Electro-Optics:

Electromagnetic Propagation in Periodic Media (2)

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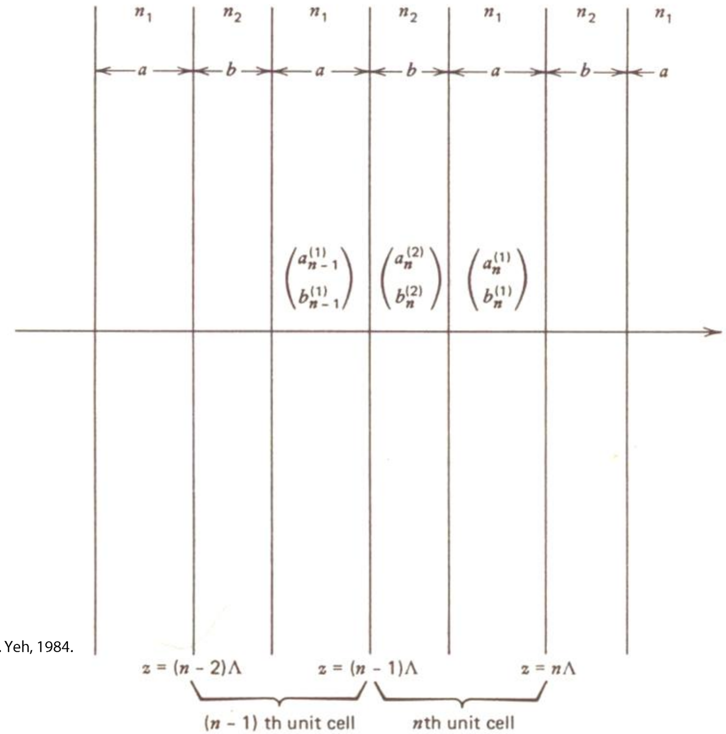
Periodic Layered Media (1)



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Alternating layers of isotropic materials:

$$n(z) = \begin{cases} n_2, & 0 < z < b \\ n_1, & b < z < \Lambda \end{cases}$$

$$\rightarrow \; n(z) = n(z + \Lambda)$$

Electric-field vectors:

$$\mathbf{E}(z) e^{i(\omega t - k_y y)}$$

$$\rightarrow \begin{pmatrix} a_n^{(\alpha)} \\ b_n^{(\alpha)} \end{pmatrix}, \quad \alpha = 1, 2$$

← Incident & reflected plane waves in layer α of the nth unit cell

Electric-field distribution:

$$\rightarrow \; E(y,z) = \left[a_n^{(\alpha)} e^{-ik_{\alpha z}(z - n\Lambda)} + b_n^{(\alpha)} e^{ik_{\alpha z}(z - n\Lambda)} \right] e^{-ik_y y}$$

$$\leftarrow \; k_{\alpha z} = \left[\left(\frac{n_\alpha \omega}{c} \right)^2 - k_y^2 \right]^{1/2}, \quad \alpha = 1, 2$$

Periodic Layered Media (2)

TE waves:

$$\rightarrow E_x \;\&\; H_y \;(\partial E_x / \partial z): \text{ Continuous at the interfaces}$$

$$\rightarrow z = (n-1)\Lambda$$

$$a_{n-1} + b_{n-1} = e^{ik_{2z}\Lambda} c_n + e^{-ik_{2z}\Lambda} d_n$$

$$ik_{1z}(a_{n-1} - b_{n-1}) = ik_{2z}(e^{ik_{2z}\Lambda} c_n - e^{-ik_{2z}\Lambda} d_n)$$

$$\rightarrow z = (n-1)\Lambda + b$$

$$e^{ik_{2z}a} c_n + e^{-ik_{2z}a} d_n = e^{ik_{1z}a} a_n + e^{-ik_{1z}a} b_n$$

$$ik_{2z}(e^{ik_{2z}a} c_n - e^{-ik_{2z}a} d_n) = ik_{1z}(e^{ik_{1z}a} a_n - e^{-ik_{1z}a} b_n)$$

