

# Electro-Optics:

## Jones Calculus and its Application to Birefringent Optical Systems (3)

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# Twisted Anisotropic Media (1)

Azimuth angle:  $\psi(z) = \alpha z$

Phase retardation when untwisted:

$$\Gamma = \frac{2\pi}{\lambda} (n_e - n_o) l$$

Total twist angle:

$$\rightarrow \phi = \psi(l) = \alpha l$$

Overall Jones matrix:

$$M = R(-\phi)W_0R(\phi)\cdots$$

$$\times R(-3\rho)W_0R(3\rho)R(-2\rho)W_0R(2\rho)R(-\rho)W_0R(\rho)$$

$$\rightarrow M = \prod_{m=1}^N R(-m\rho)W_0R(m\rho) = R(-\phi)\left[W_0R\left(\frac{\phi}{N}\right)\right]^N$$

$$\leftarrow W_0 = \begin{pmatrix} e^{-i\Gamma/2N} & 0 \\ 0 & e^{i\Gamma/2N} \end{pmatrix}$$

# Twisted Anisotropic Media (2)

Overall Jones matrix:

$$\rightarrow M = R(-\phi) \begin{pmatrix} \cos \frac{\phi}{N} e^{-i\Gamma/2N} & \sin \frac{\phi}{N} e^{-i\Gamma/2N} \\ -\sin \frac{\phi}{N} e^{i\Gamma/2N} & \cos \frac{\phi}{N} e^{i\Gamma/2N} \end{pmatrix}^N$$

$$\leftarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix}^m = \begin{pmatrix} \frac{A \sin mK\Lambda - \sin(m-1)K\Lambda}{\sin K\Lambda} & B \frac{\sin mK\Lambda}{\sin K\Lambda} \\ C \frac{\sin mK\Lambda}{\sin K\Lambda} & \frac{D \sin mK\Lambda - \sin(m-1)K\Lambda}{\sin K\Lambda} \end{pmatrix}$$

$$\leftarrow K\Lambda = \cos^{-1} \left[ \frac{1}{2}(A+D) \right] \rightarrow K\Lambda = \cos^{-1} \left[ \cos \frac{\phi}{N} \cos \frac{\Gamma}{2N} \right] \cong \frac{1}{N} \sqrt{\phi^2 + \left( \frac{\Gamma}{2} \right)^2} \equiv \frac{X}{N}$$

( $N \rightarrow \infty$ )

$$\rightarrow M = R(-\phi) \begin{pmatrix} \cos X - i \frac{\Gamma}{2} \frac{\sin X}{X} & \phi \frac{\sin X}{X} \\ -\phi \frac{\sin X}{X} & \cos X + i \frac{\Gamma}{2} \frac{\sin X}{X} \end{pmatrix} \rightarrow \mathbf{V}' = M\mathbf{V}$$

# Adiabatic Following

Twisted nematic liquid crystal:

$$M = R(-\phi) \begin{pmatrix} \cos X - i \frac{\Gamma \sin X}{2X} & \phi \frac{\sin X}{X} \\ -\phi \frac{\sin X}{X} & \cos X + i \frac{\Gamma \sin X}{2X} \end{pmatrix} \leftarrow X = \sqrt{\phi^2 + \left(\frac{\Gamma}{2}\right)^2}$$

In case:  $\Gamma \gg \phi$

$$\rightarrow M = R(-\phi) \begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix}$$

→ The polarization vector follows the rotation of the local axes, provided it is along one of the axes.

For example:  $\phi = \pi/2$  &  $\mathbf{V} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\rightarrow \mathbf{V}' = M\mathbf{V} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} M'_{11} & M'_{12} \\ M'_{21} & M'_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -\phi \frac{\sin X}{X} \\ \cos X - i \frac{\Gamma \sin X}{2X} \end{pmatrix}$$

→ Transmissivity will vary with the orientation of the second polarizer.

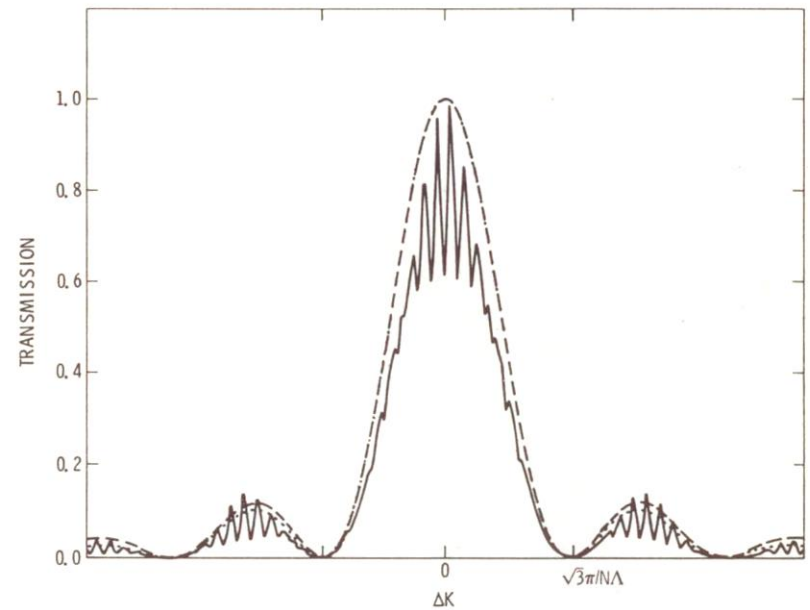
# 4×4 Matrix Method

Jones-calculus method:

→ Neglect reflected waves!

4×4 matrix method:

→ Consider reflected waves!



*Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.*