Basis of Equivalent Static Method

Dynamic Theory

dynamic equilibrium : $\underset{\sim}{M}\ddot{\underset{\sim}{u}} + \underset{\sim}{C}\dot{\underset{\sim}{u}} + \underset{\sim}{K}\underset{\sim}{u} = -\underset{\sim}{M}\underset{\sim}{r}\ddot{u}_g(t)$

eigenvalues and eigenvectors : $(\underset{\sim}{K} - \omega^2 \underset{\sim}{M})\underset{\sim}{u} = 0 \Rightarrow \omega_i,\ \underset{\sim}{\phi_i}$

total response = sum of the modal responses : $\underset{\sim}{u} = \sum_i \phi_i Y_i = \underset{\sim}{\Phi}\underset{\sim}{Y}$

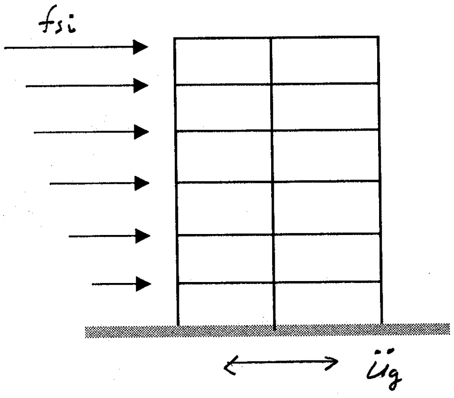
Modal decoupling

$$\underset{\sim}{\phi_i}^T \underset{\sim}{M}\underset{\sim}{\phi_i}\ddot{Y}_i + \underset{\sim}{\phi_i}^T \underset{\sim}{C}\underset{\sim}{\phi_i}\dot{Y}_i + \underset{\sim}{\phi_i}^T \underset{\sim}{K}\underset{\sim}{\phi_i}Y_i = \underset{\sim}{\phi_i}^T \underset{\sim}{M}\underset{\sim}{r}\ddot{u}_g(t)$$

Dynamic equation of each mode

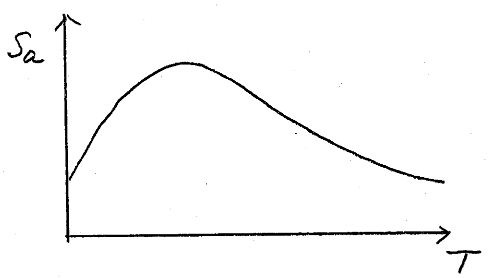
$$M_i^*\ddot{Y}_i + 2\zeta_i\omega_i M_i^*\dot{Y}_i + \omega_i^2 M^*_i Y_i = -L_i\ddot{u}_g(t)$$

$$\ddot{Y}_i + 2\zeta_i\omega_i\dot{Y}_i + \omega_i^2 Y_i = -\frac{L_i}{M_i^*}\ddot{u}_g(t)$$



Response spectrum

$$\ddot{y}_i + 2\zeta_i\omega_i\dot{y}_i + \omega_i^2 y_i = -\ddot{u}_g(t)$$



Maximum displacement of each mode estimated from response spectrum

$$Y_{i,\max} = \frac{L_i}{M_i^*} y_{i,\max} = \frac{L_i}{M_i^*} Sd(\zeta_i, W_i)$$

Story forces $\underset{\sim}{f}_{si} = \underset{\sim}{M}(\underset{\sim}{\phi_i}\ddot{Y}_{i\max}) = \underset{\sim}{M}\underset{\sim}{\phi_i}\omega^2 Y_{i\max}$

Base shear = sum of the story forces

$$V_{\substack{base,i\\ \max}} = \underbrace{\underset{\sim}{r}^T \underset{\sim}{M}\underset{\sim}{\phi_i}}_{L_i} \ \underbrace{\omega^2 Y_{i\max}}_{\frac{L_i}{M_i^*} S_a(\zeta_i, \omega_i)}$$

$$= \frac{L_i^2}{M_i^*} S_a(\zeta_i, \omega_i)$$

Effective weight of ith mode can be defined as follows.

$$W_i^{**} = \frac{L_i^2}{M_i^*} g$$

$$V_{base,i} = \frac{W_i^{**}}{g} S_a(\zeta_i, T_i)$$

Overall base shear

$$V_{base} = \sqrt{\sum V_{base,i}^2}$$

If i[st] mode is dominant,

$$V_{base} \approx V_{base,i} \quad \text{and} \quad W_i^{**} \approx W$$

Rayleigh Principle

For a continuum system,
Using energy conservation in free vibration,

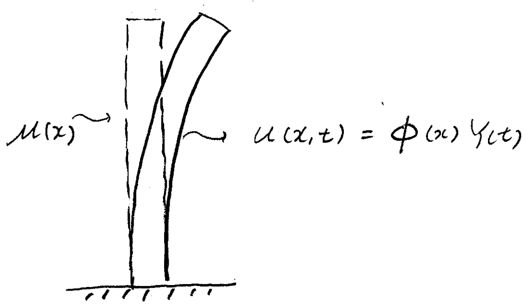
Energy = KE + PE = constant
T(t) + V(t) = constant

T(t) = kinetic energy
V(t) = potential energy

Assuming $u(x,t) = \phi(x)Y(t)$, and $Y(t) = Y_o \sin wt$

$$T(t) = \int_o^H \frac{1}{2}\mu(x)\left[\dot{u}(x,t)\right]^2 dx = \frac{1}{2}M^*\dot{Y}(t)^2$$

$$(M^* = \int_o^H \mu(x)\phi^2(x)dx)$$



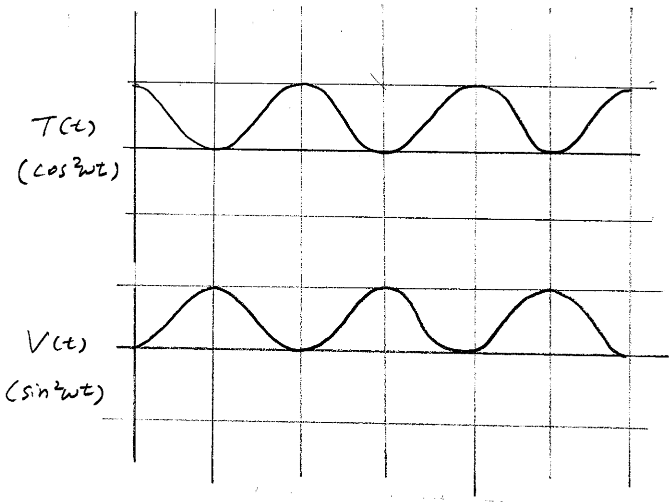
$$V(t) = \int_o^H \frac{1}{2}EI(x)\left[u''(x,t)\right]^2 dx = \frac{1}{2}K^*Y^2(t)$$

$$(K^* = \int_o^H EI(x)\phi''^2(x)dx)$$

In general ,

$$T(t) = \frac{1}{2}M^*\dot{Y}^2(t) = \frac{1}{2}M^*\omega^2 Y_o^2 \cos^2 \omega t$$

$$V(t) = \frac{1}{2}K^*Y^2(t) = \frac{1}{2}K^*Y_o^2 \sin^2 \omega t$$



T is a maximum when V is a minimum and vice-versa.