In a matrix form:

$$\rightarrow \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} e^{ik_{2z}\Lambda} & e^{-ik_{2z}\Lambda} \\ \dfrac{k_{2z}}{k_{1z}} e^{ik_{2z}\Lambda} & -\dfrac{k_{2z}}{k_{1z}} e^{-ik_{2z}\Lambda} \end{pmatrix} \begin{pmatrix} c_n \\ d_n \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} e^{ik_{2z}a} & e^{-ik_{2z}a} \\ e^{ik_{2z}a} & -e^{-ik_{2z}a} \end{pmatrix} \begin{pmatrix} c_n \\ d_n \end{pmatrix} = \begin{pmatrix} e^{ik_{1z}a} & e^{-ik_{1z}a} \\ \dfrac{k_{1z}}{k_{2z}} e^{ik_{1z}a} & -\dfrac{k_{1z}}{k_{2z}} e^{-ik_{1z}a} \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

Periodic Layered Media (3)

TE waves:

$$\rightarrow \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} \quad \leftarrow$$

$$A = e^{ik_{1z}a}\left[\cos k_{2z}b + \frac{1}{2}i\left(\frac{k_{2z}}{k_{1z}} + \frac{k_{1z}}{k_{2z}}\right)\sin k_{2z}b\right]$$

$$B = e^{-ik_{1z}a}\left[\frac{1}{2}i\left(\frac{k_{2z}}{k_{1z}} - \frac{k_{1z}}{k_{2z}}\right)\sin k_{2z}b\right]$$

$$C = e^{ik_{1z}a}\left[-\frac{1}{2}i\left(\frac{k_{2z}}{k_{1z}} - \frac{k_{1z}}{k_{2z}}\right)\sin k_{2z}b\right]$$

$$D = e^{-ik_{1z}a}\left[\cos k_{2z}b - \frac{1}{2}i\left(\frac{k_{2z}}{k_{1z}} + \frac{k_{1z}}{k_{2z}}\right)\sin k_{2z}b\right]$$

TM waves:

$$\leftarrow \quad A = e^{ik_{1z}a}\left[\cos k_{2z}b + \frac{1}{2}i\left(\frac{n_2^2 k_{1z}}{n_1^2 k_{2z}} + \frac{n_1^2 k_{2z}}{n_2^2 k_{1z}}\right)\sin k_{2z}b\right]$$

$$B = e^{-ik_{1z}a}\left[\frac{1}{2}i\left(\frac{n_2^2 k_{1z}}{n_1^2 k_{2z}} - \frac{n_1^2 k_{2z}}{n_2^2 k_{1z}}\right)\sin k_{2z}b\right]$$

$$C = e^{ik_{1z}a}\left[-\frac{1}{2}i\left(\frac{n_2^2 k_{1z}}{n_1^2 k_{2z}} - \frac{n_1^2 k_{2z}}{n_2^2 k_{1z}}\right)\sin k_{2z}b\right]$$

$$D = e^{-ik_{1z}a}\left[\cos k_{2z}b - \frac{1}{2}i\left(\frac{n_2^2 k_{1z}}{n_1^2 k_{2z}} + \frac{n_1^2 k_{2z}}{n_2^2 k_{1z}}\right)\sin k_{2z}b\right]$$

$$\rightarrow \quad AD - BC = 1$$

→ Unimodular matrix

$$\rightarrow \begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} D & -B \\ -C & A \end{pmatrix}^n \begin{pmatrix} a_0 \\ b_0 \end{pmatrix}$$

Bloch Waves and Band Structures (1)

Bloch wave representation:

$$\rightarrow \mathbf{E} = \mathbf{E}_{\mathbf{K}}(z) e^{-iKz} e^{i(\omega t - k_y y)}$$

$$\leftarrow \mathbf{E}_{\mathbf{K}}(z) = \mathbf{E}_{\mathbf{K}}(z+\Lambda)$$

$$\rightarrow \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{-iK\Lambda} \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} \quad \rightarrow \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{iK\Lambda} \begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

$$\rightarrow \begin{vmatrix} A - e^{iK\Lambda} & B \\ C & D - e^{iK\Lambda} \end{vmatrix} = 0$$

