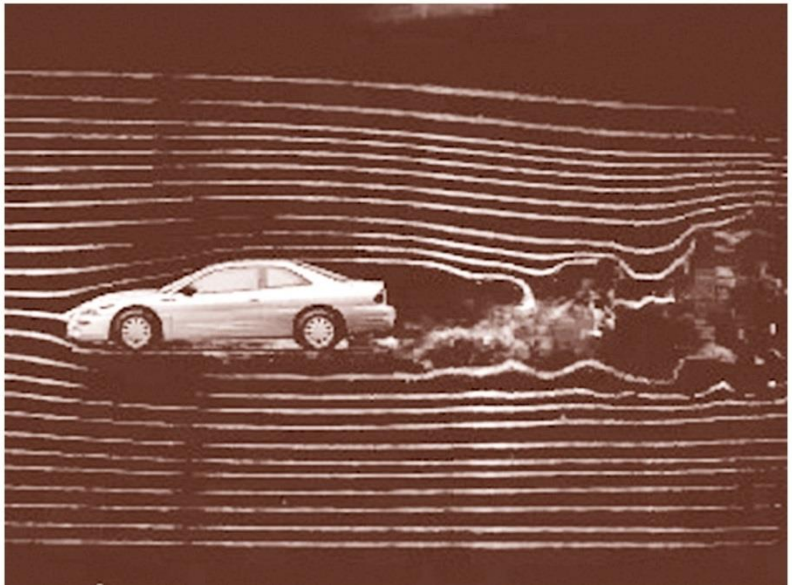
Lecture 20

Origin of Turbulence (3)



Lecture 20 Origin of Turbulence (3)

Contents

Objectives

- Learn fundamental concept of turbulence
- Study Reynolds decomposition
- Derive Reynolds equation from Navier-Stokes equation
- Study eddy viscosity model and mixing length model

20.1 Velocities and Energies in Turbulence

20.1.1 Reynolds decomposition

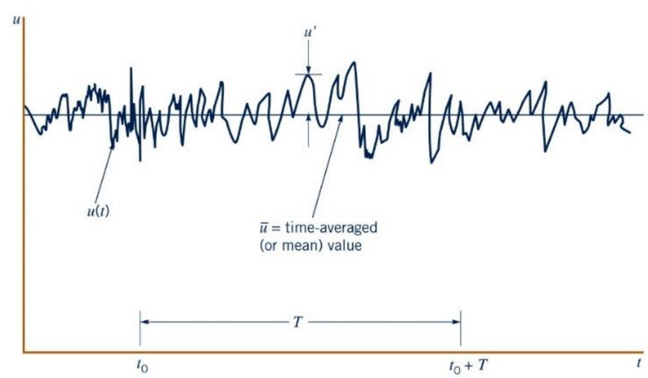
(1) Velocity decomposition

- The variables in a turbulent flow are described using the theory of stochastic process and random variables even though fluid-dynamic turbulence is not entirely random.

- instantaneous velocity = time-averaged value + fluctuating components

$$u_i(x,y,z,t) = \overline{u_i}(x,y,z) + u_i'(x,y,z,t) \qquad (20.1)$$

$$\overline{u_i} = \frac{1}{T}\int_0^T u_i \, dt \qquad (20.2)$$



20.1 Velocities and Energies in Turbulence

T = long time compared to the time scale of the turbulence

- *T* should include (smooth out) all the effects of turbulence fluctuation

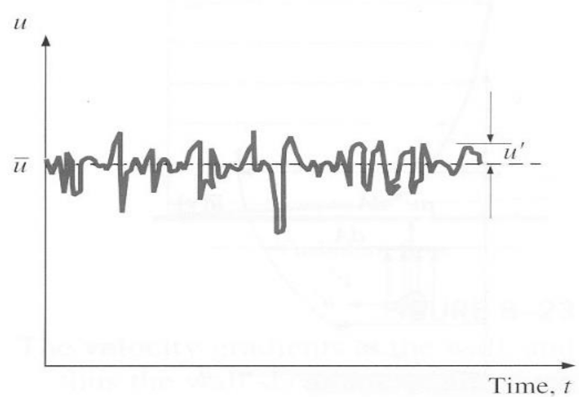
- pipe flow: $10^{-1} \sim 10^{0}$ s; channel/river flow: $10^{1} \sim 10^{2}$ s

