Electro-Optics:

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Electro-Optic Effect

Dielectric impermeability tensor:

$$\eta = \varepsilon_0 \varepsilon^{-1}$$

Index ellipsoid in the principal coordinate system:

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

Electro-optic coefficients:

Linear (or Pockels) EO coeffs.

Quadratic (or Kerr) EO coeffs.

$$\eta_{ij}(\mathbf{E}) - \eta_{ij}(0) = r_{ijk} E_k + s_{ijkl} E_k E_l$$

Index ellipsoid in the presence of an applied electric field:

$$\rightarrow \ \eta_{ij}(\mathbf{E}) x_i x_j = 1$$

Permutation symmetries:

$$\rightarrow \ r_{ijk} = r_{jik}$$

$$\rightarrow \ s_{ijkl} = s_{jikl}$$

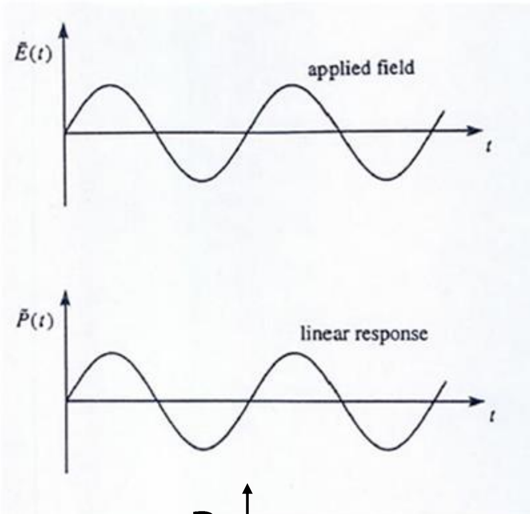
← Lossless

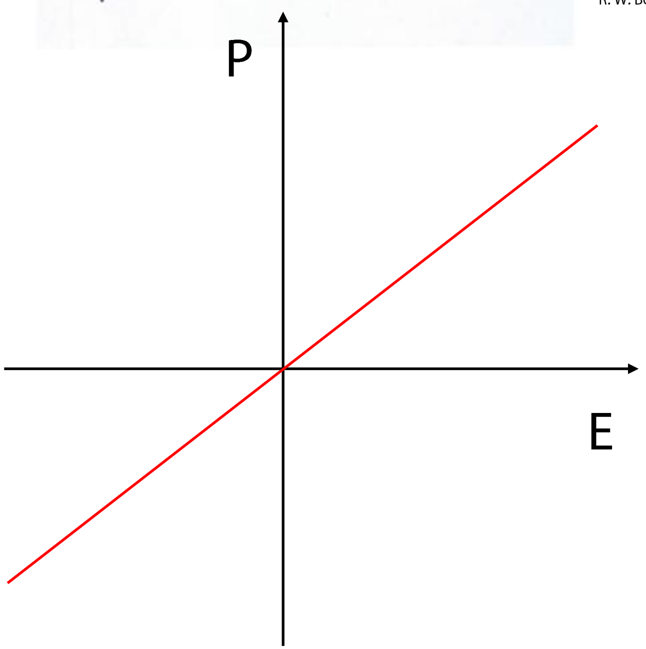
$$\rightarrow \ s_{ijkl} = s_{ijlk}$$

$$\leftarrow \ s_{ijkl} = \frac{1}{2}\left(\frac{\partial^2 \eta_{ij}}{\partial E_k \partial E_l}\right)_{E=0}$$

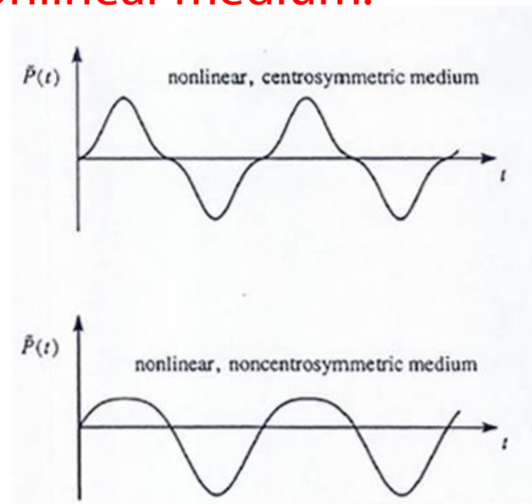
Linear and Nonlinear Atomic Response

Linear medium:

