Review of Dynamic Response of single – degree- of – freedom system

Dynamic equilibrium Equation

$$f_i + f_d + f_s = p(t)$$

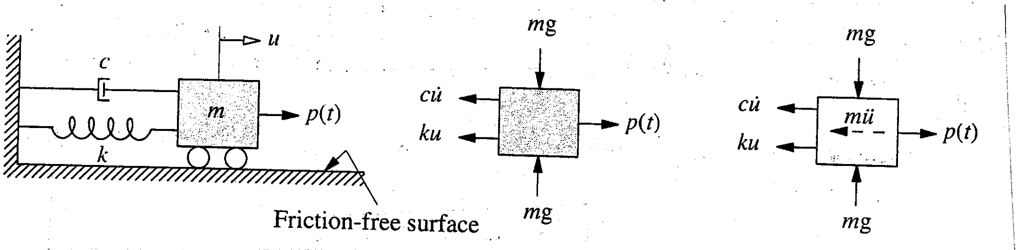
f_i = inertia force = $m\ddot{u} = m\dfrac{d^2u}{dt^2}$

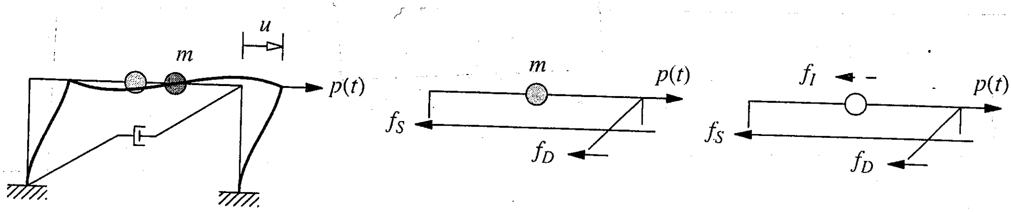
f_d = damping force (dissipative force) = $c\dot{u} = c\dfrac{du}{dt}$

f_s = spring force = ku

$p(t)$ = time dependent applied force

$$m\ddot{u} + c\dot{u} + ku = p(t)$$





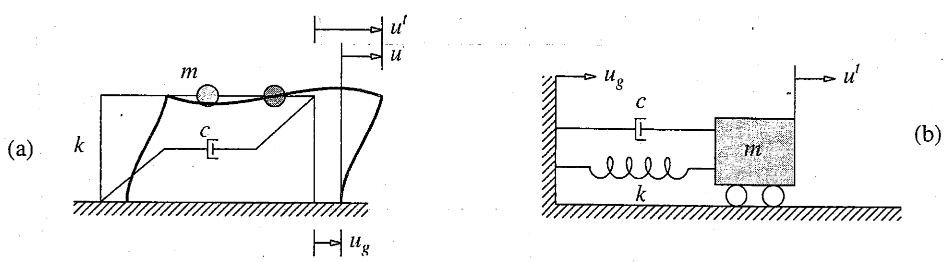
(a) (b) (c)

EQ ground motion

$$m(\ddot{u} + \ddot{u}_g) + c\dot{u} + ku = 0$$

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g = p_e(t)$$ = effective time-dependent force

$$u_{total} = u + u_g$$

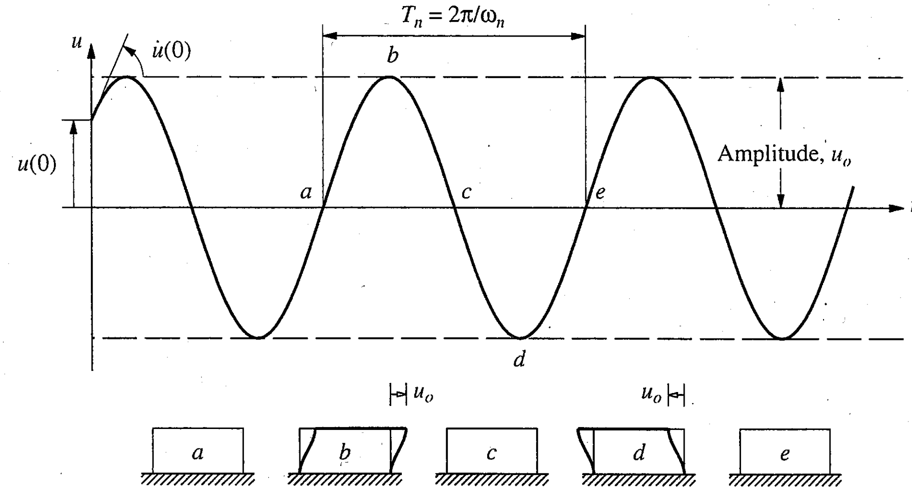


Undamped free vibration

$m\ddot{u} + ku = 0$ or $\ddot{u}(t) + \omega^2 u(t) = 0 \quad \omega^2 = k/m$

$u(t) = A\sin\omega t + B\cos\omega t \qquad T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}} = \dfrac{1}{f}$

$m\ddot{u}$ and ku have the same phase angle

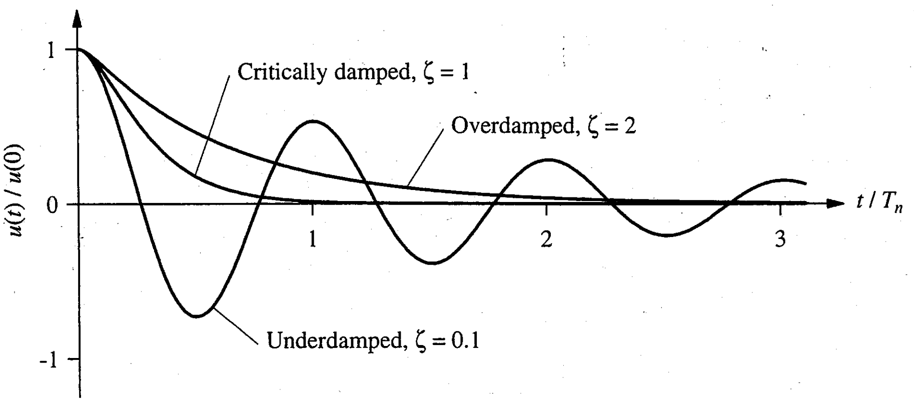


Damped free vibration

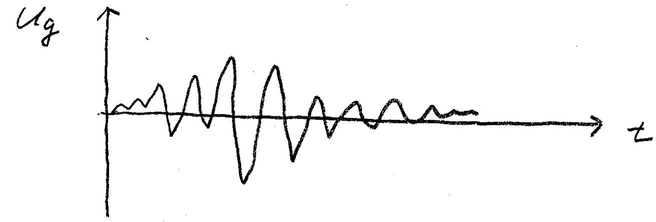
$m\ddot{u} + c\dot{u} + ku = 0$

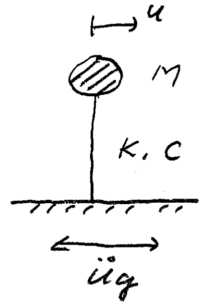
$m\ddot{u}$ and ku have different phase angle from $c\dot{u}$