Eigenvalues:

$$\rightarrow e^{iK\Lambda} = \frac{1}{2}(A+D) \pm \left\{ \left[\frac{1}{2}(A+D) \right]^2 - 1 \right\}^{1/2}$$

Eigenvectors:

$$\rightarrow \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} B \\ e^{iK\Lambda} - A \end{pmatrix} \quad \rightarrow \begin{pmatrix} a_n \\ b_n \end{pmatrix} = e^{-inK\Lambda} \begin{pmatrix} B \\ e^{iK\Lambda} - A \end{pmatrix}$$

Bloch wave vector:

$$\rightarrow K(k_y, \omega) = \frac{1}{\Lambda} \cos^{-1} \left[\frac{1}{2}(A+D) \right]$$

Bloch Waves and Band Structures (2)

In case:

$$\left|\frac{1}{2}(A+D)\right| > 1 \quad \rightarrow \quad K = m\pi/\Lambda + iK_i$$ → Forbidden bands

Bloch wave (layer 1 of the *n*th unit cell):

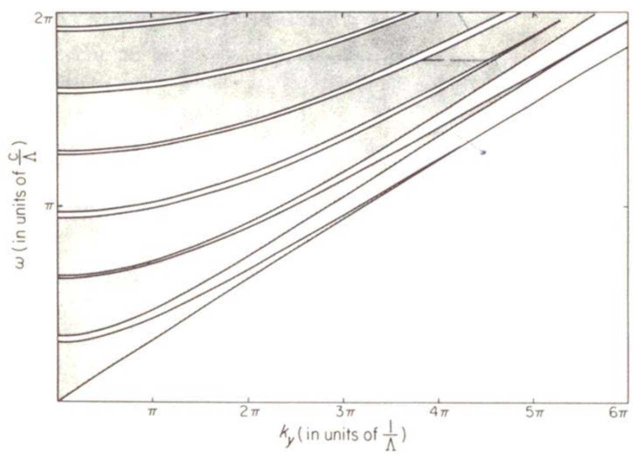
$$\rightarrow E(y,z) = \left[a_n^{(1)} e^{-ik_{1z}(z-n\Lambda)} + b_n^{(1)} e^{ik_{1z}(z-n\Lambda)}\right] e^{-ik_y y}$$

$$\rightarrow E(y,z) = \left[a_0 e^{-ik_{1z}(z-n\Lambda)} + b_0 e^{ik_{1z}(z-n\Lambda)}\right] e^{-ik_y y} e^{-inK\Lambda}$$

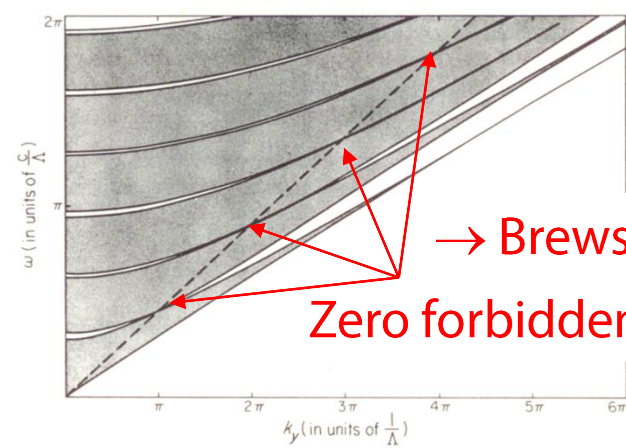
$$\rightarrow E(z) = E_K(z) e^{-iKz} = \boxed{\left[\left(a_0 e^{-ik_{1z}(z-n\Lambda)} + b_0 e^{ik_{1z}(z-n\Lambda)}\right) e^{i(Kz-nK\Lambda)}\right]} e^{-iKz}$$

$$\rightarrow E_K(z)$$

TE waves:



TM waves:



→ Brewster angle

Zero forbidden bands

Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.