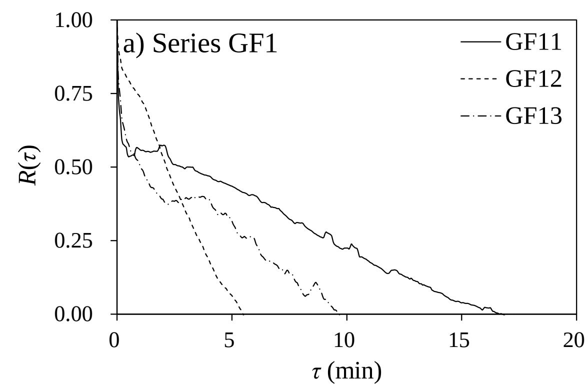
[Re] Integral time scale, T_I

- Time over which a turbulent fluctuation is correlated with itself
- A measure of the memory of the turbulence

$$T_I = \int_0^{\infty} R_I(\tau)\, d\tau$$

$$R_I(\tau) = \frac{\langle U(t)U(t+\tau)\rangle}{\langle U^2 \rangle} \qquad (20.3)$$





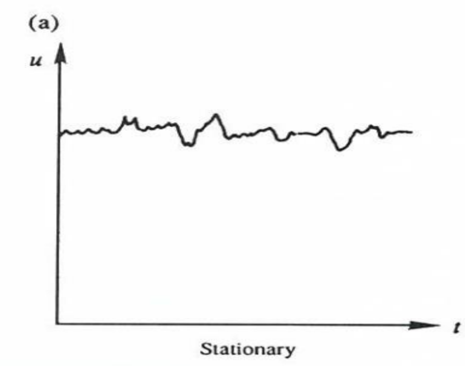
20.1 Velocities and Energies in Turbulence

[Re] Averaging of stationary vs nonstationary turbulence

1) Stationary turbulence

- Use time average of a single time series data

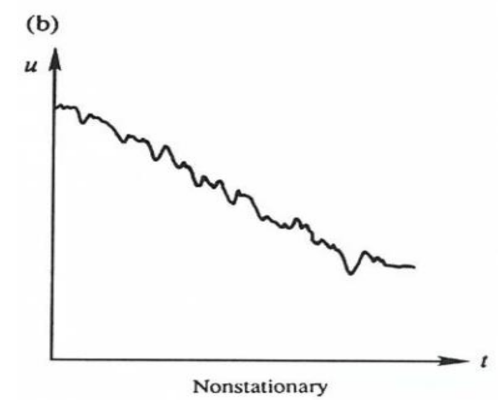
$$\bar{u} = \frac{1}{T}\int_0^T u\,dt \qquad (20.4)$$



2) Nonstationary turbulence

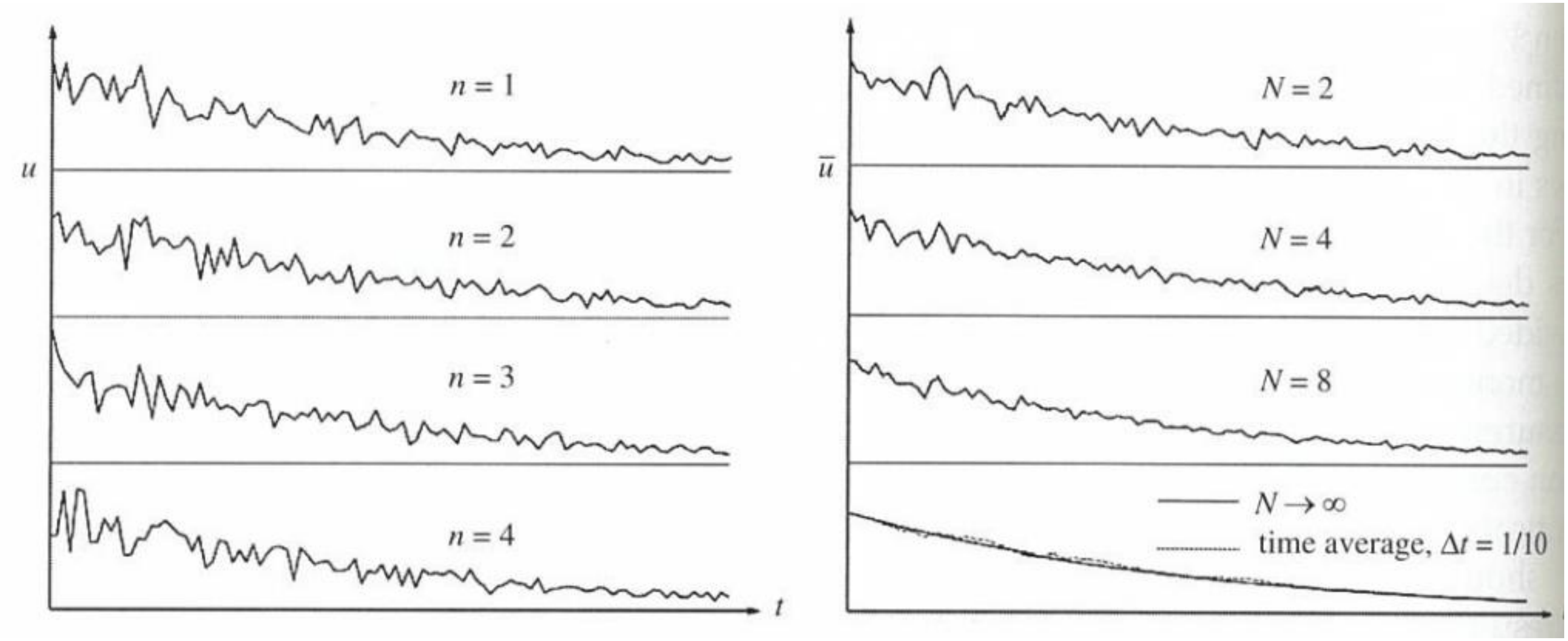
- Use ensemble average of N independent realization of time series data