At any time, $T(t)+V(t) = \text{constant} = T_{\max} = V_{\max}$

Therefore, $\frac{1}{2}M^*\omega^2 = \frac{1}{2}K^*$

$$\omega^2 \approx \frac{K^*}{M^*}$$

When the shape function is an approximate, the angular frequency is an approximate value. The value of ω depends on the assumed shape function.

Rayleigh's Principle.

1)The Rayleigh Quotient is an upper bound to ω.

$$\Lambda = \sqrt{\frac{K^*}{M^*}} \geq \omega_{\text{actual}} \qquad \Rightarrow \text{ inverse iteration method / stodola method}$$

This is because the order of K^* is lower than the order of M^*.
Thus, K^*/M^* is overestimated.

2) The fundamental frequency is a true minimum point.
First-order change in $\phi(x)$ leads to second order changes in ω.

As a result, $\omega_{\text{approx.}}$ will be much closer to ω_{actual},

than $\phi(x)$ is to the true mode shape.

For a discrete system

Maximum kinetic energy of the structure

$$T_{max} = \frac{1}{2} \dot{\underset{\sim}{u}}^{T}_{max} \underset{\sim}{m} \dot{\underset{\sim}{u}}_{max} \qquad \left(\frac{1}{2} m\dot{u}^2 \right)$$

Maximum potential energy

$$V_{max} = \frac{1}{2} \underset{\sim}{u}^{T}_{max} \underset{\sim}{K} \underset{\sim}{u}_{max} \qquad \left(\frac{1}{2} ku^2 \right)$$

Set $\underset{\sim}{u}_{max} = \underset{\sim}{\phi} Y_{max}$, and $\dot{\underset{\sim}{u}}_{max} = \omega \underset{\sim}{\phi} Y_{max}$

$$T_{max} = \frac{1}{2} Y^2{}_{max} \omega^2 \underset{\sim}{\phi}^T \underset{\sim}{M} \underset{\sim}{\phi}$$

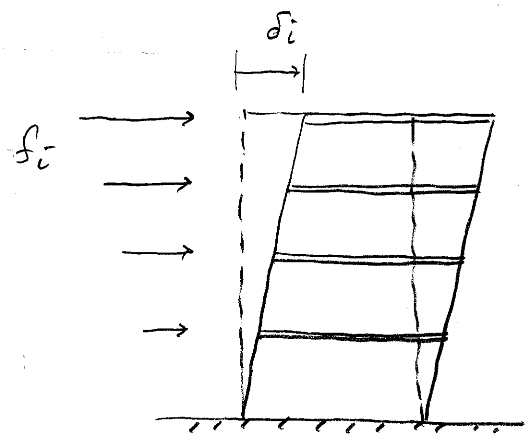
$$V_{max} = \frac{1}{2} Y^2_{max} \underset{\sim}{\phi}^T \underset{\sim}{K} \underset{\sim}{\phi}$$

From $T_{max} = V_{max}$

$$\omega^2 = \frac{\underset{\sim}{\phi}^T \underset{\sim}{K} \underset{\sim}{\phi}}{\underset{\sim}{\phi}^T \underset{\sim}{M} \underset{\sim}{\phi}} \quad \text{or} \quad T = 2\pi \sqrt{\frac{\underset{\sim}{\phi}^T \underset{\sim}{M} \underset{\sim}{\phi}}{\underset{\sim}{\phi}^T \underset{\sim}{K} \underset{\sim}{\phi}}}$$

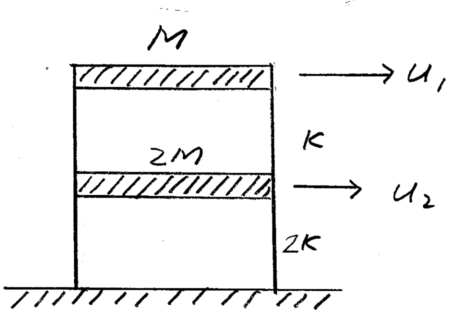
$$T \approx 2\pi \sqrt{\frac{\sum w_i \delta_i^2}{g \sum f_i \delta_i}}$$

$$(\underset{\sim}{K} \underset{\sim}{\delta} = \underset{\sim}{f})$$



Exact solution for multi-degree of system (Iterative method)

$$\underset{\sim}{\mathbf{M}} = \begin{bmatrix} M & 0 \\ 0 & 2M \end{bmatrix} \qquad \underset{\sim}{\mathbf{K}} = \begin{bmatrix} K & -K \\ -K & 3K \end{bmatrix}$$



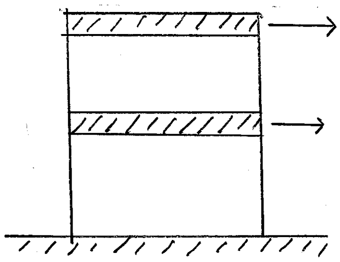
shear building
⇒ only shear deformation is considered.

$$\underset{\sim}{u} = \underset{\sim}{\phi} Y(t)$$

1) Assume $\phi^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\wedge^2 = \frac{\underset{\sim}{\phi}^T \underset{\sim}{K} \underset{\sim}{\phi}}{\underset{\sim}{\phi}^T \underset{\sim}{M} \underset{\sim}{\phi}} = \frac{\langle 1 \;\; 1 \rangle \begin{bmatrix} K & -K \\ -K & 3K \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\langle 1 \;\; 1 \rangle \begin{bmatrix} M & 0 \\ 0 & 2M \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}} = \frac{2K}{3M} = \omega_1^2$$

