Electro-Optics

Noise in Optical Detection and Generation

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Limitations Due to Noise Power (1)

Noise in optical detection:

The noise causes fluctuations in the measurement, thus placing a lower limit on the smallest amount of power that can be measured.

Thermal noise:

Generated by thermally agitated charge carriers

$\rightarrow$ Thermodynamic treatment & statistical analysis

Shot noise:

Generation-recombination noise in photoconductive detectors

$\rightarrow$ Uncertainty principle in quantum mechanics

Measurement of optical power:

Optical signal field: $v_S(t) = V_s \cos \omega t$

Noise field: $v_N(t) = V_{NC}(t) \cos \omega t + V_{NS}(t) \sin \omega t$

Total field at the detector: $v(t) = v_S(t) + v_N(t)$

$$= \mathrm{Re}\{[V_s + V_{NC}(t) - i V_{NS}(t)] e^{i\omega t}\}$$

$$\equiv \mathrm{Re}[V(t) e^{i\omega t}]$$

Limitations Due to Noise Power (2)

Probability function (Gaussian) :

$$p(V_{NC})dV_{NC} = \frac{1}{(2\pi)^{1/2}\sigma} e^{-V_{NC}^2/2\sigma^2} dV_{NC} \quad \rightarrow \int_{-\infty}^{+\infty} p(V_{NC})dV_{NC} = 1$$

Ensemble average:

$$\rightarrow \overline{V_{NC}^2} = \int_{-\infty}^{+\infty} V_{NC}^2 p(V_{NC})dV_{NC} = \sigma^2 \quad \rightarrow \overline{V_{NS}^2} = \sigma^2$$

$$\rightarrow \overline{V_{NC}} = 0 \quad \rightarrow \overline{V_{NS}} = 0$$

Field "power":

$$P(t) \equiv [V(t)e^{i\omega t}][V^*(t)e^{-i\omega t}] = V_S^2 + 2V_S V_{NC} + V_{NC}^2 + V_{NS}^2$$

$$\rightarrow \overline{P} \equiv \overline{P(t)} = \overline{V_S^2} + \overline{V_{NC}^2} + \overline{V_{NS}^2} = \overline{V_S^2} + 2\sigma^2$$

Uncertainty in power measurement: → Root mean square power deviation

$$\Delta P \equiv \{[\overline{P(t) - \overline{P}]^2}\}^{1/2} = (4\overline{V_S^2}\,\overline{V_{NC}^2} + 2\overline{V_{NC}^4} - 2\overline{V_{NS}^2}\,\overline{V_{NC}^2})^{1/2}$$

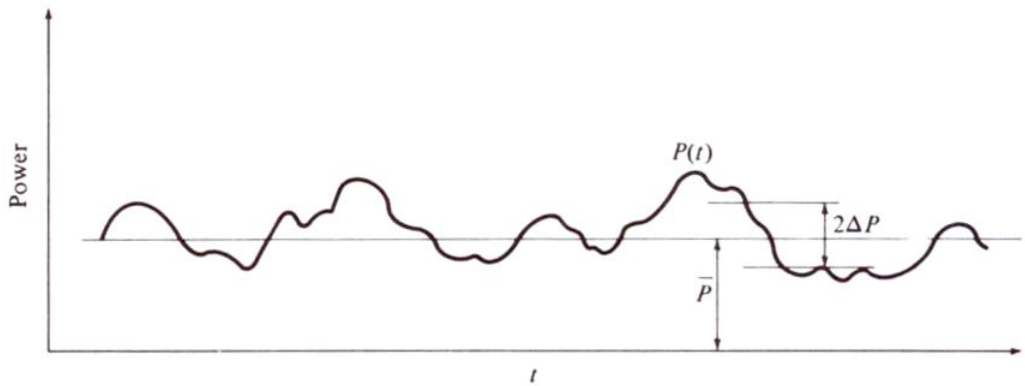
$$= 2\sigma(\overline{V_S^2} + \sigma^2)^{1/2} \qquad \leftarrow \overline{V_{NC}^4} = \int_{-\infty}^{+\infty} V_{NC}^4 p(V_{NC})dV_{NC} = 3\sigma^4$$

$$= 2\sigma(P_s + \sigma^2)^{1/2}$$

Limitations Due to Noise Power (3)

