

Electro-Optics:

Jones Calculus and its Application to Birefringent Optical Systems (1)

Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: yunchan@snu.ac.kr

Jones Matrix Formulation (1)

Jones vector:

$$\mathbf{V} = \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

Coordinate transformation:

$$\begin{pmatrix} V_s \\ V_f \end{pmatrix} = \begin{pmatrix} \cos \psi & \sin \psi \\ -\sin \psi & \cos \psi \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix} \equiv R(\psi) \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

Phase retardation:

$$\begin{pmatrix} V'_s \\ V'_f \end{pmatrix} = \begin{pmatrix} \exp\left(-in_s \frac{\omega l}{c}\right) & 0 \\ 0 & \exp\left(-in_f \frac{\omega l}{c}\right) \end{pmatrix} \begin{pmatrix} V_s \\ V_f \end{pmatrix}$$

$$\rightarrow \Gamma = (n_s - n_f) \frac{\omega l}{c} \quad \rightarrow \phi = \frac{1}{2} (n_s + n_f) \frac{\omega l}{c}$$

$$\rightarrow \begin{pmatrix} V'_s \\ V'_f \end{pmatrix} = e^{-i\phi} \begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix} \begin{pmatrix} V_s \\ V_f \end{pmatrix} \equiv W_0 \begin{pmatrix} V_s \\ V_f \end{pmatrix}$$

Jones Matrix Formulation (2)

Coordinate transformation:

$$\begin{pmatrix} V'_x \\ V'_y \end{pmatrix} = \begin{pmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{pmatrix} \begin{pmatrix} V_s \\ V_f \end{pmatrix}$$

Whole formulation:

$$\rightarrow \begin{pmatrix} V'_x \\ V'_y \end{pmatrix} = R(-\psi)W_0R(\psi) \begin{pmatrix} V_x \\ V_y \end{pmatrix} \equiv W \begin{pmatrix} V_x \\ V_y \end{pmatrix} \rightarrow W^\dagger W = I$$

→ Unitary matrix

Polarizers:

$$P_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

A half-wave retardation plate with $\psi = \pi/4$:

$$\rightarrow W = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$$

$$\rightarrow \mathbf{V} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \mathbf{V}' = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \text{Rotation of the polarization}$$

Jones Matrix Formulation (3)

A quarter-wave plate with $\psi = \pi/4$:

$$\rightarrow W = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

$$\rightarrow \mathbf{V} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \rightarrow \mathbf{V}' = \frac{1}{\sqrt{2}} \begin{pmatrix} -i \\ 1 \end{pmatrix} = \frac{-i}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$$

→ Left-handed circularly polarized light

Intensity Transmission (1)

Electric-field vector and transmissivity:

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} \rightarrow I = \mathbf{E}^\dagger \cdot \mathbf{E}$$

$$\mathbf{E}' = \begin{pmatrix} E'_x \\ E'_y \end{pmatrix} \rightarrow T = \frac{|E'_x|^2 + |E'_y|^2}{|E_x|^2 + |E_y|^2}$$

A birefringent plate sandwiched between two parallel polarizers with $\psi = \pi/4$:

$$\rightarrow \Gamma = \frac{2\pi}{\lambda} (n_e - n_o) d$$

$$\rightarrow W = R(-\psi) W_0 R(\psi)$$

$$\rightarrow \mathbf{E}' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \frac{1}{2} \Gamma & -i \sin \frac{1}{2} \Gamma \\ -i \sin \frac{1}{2} \Gamma & \cos \frac{1}{2} \Gamma \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \cos \frac{1}{2} \Gamma \end{pmatrix}$$

Un-polarized light

$$\rightarrow I = \frac{1}{2} \cos^2 \frac{1}{2} \Gamma = \frac{1}{2} \cos^2 \left[\frac{\pi (n_e - n_o) d}{\lambda} \right]$$

Intensity Transmission (2)

A birefringent plate sandwiched between a pair of crossed polarizers with $\psi = \pi/4$:

$$\begin{aligned}\rightarrow \mathbf{E}' &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos \frac{1}{2} \Gamma & -i \sin \frac{1}{2} \Gamma \\ -i \sin \frac{1}{2} \Gamma & \cos \frac{1}{2} \Gamma \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= \frac{-i}{\sqrt{2}} \begin{pmatrix} \sin \frac{1}{2} \Gamma \\ 0 \end{pmatrix} \\ \rightarrow I &= \frac{1}{2} \sin^2 \frac{1}{2} \Gamma = \frac{1}{2} \sin^2 \left[\frac{\pi(n_e - n_o)d}{\lambda} \right]\end{aligned}$$