Electro-Optics:

Jones Calculus and its Application to Birefringent Optical Systems (2)

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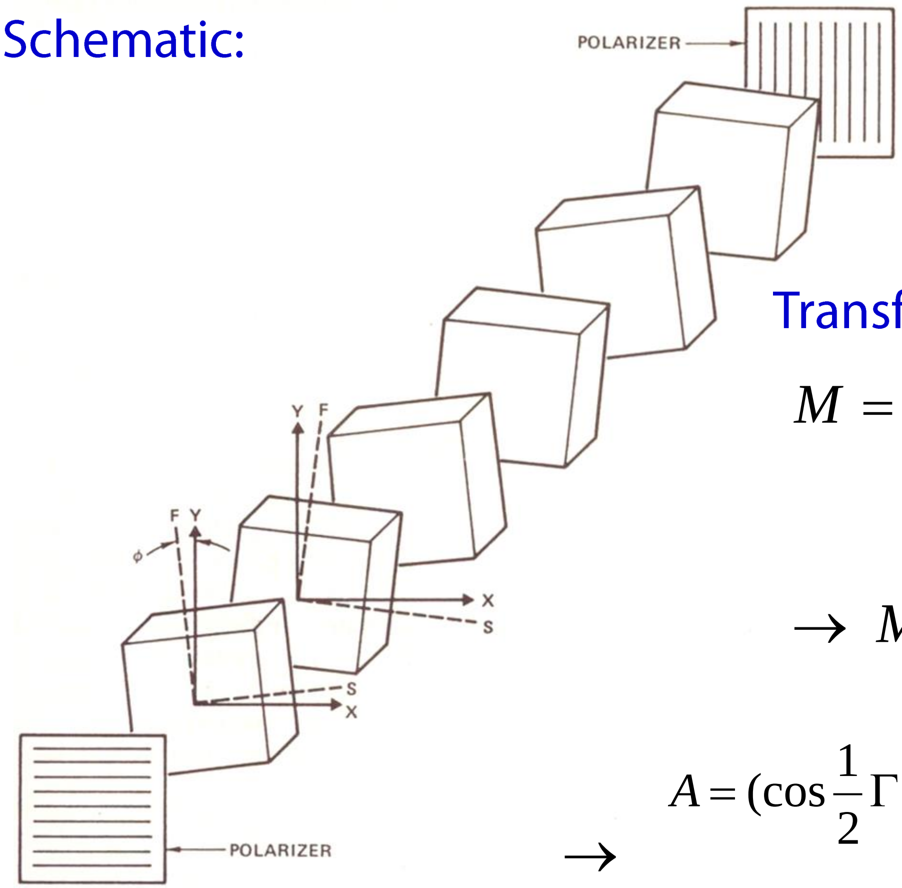
Folded Šolc Filters (1)

Schematic:



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Transfer matrix:

$$M = [R(\rho)W_0R(-\rho)R(-\rho)W_0R(\rho)]^m \quad \leftarrow \quad N = 2m$$

$$\rightarrow \quad M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^m$$

$\rightarrow$

$$A = (\cos\frac{1}{2}\Gamma - i\cos 2\rho \sin\frac{1}{2}\Gamma)^2 + \sin^2 2\rho \sin^2\frac{1}{2}\Gamma$$

$$B = -\sin 4\rho \sin^2\frac{1}{2}\Gamma$$

$$C = -B$$

$$D = (\cos\frac{1}{2}\Gamma + i\cos 2\rho \sin\frac{1}{2}\Gamma)^2 + \sin^2 2\rho \sin^2\frac{1}{2}\Gamma$$

Folded Šolc Filters (2)

Chebyshev's identity:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^m = \begin{pmatrix} \dfrac{A\sin mK\Lambda - \sin(m-1)K\Lambda}{\sin K\Lambda} & B\dfrac{\sin mK\Lambda}{\sin K\Lambda} \\ C\dfrac{\sin mK\Lambda}{\sin K\Lambda} & \dfrac{D\sin mK\Lambda - \sin(m-1)K\Lambda}{\sin K\Lambda} \end{pmatrix}$$

$$\leftarrow \; K\Lambda = \cos^{-1}[\frac{1}{2}(A+D)]$$

Input and output electric fields:

$$\rightarrow \begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = P_y M P_x \begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad \rightarrow \; E'_y = M_{21} E_x$$