2) Inertia forces from $\underset{\sim}{u}^{(0)}$



$$M\ddot{u}_1^{(0)} = M\omega^2 Y(t) \qquad \underset{\sim}{p}^{(0)} = \omega^2 \begin{bmatrix} M \\ 2M \end{bmatrix} Y(t)$$

$$2M\ddot{u}_2^{(0)} = 2M\omega^2 Y(t)$$

3) Find deflected shape from the inertia forces:

$$\underset{\sim}{K}\underset{\sim}{u}^{(1)} = \underset{\sim}{p}^{(0)}, \qquad \underset{\sim}{K}\underset{\sim}{u}^{(1)} = \omega^2 \underset{\sim}{M}\underset{\sim}{u}^{(0)}$$

$$\begin{bmatrix} K & -K \\ -K & 3K \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} M \\ 2M \end{bmatrix} Y(t)\omega^2 \qquad \begin{bmatrix} K & -K \\ 0 & 2K \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} M \\ 3M \end{bmatrix} Y(t)\omega^2$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 3/2 \end{bmatrix} \frac{M}{K} Y(t)\omega^2 \Rightarrow \underset{\sim}{\phi}^{(1)} = \begin{bmatrix} 5/2 \\ 3/2 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ 3/5 \end{bmatrix}$$

$$\wedge^2 = \frac{\underset{\sim}{\phi}^T \underset{\sim}{K} \underset{\sim}{\phi}}{\underset{\sim}{\phi}^T \underset{\sim}{M} \underset{\sim}{\phi}} = \frac{<1 \quad 3/5> \begin{bmatrix} K & -K \\ -K & 3K \end{bmatrix} \begin{bmatrix} 1 \\ 3/5 \end{bmatrix}}{<1 \quad 3/5> \begin{bmatrix} M & 0 \\ 0 & 2M \end{bmatrix} \begin{bmatrix} 1 \\ 3/5 \end{bmatrix}} = \frac{0.88K}{1.72M}$$

$$\omega_1^2 = 0.5116 K/M$$

Iterative calculations until $\underset{\sim}{\phi}^{(0)} = \underset{\sim}{\phi}^{(1)}$ $(\underset{\sim}{\phi}^{(1)} // \underset{\sim}{\phi}^{(1)})$

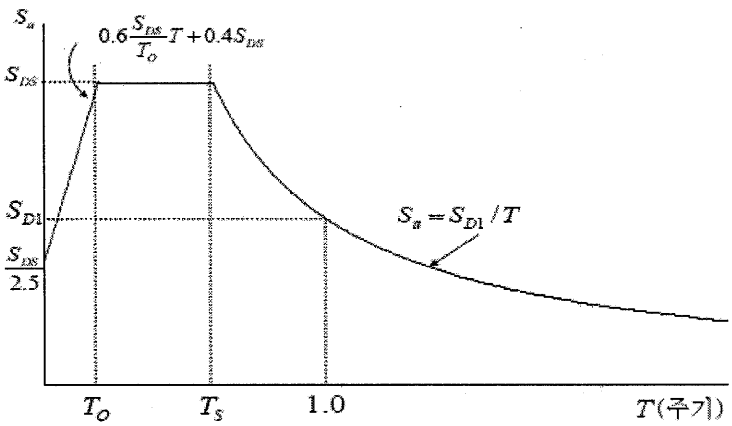
Exact solution : $\omega_1^2 = 0.5 K/M \qquad \underset{\sim}{\phi}_1 = \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$

Modal Analysis $\quad (K - \omega^2 \underset{\sim}{M}) \underset{\sim}{\phi} = \underset{\sim}{0}$ $\quad$ ➔ exact solution

$$|\underset{\sim}{K} - \omega^2 \underset{\sim}{M}| = 0$$

Modal superposition method using design response spectrum

1) Design Response Spectrum



$T_o = 0.2 \quad S_{D1} / S_{DS}$
$T_S = \quad S_{D1} / S_{DS}$

2) the number of modes required to be considered

All significant modes to reach 90% of the participating mass of the structure

3) modal combination

SRSS (Square Root of Sum of Square)

CQC (Complete Quadratic Combination)

4) Modification

$$C_m = 0.85\frac{V}{V_t} \geq 1.0$$

V = base shear calculated by equivalent static method,
based on $C_u T_a$ $(1.4 \leq C_u \leq 1.7)$

Liner Elastic Dynamic Response of Multi-degree of freedom (MDOF) Systems subjected to earthquake motion

$$\underset{\sim}{M}\ddot{\underset{\sim}{u}} + \underset{\sim}{C}\dot{\underset{\sim}{u}} + \underset{\sim}{K}\underset{\sim}{u} = \underset{\sim}{P} = -\underset{\sim}{M}\underset{\sim}{r}\ddot{u}_g(t) \qquad (1)$$

Eigenvalue Problem : $(\underset{\sim}{K} - \omega^2 \underset{\sim}{M})\underset{\sim}{u} = \underset{\sim}{0}$

=> ω_i : i^{th} eigenvalue

ϕ_i : corresponding eigenvector

$$\underset{\sim}{u} = \sum_i \underset{\sim}{\phi}_i Y_i = \underset{\sim}{\Phi}\underset{\sim}{Y} \quad (\therefore Y_i \text{ are bases }) \qquad (2)$$

From (1) and (2)

$$\underset{\sim}{\Phi}^T \underset{\sim}{M}\underset{\sim}{\Phi}\ddot{\underset{\sim}{Y}} + \underset{\sim}{\Phi}^T \underset{\sim}{C}\underset{\sim}{\Phi}\dot{\underset{\sim}{Y}} + \underset{\sim}{\Phi}^T \underset{\sim}{K}\underset{\sim}{\Phi}\underset{\sim}{Y} = -\underset{\sim}{\Phi}^T \underset{\sim}{M}\underset{\sim}{r}\ddot{u}_g(t) \qquad (3)$$

Using the orthogonality of eigenvectors,

$$\underset{\sim}{\phi}_i^T \underset{\sim}{M}\underset{\sim}{\phi}_j = 0, \qquad \underset{\sim}{\phi}_i^T \underset{\sim}{K}\underset{\sim}{\phi}_j = 0$$

Also it is assumed $\underset{\sim}{\phi}_i^T \underset{\sim}{C}\underset{\sim}{\phi}_j = 0$

The dynamic equilibrium equation (3) of a n dof system can be decoupled into n sdof systems.