$\zeta = c/2\omega m$: critical damping ratio



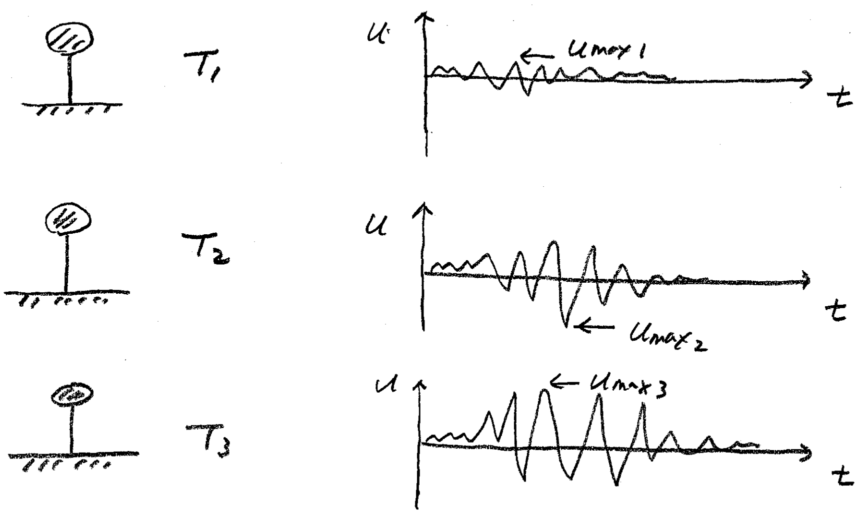
EQ Response Spectra

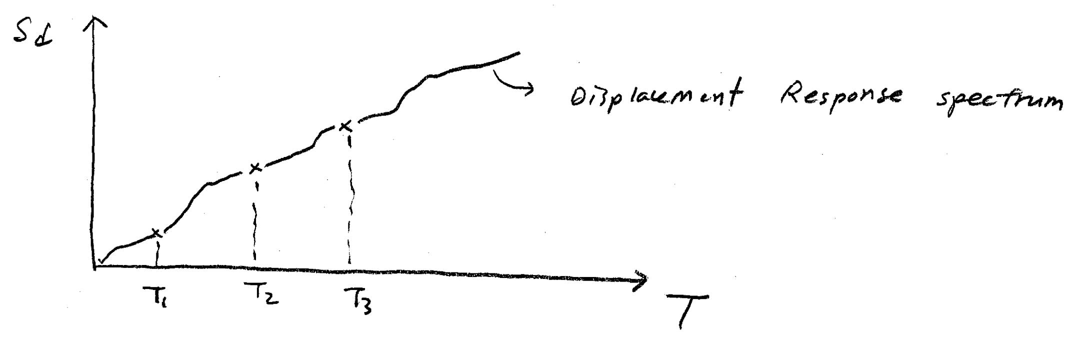




$$T = 2\pi\sqrt{\frac{M}{K}}$$

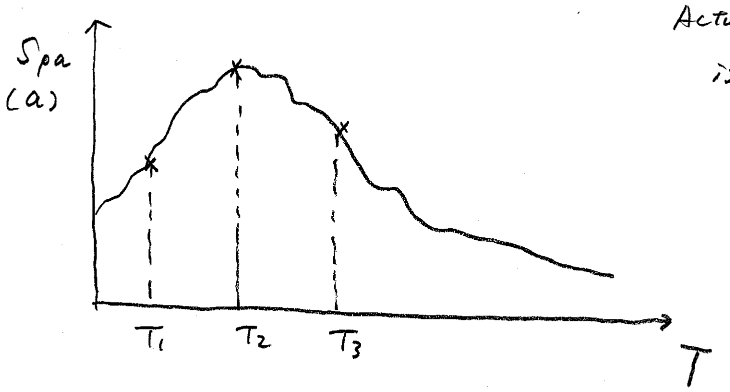
$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g$$





$$S_{pa} = \omega^2 S_d$$

$F = ku$ (EQ Load)



Actual system
is tranformed to artificial
undamped free vibration,
to evaluate the EQ load

$$m S_a + k S_d = 0$$

$$S_a = \omega^2 S_d$$

EQ Response Spectra

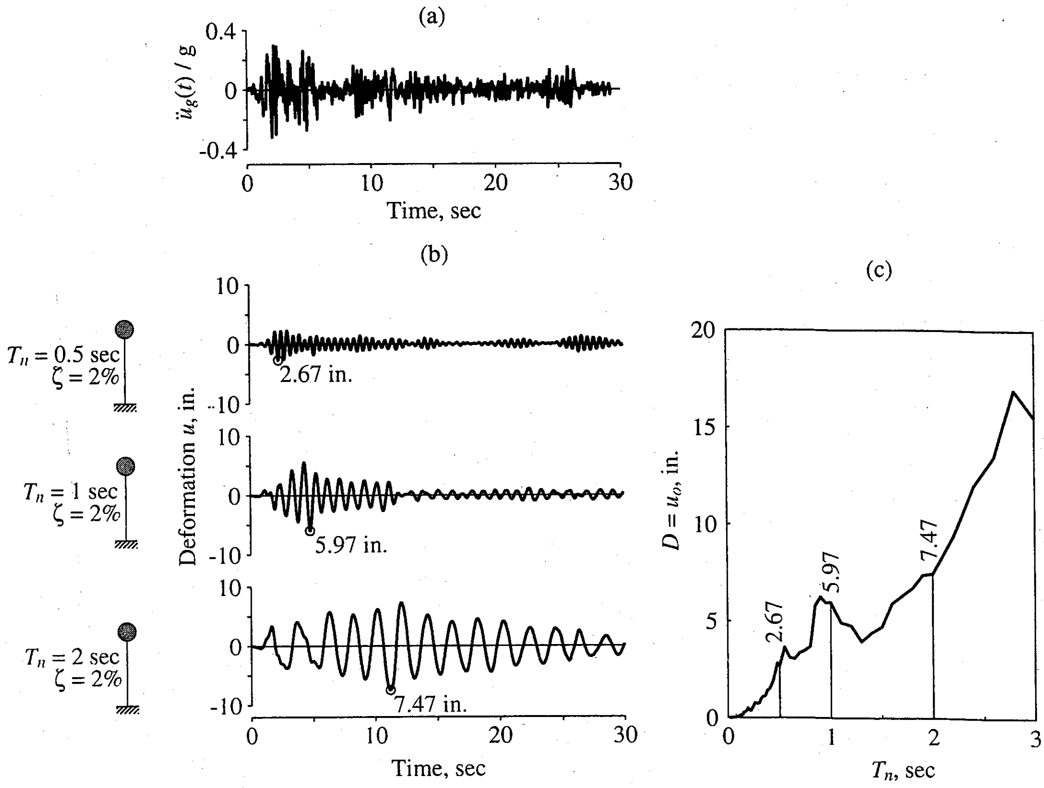


Figure 6.6.1 (a) Ground acceleration; (b) deformation response of three SDF systems with $\zeta = 2\%$ and $T_n = 0.5$, 1, and 2 sec; (c) deformation response spectrum for $\zeta = 2\%$.

