Electro-Optics:

Jones Calculus and its Application to Birefringent Optical Systems (3)

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Twisted Anisotropic Media (1)

Azimuth angle: $\psi(z) = \alpha z$

Phase retardation when untwisted:

$$\Gamma = \frac{2\pi}{\lambda}(n_e - n_o)l$$

Total twist angle:

$$\rightarrow \ \phi = \psi(l) = \alpha l$$

Overall Jones matrix:

$$\leftarrow \ W_0 = \begin{pmatrix} e^{-i\Gamma/2N} & 0 \\ 0 & e^{i\Gamma/2N} \end{pmatrix}$$

$$M = R(-\phi)W_0R(\phi)\cdots$$

$$\times R(-3\rho)W_0R(3\rho)R(-2\rho)W_0R(2\rho)R(-\rho)W_0R(\rho)$$

$$\rightarrow \ M = \prod_{m=1}^{N} R(-m\rho)W_0R(m\rho) = R(-\phi)[W_0R(\frac{\phi}{N})]^N$$

Twisted Anisotropic Media (2)

Overall Jones matrix:

$$\rightarrow\ M = R(-\phi)\begin{pmatrix} \cos\frac{\phi}{N}e^{-i\Gamma/2N} & \sin\frac{\phi}{N}e^{-i\Gamma/2N} \\ -\sin\frac{\phi}{N}e^{i\Gamma/2N} & \cos\frac{\phi}{N}e^{i\Gamma/2N} \end{pmatrix}^N$$

$$\leftarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix}^m = \begin{pmatrix} \frac{A\sin mK\Lambda - \sin(m-1)K\Lambda}{\sin K\Lambda} & B\frac{\sin mK\Lambda}{\sin K\Lambda} \\ C\frac{\sin mK\Lambda}{\sin K\Lambda} & \frac{D\sin mK\Lambda - \sin(m-1)K\Lambda}{\sin K\Lambda} \end{pmatrix}$$

$$\leftarrow\ K\Lambda = \cos^{-1}[\frac{1}{2}(A+D)] \quad \rightarrow\ K\Lambda = \cos^{-1}[\cos\frac{\phi}{N}\cos\frac{\Gamma}{2N}] \cong \frac{1}{N}\sqrt{\phi^2 + \left(\frac{\Gamma}{2}\right)^2} \equiv \frac{X}{N}$$

$$(N \rightarrow \infty)$$

$$\rightarrow\ M = R(-\phi)\begin{pmatrix} \cos X - i\frac{\Gamma}{2}\frac{\sin X}{X} & \phi\frac{\sin X}{X} \\ -\phi\frac{\sin X}{X} & \cos X + i\frac{\Gamma}{2}\frac{\sin X}{X} \end{pmatrix} \quad \rightarrow\ \mathbf{V}' = M\mathbf{V}$$

Adiabatic Following

Twisted nematic liquid crystal:

$$M = R(-\phi)\begin{pmatrix} \cos X - i\frac{\Gamma}{2}\frac{\sin X}{X} & \phi\frac{\sin X}{X} \\ -\phi\frac{\sin X}{X} & \cos X + i\frac{\Gamma}{2}\frac{\sin X}{X} \end{pmatrix} \leftarrow X = \sqrt{\phi^2 + \left(\frac{\Gamma}{2}\right)^2}$$

In case: $\Gamma >> \phi$

$$\rightarrow M = R(-\phi)\begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix}$$

$\rightarrow$ The polarization vector follows the rotation of the local axes, provided it is along one of the axes.

For example: $\phi = \pi/2$ & $\mathbf{V} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\rightarrow \mathbf{V}' = M\mathbf{V} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} M'_{11} & M'_{12} \\ M'_{21} & M'_{22} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -\phi\frac{\sin X}{X} \\ \cos X - i\frac{\Gamma}{2}\frac{\sin X}{X} \end{pmatrix}$$

$\rightarrow$ Transmissivity will vary with the orientation of the second polarizer.

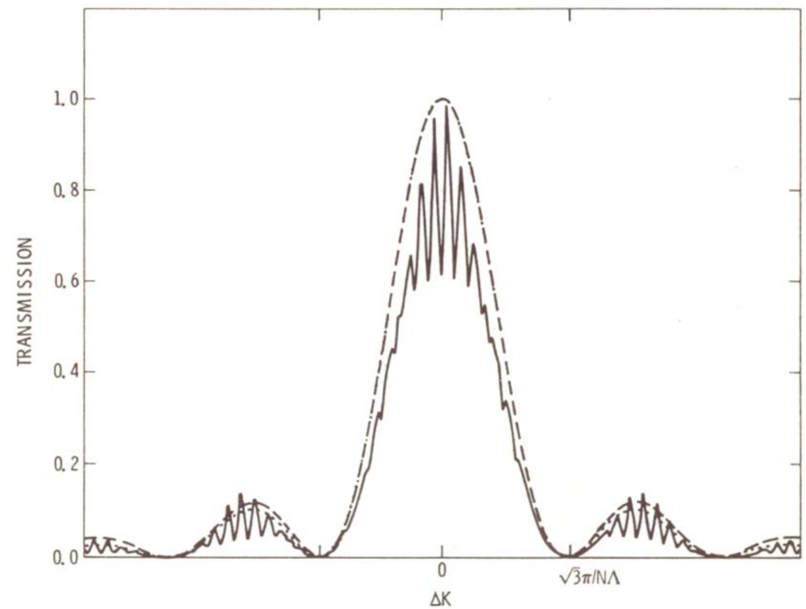
4×4 Matrix Method

Jones-calculus method:

→ Neglect reflected waves!

4×4 matrix method:

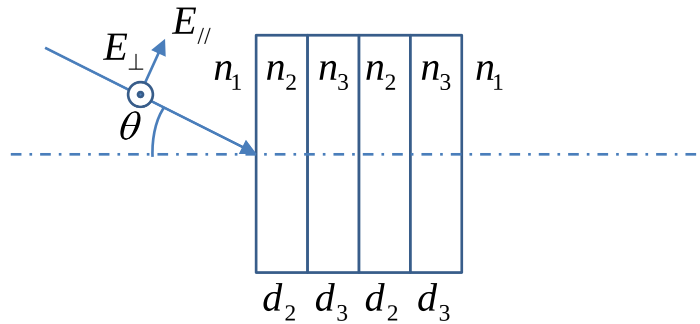
→ Consider reflected waves!



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

2×2 matrix method for isotropic media:

→ H.W. Find the transmission spectrum of the following multi-layered medium with respect to θ for parallel- or perpendicular-polarized waves at 1060 nm (Due on 07/May/2013).



$$n_1 = 1,\ \ n_2 = 1.5,\ \ n_3 = 1.6$$

$$d_2 = 150\ nm,\ \ d_3 = 200\ nm$$

$$\lambda = 1060\ nm$$