$$\langle u \rangle = \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N} u \qquad (20.5)$$



20.1 Velocities and Energies in Turbulence

- Ensemble average of nonstationary turbulence



20.1 Velocities and Energies in Turbulence

$$\overline{u'} = \frac{1}{T}\int_0^T u'\,dt \equiv 0 \qquad (\because \text{fluctuations are both plus and minus}) \qquad (20.6)$$

$$\left(\frac{1}{T}\int_0^T (u-\overline{u})dt = \overline{u} - \overline{u} = 0\right)$$

(2) Pressure and stress decomposition

$$p = \overline{p} + p'$$

$$\sigma_{ij} = \overline{\sigma_{ij}} + \sigma'_{ij} \qquad (20.7)$$

20.1 Velocities and Energies in Turbulence

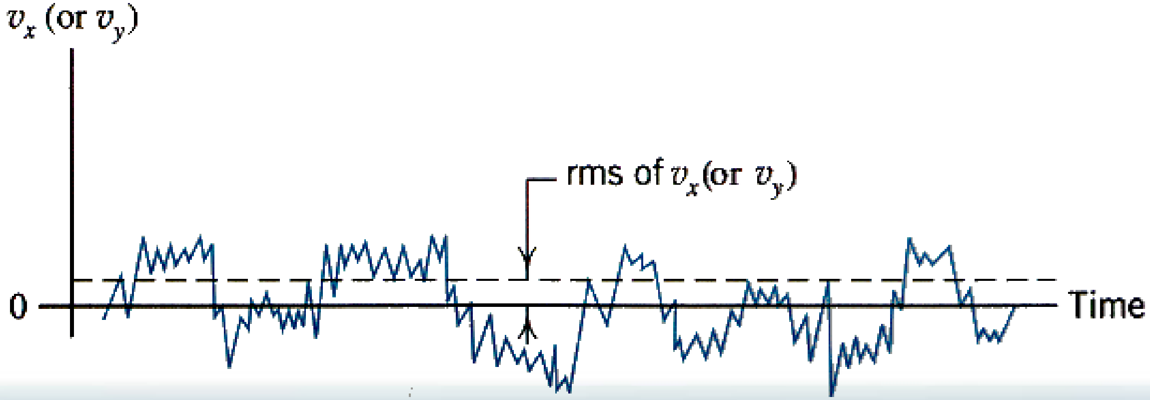
(3) Turbulence intensity – show turbulence effects