$S_d = u(t)_{\max}$ = spectral displacement

$S_{pv} = \omega S_d$ = spectral pseudo velocity

$S_{pa} = \omega^2 S_a$ = spectral pseudo acceleration

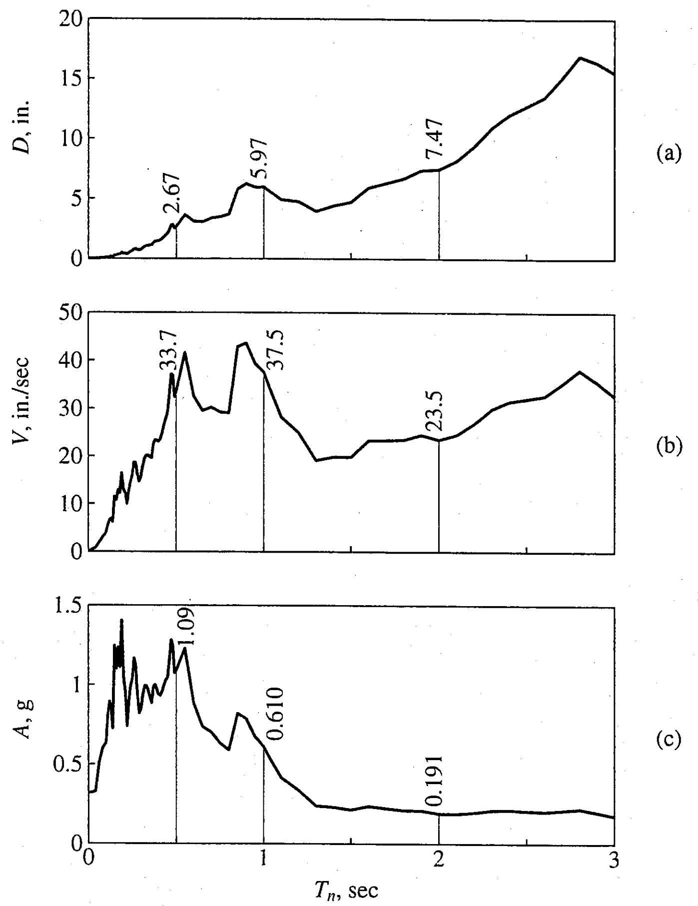


Figure 6.6.2 Response spectra ($\zeta = 0.02$) for El Centro ground motion: (a) deformation response spectrum; (b) pseudo-velocity response spectum; (c) pseudo-acceleration response spectrum.

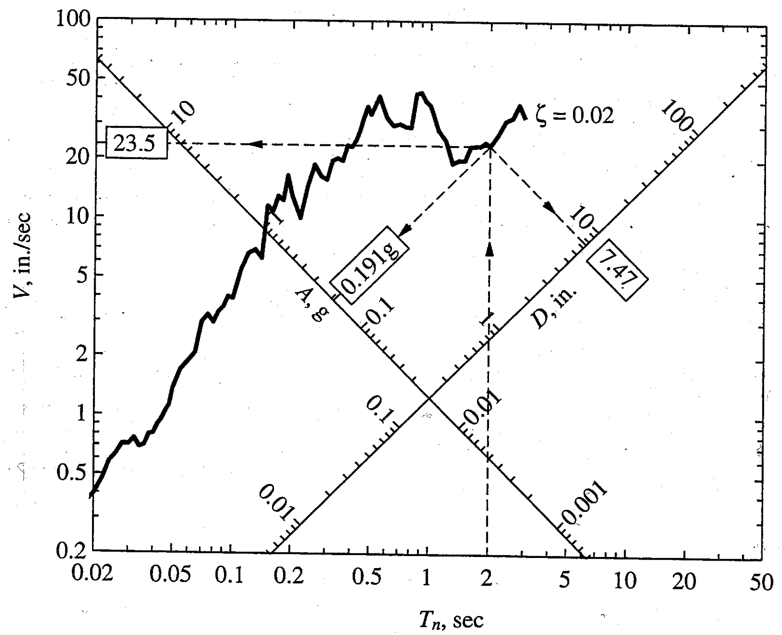


Figure 6.6.3 Combined D–V–A response spectrum for El Centro ground motion; $\zeta = 2\%$.

At small frequencies, the spectral displacement approaches the peak ground displacement, indicating that for very flexible systems (systems with small frequencies or long periods) the maximum displacement is equal to the ground displacement. At large frequencies, such as 25-30 Hz, the acceleration approaches the peak ground acceleration, indicating that for rigid systems the absolute acceleration of the mass is the same as that of the ground.

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g$$

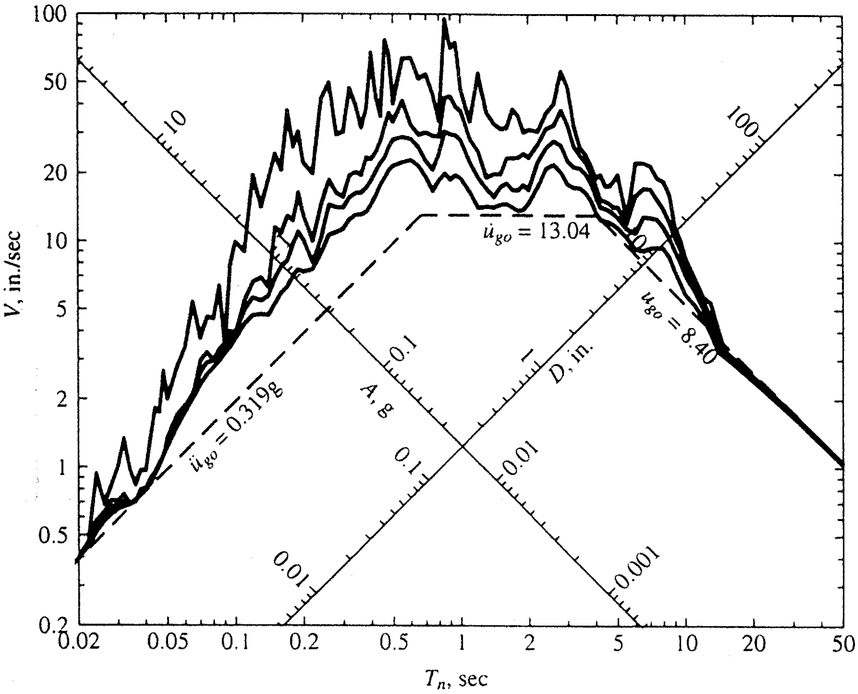


Figure 6.8.1 Response spectrum (ζ = 0, 2, 5, and 10%) and peak values of ground acceleration, ground velocity, and ground displacement for El Centro ground motion.