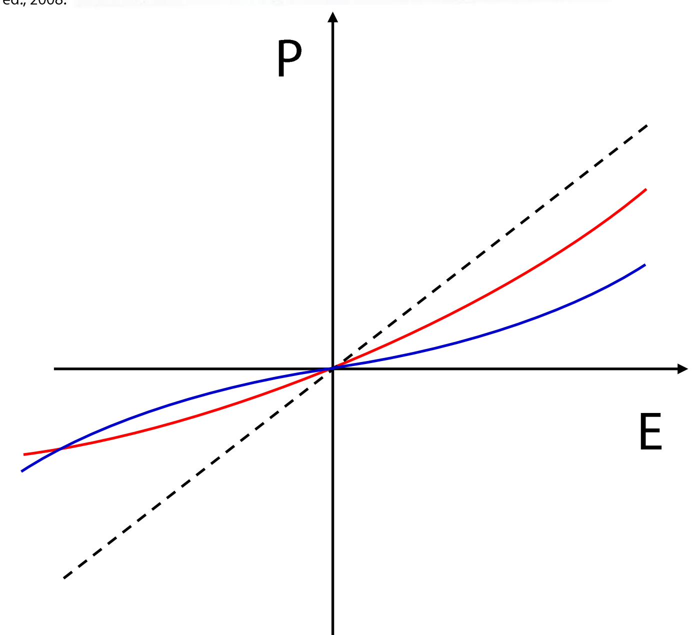




Nonlinear medium:



R. W. Boyd, Nonlinear Optics, 3rd ed., 2008.



Linear Electro-Optic Effect

Permutation symmetries:

$$\rightarrow \; r_{ijk} = r_{jik}$$

$$\rightarrow \; r_{1k} = r_{11k}, \quad r_{2k} = r_{22k}, \quad r_{3k} = r_{33k}$$

$$\rightarrow \; r_{4k} = r_{23k} = r_{32k}, \quad r_{5k} = r_{13k} = r_{31k}, \quad r_{6k} = r_{12k} = r_{21k}$$

Inversion symmetry (Centrosymmetry):

$$I r_{ijk} = r'_{ijk} = -r_{ijk}$$

$$\rightarrow r'_{ijk} = r_{ijk} = 0$$

Index ellipsoid:

$$\rightarrow \; \eta_{ij}(\mathbf{E}) x_i x_j = 1$$

$$\rightarrow \; \left(\frac{1}{n_x^2} + r_{1k} E_k\right) x^2 + \left(\frac{1}{n_y^2} + r_{2k} E_k\right) y^2 + \left(\frac{1}{n_z^2} + r_{3k} E_k\right) z^2$$

$$+ 2yz r_{4k} E_k + 2zx r_{5k} E_k + 2xy r_{6k} E_k = 1$$

Example: Electro-Optic Effect in KH_2PO_4 (KDP)

Electro-optic tensor:

$$r_{ij} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \rightarrow r_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

Symmetry group: $\bar{4}2m$

(uniaxial)

Index ellipsoid:

$$\rightarrow \mathbf{E}(E_x, E_y, E_z): \rightarrow \frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41}E_x yz + 2r_{41}E_y zx + 2r_{63}E_z xy = 1$$

$$\rightarrow \mathbf{E}(0,0,E_z): \quad \rightarrow \frac{x^2 + y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}E_z xy = 1 \rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

In the new principal coordinate system:

$$\rightarrow \left(\frac{1}{n_o^2} + r_{63}E_z \right) x'^2 + \left(\frac{1}{n_o^2} - r_{63}E_z \right) y'^2 + \frac{z^2}{n_e^2} = 1$$

$$\rightarrow n_{x'} = n_o - \frac{1}{2} n_o^3 r_{63} E_z, \quad n_{y'} = n_o + \frac{1}{2} n_o^3 r_{63} E_z, \quad n_z = n_e$$

Example: Electro-Optic Effect in $LiNbO_3$

Electro-optic tensor:

$$r_{ij} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}$$

Symmetry group: $3m$ (uniaxial)

Index ellipsoid:

$$\rightarrow \mathbf{E}(0,0,E):$$

$$\rightarrow x^2\left(\frac{1}{n_o^2} + r_{13}E\right) + y^2\left(\frac{1}{n_o^2} + r_{13}E\right) + z^2\left(\frac{1}{n_e^2} + r_{33}E\right) = 1$$

$$\rightarrow n_x = n_o - \frac{1}{2}n_o^3 r_{13}E, \quad n_y = n_o - \frac{1}{2}n_o^3 r_{13}E, \quad n_z = n_e - \frac{1}{2}n_e^3 r_{33}E$$