

Electro-Optics

Theory of Lasers (1)

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Spontaneous Transitions between Atomic Levels

Atomic systems:

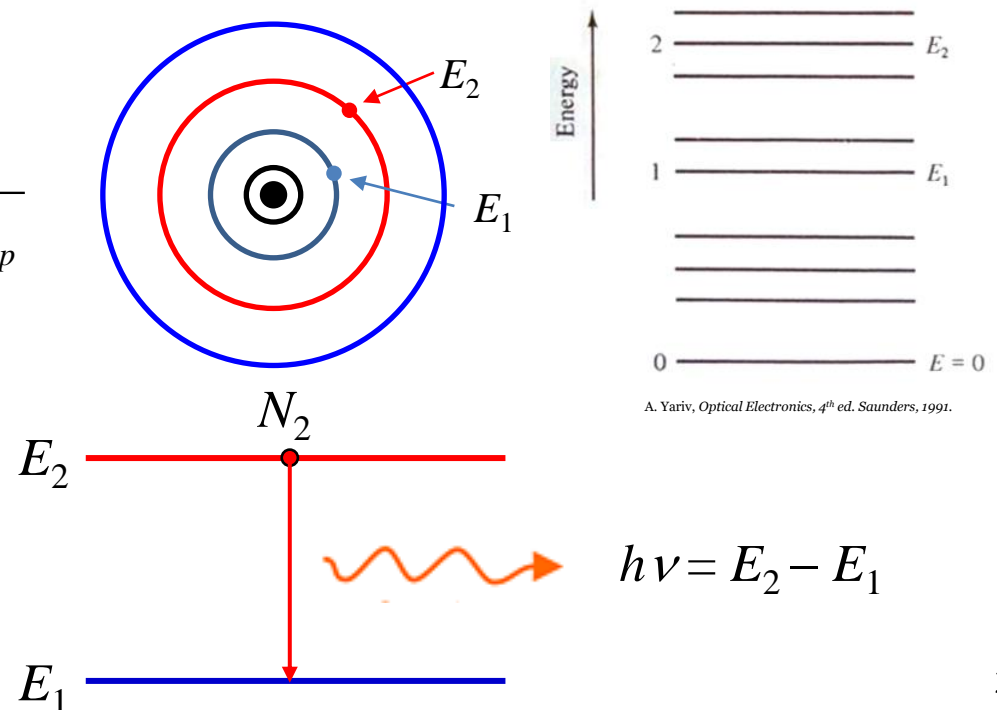
Predetermined set of energy states: **Eigenstates**

Spontaneous emission:

Photon emitting process without the inducement of a radiation field: $h\nu = E_2 - E_1$

Spontaneous emission rate:

$$\frac{dN_2}{dt} = -A_{21}N_2 \equiv -\frac{N_2}{\tau_{21,sp}}$$



Lineshape Function (1)

“Atomic” lineshape function $g(\nu)$:

Spectral distribution of emitted intensity vs. frequency

$$\int_{-\infty}^{+\infty} g(\nu) d\nu = 1$$

Homogeneous broadening: Uniform and identical frequency response

Incl. lifetime, collision, dipolar, and thermal broadening

→ Lifetime broadening:

$$E(t) = E_0 e^{-t/2\tau} e^{i\omega_0 t} = E_0 e^{i(\omega_0 + i\sigma/2)t} \quad \leftarrow \sigma = \frac{1}{\tau} \quad \leftarrow t \geq 0$$

$$\rightarrow F(\omega) = \int_0^{+\infty} E(t) e^{-i\omega t} dt = \frac{-iE_0}{\omega - \omega_0 - i\sigma/2}$$

$$\rightarrow |F(\omega)|^2 \propto \frac{1}{(\omega - \omega_0)^2 + (\sigma/2)^2}$$

$$\rightarrow g(\nu) = \frac{\Delta\nu}{2\pi[(\nu - \nu_0)^2 + (\Delta\nu/2)^2]} \quad \leftarrow \text{Lorentzian lineshape function}$$

$$\rightarrow \Delta\nu = \frac{\sigma}{2\pi} = \frac{1}{2\pi\tau} \quad \leftarrow \text{Linewidth (FWHM)}$$