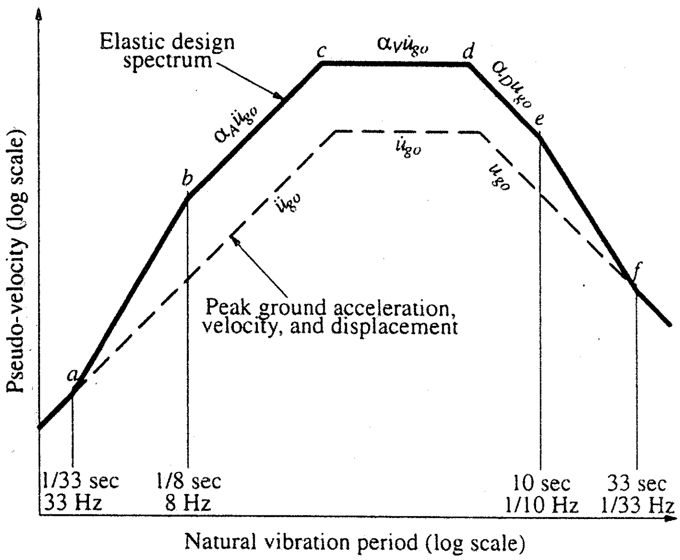
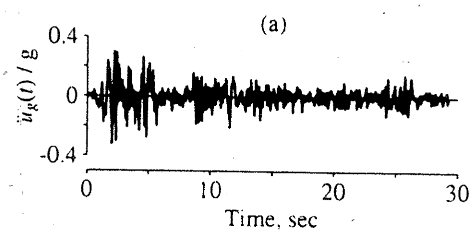
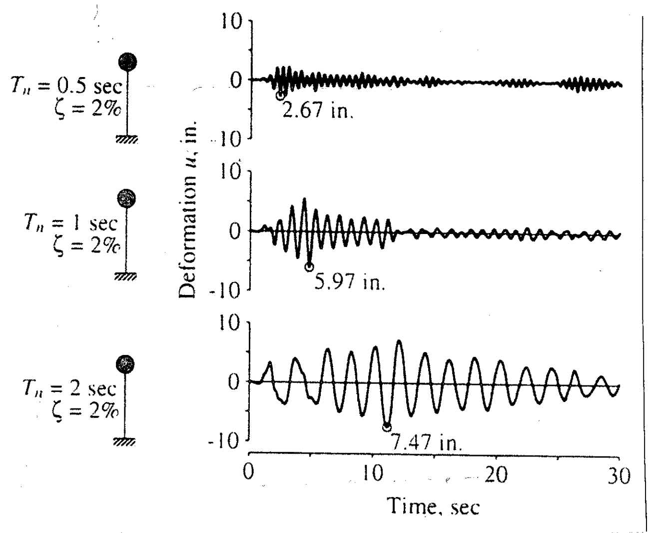


Figure 6.9.3 Construction of elastic design spectrum.

Time history Analysis of a SDF system





$$M\ddot{u} + C\dot{u} + Ku = -M\ddot{u}_g(t) \qquad (1)$$

If we use the central difference method,

$$\dot{u}_i = \frac{u_{i+1} - u_{i-1}}{2\Delta t} \qquad (2)$$

$$\ddot{u}_i = \left(\frac{u_{i+1} - u_i}{\Delta t} - \frac{u_i - u_{i-1}}{\Delta t} \right) \frac{1}{\Delta t} = \frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta t^2} \qquad (3)$$

From Eqns. (1), (2), and (3)

$$M\left(\frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta t^2} \right) + C\left(\frac{u_{i+1} - u_{i-1}}{2\Delta t} \right) + Ku_i = -M\ddot{u}_{g,i}$$

$$u_{i+1}\left(\frac{M}{\Delta t^2} + \frac{C}{2\Delta t} \right) + u_i\left(K - \frac{2M}{2\Delta t^2} \right) + u_{i-1}\left(\frac{M}{\Delta t^2} - \frac{C}{2\Delta t} \right) = -M\ddot{u}_{g,i}$$

$$u_{i+1} = \frac{\left[-M\ddot{u}_{g,i} - u_i\left(K - \dfrac{2M}{2\Delta t^2} \right) - u_{i-1}\left(\dfrac{M}{\Delta t^2} - \dfrac{C}{2\Delta t} \right) \right]}{\left(\dfrac{M}{\Delta t^2} + \dfrac{C}{2\Delta t} \right)}$$

With known values of u_i and u_{i-1}, u_{i+1} can be calculated.

Basic Principles of Earthquake-Resistance Design

Because earthquake motions are much less predictable than other kinds of lateral loads, and because strong earthquakes can produce very large forces in linear elastic structures, it is impractical to design a structure for a single design earthquake.

Instead, we design for several selective limit states:

a) service limit state (operational) : building continues to function

b) damage limit state (immediate occupancy): damage is limited.

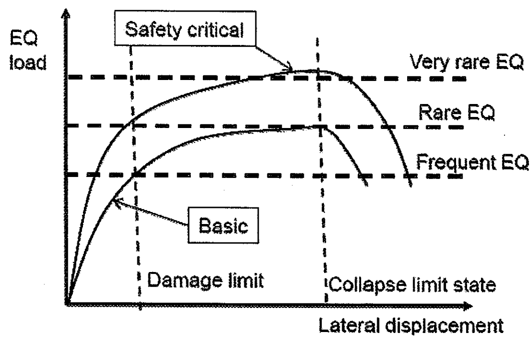
c) Collapse limit state (Life safety or collapse prevention) : building remains standing

Any or all of these limit states could control, depending on the type of building.

Hospital : must function under strong EQ.

Office Tower : must continue functioning under light EQ.
must limit damage under moderate EQ.
must not collapse under strong EQ.

Single Family Home : can stop functioning under light EQ.
damage permitted under moderate EQ.
must not collapse under strong EQ.



Relationship Between Performance Objectives and Earthquake Probability

Earthquake Probability	**Performance objective**			
	operational	**Immediate occupancy**	**Life safety**	**Collapse prevention**
Frequent				
Occasional		Basic facilities		
Rare		Essential		
Very Rare	Safety control			

Performance limit States

Operational

Continuous service. Negligible structural and non-structural damage

Immediate occupancy

Most operations and functions can resume immediately. Structure safe for occupancy. Essential operations protected, non-essential operations disrupted. Repair required to restore some non-essential services. Damage is light.

Life Safe

Damage is moderate, but structure remains stable. Selected building systems, features or contents may be protected from damage. Life safety is generally protected. Building may be evacuated following earthquake. Repair possible, but may be economically impractical.

Collapse prevention

Damage severe, but structural collapse prevented. Non-structural elements may fall.

Occupancy types

Safety Critical Facilities:

Large quantities of hazardous materials (toxins, radioactive materials, explosives) with significant external effects of damage to building.

Essential/Hazardous Facilities

Critical post-earthquake facilities (hospitals, communications centers, police, fire stations, etc.)

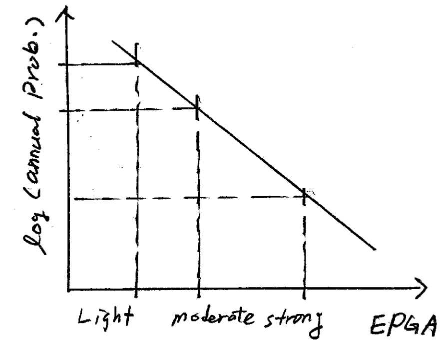
Hazardous materials with limited impact outside of immediate vicinity of building. (Refineries, etc.)

Basic Facilities

All other structures.

Earthquake Probability

Type of earthquake (probability)	Return period (year)	10% probability of exceedance in ? years
Light (frequent)	10	1
Moderate (occasional)	48	5
Strong (rare)	475	50



Sample Calculation

Let T = return period in years

Then the probability that the event will be exceeded in a single year is 1/T

The probability that the event will not be exceeded in n years is $(1-1/T)^n$

The probability that the event will be exceeded in n years is $1-(1-1/T)^n$

For T = 475 years and n = 50 years

$$1-(1-1/T)^n = 1-(1-1/475)^{50} = 0.10$$

In a highly seismic area, each type of earthquake would have about the following maximum ground acceleration:

Light EQ : a = 0.1 g

Moderate EQ : a = 0.2 g

Strong EQ : a = 0.4 ~ 0.5 g

Corresponding values of v_{max} and d_{max} will depend on the earthquake frequency content.