→ root-mean-square (rms) = square root of variance = standard deviation

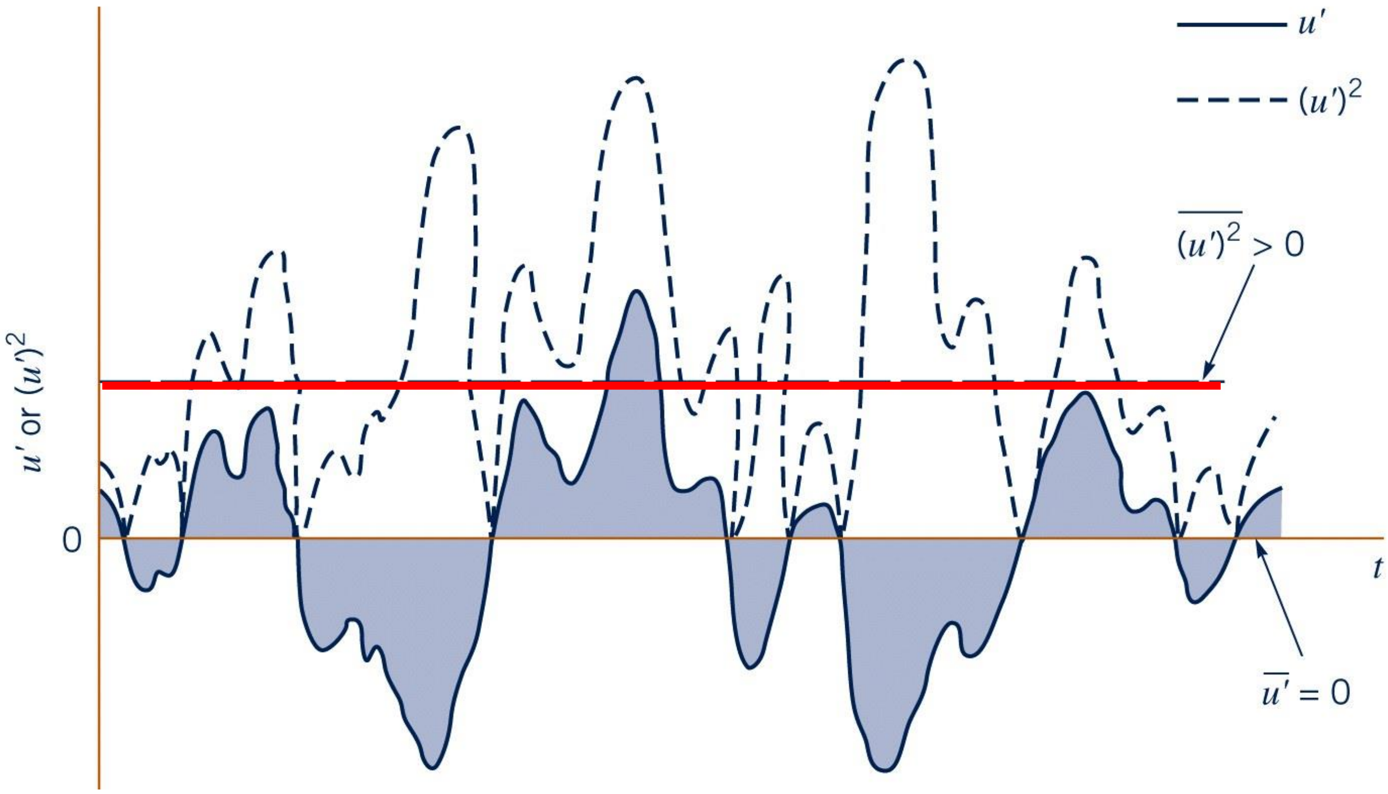
- average intensity of the turbulence *= rms* of *u'*

$$TI = \tilde{u} = \sqrt{\overline{u'^2}} = \left\{ \frac{1}{T} \int_0^T u'^2 \, dt \right\}^{\frac{1}{2}} \qquad (20.8)$$

• Relative Turbulence Intensity (RTI) = $\frac{\sqrt{\overline{u'^2}}}{\bar{u}}$



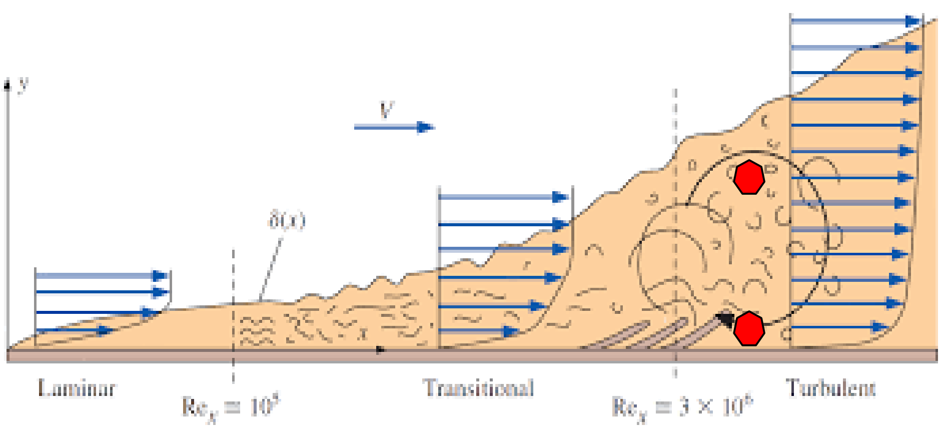
20.1 Velocities and Energies in Turbulence