Minimum signal power measurable in the presence of noise:

$$\Delta P = 2\sigma(P_s + \sigma^2)^{1/2}$$



A. Yariv, *Optical Electronics, 4th ed. Saunders, 1991.*

$$\rightarrow \; P_{\text{limit}} = 2\sigma(P_{\text{limit}} + \sigma^2)^{1/2}$$

$$= 2\sigma^2(1+\sqrt{2}) \; = P_N(1+\sqrt{2})$$

Noise power

Noise – Basic Definitions and Theorems

A real function and its Fourier transform:

$$V(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} v(t)e^{-i\omega t}dt \quad \rightarrow \quad v(t) = \int_{-\infty}^{+\infty} V(\omega)e^{i\omega t}d\omega$$

In case: $v(t) = 0 \quad for \quad t \leq -T/2 \;\; \& \;\; t \geq T/2$

$$V_T(\omega) = \frac{1}{2\pi}\int_{-T/2}^{+T/2} v(t)e^{-i\omega t}dt \quad \rightarrow \quad V_T(\omega) = V_T^*(-\omega)$$

Average power:

$$P = \frac{1}{T}\int_{-T/2}^{+T/2} v^2(t)dt = \frac{1}{T}\int_{-T/2}^{+T/2} v(t)\left[\int_{-\infty}^{+\infty} V_T(\omega)e^{i\omega t}d\omega\right]dt$$

$$= \frac{1}{T}\int_{-T/2}^{+T/2}\left[\int_{-\infty}^{+\infty} V_T(\omega')e^{i\omega' t}d\omega'\right]\left[\int_{-\infty}^{+\infty} V_T(\omega)e^{i\omega t}d\omega\right]dt$$

$$\leftarrow \lim_{T\to\infty}\frac{1}{2\pi}\int_{-T/2}^{+T/2}\exp[i(\omega+\omega')t]dt = \delta(\omega+\omega')$$

$$= \frac{2\pi}{T}\int_{-\infty}^{+\infty}\left|V_T(\omega)\right|^2 d\omega = \frac{4\pi}{T}\int_{0}^{+\infty}\left|V_T(\omega)\right|^2 d\omega$$

Spectral density function:

$$S_v(\omega) = \lim_{T\to\infty}\frac{4\pi\left|V_T(\omega)\right|^2}{T}$$

Wiener-Khintchine Theorem

Autocorrelation function:

$$C_v(\tau) = \overline{v(t)v(t+\tau)} = \frac{1}{T}\int_{-T/2}^{+T/2} v(t)v(t+\tau)dt$$

$$= \frac{1}{T}\int_{-T/2}^{+T/2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} V_T(\omega)V_T(\omega')e^{i(\omega+\omega')t}e^{i\omega\tau}(t)d\omega d\omega' dt$$

$$= \frac{1}{2}\int_{-\infty}^{+\infty} \frac{4\pi\left|V_T(\omega)\right|^2}{T} e^{i\omega\tau} d\omega$$

Spectral density function

Wiener-Khintchine theorem:

$$\rightarrow\ C_v(\tau) = \frac{1}{2}\int_{-\infty}^{+\infty} S_v(\omega)e^{i\omega\tau} d\omega$$

$$\rightarrow\ S_v(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} C_v(\tau)e^{-i\omega\tau} d\tau$$

Fourier transform pair

Spectral Density Function of Random Events (1)