Lineshape Function (2)

Inhomogeneous broadening: Non-uniform and detuned frequency response
e.g. Doppler broadening, impurities

→ Doppler broadening:

Doppler shift: $\rightarrow \nu = \nu_0 + \frac{v_x}{c} \nu_0$ ← Non-relativistic collision

Maxwell velocity distribution of a gas:

$$f(v_x, v_y, v_z) = \left(\frac{M}{2\pi kT} \right)^{3/2} \exp \left[-\frac{M}{2kT} (v_x^2 + v_y^2 + v_z^2) \right]$$

$$\rightarrow g(\nu) d\nu = \left(\frac{M}{2\pi kT} \right)^{3/2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-M/2kT(v_y^2 + v_z^2)} dv_y dv_z \times e^{-M/2kT(c^2/\nu_0^2)(\nu - \nu_0)^2} \left(\frac{c}{\nu_0} \right) d\nu$$

$$\rightarrow g(\nu) = \frac{c}{\nu_0} \left(\frac{M}{2\pi kT} \right)^{1/2} e^{-M/2kT(c^2/\nu_0^2)(\nu - \nu_0)^2}$$

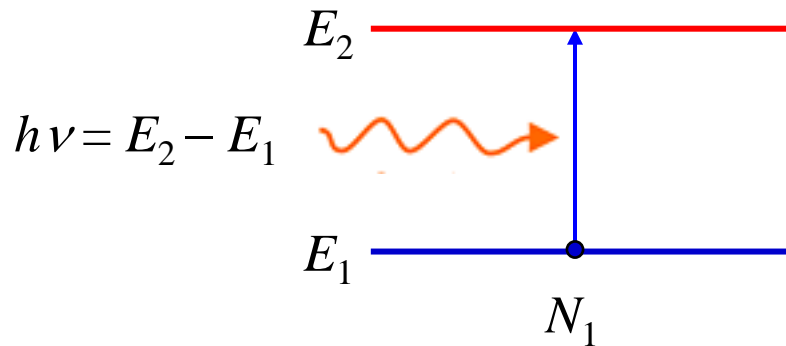
← Gaussian lineshape function

$$\rightarrow \Delta \nu_D = 2\nu_0 \sqrt{\frac{2kT}{Mc^2} \ln 2} \quad \leftarrow \text{Linewidth (FWHM)}$$

Induced Transitions (1)

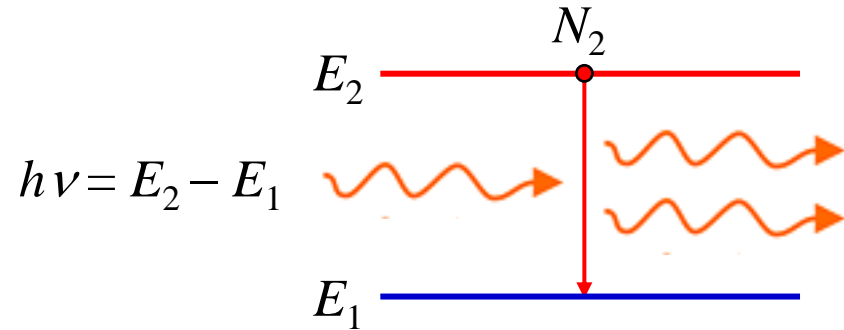
Induced transitions and their rates per atom:

Absorption



$$(W'_{12})_{induced} = B_{12}\rho(\nu)$$

Stimulated emission



$$(W'_{21})_{induced} = B_{21}\rho(\nu)$$

Energy density per unit frequency

Total upward (1→2) transition rate:

$$W'_{12} = (W'_{12})_{induced} = B_{12}\rho(\nu)$$

Total downward (2→1) transition rate:

$$W'_{21} = B_{21}\rho(\nu) + A_{21}$$

Induced Transitions (2)