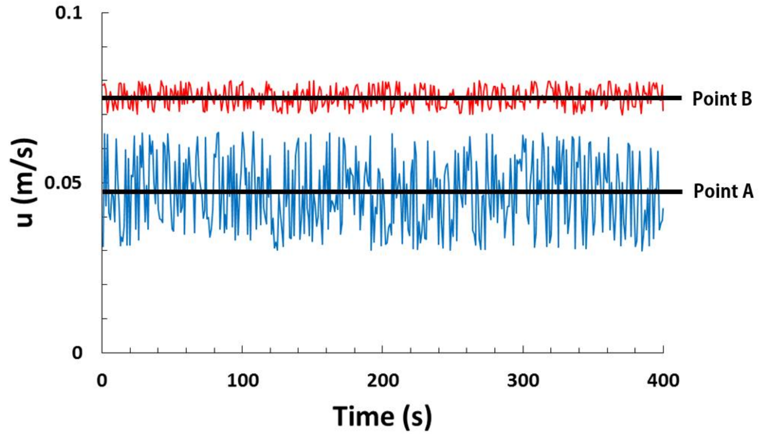


20.1 Velocities and Energies in Turbulence

[Ex] Actual velocity records obtained at two depths in the open channel flow (stationary turbulence)

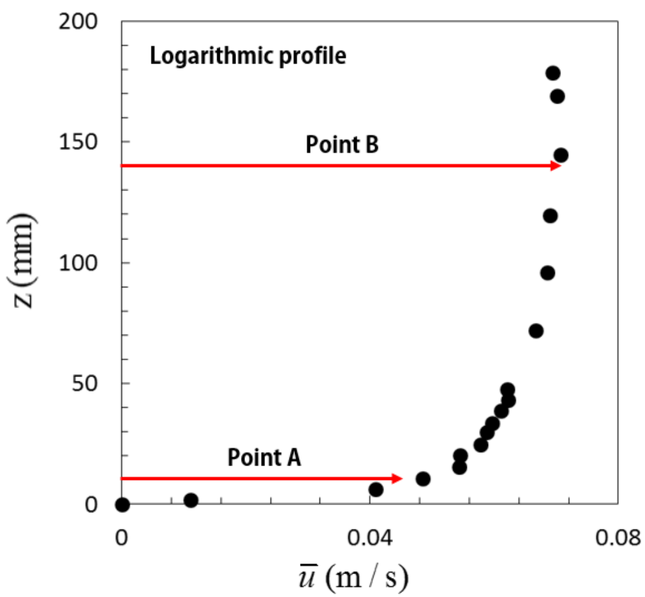
- The time-averaged velocity is greater farther from the wall, but the <u>turbulence intensity is significantly larger near the wall</u>.



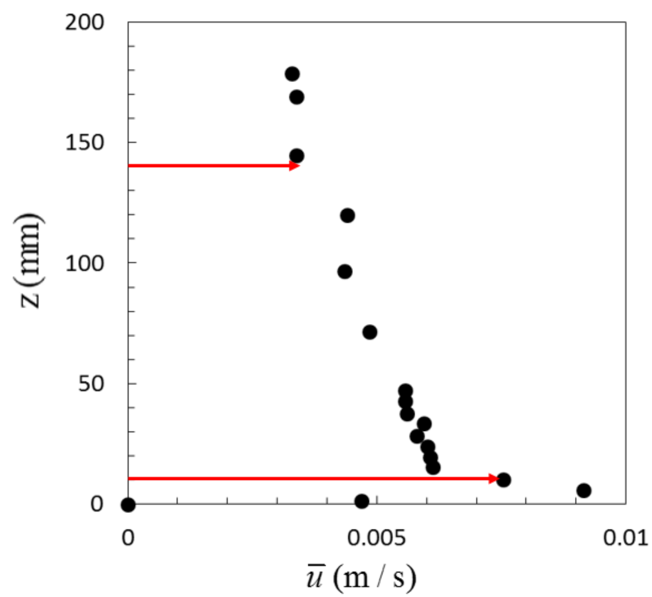


20.1 Velocities and Energies in Turbulence

- The time-averaged velocity increases monotonically from zero at the wall to be approximated into the logarithmic profile.
- The turbulence intensity increases rapidly from zero at the wall to a local maximum near the wall and then monotonically decreases.



a) Mean velocity



b) Turbulence intensity

20.1 Velocities and Energies in Turbulence

(4) Average kinetic energy of turbulence per unit mass

~ average KE of turbulence / mass

$$TKE = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}) = \frac{1}{2}\sum (intensity)^2 \qquad (20.9)$$

(5) Energy density, $\phi(f)$

The kinetic energy is decomposed into an energy spectrum (density) vs. frequency.