Time-dependent random variable:

$$i_T(t) = \sum_{i=1}^{N_T} f(t - t_i) \quad 0 \le t \le T$$

$$\rightarrow I_T(\omega) = \sum_{i=1}^{N_T} F_i(\omega) \leftarrow F_i(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t - t_i) e^{-i\omega t} dt$$

$$= \frac{e^{-i\omega t_i}}{2\pi}\int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$= e^{-i\omega t_i} F(\omega)$$

Ensemble average:

$$\rightarrow \overline{|I_T(\omega)|^2} = \overline{|F(\omega)|^2 \sum_{i=1}^{N_T}\sum_{j=1}^{N_T} e^{-i\omega(t_i - t_j)}}$$

$$= |F(\omega)|^2 \left(\overline{N_T + \sum_{i \ne 1}^{N_T}\sum_{j=1}^{N_T} e^{-i\omega(t_i - t_j)}} \right) = \overline{N}_T |F(\omega)|^2 = \overline{N}T|F(\omega)|^2$$

Average rate

Recall: Spectral density function

$$S_v(\omega) = \frac{4\pi |V_T(\omega)|^2}{T} \qquad \rightarrow S(\omega) = 4\pi\overline{N}|F(\omega)|^2$$

Spectral Density Function of Random Events (2)

Carson's theorem:

$$S(\omega) = 4\pi \overline{N} |F(\omega)|^2$$

$$\rightarrow \; S(\nu) = 8\pi^2 \overline{N} |F(2\pi\nu)|^2 \quad \leftarrow \; S(\nu)d\nu = S(\omega)d\omega$$

For subclass events: $f_\alpha(t - t_i)$

$$\rightarrow \; S_\alpha(\nu)d\alpha = 8\pi^2 \overline{N}(\alpha) |F_\alpha(2\pi\nu)|^2 d\alpha \quad \leftarrow \; \int_{-\infty}^{+\infty} \overline{N}(\alpha)d\alpha = \overline{N}$$

Probability distribution function:

$$\rightarrow \; \int_{-\infty}^{+\infty} p(\alpha)d\alpha = \frac{1}{\overline{N}} \int_{-\infty}^{+\infty} \overline{N}(\alpha)d\alpha = 1$$

Overall "spectral density function":

$$\rightarrow \; S(\nu) = \int_{-\infty}^{+\infty} S_\alpha(\nu)d\alpha = \int_{-\infty}^{+\infty} 8\pi^2 \overline{N}(\alpha) |F_\alpha(2\pi\nu)|^2 d\alpha$$

$$= 8\pi^2 \overline{N} \int_{-\infty}^{+\infty} |F_\alpha(2\pi\nu)|^2 p(\alpha)d\alpha$$

$$= 8\pi^2 \overline{N} \overline{|F_\alpha(2\pi\nu)|^2}$$

Shot Noise

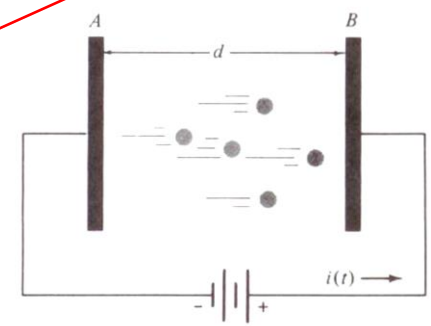
Random electron flow between two electrodes:

Instantaneous velocity

$$\overline{N} = \overline{I} / q \quad \leftarrow \text{Average rate of electron emission}$$

Current pulse due to a "single" electron:

$$i_e(t) = \frac{qv(t)}{d}$$



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Fourier transform of a single current pulse:

Arrival time

$$F(\omega) = \frac{q}{2\pi d} \int_0^{t_a} v(t) e^{-i\omega t} dt \quad \leftarrow \quad \omega t_a << 1$$

(Small transit time)

$$\cong \frac{q}{2\pi d} \int_0^{t_a} \frac{dx}{dt} dt = \frac{q}{2\pi}$$

Spectral density function:

$$S(\nu) = 8\pi^2 \overline{N} \left| F(2\pi\nu) \right|^2 = 8\pi^2 \overline{N} \left(\frac{q}{2\pi} \right)^2 = 2q\overline{I}$$