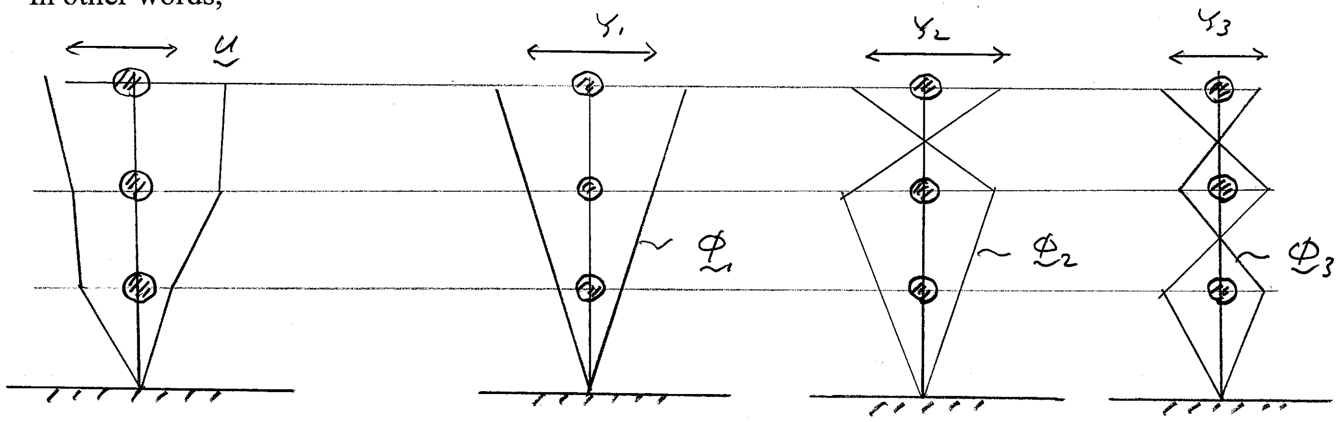
$$\underset{\sim}{\phi}_1^T \underset{\sim}{M}\underset{\sim}{\phi}_1 \ddot{Y}_1 + \underset{\sim}{\phi}_1^T \underset{\sim}{C}\underset{\sim}{\phi}_1 \dot{Y}_1 + \underset{\sim}{\phi}_1^T \underset{\sim}{K}\underset{\sim}{\phi}_1 \underset{\sim}{Y}_1 = -\underset{\sim}{\phi}_1^T \underset{\sim}{M}\underset{\sim}{r}\ddot{u}_g$$

$$\underset{\sim}{\phi}_2^T \underset{\sim}{M}\underset{\sim}{\phi}_2 \ddot{Y}_2 + \underset{\sim}{\phi}_2^T \underset{\sim}{C}\underset{\sim}{\phi}_2 \dot{Y}_2 + \underset{\sim}{\phi}_2^T \underset{\sim}{K}\underset{\sim}{\phi}_2 \underset{\sim}{Y}_2 = -\underset{\sim}{\phi}_2^T \underset{\sim}{M}\underset{\sim}{r}\ddot{u}_g$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\underset{\sim}{\phi}_n^T \underset{\sim}{M}\underset{\sim}{\phi}_n \ddot{Y}_n + \underset{\sim}{\phi}_n^T \underset{\sim}{C}\underset{\sim}{\phi}_n \dot{Y}_n + \underset{\sim}{\phi}_n^T \underset{\sim}{K}\underset{\sim}{\phi}_n \underset{\sim}{Y}_n = -\underset{\sim}{\phi}_n^T \underset{\sim}{M}\underset{\sim}{r}\ddot{u}_g$$

In other words,



By eigenvalue problem, a n dof system is decomposed into n number of one dof system. The dynamic motion of the original system is calculated by superposition of the dynamic motions of the decomposed n-one dof systems.

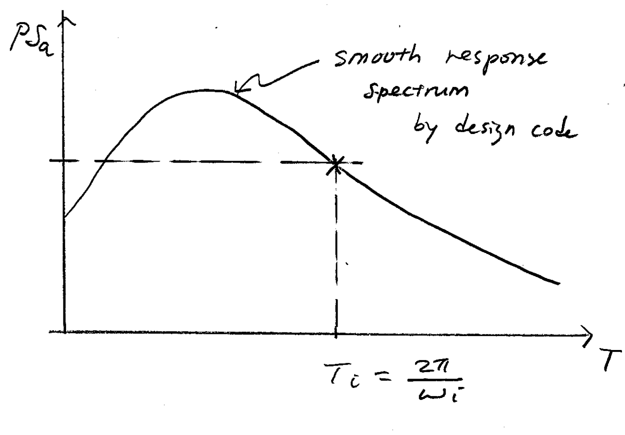
For ith mode,

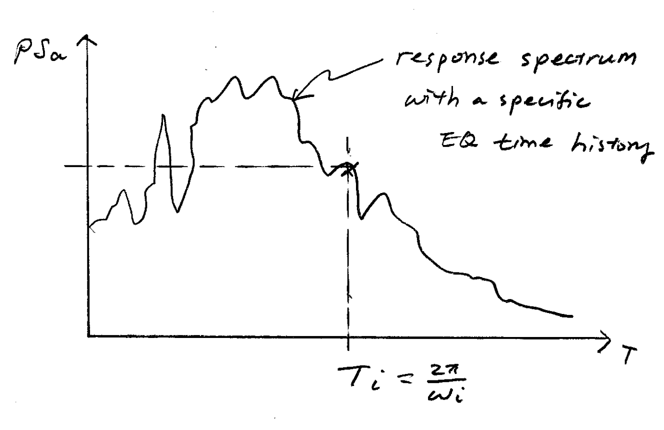
$$\Phi_i^T M \Phi_i \ddot{Y}_i + \Phi_i^T C \Phi_i \dot{Y}_i + \Phi_i^T K \Phi_i Y_i = -\Phi_i^T M r \ddot{u}_g \qquad (4)$$

$$M^*_i \ddot{Y}_i + 2\zeta_i \omega_i m_i^* \dot{Y}_i + \omega_i^2 m_i^* Y_i = -L_i \ddot{u}_g$$

$$\text{or} \quad \ddot{Y}_i + 2\zeta_i \omega_i \dot{Y}_i + \omega_i^2 m_i^* Y_i = -\frac{L_i}{M_i^*} \ddot{u}_g \qquad (5)$$

If a response spectrum for one dof system with $\zeta = \zeta_i$ is provided by design code, or if a response spectrum is readily made with a specified EQ time history.





Then $Y_{i\max} = \frac{L_i}{M_i^*} S_d(\zeta_i, T_i)$

or $Y_{i\max} \approx \frac{L_i}{M_i^* \omega_i^2} PS_a(\zeta_i, T_i)$ $\quad \underset{\sim}{u}_i = \Sigma \underset{\sim}{\phi}_i Y_i$ (Modal analysis)

$\underset{\sim}{u}_{i,\max} = \underset{\sim}{\phi}_i Y_{i,\max}$ if we use SRSS combination method.

$u_{\max}^{\text{level k}} = \sqrt{u_{1,\max}^{2\ \text{level k}} + u_{2,\max}^{2\ \text{level k}} + \cdots + u_{n,\max}^{2\ \text{level k}}}$

Member actions for j[th] member

$\underset{\sim}{S}^j_{i,\max} = \underset{\sim}{K}^j \underset{\sim}{u}^j_{i,\max}$

$S^{j,k}_{\max} = \sqrt{S^{2\ j,k}_{1\max} + S^{2\ j,k}_{2\max} + \ldots + S^{2\ j,k}_{n\max}}$ $\quad$ For k[th] dof of j[th] member

Generally, we do not consider the total number of eigenmode that is the same as the total number of d.o.f..
Since the contribution of the lower modes to the base shear is much larger than that of the higher mode, we usually use several low modes.

