Electro-Optics

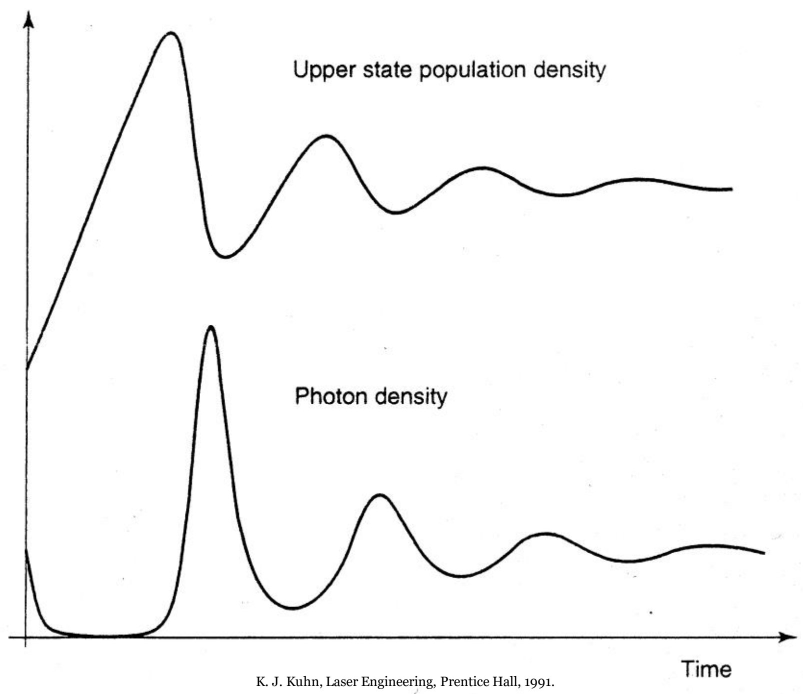
Theory of Lasers (4)

Yoonchan Jeong
School of Electrical Engineering, Seoul National University
Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953
Email: yoonchan@snu.ac.kr

Relaxation Oscillation

Basic physical mechanism:

An interplay between the oscillation field in the resonator and the atomic inversion



K. J. Kuhn, Laser Engineering, Prentice Hall, 1991.

"Relaxation oscillations occurs in lasers when the upper stale lifetime is significantly greater than the lifetime of a photon in the cavity."

Relaxation Oscillation in Lasers (1)

Inversion density for an ideal homogeneously broadened 4-level laser:

$$N \equiv N_2 - N_1 = N_2$$

$$\rightarrow \frac{dN}{dt} = R - W_i N - \frac{N}{\tau} \qquad \leftarrow W_i \propto I \propto q \leftarrow \text{Photon density}$$

$$W_i \equiv Bq$$

$$\rightarrow \frac{dN}{dt} = R - qBN - \frac{N}{\tau}$$

Photon density:

$$\rightarrow \frac{dq}{dt} = qBN - \frac{q}{\tau_c} \qquad \leftarrow \tau_c : \text{Photon's cavity decay time}$$

In equilibrium:

$$\frac{dq}{dt} = 0, \ \frac{dN}{dt} = 0 \ \rightarrow N_0 = \frac{1}{B\tau_c}, \ q_0 = \frac{RB\tau_c - 1/\tau}{B}$$

Threshold pumping rate: $q_0 = 0$

$$R_{th} = (B\tau_c\tau)^{-1} \quad \rightarrow r \equiv R / R_{th} \leftarrow \text{Pumping factor}$$

$$\rightarrow q_0 = \frac{r-1}{B\tau}$$

Relaxation Oscillation in Lasers (2)

Small perturbation from equilibrium:

$$N(t) = N_0 + N_1(t), \;\; N_1 << N_0$$

$$q(t) = q_0 + q_1(t), \;\; q_1 << q_0$$

Rate equations:

$$\begin{cases} \dfrac{dN}{dt} = R - qBN - \dfrac{N}{\tau} \\ \dfrac{dq}{dt} = qBN - \dfrac{q}{\tau_c} \end{cases} \rightarrow \begin{cases} \dfrac{dN_1}{dt} = -RB\tau_c N_1 - \dfrac{q_1}{\tau_c} \\ \dfrac{dq_1}{dt} = \left(RB\tau_c - \dfrac{1}{\tau} \right) N_1 \end{cases}$$

$$\rightarrow \frac{d^2 q_1}{dt^2} + RB\tau_c \frac{dq_1}{dt} + \left(RB - \frac{1}{\tau_c \tau} \right) q_1 = 0$$

$$\rightarrow \frac{d^2 q_1}{dt^2} + \frac{r}{\tau}\frac{dq_1}{dt} + \frac{1}{\tau_c \tau}(r-1) q_1 = 0 \quad \rightarrow q_1 = q_{10} e^{pt}$$

Characteristic equation:

$$p^2 + \frac{r}{\tau} p + \frac{1}{\tau_c \tau}(r-1) = 0$$ ← Damped harmonic oscillator

Relaxation Oscillation in Lasers (3)

Characteristic equation:

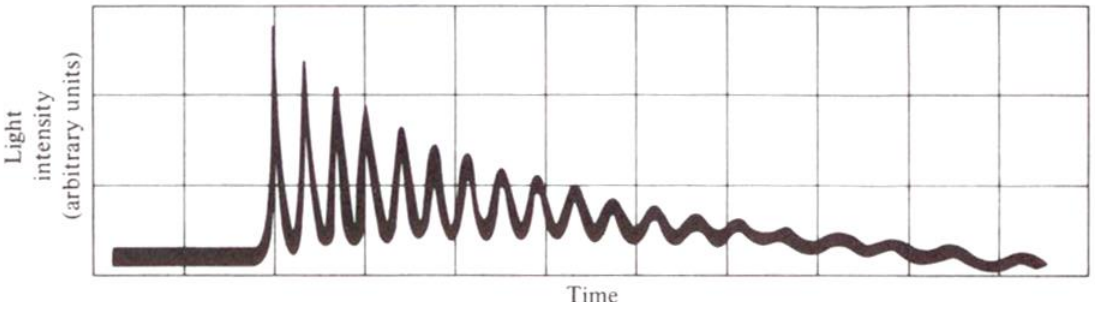
$$p^2 + \frac{r}{\tau} p + \frac{1}{\tau_c \tau}(r-1) = 0$$

Solutions:

$$p_{\pm} = -\alpha \pm i\omega_m \quad \leftarrow \quad \alpha = \frac{r}{2\tau}$$

$$\omega_m = \sqrt{\frac{1}{\tau_c \tau}(r-1) - \left(\frac{r}{2\tau}\right)^2} \cong \sqrt{\frac{1}{\tau_c \tau}(r-1)}$$

$$\rightarrow \quad q_1(t) = q_{10} e^{-\alpha t} \cos \omega_m t$$ ← Damped oscillation



A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.