California $\dfrac{a\,d}{v^2} \approx 6$

If $a_{max} = 386.4 \text{ in/sec}^2 = 1g = 981 \text{ cm / sec}^2$

v_{max} = 48 in/sec x a_{max} = 122 cm/sec x a_{max} = 122 cm / sec

$d_{max} = 6\, v_{max}^{\ 2} / a_{max}$ = 36 in = 92 cm

Chile $\dfrac{a\,d}{v^2} \approx 6$,

Mexico City $\dfrac{a\,d}{v^2} \approx 1$ Unusual soft soil conditions produce nearly harmonic motion.

Linear Elastic Design Response Spectra

Given values of a_{max} v_{max} d_{max}, how can we get response spectra for design in the linear, elastic range (service limit state) ? One of the most widely used procedures is that of Newmark & Hall :

Newmark, N. M. and Hall, W. J., "Procedures and criteria for earthquake resistant design," Building Practice for Disaster Mitigation, Building Science Series 45, National Bureau of Standards, Washington, D. C. Feb. 1973, pp 209-236.

After plotting the lines corresponding to ground motion maxima on tripartite log paper, the response spectrum ordinates are obtained by multiplying these by factors which depend on the structure's damping

Tripartite log spectrum

$S_d = S_v / \omega$ $\quad\quad$ $S_a = \omega S_v$

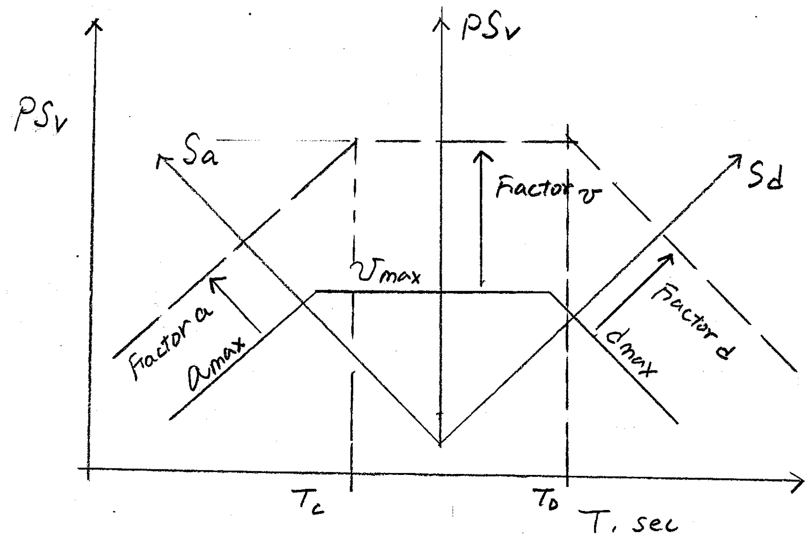
$\log S_d = \log S_v - \log \omega$ or $\log S_v = \log S_d + \log \omega$

$\log S_a = \log S_v + \log \omega$ or $\log S_v = \log S_a - \log \omega$

$\log S_v$: x axis

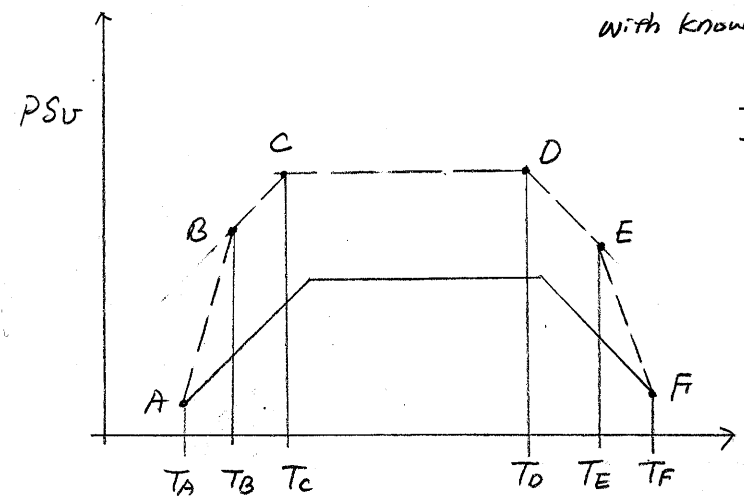
$\log \omega$: y axis $\quad T = 2\pi / \omega$

log Sd is proportional to (log Sv + log T) ⇒ 45° Axis



Damping ratio ξ	Amplification factors for		
	a_{max}	v_{max}	d_{max}
0	6.4	4.0	2.5
0.5	5.8	3.6	2.2
1	5.2	3.2	2.0
2	4.3	2.8	1.8
3	3.7	2.5	1.7
5	2.6	1.9	1.4
7	1.9	1.5	1.2
10	1.5	1.3	1.1
20	1.2	1.1	1.0

Then modify shape at ends :



with known T_C and T_D

$$\frac{T_C}{T_B} = \frac{T_E}{T_D} = 4$$

$$\frac{T_C}{T_A} = \frac{T_F}{T_D} = 10$$

Example :
Using the procedure of Newmark & Hall, compute linear elastic design response spectra (LEDRS) for a structure on rock or firm soil, with the following ground motion parameters and 2 % damping :

a_{max} = 0.40 g

v_{max} = 19.2 in/sec

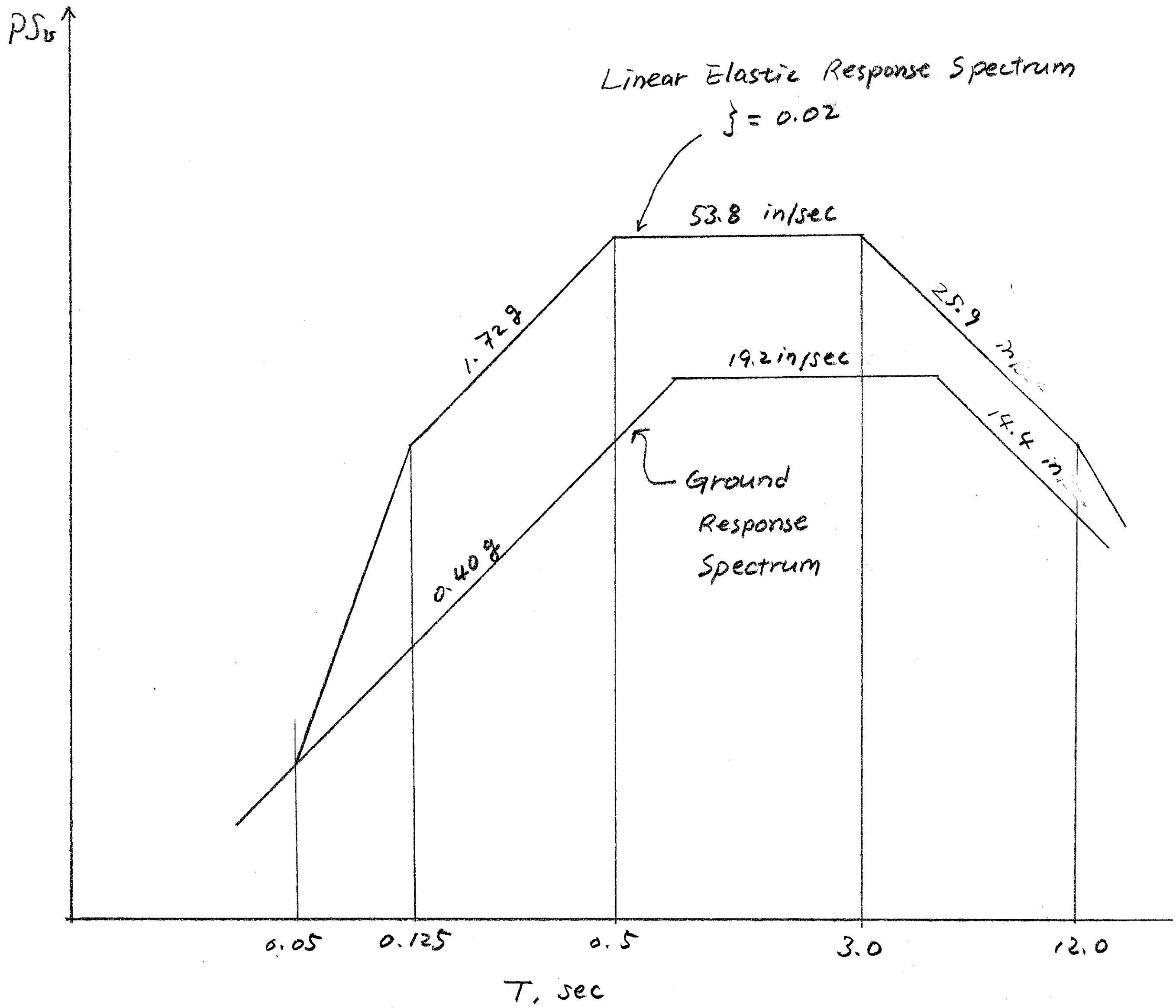
d_{max} = 14.4 in.