From (5)

$$\ddot{Y}_i + 2\zeta_i \omega_i \dot{Y}_i + \omega_i^2 m_i^* Y_i = -\frac{L_i}{M_i^*} \ddot{u}_g$$

The response Y_i depends on L_i $(= \underset{\sim}{\phi}_i^T \underset{\sim}{M} \underset{\sim}{r})$

$$L_1 = \phi_1 m_1 + \phi_2 m_2 + \phi_3 m_3$$
$$L_3 = \phi_1 m_1 - \phi_2 m_2 + \phi_3 m_3$$

If $m_1 = m_2 = m_3$, $L_1 > L_3$

$\Rightarrow V_{1base} > V_{3base}$ $\quad$ (base shear is proportional to $\frac{L_i^2}{M_i^*} S_a$)

Number of modes considered for EQ Response

Use of modal analysis also permits an estimate of the number of modes which must be included to capture the essential part of the response. First, review some definitions:

Modal mass participation factor:

$$PF_i = \frac{\underset{\sim}{\phi}_i^T \underset{\sim}{M} \underset{\sim}{r}}{\underset{\sim}{\phi}_i^T \underset{\sim}{M} \underset{\sim}{\phi}_i} = \frac{\phi_i^T \underset{\sim}{M} \underset{\sim}{r}}{M_i^*} = \frac{L_i}{M_i^*}$$

If the eigenvectors are normalized so that $M_i^* = 1$

$$PF_i = \underset{\sim}{\phi}_i^T \underset{\sim}{M} \underset{\sim}{r}$$

Resulted from response spectrum analysis

$$V_{base} = \frac{L_i^2}{M_i^*} S_a = L_i^2 S_a$$

Which will be compared with $V_{base} = \sum M\ S_a$

Modal effective mass factor

$$EM_i = \frac{PF_i^2}{\sum M} \qquad (\sum_{all} PF_i^2 = \sum_{all} M)$$

If we use k number of modes and if $\sum_i^k EM_i$ is close to 1, we can say that the analytical result is accurate enough.

Generally, we use k number of modes so that $\sum_i^k EM_i \geq 0.9$

$$\sum \frac{L_i^2}{M_i^*} = \underset{\sim}{L}^T \underset{\sim}{M}^{*-1} \underset{\sim}{L} \qquad \underset{\sim}{L}^T = \left[L_1 L_2 L_3 L_4 \cdots L_n \right]$$

$$= \left(\underset{\sim}{\Phi}^T \underset{\sim}{M} \underset{\sim}{r} \right)^T \left(\underset{\sim}{\Phi}^T \underset{\sim}{M} \underset{\sim}{\Phi} \right)^{-1} \left(\underset{\sim}{\Phi}^T \underset{\sim}{M} \underset{\sim}{r} \right)$$

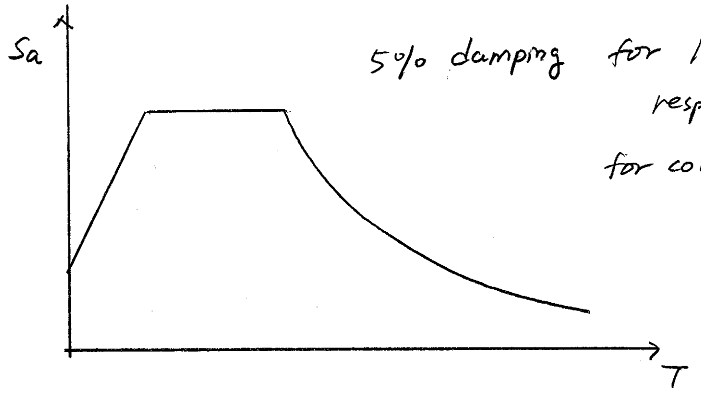
$$= \left(\underset{\sim}{r}^T \underset{\sim}{M} \underset{\sim}{\Phi} \right) \left(\underset{\sim}{\Phi}^{-1} \underset{\sim}{M}^{-1} \underset{\sim}{\Phi}^{-T} \right) \left(\underset{\sim}{\Phi}^T \underset{\sim}{M} \underset{\sim}{r} \right)$$

$$= \underset{\sim}{r}^T \underset{\sim}{M} \underset{\sim}{r} = \sum \underset{\sim}{M}$$

Sources of damping in structures

1) viscous effects – fluid friction   proportional to velocity
2) coulomb friction (constant drag) – dry friction   proportional to pressure
3) structural damping (friction proportional to displacement)
4) elastic hysteresis caused by rote effects
5) inelastic effects (cracking, yielding, etc)

Material	Damping coefficient in fundamental mode		
	Undamaged	Some damage	Severe damage
Steel	1%	3%	5%
RC	3%	5%	7%
Masonry	5%	7%	10%
wood	3%	5%	7%



5% damping for linear elastic response spectrum for collapse limit state

Time history Analysis

Considering the uncertainties of time history of EQ ground motions, multiple records should be used to assure the safety of structures

Number of time history records = 3

when earthquake design is performed for the maximum response values.

= 7

when earthquake design is performed for the average response values.

A pair of x and y directions should be used per each record.

The amplitude of the records should be greater than the demand required in the code-specified design response spectrum :

SRSS of the two (x and y) response spectral values of EQ > 0.9 x 1.3 x response spectrum of design code, in the range of 0.2T < T < 1.5 T where T = fundamental dynamic period of the structure.

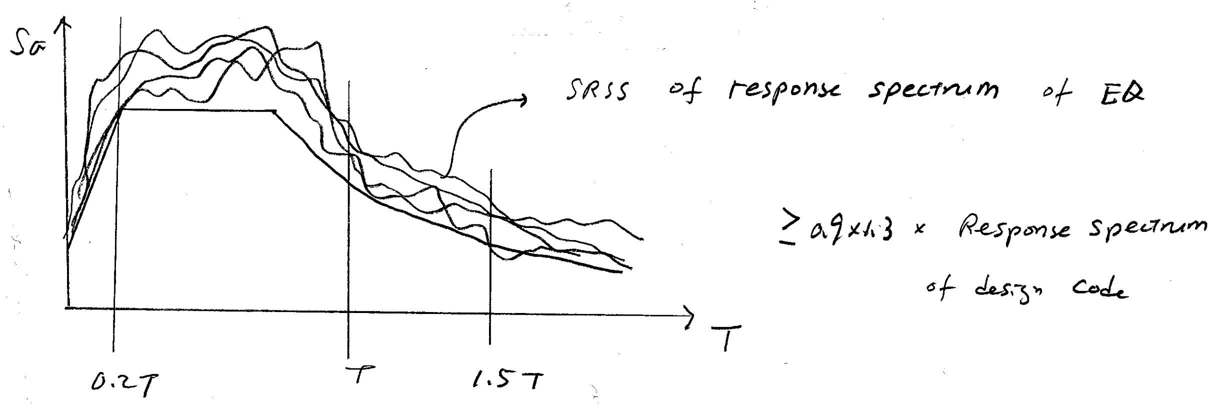
Artificial time history

Based on the previous histories of earthquakes, artificial time history can be made using assumptions of frequency and durations of earthquakes.