Transmissivity:

$$\rightarrow \; T = \left|M_{21}\right|^2 = \left|\sin 4\rho \sin^2 \frac{1}{2}\Gamma \frac{\sin mK\Lambda}{\sin K\Lambda}\right|^2$$

$$\rightarrow \; T = \left|\tan 2\rho \cos\chi \frac{\sin N\chi}{\sin \chi}\right|^2 \quad \leftarrow \; \cos K\Lambda = 1 - 2\cos^2 2\rho \sin^2 \frac{1}{2}\Gamma$$

$$\leftarrow \; K\Lambda = \pi - 2\chi$$

$$\rightarrow \; \cos\chi = \cos 2\rho \sin \frac{1}{2}\Gamma$$

Folded Šolc Filters (3)

Transmissivity:

$$T = \left| \tan 2\rho \cos\chi \frac{\sin N\chi}{\sin\chi} \right|^2 \qquad \leftarrow \ \cos\chi = \cos 2\rho \sin\frac{1}{2}\Gamma$$

$$\leftarrow \ \Gamma = \pi, 3\pi, 5\pi, \cdots$$

$$\rightarrow \ T = \sin^2 2N\rho$$

Maximum transmission: $\rightarrow \ \rho = \dfrac{\pi}{4N}$

Phase retardation:

$$\Gamma = \frac{2\pi}{\lambda}(n_e - n_o)d \qquad \leftarrow \ \frac{2\pi}{\lambda_\nu}(n_e - n_o)d \equiv (2\nu+1)\pi$$

$$\rightarrow \ \Gamma = (2\nu+1)\pi + \Delta\Gamma$$

$$\rightarrow \ \Delta\Gamma = -\frac{(2\nu+1)\pi}{\lambda_\nu}(\lambda - \lambda_\nu)$$

$$\leftarrow \ \rho = \frac{\pi}{4N} \ \leftarrow \ N >> 1$$

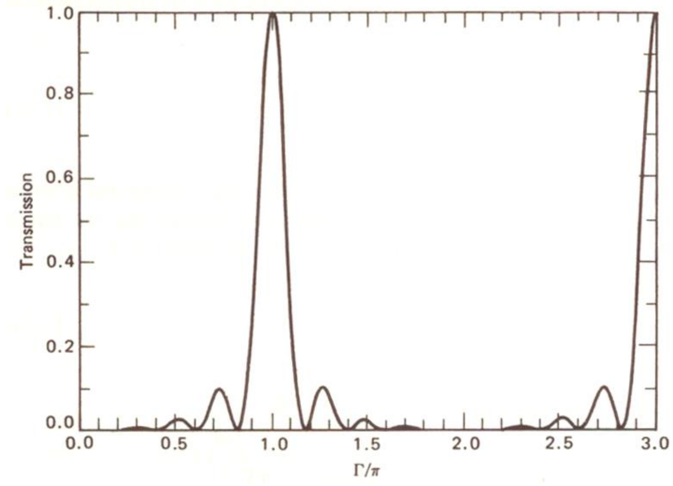
$$\rightarrow \ \chi \cong \frac{\pi}{2N}\left[1 + \left(\frac{N\Delta\Gamma}{\pi}\right)^2\right]^{1/2}$$

Folded Šolc Filters (4)

Transmissivity:

$$T = \left| \tan 2\rho \cos\chi \frac{\sin N\chi}{\sin\chi} \right|^2 \leftarrow \chi \cong \frac{\pi}{2N}\left[1+\left(\frac{N\Delta\Gamma}{\pi}\right)^2\right]^{1/2}$$

$$\to T = \left[\frac{\sin\left(\frac{1}{2}\pi\sqrt{1+\left(N\Delta\Gamma/\pi\right)^2}\right)}{\sqrt{1+\left(N\Delta\Gamma/\pi\right)^2}} \right]^2$$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Full bandwidth at half maximum:

$$\to \Delta\Gamma_{1/2} \cong 1.60\pi / N$$

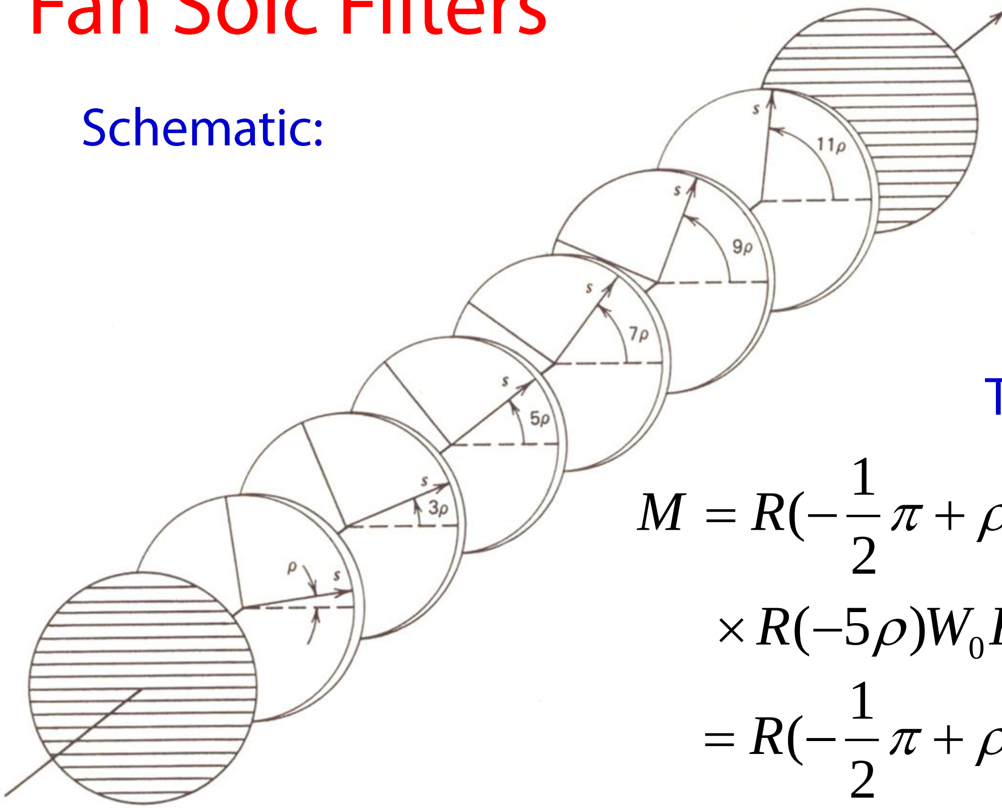
$$\to \Delta\lambda_{1/2} \cong 1.60\left[\frac{\lambda_\nu}{(2\nu+1)N}\right]$$

Secondary maxima:

$$\to \sqrt{1+\left(\frac{N\Delta\Gamma}{\pi}\right)^2} \cong 2l+1, \quad l = 1, 2, 3, \cdots \quad \to T \cong \frac{1}{(2l+1)^2}$$

Fan Šolc Filters

Schematic:



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Transfer matrix:

$$M = R(-\frac{1}{2}\pi + \rho)W_0 R(\frac{1}{2}\pi - \rho)\cdots$$

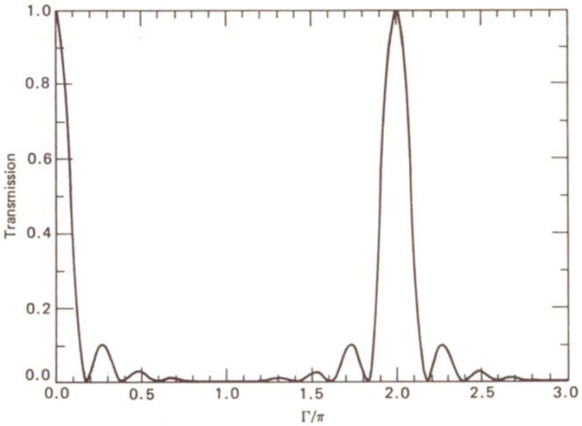
$$\times R(-5\rho)W_0 R(5\rho)R(-3\rho)W_0 R(3\rho)R(-\rho)W_0 R(\rho)$$

$$= R(-\frac{1}{2}\pi + \rho)[W_0 R(2\rho)]^N R(-\rho)$$

Transmissivity:

$$\rightarrow \begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = P_x M P_x \begin{pmatrix} E_x \\ E_y \end{pmatrix} \rightarrow E'_x = M_{11} E_x$$

$$\rightarrow \Delta\lambda_{1/2} \cong 1.60 \frac{\lambda_\nu}{2\nu N}, \quad \nu = 1, 2, 3, \cdots$$



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