Sa_{max} = 0.40 g x 4.3 = 1.72 g

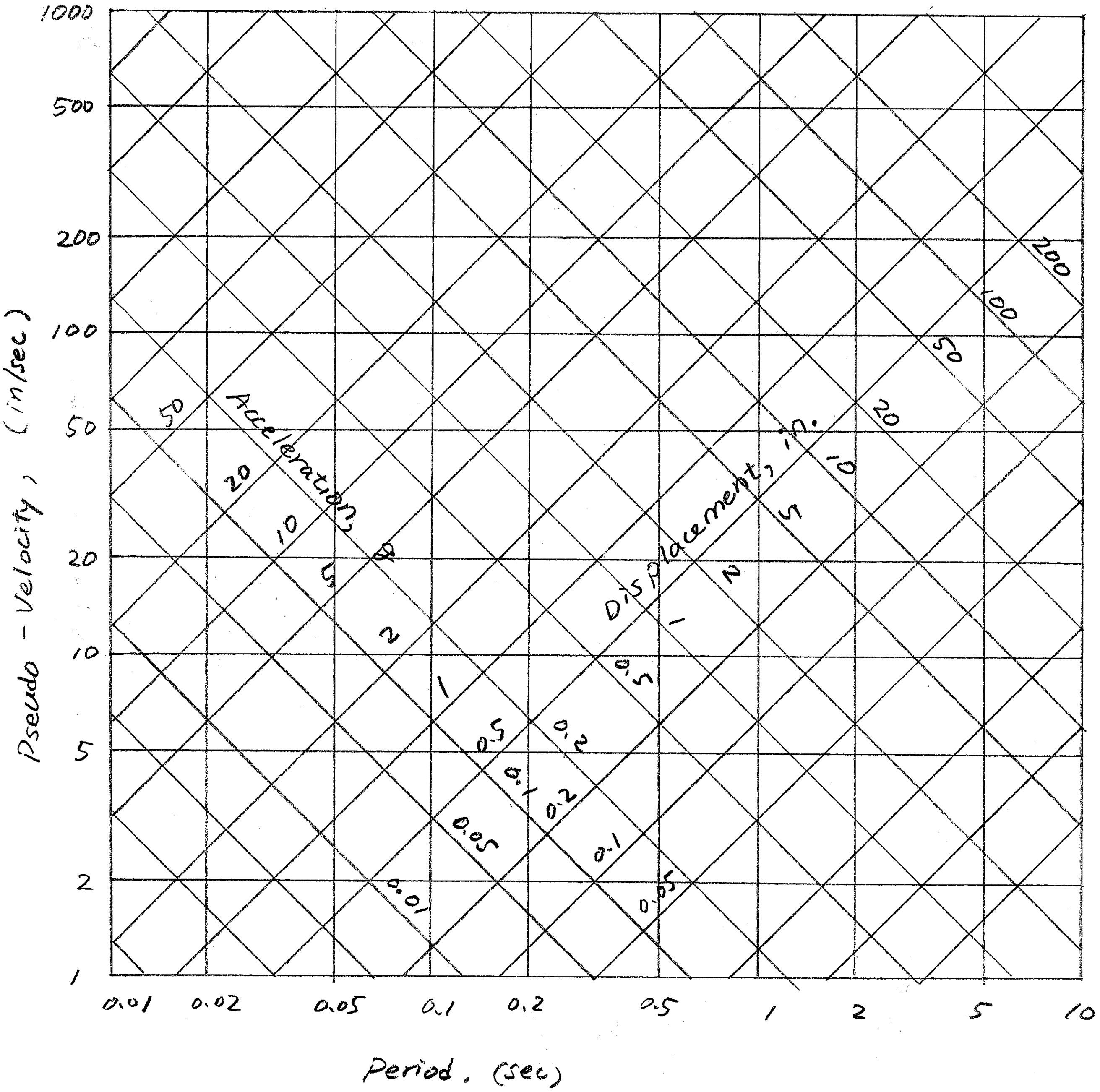
Sv_{max} = 19.2 x 2.8 = 53.8 in/sec

Sd_{max} = 14.4 x 1.8 = 25.9 in

Plot these on a tripartite graph.



PS_v
Linear Elastic Response Spectrum
$\xi = 0.02$
53.8 in/sec
1.72g
25.9 in
19.2 in/sec
14.4 in
Ground Response Spectrum
0.40g
0.05
0.125
0.5
3.0
12.0
T, sec


1000
500
200
100
50
20
10
5
2
1
Pseudo - Velocity, (in/sec)
0.01
0.02
0.05
0.1
0.2
0.5
1
2
5
10
Period, (sec)
Acceleration, g
50
20
10
5
2
1
0.5
0.2
0.1
0.05
0.01
Displacement, in.
200
100
50
20
10
5
2
1
0.5
0.2
0.1
0.05

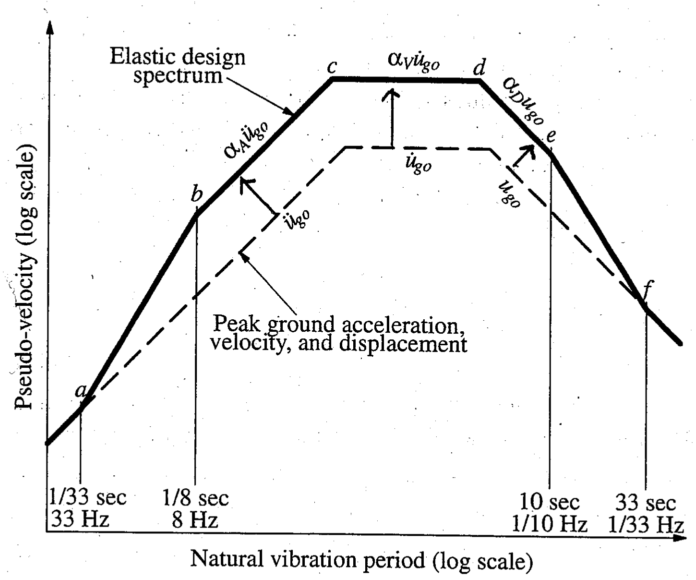


Figure 6.9.3 Construction of elastic design spectrum.

