

# Electro-Optics:

## Electro-Optics (1)

Dr Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: [yunchan@snu.ac.kr](mailto:yunchan@snu.ac.kr)

# Electro-Optic Effect

Dielectric impermeability tensor:

$$\eta = \varepsilon_0 \varepsilon^{-1}$$

Index ellipsoid in the principal coordinate system:

$$\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

Linear (or Pockels) EO coeffs.

Electro-optic coefficients:

$$\eta_{ij}(\mathbf{E}) - \eta_{ij}(0) = r_{ijk} E_k + s_{ijkl} E_k E_l$$

Quadratic (or Kerr) EO coeffs.

Index ellipsoid in the presence of an applied electric field:

$$\rightarrow \eta_{ij}(\mathbf{E}) x_i x_j = 1$$

Permutation symmetries:

$$\rightarrow r_{ijk} = r_{jik}$$

$$\rightarrow s_{ijkl} = s_{jikl}$$

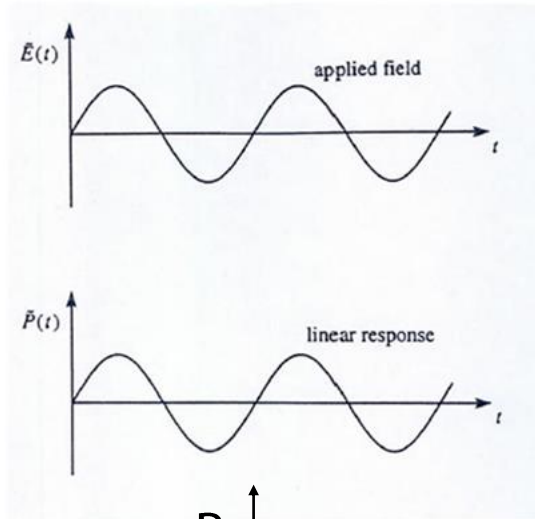
$$\rightarrow s_{ijkl} = s_{ijlk}$$

← Lossless

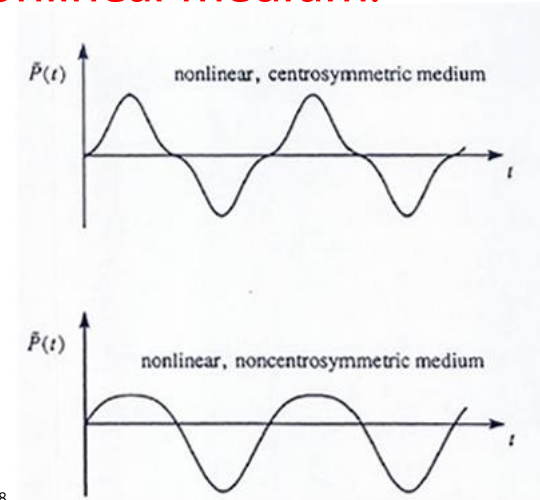
$$\leftarrow s_{ijkl} = \frac{1}{2} \left( \frac{\partial^2 \eta_{ij}}{\partial E_k \partial E_l} \right)_{E=0}$$

# Linear and Nonlinear Atomic Response

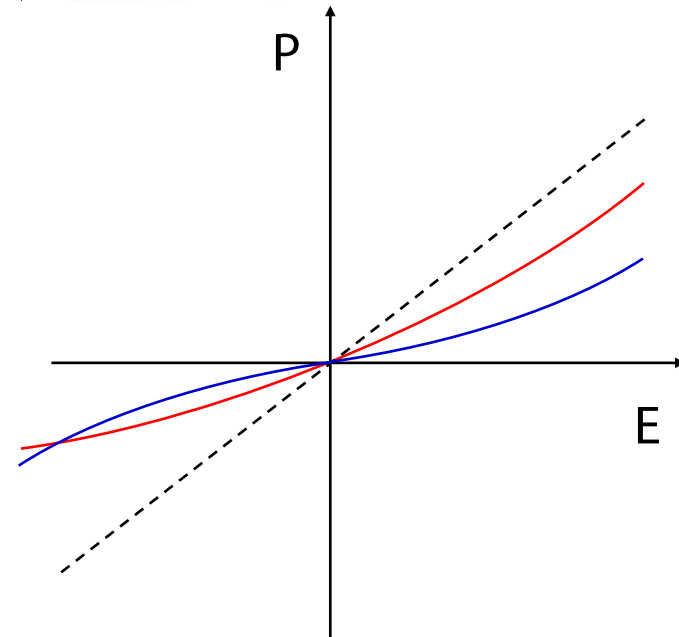
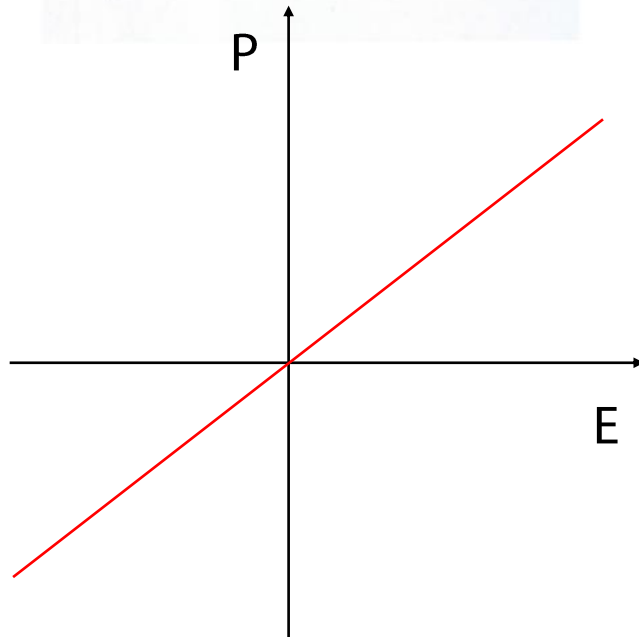
Linear medium:



Nonlinear medium:



R. W. Boyd, Nonlinear Optics, 3<sup>rd</sup> ed., 2008.



# Linear Electro-Optic Effect

Permutation symmetries:

$$\rightarrow r_{ijk} = r_{jik}$$

$$\rightarrow r_{1k} = r_{11k}, \quad r_{2k} = r_{22k}, \quad r_{3k} = r_{33k}$$

$$\rightarrow r_{4k} = r_{23k} = r_{32k}, \quad r_{5k} = r_{13k} = r_{31k}, \quad r_{6k} = r_{12k} = r_{21k}$$

Inversion symmetry (Centrosymmetry):

$$I r_{ijk} = r'_{ijk} = -r_{ijk}$$

$$\rightarrow r'_{ijk} = r_{ijk} = 0$$

Index ellipsoid:

$$\rightarrow \eta_{ij}(\mathbf{E}) x_i x_j = 1$$

$$\begin{aligned} \rightarrow & \left( \frac{1}{n_x^2} + r_{1k} E_k \right) x^2 + \left( \frac{1}{n_y^2} + r_{2k} E_k \right) y^2 + \left( \frac{1}{n_z^2} + r_{3k} E_k \right) z^2 \\ & + 2 y z r_{4k} E_k + 2 z x r_{5k} E_k + 2 x y r_{6k} E_k = 1 \end{aligned}$$

# Example: Electro-Optic Effect in $\text{KH}_2\text{PO}_4$ (KDP)

Electro-optic tensor:

$$r_{ij} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \rightarrow r_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} \quad \text{Symmetry group: } \bar{4}2m \quad \text{(uniaxial)}$$

Index ellipsoid:

$$\rightarrow \mathbf{E}(E_x, E_y, E_z): \rightarrow \frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41}E_x yz + 2r_{41}E_y zx + 2r_{63}E_z xy = 1$$

$$\rightarrow \mathbf{E}(0, 0, E_z): \rightarrow \frac{x^2 + y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}E_z xy = 1 \rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos \frac{\pi}{4} & -\sin \frac{\pi}{4} \\ \sin \frac{\pi}{4} & \cos \frac{\pi}{4} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

In the new principal coordinate system:

$$\rightarrow \left( \frac{1}{n_o^2} + r_{63}E_z \right) x'^2 + \left( \frac{1}{n_o^2} - r_{63}E_z \right) y'^2 + \frac{z^2}{n_e^2} = 1$$

$$\rightarrow n_{x'} = n_o - \frac{1}{2} n_o^3 r_{63} E_z, \quad n_{y'} = n_o + \frac{1}{2} n_o^3 r_{63} E_z, \quad n_z = n_e \quad 5$$

# Example: Electro-Optic Effect in $\text{LiNbO}_3$

Electro-optic tensor:

$$r_{ij} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}$$

Symmetry group:  $3m$  (uniaxial)

Index ellipsoid:

→  $\mathbf{E}(0,0,E)$ :

$$\rightarrow x^2 \left( \frac{1}{n_o^2} + r_{13} E \right) + y^2 \left( \frac{1}{n_o^2} + r_{13} E \right) + z^2 \left( \frac{1}{n_e^2} + r_{33} E \right) = 1$$

$$\rightarrow n_x = n_o - \frac{1}{2} n_o^3 r_{13} E, \quad n_y = n_o - \frac{1}{2} n_o^3 r_{13} E, \quad n_z = n_e - \frac{1}{2} n_e^3 r_{33} E$$