Relevant web site : http://www.daveboore.com/software_online.html (Prof. David Boore)

SIMQKE (1999) : Gasparini & Vanmarche

(artificial earthquake based on random vibration)



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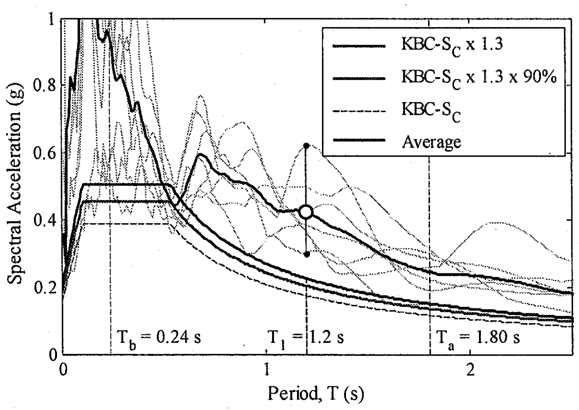
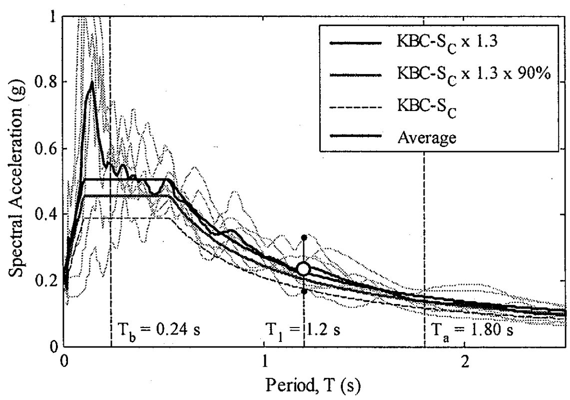


Bad choice and scaling



good choice and scaling

Direct Integration Method

Incremental Dynamic Equilibrium Equation (for MDOF)

$$\underset{\sim}{M}\Delta\ddot{\underset{\sim}{u}}(t)+\underset{\sim}{c}\Delta\dot{\underset{\sim}{u}}(t)+\underset{\sim}{K}\Delta\underset{\sim}{u}(t)=\Delta\underset{\sim}{P}(t)$$

$$=-\underset{\sim}{M}\underset{\sim}{r}\Delta\ddot{u}_g(t)$$

Here, we have 3 unknown vectors $\Delta\ddot{\underset{\sim}{u}}$, $\Delta\dot{\underset{\sim}{u}}$, $\Delta\underset{\sim}{u}$ and one equation vector. Therefore we need to reduce 3 unknown vectors to one unknown vector.

We assume $\Delta\ddot{\underset{\sim}{u}}(t) =$ function of $(\dot{\underset{\sim}{u}}(t), \ddot{\underset{\sim}{u}}(t), u(t), \Delta\underset{\sim}{u}(t))$

$\Delta\dot{\underset{\sim}{u}}(t) =$ function of $(\dot{\underset{\sim}{u}}(t), \ddot{\underset{\sim}{u}}(t), u(t), \Delta\underset{\sim}{u}(t))$

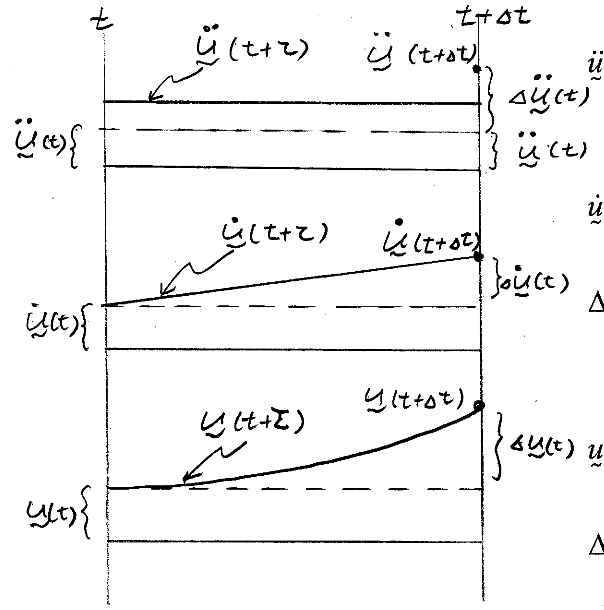
The assumptions are constant acceleration method
Linear acceleration method
Wilson - θ method etc.

. linear acceleration method : conditionally stable, $\Delta t/T_n \le 1/1.8$

. constant acceleration method: unconditionally stable

Constant Average acceleration Method (Newmark)

Assume $\underset{\sim}{\ddot{u}}$ (t) in constant over interval.



$$\underset{\sim}{\ddot{u}}(t+\tau)=\underset{\sim}{\ddot{u}}(t)+\frac{\Delta\underset{\sim}{\ddot{u}}(t)}{2} \quad (1)$$

$$\underset{\sim}{\dot{u}}(t+\tau)=\underset{\sim}{\dot{u}}(t)+\underset{\sim}{\ddot{u}}(t)\tau+\frac{\Delta\ddot{u}(t)\tau}{2}$$

$$\Delta\underset{\sim}{\dot{u}}(t)=\underset{\sim}{\ddot{u}}\Delta t+\frac{\Delta\ddot{u}(t)}{2}\Delta t \quad (2)$$

$$\underset{\sim}{u}(t+\tau)=\underset{\sim}{u}(t)+\underset{\sim}{\dot{u}}(t)\tau+\underset{\sim}{\ddot{u}}(t)\frac{\tau^2}{2}+\frac{\Delta\ddot{u}(t)\tau^2}{4}$$

$$\Delta\underset{\sim}{u}(t)=\underset{\sim}{\dot{u}}(t)\Delta t+\frac{\underset{\sim}{\ddot{u}}(t)\Delta t^2}{2}+\frac{\Delta\ddot{u}(t)\Delta t^2}{4} \quad (3)$$