TABLE 6.9.1 AMPLIFICATION FACTORS: ELASTIC DESIGN SPECTRA

Damping, ζ (%)	Median (50th percentile) α_A	α_V	α_D	One Sigma (84.1th percentile) α_A	α_V	α_D
1	3.21	2.31	1.82	4.38	3.38	2.73
2	2.74	2.03	1.63	3.66	2.92	2.42
5	2.12	1.65	1.39	2.71	2.30	2.01
10	1.64	1.37	1.20	1.99	1.84	1.69
20	1.17	1.08	1.01	1.26	1.37	1.38

Source: N. M. Newmark and W. J. Hall, *Earthquake Spectra and Design*, Earthquake Engineering Research Institute, Berkeley, Calif., 1982, pp. 35 and 36.

TABLE 6.9.2 AMPLIFICATION FACTORS: ELASTIC DESIGN SPECTRA[a]

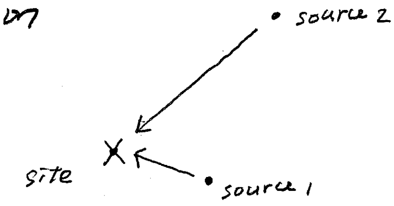
	Median (50th percentile)	One Sigma (84.1th percentile)
α_A	$3.21 - 0.68 \ln \zeta$	$4.38 - 1.04 \ln \zeta$
α_V	$2.31 - 0.41 \ln \zeta$	$3.38 - 0.67 \ln \zeta$
α_D	$1.82 - 0.27 \ln \zeta$	$2.73 - 0.45 \ln \zeta$

Source: N. M. Newmark and W. J. Hall, *Earthquake Spectra and Design*, Earthquake Engineering Research Institute, Berkeley, Calif., 1982, pp. 35 and 36.

[a]Damping ratio in percent.

1982 Newmark & Hall

Effect of frequency of earthquake motion



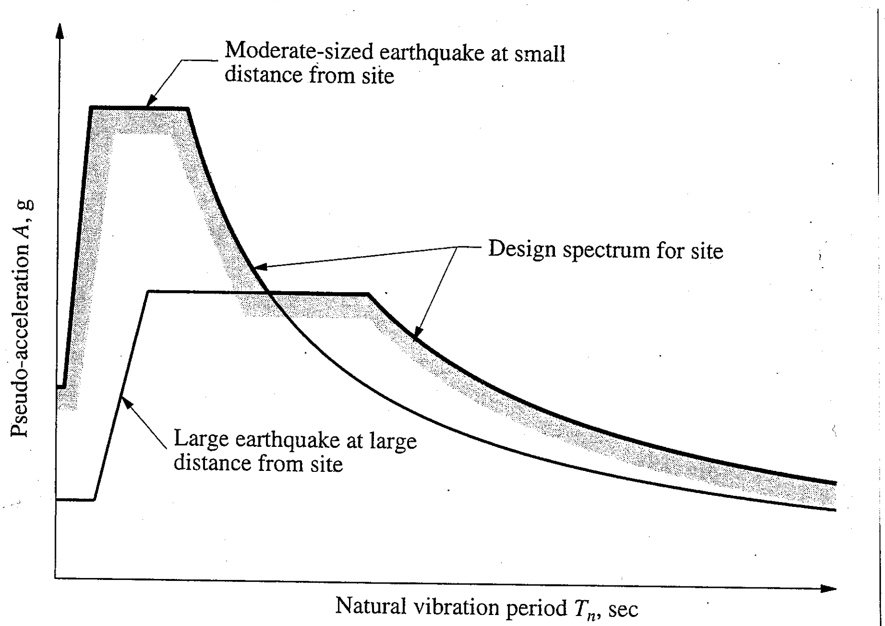
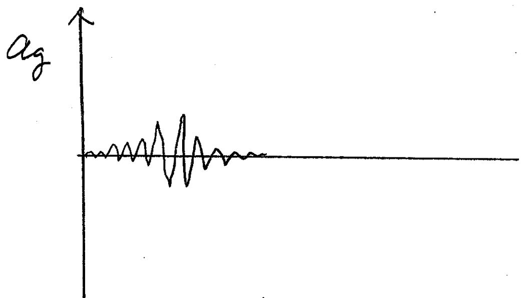


Figure 6.11.1 Design spectrum defined as the envelope of design spectra for earthquakes originating on two different faults.

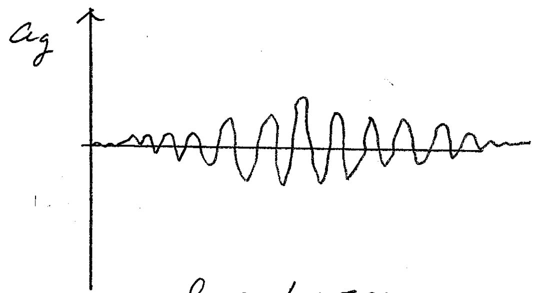


small duration

high frequency

Moderate sized EQ

at small distance



long duration

low frequency

Large EQ

at large distance

Influence of Soil Condition

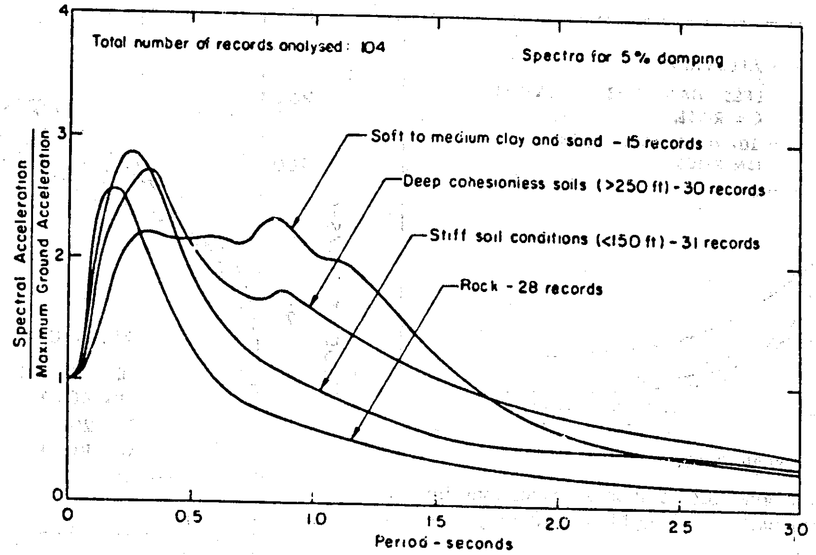


Figure 2-37 Average acceleration spectra for different site conditions. [After Seed et al. (2-68).]