≡ limit of average kinetic energy per unit mass divided by the bandwidth Δf

$$\phi(f) = \lim_{\Delta f \to 0} \frac{average\ KE / mass\ contained\ in\ \Delta f}{\Delta f} = \frac{\partial KE}{\partial f} \qquad (20.10)$$

20.1 Velocities and Energies in Turbulence

where f = ordinary frequency in cycles per second

$\therefore$ average KE of turbulence / mass = $\int_0^{\infty} \phi(f)df = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$

(6) Correlation between u', v', and w'

exact correlation = one-to-one correlation

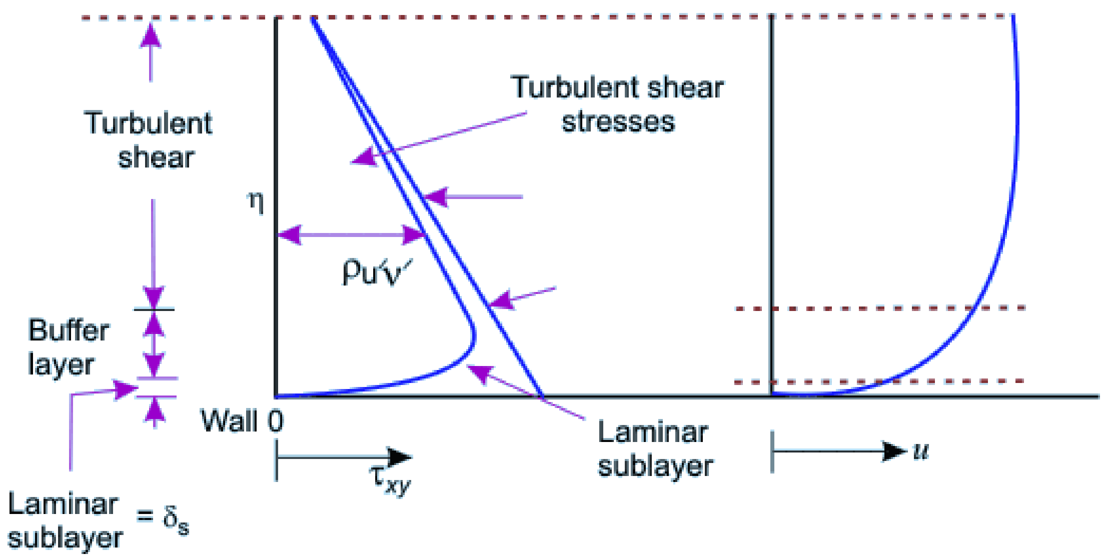
zero correlation = completely independent

$$\overline{u'v'} = \frac{1}{T}\int_0^T u'v'dt \quad \begin{cases} \neq 0 & correlated \\ = 0 & uncorrelated \end{cases} \qquad (20.11)$$

20.1 Velocities and Energies in Turbulence

- In a shear flow in an *xy*-plane, $\overline{u'v'}$ is finite, and it is related to the magnitude of the <u>turbulent shear stress</u>.

$$\tau_{tur} = -\rho \overline{u'v'}$$



20.1 Velocities and Energies in Turbulence

[Re] Correlated variables

1) Averages of products *u*

$$\overline{u_i u_j} = \overline{(\overline{u_i} + u_i')(\overline{u_j} + u_j')}$$

$$= \overline{\overline{u_i}\,\overline{u_j}} + \overline{u_i' u_j'} + \cancel{\overline{\overline{u_i} u_j'}} + \cancel{\overline{\overline{u_j} u_i'}}$$

$$= \overline{u_i}\,\overline{u_j} + \overline{u_i' u_j'} \qquad (20.12)$$

If $\overline{u_i' u_j'} \neq 0 \rightarrow$ u_i' and u_j' are said to be correlated.

If $\overline{u_i' u_j'} = 0 \rightarrow$ u_i' and u_j' are uncorrelated.

