Electro-Optics:

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Electro-Optic Effect

Dielectric impermeability tensor:

$$\eta = \varepsilon_0 \varepsilon^{-1}$$

Index ellipsoid in the principal coordinate system:

$$\eta_{ij} x_i x_j = 1 \quad \rightarrow \frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

Linear (or Pockels) EO coeffs.

Quadratic (or Kerr) EO coeffs.

Electro-optic coefficients:

$$\eta_{ij}(\mathbf{E}) - \eta_{ij}(0) = r_{ijk} E_k + s_{ijkl} E_k E_l$$

Index ellipsoid in the presence of an applied electric field:

$$\rightarrow \quad \eta_{ij}(\mathbf{E}) x_i x_j = 1$$

Permutation symmetries:

$$\rightarrow \quad r_{ijk} = r_{jik}$$

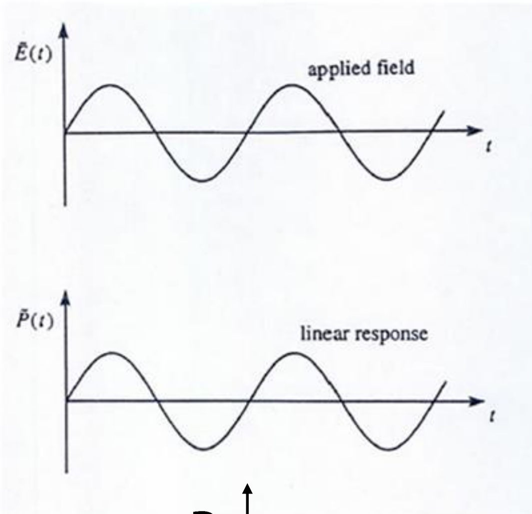
$\leftarrow$ Lossless

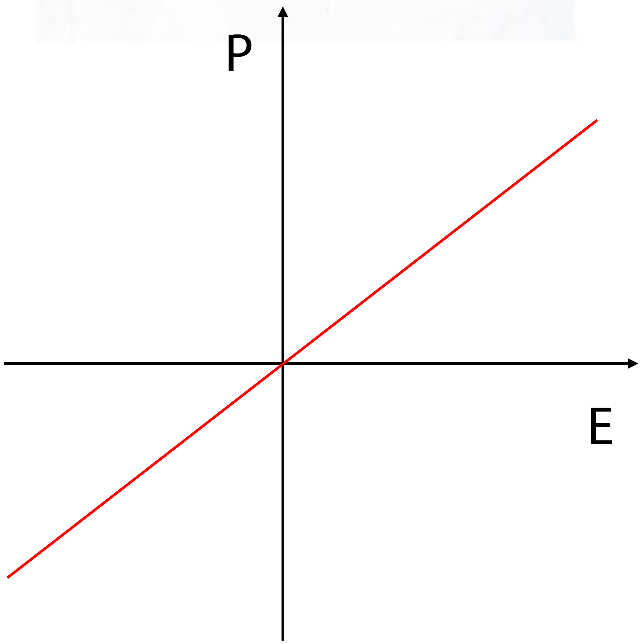
$$\rightarrow \quad s_{ijkl} = s_{jikl}$$

$$\rightarrow \quad s_{ijkl} = s_{ijlk} \qquad \leftarrow \quad s_{ijkl} = \frac{1}{2} \left(\frac{\partial^2 \eta_{ij}}{\partial E_k \partial E_l} \right)_{E=0}$$

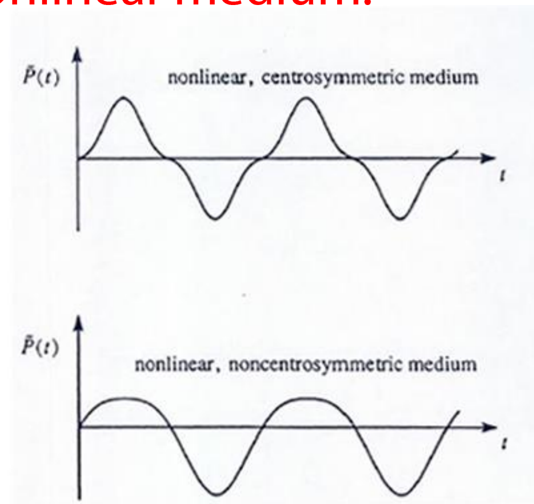
Linear and Nonlinear Atomic Response

Linear medium:

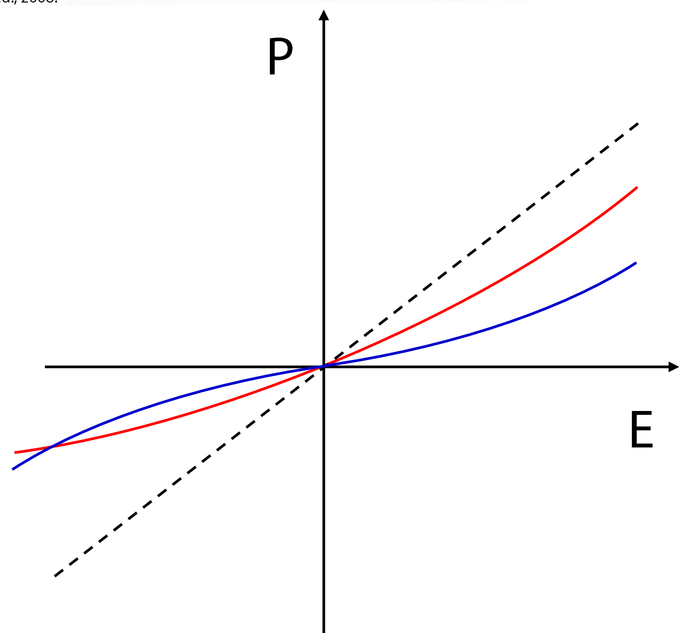




Nonlinear medium:



R. W. Boyd, Nonlinear Optics, 3rd ed., 2008.



Linear Electro-Optic Effect

Permutation symmetries:

$$\rightarrow\ r_{ijk} = r_{jik}$$

$$\rightarrow\ r_{1k} = r_{11k},\quad r_{2k} = r_{22k},\quad r_{3k} = r_{33k}$$

$$\rightarrow\ r_{4k} = r_{23k} = r_{32k},\quad r_{5k} = r_{13k} = r_{31k},\quad r_{6k} = r_{12k} = r_{21k}$$

Inversion symmetry (Centrosymmetry):

$$Ir_{ijk} = r'_{ijk} = -r_{ijk}$$

$$\rightarrow r'_{ijk} = r_{ijk} = 0$$

Index ellipsoid:

$$\rightarrow\ \eta_{ij}(\mathbf{E})x_i x_j = 1 \qquad \leftarrow \eta_{ij}(\mathbf{E}) = \eta_{ij}(0) + r_{ijk}E_k + \cancel{s_{ijkl}E_k E_l}$$

$$\rightarrow\ \left(\frac{1}{n_x^2} + r_{1k}E_k\right)x^2 + \left(\frac{1}{n_y^2} + r_{2k}E_k\right)y^2 + \left(\frac{1}{n_z^2} + r_{3k}E_k\right)z^2$$

$$+ 2yzr_{4k}E_k + 2zxr_{5k}E_k + 2xyr_{6k}E_k = 1$$

Example: Electro-Optic Effect in $LiNbO_3$

Electro-optic tensor:

$$r_{ij} = \begin{pmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{pmatrix}$$

←Symmetry group: $3m$ (uniaxial)

Index ellipsoid:

$$\rightarrow \mathbf{E}(0,0,E):$$

$$\rightarrow x^2\left(\frac{1}{n_o^2}+r_{13}E\right)+y^2\left(\frac{1}{n_o^2}+r_{13}E\right)+z^2\left(\frac{1}{n_e^2}+r_{33}E\right)=1$$

$$\rightarrow n_x = n_o - \frac{1}{2}n_o^3 r_{13}E, \quad n_y = n_o - \frac{1}{2}n_o^3 r_{13}E, \quad n_z = n_e - \frac{1}{2}n_e^3 r_{33}E$$