For thermal equilibrium: $N_2 W'_{21} = N_1 W'_{12}$

$$\rightarrow N_2 [B_{21} \rho(\nu) + A_{21}] = N_1 B_{12} \rho(\nu)$$

$$\rightarrow \rho(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

Boltzmann distribution:

(with no degeneracy)

$$\frac{N_i}{N} = \frac{e^{-E_i/kT}}{\sum_i e^{-E_i/kT}} \rightarrow \frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-h\nu/kT}$$

$$\rightarrow \rho(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{(B_{12}/B_{21})e^{h\nu/kT} - 1} = \frac{8\pi n^3 h \nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

Energy density for thermal (blackbody) radiation

Einstein's A and B coefficients:

$$\rightarrow B_{12} = B_{21} \quad \rightarrow \frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h \nu^3}{c^3}$$

$$\rightarrow W'_i = \frac{A_{21} c^3}{8\pi n^3 h \nu^3} \rho(\nu) = \frac{c^3}{8\pi n^3 h \nu^3 \tau_{sp}} \rho(\nu)$$

Induced Transitions (3)

Monochromatic transition rate:

$$W_i(\nu_k) = \frac{c^3 \rho_{\nu_k}}{8\pi n^3 h \nu_k^3 \tau_{sp}} g(\nu_k)$$

Energy density at ν_k

Atomic lineshape function

"Probability density of the transition tuned at ν_k "

Total transition rate:

$$W_i' = \sum_{\nu_k} W_i(\nu_k) = \frac{c^3}{8\pi n^3 h \tau_{sp}} \sum_{\nu_k} \frac{\rho_{\nu_k}}{\nu_k^3} g(\nu_k)$$
$$\rightarrow W_i' = \frac{c^3}{8\pi n^3 h \tau_{sp}} \int_{-\infty}^{+\infty} \frac{\rho(\nu)}{\nu^3} g(\nu) d\nu \leftarrow \int_{-\infty}^{+\infty} g(\nu) d\nu = 1$$

Transition rate in terms of the intensity:

$$I_\nu = \frac{c}{n} \rho_\nu$$

$$\rightarrow W_i(\nu) = \frac{c^2 I_\nu}{8\pi n^2 h \nu^3 \tau_{sp}} g(\nu) = \frac{\lambda^2 I_\nu}{8\pi n^2 h \nu \tau_{sp}} g(\nu)$$

Absorption and Amplification

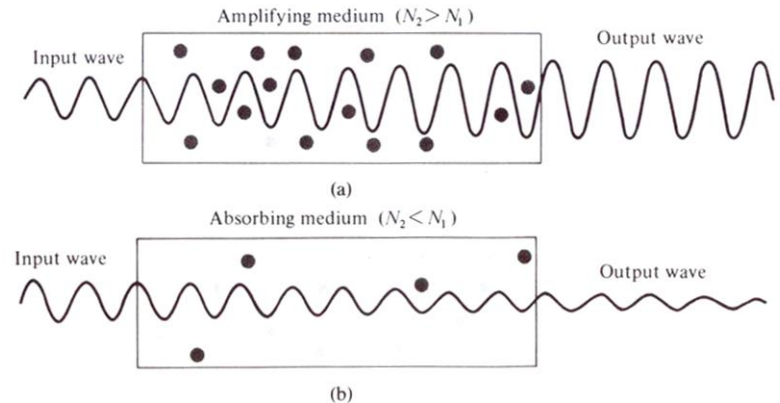
Net power generated per unit volume:

$$\frac{P_\nu}{\text{Volume}} = (N_2 - N_1)W_i h\nu$$

$$\rightarrow \frac{dI_\nu}{dz} = (N_2 - N_1) \frac{c^2 g(\nu)}{8\pi n^2 \nu^2 \tau_{sp}} I_\nu$$

$$\rightarrow I_\nu(z) = I_\nu(0) e^{\gamma(\nu)z}$$

$$\leftarrow \gamma(\nu) = (N_2 - N_1) \frac{c^2 g(\nu)}{8\pi n^2 \nu^2 \tau_{sp}}$$



A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.