20.1 Velocities and Energies in Turbulence

2) Correlation coefficient

$$c_{ij} = \frac{\overline{u_i' u_j'}}{\left(\overline{u_i'^2} \cdot \overline{u_j'^2}\right)^{1/2}} \qquad (20.13)$$

in which $\overline{u_i'^2}, \ \overline{u_j'^2}$ = variances

$c_{ij} = \pm 1 \rightarrow$ perfect correlation (같이 움직임)

$c_{ij} = 0 \rightarrow$ independent (독립적 운동)

$c_{ij} = 0.45 \sim 0.55$ for general turbulence

[Re] Classification of turbulence

1) General turbulence

$$\bar{u} \neq \bar{v} \neq \bar{w}$$

$$\overline{u'^2} \neq \overline{v'^2} \neq \overline{w'^2} \qquad \overline{u'v'} \neq \overline{v'w'} \neq \overline{w'u'}$$

20.1 Velocities and Energies in Turbulence

2) Homogeneous turbulence

~ statistically independent of the location

$$(\overline{u_i' u_j'})_a = (\overline{u_i' u_j'})_b$$

3) Isotropic turbulence

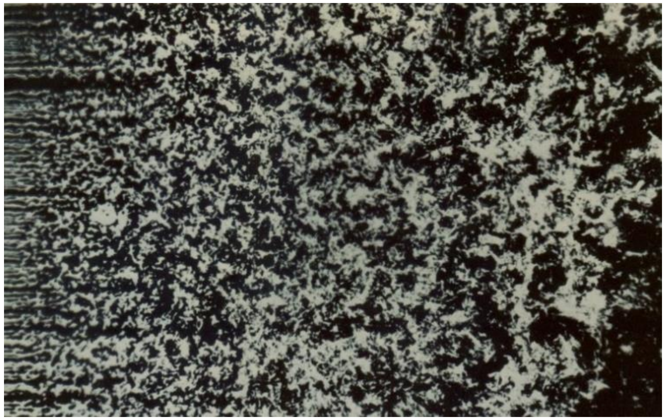
~ statistically independent of the orientation and location of the coordinate axes

$$\overline{u'^2} = \overline{v'^2} = \overline{w'^2} = \text{constant}$$

$$\overline{u'v'} = \overline{v'w'} = \overline{w'u'} = 0$$

~ uncorrelated

~ not coherent structures - small scale eddies



20.1 Velocities and Energies in Turbulence

20.1.2 Measurement of turbulence

~ measure turbulent fluctuations

(1) Hot-wire (hot-film) anemometer

~ Hot-film is usable in contaminated water.

~ Change of temperature affects the electric current flow or voltage drop through wire. Fine platinum wire (film) is heated electrically by a circuit that maintains voltage drop constant.

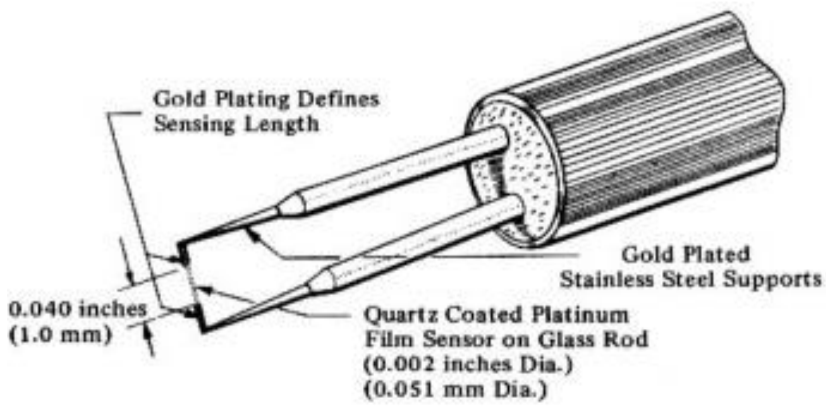
~ When inserted into the stream, the cooling, which is a function of the velocity, can be detected as variations in voltage.

20.1 Velocities and Energies in Turbulence

~ Use two or more wires at one point in the flow to make simultaneous measurements of different velocity components.

→ After subtracting mean value, rms-values, correlations, and energy spectra can be computed using fluctuation.

→ These operations can be performed electronically.



20.1 Velocities and Energies in Turbulence

(2) Laser Doppler Velocimeter (LDV)

~ use Doppler effect

~ A laser (ultrasonic) beam transmitted into the fluid will be reflected by impurities or bubbles in the fluid to a receiving sensor at a different frequency.