Example: Electro-Optic Effect in KH_2PO_4 (KDP)

Electro-optic tensor:

$$r_{ij} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix} \rightarrow r_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

← Symmetry group: $\bar{4}2m$ (uniaxial)

Index ellipsoid:

$$\rightarrow \mathbf{E}(E_x, E_y, E_z): \rightarrow \frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41}E_x yz + 2r_{41}E_y zx + 2r_{63}E_z xy = 1$$

$$\rightarrow \mathbf{E}(0,0,E_z): \quad \rightarrow \frac{x^2 + y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}E_z xy = 1 \rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

In the new principal coordinate system:

$$\rightarrow \left(\frac{1}{n_o^2} + r_{63}E_z\right) x'^2 + \left(\frac{1}{n_o^2} - r_{63}E_z\right) y'^2 + \frac{z^2}{n_e^2} = 1$$

$$\rightarrow n_{x'} = n_o - \frac{1}{2} n_o^3 r_{63} E_z, \quad n_{y'} = n_o + \frac{1}{2} n_o^3 r_{63} E_z, \quad n_z = n_e$$

Electro-Optic Modulation

Electro-optic effect in KH_2PO_4 (KDP):

$$n_{x'} = n_o - \frac{1}{2} n_o^3 r_{63} E_z , \quad n_{y'} = n_o + \frac{1}{2} n_o^3 r_{63} E_z , \quad n_z = n_e$$

$$\rightarrow \; n_{y'} - n_{x'} = n_o^3 r_{63} E_z$$

$$\rightarrow \; \Gamma = \frac{\omega}{c}(n_{y'} - n_{x'})d = \frac{2\pi}{\lambda} n_o^3 r_{63} V$$

Rotation of the polarization state:

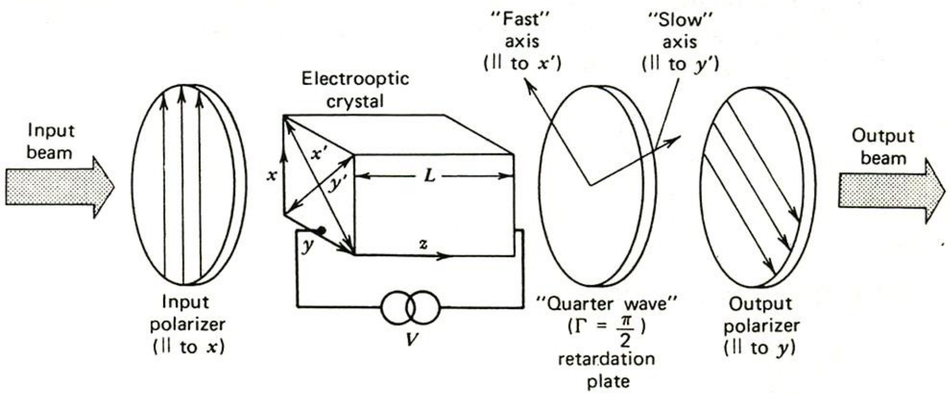
Input: $\rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ Output: $\rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\Gamma/2} \\ e^{-i\Gamma/2} \end{pmatrix}$

Half-wave voltage:

$$\rightarrow \; V_\pi = \frac{\lambda}{2 n_o^3 r_{63}}$$

Amplitude Modulation

Electro-optic amplitude modulator:



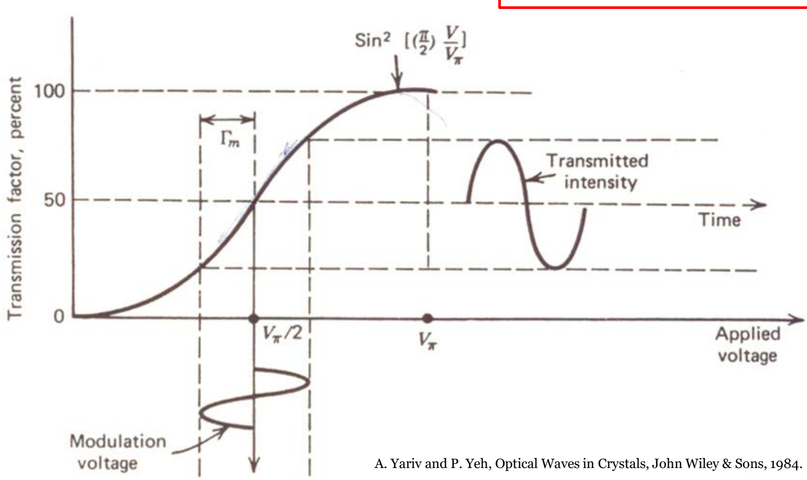
A. Yariv and P. Yeh, Optical Waves in Crystals, John Wiley & Sons, 1984.

$$\rightarrow \mathbf{E}' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\frac{1}{2}\Gamma & -i\sin\frac{1}{2}\Gamma \\ -i\sin\frac{1}{2}\Gamma & \cos\frac{1}{2}\Gamma \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{-i}{\sqrt{2}} \begin{pmatrix} \sin\frac{1}{2}\Gamma \\ 0 \end{pmatrix}$$

Recall: $\rightarrow \begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = R(-\psi) W_0 R(\psi) \begin{pmatrix} V_x \\ V_y \end{pmatrix} \equiv W \begin{pmatrix} V_x \\ V_y \end{pmatrix}$

Transmission:



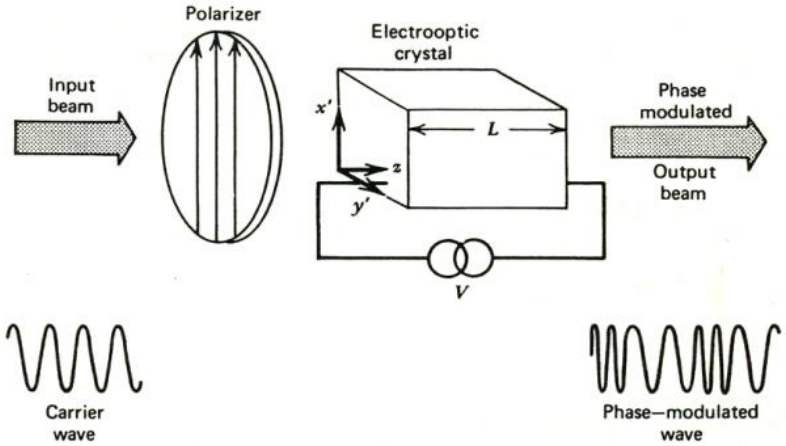
A. Yariv and P. Yeh, Optical Waves in Crystals, John Wiley & Sons, 1984.

$$\rightarrow T = \sin^2 \frac{\Gamma}{2} = \sin^2 \left(\frac{\pi}{2} \frac{V}{V_\pi} \right)$$

$$\rightarrow \Gamma = \frac{\pi}{2} + \Gamma_m \sin \omega_m t$$

Phase Modulation

Electro-optic phase modulator:



A. Yariv and P. Yeh, Optical Waves in Crystals, John Wiley & Sons, 1984.

Phase change:

$$\rightarrow \Delta\phi_{x'} = -\frac{\omega d}{c}\Delta n_{x'} = -\frac{\omega n_o^3 r_{63}}{2c} E_z d \quad \rightarrow E_z = E_m \sin\omega_m t$$

$$\rightarrow E_{out} = A\cos\left[\omega t - \frac{\omega}{c}\left(n_o - \frac{n_o^3}{2} r_{63} E_m \sin\omega_{mt} t\right) d\right]$$

$$\rightarrow E_{out} = A\cos[\omega t + \delta \sin\omega_m t]$$

$$\leftarrow \delta = \frac{\omega n_o^3 r_{63} E_m d}{2c} = \frac{\pi n_o^3 r_{63} E_m d}{\lambda}$$