From (3) $$\Delta\underset{\sim}{\ddot{u}}(t)=\frac{4}{\Delta t^2}\left(\Delta\underset{\sim}{u}(t)-\underset{\sim}{\dot{u}}(t)\Delta t-\underset{\sim}{\ddot{u}}(t)\frac{\Delta t^2}{2}\right) \quad (4)$$

Substitute (4) into (2)

$$\Delta\underset{\sim}{\dot{u}}(t)=\underset{\sim}{\ddot{u}}(t)\Delta t+\frac{2}{\Delta t}\left(\Delta\underset{\sim}{u}-\underset{\sim}{\dot{u}}(t)\Delta t-\underset{\sim}{\ddot{u}}(t)\frac{\Delta t^2}{2}\right) \quad (5)$$

Now plug (4) and (5) into

$$\underset{\sim}{m}\Delta\underset{\sim}{\ddot{u}}(t)+\underset{\sim}{c}\Delta\underset{\sim}{\dot{u}}(t)+\underset{\sim}{k}\Delta\underset{\sim}{u}(t)=-\underset{\sim}{M}\underset{\sim}{r}\Delta\ddot{u}_g(t)$$

$$\underset{\sim}{M}\left(\frac{4}{\Delta t^2}\right)\left[\Delta\underset{\sim}{u}(t)-\dot{u}(t)\Delta t-\underset{\sim}{\ddot{u}}(t)\frac{\Delta t^2}{2}\right]+\underset{\sim}{C}\left[\ddot{u}(t)\Delta t+\frac{2}{\Delta t}\left(\Delta\underset{\sim}{u}(t)-\underset{\sim}{\dot{u}}(t)\Delta t-\underset{\sim}{\ddot{u}}(t)\frac{\Delta t^2}{2}\right)\right]+\underset{\sim}{K}\Delta\underset{\sim}{u}(t)=-\underset{\sim}{M}\underset{\sim}{r}\Delta\ddot{u}_g(t)$$

Regrouping the terms for $\Delta\underset{\sim}{u}(t)$

$$\Delta\underset{\sim}{u}(t)\underbrace{\left[\frac{4}{\Delta t^2}\underset{\sim}{M}+\frac{2}{\Delta T}\underset{\sim}{C}+\underset{\sim}{K}\right]}_{\overline{\underset{\sim}{K}}}=\underbrace{-\underset{\sim}{M}\underset{\sim}{r}\Delta\ddot{u}_g+\underset{\sim}{M}\left[\frac{4}{\Delta t}\underset{\sim}{\dot{u}}(t)+2\underset{\sim}{\ddot{u}}(t)\right]+\underset{\sim}{C}\left[2\underset{\sim}{\dot{u}}(t)\right]}_{\overline{\underset{\sim}{P}}}$$

$\overline{\underset{\sim}{K}}\Delta\underset{\sim}{u}(t)=\Delta\overline{\underset{\sim}{P}}$ solve $\Delta\underset{\sim}{u}(t)$

Evaluation of Damping for MDOF systems solution by Direct Integration.

When a MDOF system is solved by direct Integration, we cannot work directly with ζ_i (damping ratio), since the eigenvalue problem is never solved. We must compute $\underset{\sim}{C}$ directly.

Rayleigh Damping

Assume $\underset{\sim}{C}=a_0\underset{\sim}{M}+a_1\underset{\sim}{K}$

Since the orthogonality of eigenvectors are effective for $\underset{\sim}{M}$ and $\underset{\sim}{K}$

$$C_1^* = 2\zeta_1\omega_1 M_1^* = a_0 M_1^* + a_1 K_1^* = a_0 M_1^* + a_1\omega_1^2 M_1^* \qquad (1)$$

$$C_2^* = 2\zeta_2\omega_2 M_2^* = a_0 M_2^* + a_1 K_2^* = a_0 M_2^* + a_1\omega_2^2 M_2^* \qquad (2)$$

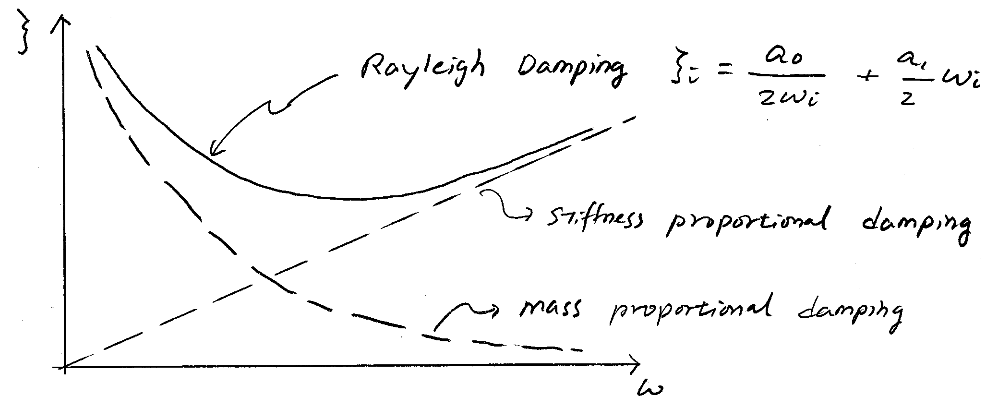
This gives us 2 equations in a_0 and a_1

$$\zeta_1 = \frac{a_0}{2\omega_1} + \frac{a_1}{2}\omega_1$$

$$\zeta_2 = \frac{a_0}{2\omega_2} + \frac{a_1}{2}\omega_2$$

Solve for a_0 in terms of ζ_1, ζ_2

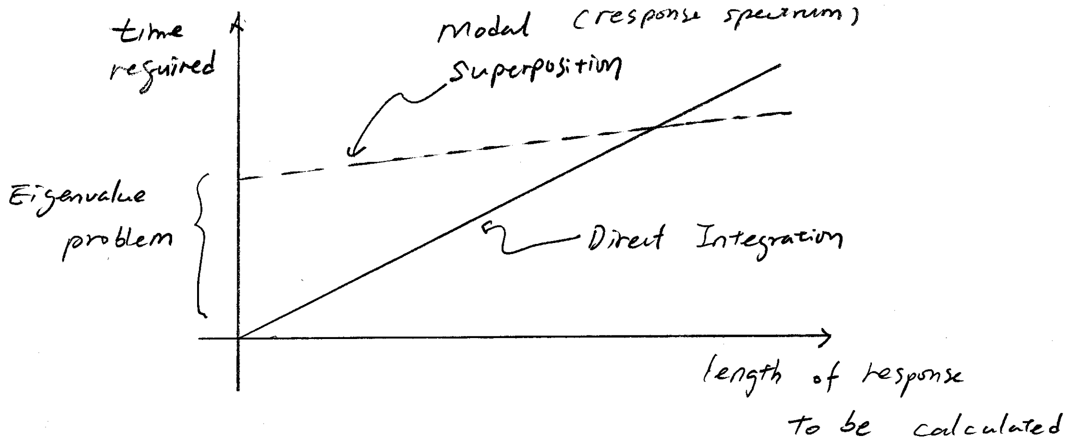
This kind of damping can determine ζ_i for other modes



ζ increase for increasing ω (higher modes), roughly agreeing with experimental observation.