"Shot noise" power by an equivalent noise generator:

$$\overline{i_N^2(\nu)} = S(\nu)\Delta\nu$$ → Proportional to the "magnitude of an *individual* charge" even for the same $\overline{I}$

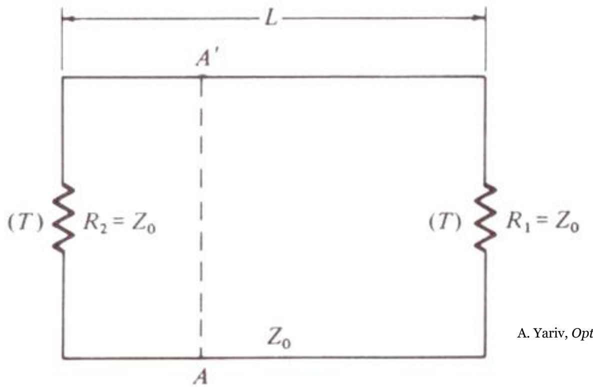
→ Caused by quantum fluctuations in the current

Thermal Noise (1)

Johnson-Nyquist noise:

Fluctuations in the voltage across a dissipative circuit element

Transmission line connected by two similar resistances at the same T:



→ Matched impedance having no reflection

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Traveling voltage wave: $v(t) = A\cos(\omega t \pm kz)$

$$\rightarrow\ v(t) = A\cos[\omega t \pm k(z+L)] = A\cos(\omega t \pm kz)$$

$$\rightarrow\ kL = 2m\pi, \quad m = 1, 2, ,3, \cdots$$

$$\rightarrow\ \Delta k = \frac{2\pi}{L}$$ → Mode spacing

Number of modes between 0 to k:

$$\rightarrow\ N_k = \frac{k}{\Delta k} = \frac{kL}{2\pi} \quad \rightarrow\ N(\nu) = \frac{\nu L}{c}$$

Thermal Noise (2)

Number of mode per unit frequency:

$$p(\nu) = \frac{dN(\nu)}{d\nu} = \frac{L}{c}$$

Power transmission:

$$power = \frac{energy}{distance}(velocity\ of\ energy)$$

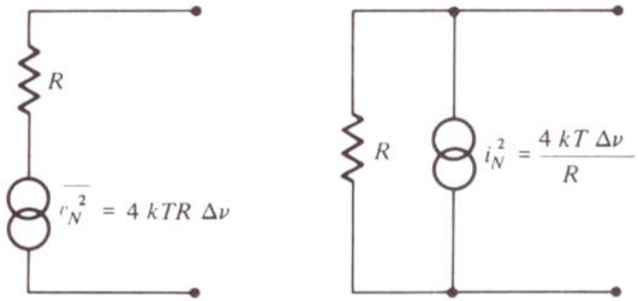
$$\rightarrow P = \left(\frac{c}{L}\right)(number\ of\ modes\ between\ \nu\ and\ \nu + \Delta\nu)(engery\ per\ mode)$$

$$= \left(\frac{c}{L}\right)\left(\frac{L}{c}\Delta\nu\right)\left(\frac{h\nu}{e^{h\nu/kT}-1}\right) = \frac{h\nu\Delta\nu}{e^{h\nu/kT}-1} \cong kT\Delta\nu \quad (kT >> h\nu)$$

Equivalent noise generator:

$$\rightarrow \overline{v_N^2(\nu)} = \frac{4h\nu R\Delta\nu}{e^{h\nu/kT}-1} \underset{kT>>h\nu}{\cong} 4kTR\Delta\nu$$

$$\rightarrow \overline{i_N^2(\nu)} = \frac{4h\nu\Delta\nu}{R(e^{h\nu/kT}-1)} \underset{kT>>h\nu}{\cong} \frac{4kT\Delta\nu}{R}$$



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