathbf{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

Coordinate transformation:

$$\begin{pmatrix} V_s \\ V_f \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix} \equiv R(\psi) \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

Phase retardation:

$$\begin{pmatrix} V_s' \\ V_f' \end{pmatrix} = \begin{pmatrix} \exp\left(-in_s \frac{\omega}{c} l\right) & 0 \\ 0 & \exp\left(-in_f \frac{\omega}{c} l\right) \end{pmatrix} \begin{pmatrix} V_s \\ V_f \end{pmatrix}$$

$$\rightarrow \ \Gamma = (n_s - n_f)\frac{\omega l}{c} \qquad \rightarrow \ \phi = \frac{1}{2}(n_s + n_f)\frac{\omega l}{c}$$

$$\rightarrow \begin{pmatrix} V_s' \\ V_f' \end{pmatrix} = e^{-i\phi} \begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix} \begin{pmatrix} V_s \\ V_f \end{pmatrix} \equiv W_0 \begin{pmatrix} V_s \\ V_f \end{pmatrix}$$

Jones Matrix Formulation (2)

Coordinate transformation:

$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} V_s' \\ V_f' \end{pmatrix}$$

Whole formulation: → Unitary matrix

$$\rightarrow \begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = R(-\psi) W_0 R(\psi) \begin{pmatrix} V_x \\ V_y \end{pmatrix} \equiv W \begin{pmatrix} V_x \\ V_y \end{pmatrix} \qquad \rightarrow W^{\dagger} W = I$$

Polarizers:

$$P_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

A half-wave retardation plate with $\psi = \pi/4$:

$$\rightarrow W = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$$

$$\rightarrow \mathbf{V} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \rightarrow \mathbf{V}' = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

→ Rotation of the polarization

Jones Matrix Formulation (3)

A quarter-wave plate with $\psi = \pi/4$:

$$\rightarrow W = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$$\rightarrow \mathbf{V} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \rightarrow \mathbf{V}' = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix} = \frac{-i}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$\rightarrow$ Left-handed circularly polarized light

Intensity Transmission (1)

Electric-field vector and transmissivity:

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad \rightarrow \quad I = \mathbf{E}^\dagger \cdot \mathbf{E} = |E_x|^2 + |E_y|^2$$

$$\mathbf{E}' = \begin{pmatrix} E'_x \\ E'_y \end{pmatrix} \quad \rightarrow \quad T = \frac{|E'_x|^2 + |E'_y|^2}{|E_x|^2 + |E_y|^2}$$

A birefringent plate sandwiched between parallel polarizers:

$$\Gamma = 2\pi(n_e - n_o)\frac{d}{\lambda}$$

Un-polarized light

$$\rightarrow \mathbf{E}' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\frac{1}{2}\Gamma & -i\sin\frac{1}{2}\Gamma \\ -i\sin\frac{1}{2}\Gamma & \cos\frac{1}{2}\Gamma \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \cos\frac{1}{2}\Gamma \end{pmatrix}$$

$$\rightarrow I = \frac{1}{2}\cos^2\frac{1}{2}\Gamma = \frac{1}{2}\cos^2\left[\frac{\pi(n_e - n_o)d}{\lambda}\right]$$

Intensity Transmission (2)

A birefringent plate sandwiched between a pair of crossed polarizers:

$$\rightarrow \mathbf{E}' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\frac{1}{2}\Gamma & -i\sin\frac{1}{2}\Gamma \\ -i\sin\frac{1}{2}\Gamma & \cos\frac{1}{2}\Gamma \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{-i}{\sqrt{2}} \begin{pmatrix} \sin\frac{1}{2}\Gamma \\ 0 \end{pmatrix}$$

$$\rightarrow I = \frac{1}{2}\sin^2\frac{1}{2}\Gamma = \frac{1}{2}\sin^2\left[\frac{\pi(n_e - n_o)d}{\lambda}\right]$$