Absorption:

$$\frac{N_2}{N_1} = e^{-h\nu/kT} \rightarrow N_2 < N_1 \quad \leftarrow \text{Thermal equilibrium}$$

Amplification: $N_2 > N_1$ \leftarrow Population inversion via a metastable state

Emission and Absorption Crosssections (1)

Boltzmann distribution:

$$\frac{N_i}{N} = \frac{e^{-E_i/kT}}{\sum_i e^{-E_i/kT}} \rightarrow \frac{N_2}{N_1} = \frac{e^{-E_2/kT}}{e^{-E_1/kT}} = e^{-h\nu/kT} \quad \leftarrow \text{With no degeneracy}$$

With degeneracy:

$$\rightarrow \frac{N_i}{N} = \frac{g_i e^{-E_i/kT}}{\sum_i g_i e^{-E_i/kT}} \rightarrow \frac{N_2}{N_1} = \frac{g_2 e^{-E_2/kT}}{g_1 e^{-E_1/kT}} = \frac{g_2}{g_1} e^{-h\nu/kT}$$

Einstein's A and B coefficients:

$$\rightarrow \frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h \nu^3}{c^3} \quad \rightarrow B_{21} = \frac{g_1}{g_2} B_{12}$$
$$\rightarrow W'_{i,e} = \frac{A_{21} c^3}{8\pi n^3 h \nu^3} \rho(\nu) \quad \rightarrow W'_{i,a} = \frac{g_2}{g_1} \frac{A_{21} c^3}{8\pi n^3 h \nu^3} \rho(\nu)$$

Transition rate in terms of the intensity:

$$\rightarrow W_{i,e}(\nu) = \frac{\lambda^2 I_\nu}{8\pi n^2 h \nu \tau_{sp}} g(\nu) \quad \rightarrow W_{i,a}(\nu) = \frac{g_2}{g_1} \frac{\lambda^2 I_\nu}{8\pi n^2 h \nu \tau_{sp}} g(\nu)$$

Emission and Absorption Crosssections (2)

Transition rate in terms of the intensity:

$$\rightarrow W_{i,e}(\nu) = \frac{\lambda^2 I_\nu}{8\pi n^2 h \nu \tau_{sp}} g(\nu) \quad \rightarrow \quad W_{i,a}(\nu) = \frac{g_2}{g_1} \frac{\lambda^2 I_\nu}{8\pi n^2 h \nu \tau_{sp}} g(\nu)$$

Emission and absorption crosssections:

$$\rightarrow \sigma_e(\nu) = \frac{\lambda^2}{8\pi n^2 \tau_{sp}} g(\nu) \quad \rightarrow \quad \sigma_a(\nu) = \frac{g_2}{g_1} \frac{\lambda^2}{8\pi n^2 \tau_{sp}} g(\nu) = \frac{g_2}{g_1} \sigma_e$$

$$\rightarrow W_{i,e}(\nu) = \frac{\sigma_e I_\nu}{h \nu} \quad \rightarrow \quad W_{i,a}(\nu) = \frac{\sigma_a I_\nu}{h \nu}$$

Gain constant with no degeneracy:

$$\gamma(\nu) = (N_2 - N_1) \frac{\lambda^2}{8\pi n^2 \tau_{sp}} g(\nu)$$

With degeneracy:

$$\rightarrow \gamma(\nu) = (N_2 - \frac{\sigma_a}{\sigma_e} N_1) \sigma_e$$

↑
"Effective population inversion"