Comparison between modal superposition and Direct Integration

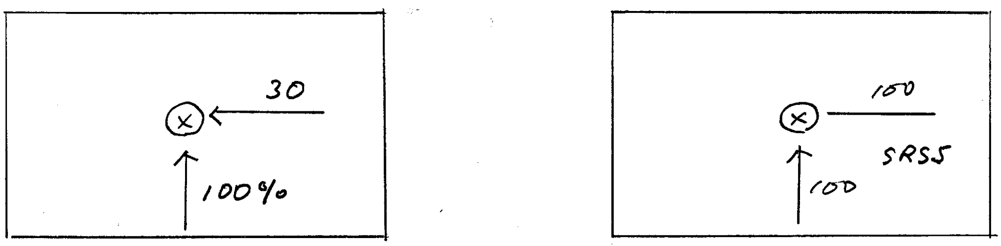
Modal superposition	Direct Integration
1) linear system only	1) linear and nonlinear systems
2) easy to specify ζ_i	2) hard to specify $\underset{\sim}{c}$ more complex than Rayleigh damping
3) must solve eigenvalue	3) no eigenvalue problem
4) can use response spectra	4) can't use spectra
5) Efficient for lengthy response	5) efficient for short-duration response



Direction of EQ load

In reality, earthquake motions are the three dimensional motion: x- directional motion should be combined with y-directional motion and, even with the vertical motion.

For design categories 'C' and 'D', the combination of two directional loadings is considered.

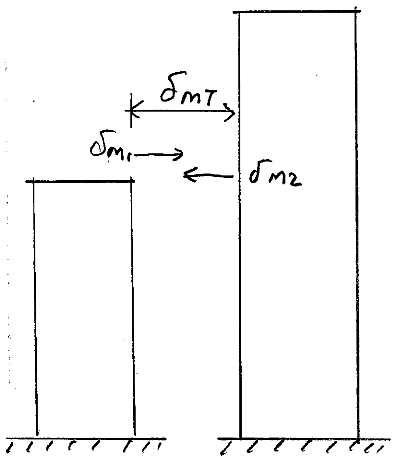


두 방향 지진이 서로 독립적 이라고 가정하여
(상관계수 0.3 이하) 두 방향 하중에 대한 독립된
응답의 조합을 사용.

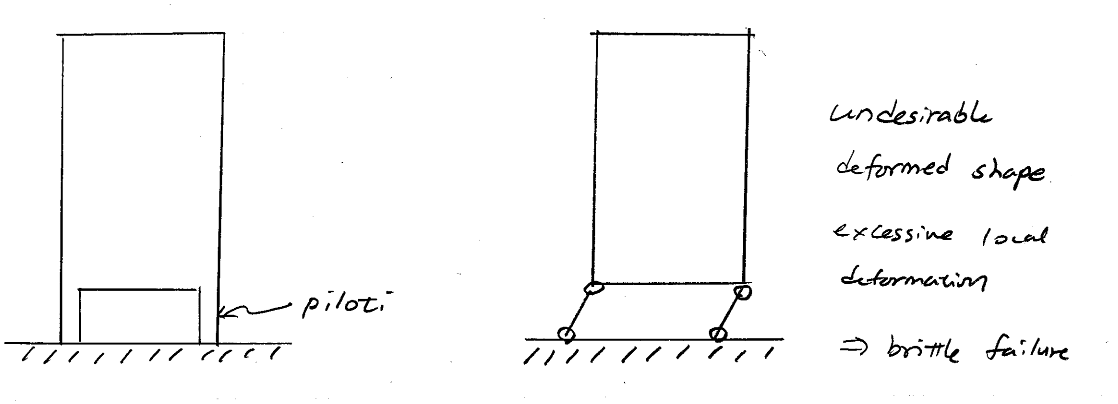
Building separation

For 'D' Type building, to avoid collision of adjacent buildings,

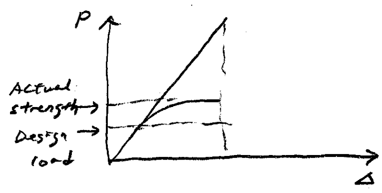
$$\delta_{MT} = \sqrt{(\delta_{M1})^2 + (\delta_{M2})^2}$$



Special EQ load

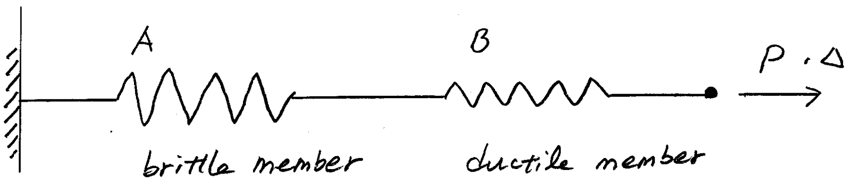


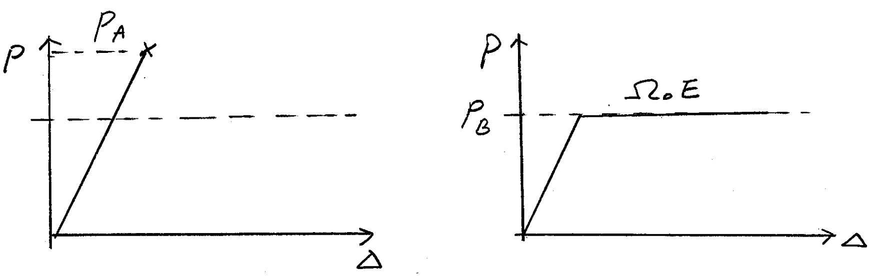
Basically, in earthquake design codes, the R factor is used, assuming a certain degree of ductility is assured for structural system. Thus, if the target ductility cannot be obtained, the R factor should be decreased. In particular, if the use of a brittle member (with extremely small ductility) is unavoidable within the load-transfer mechanism, the brittle member should be sufficiently strong to withstand the ultimate member force which can be developed by the overstrength of other ductile members.



$$E_m = \Omega_0 E \pm 0.2 S_{DS} . D$$

Ω_0 = overstrength factor





$$P_A > P_B = \Omega_0 E$$