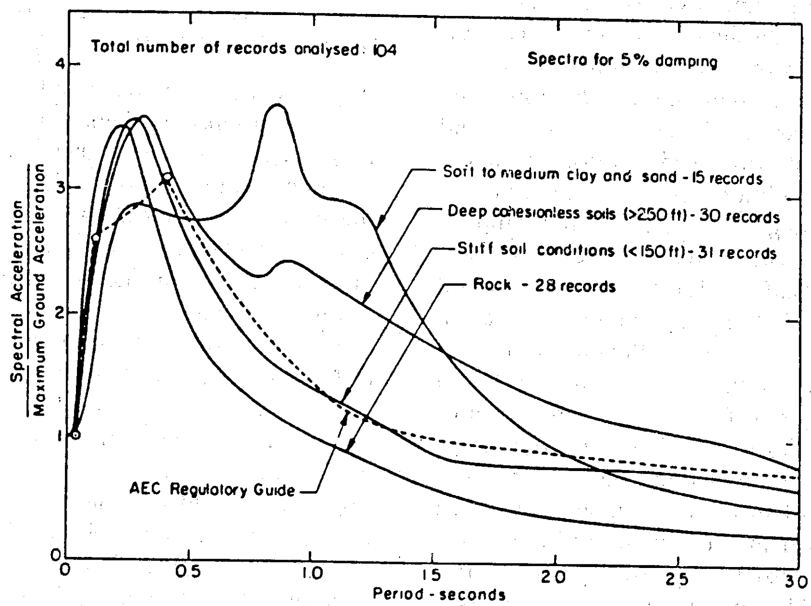


Figure 2-38 Mean plus one standard deviation acceleration spectra for different site conditions. [After Seed et al. (2-68).]

EQ in soft soil

⇒ long duration, low frequency

Effect of Depth of soft soil

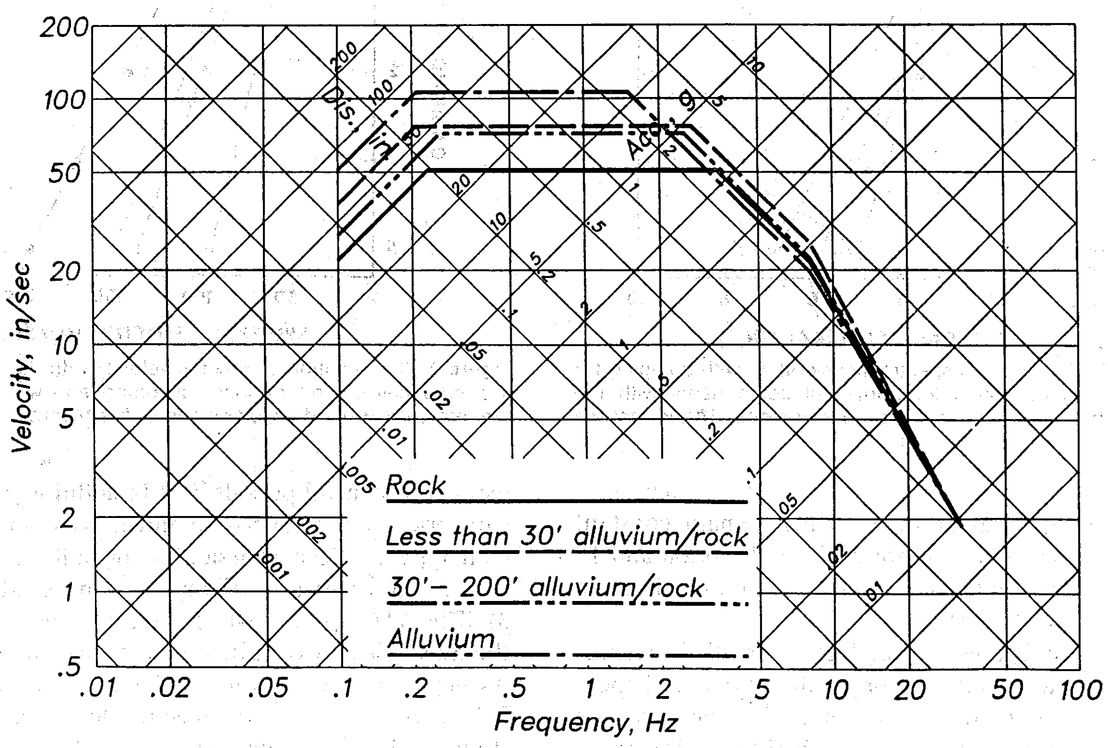
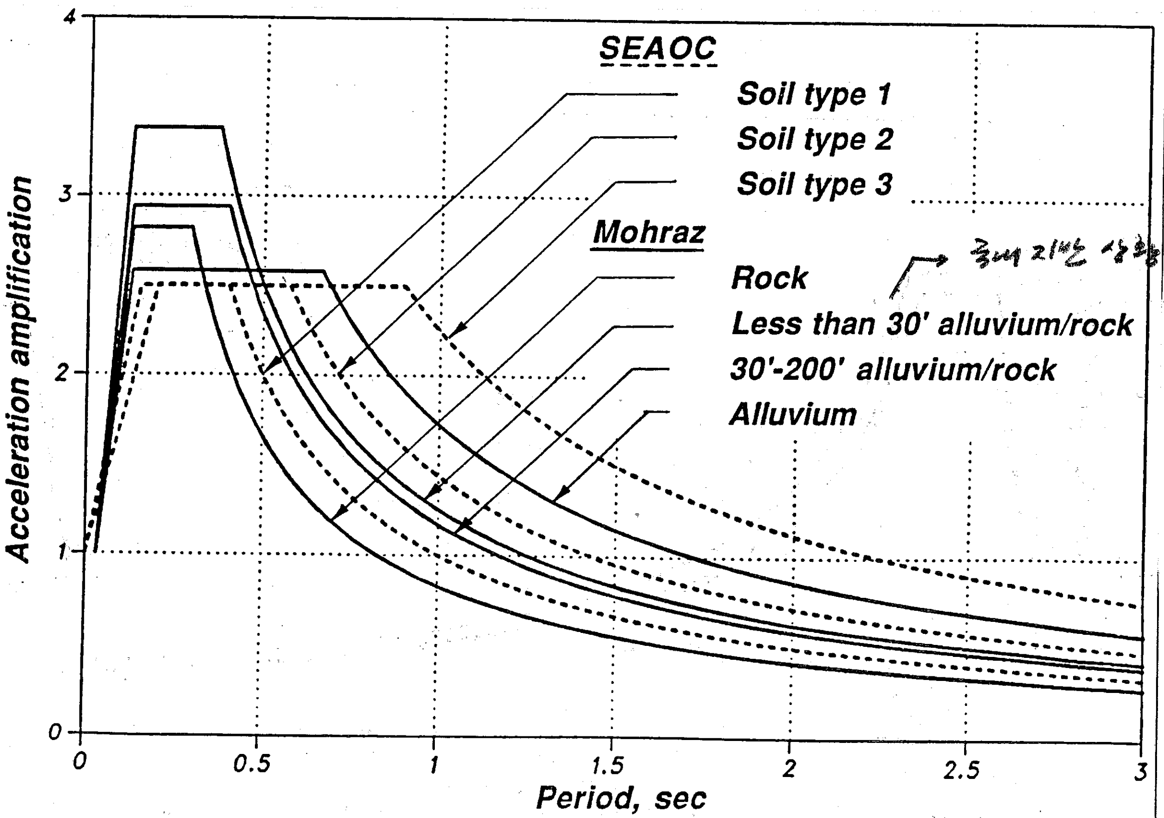


Figure 2-45 Design spectra for horizontal motion for 5% damping. [After Mohraz (2-33).]



국내 지반 상황과 비슷함

Figure 2-47 Comparison of spectral shapes for 5% damping proposed by Mohraz with those recommended by SEAOC.