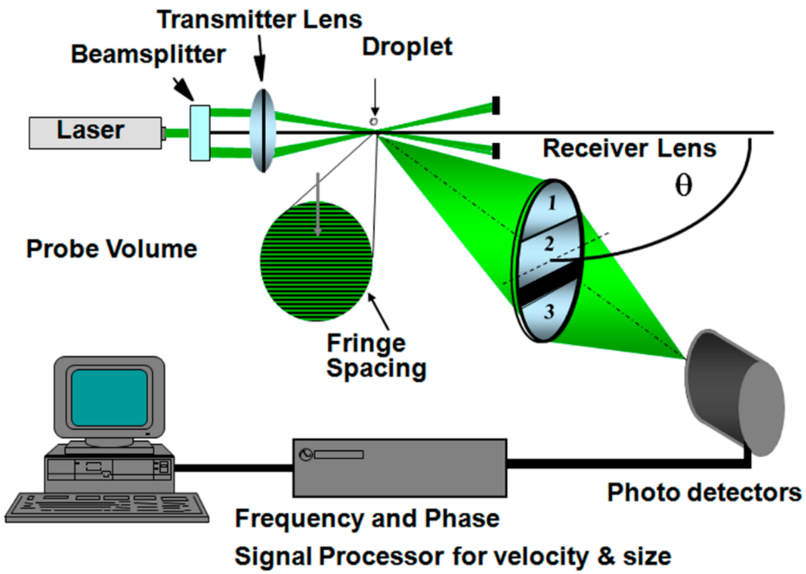
→ The transmitted and reflected signals are then compared by electronic means to calculate the Doppler shift which is proportional to the velocity.

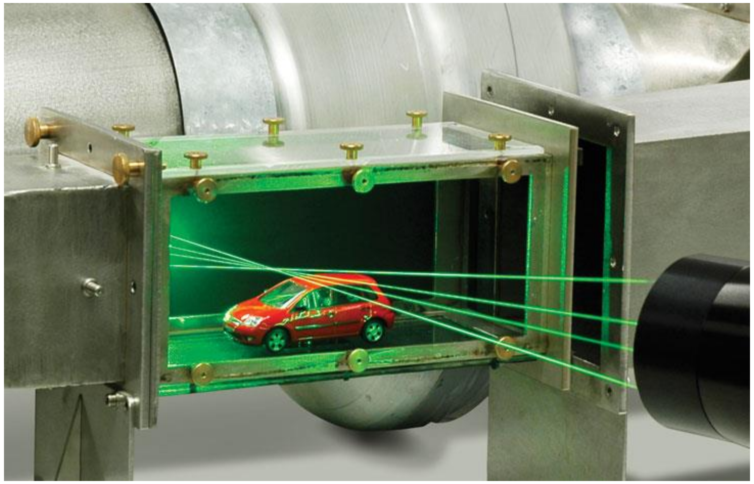
~ non-intrusive sensing (immersible LDA)

~ sampling frequency is up to 20,000 Hz

$$F_{doppler} = -F_{source} \frac{V}{C}$$

20.1 Velocities and Energies in Turbulence





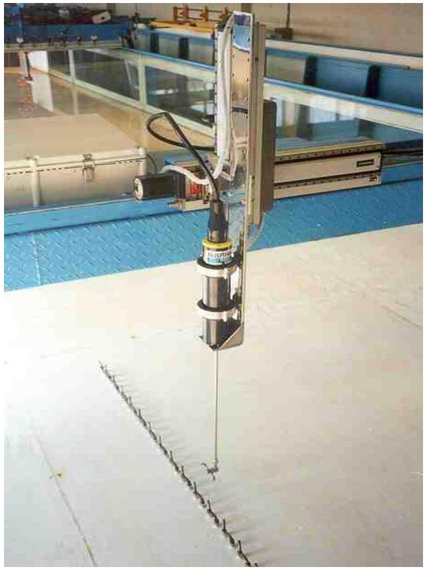
20.1 Velocities and Energies in Turbulence

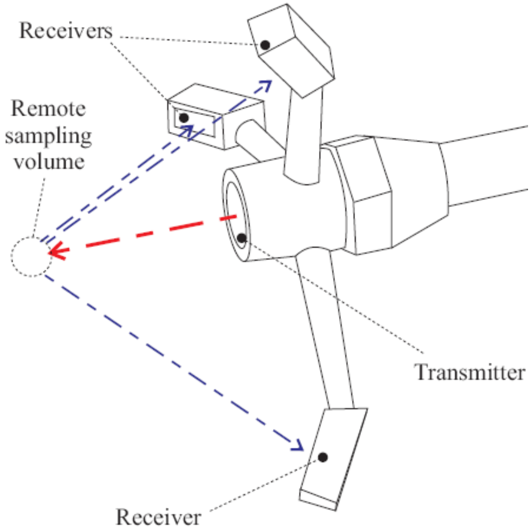
(3) Acoustic Doppler Velocimeter (ADV)

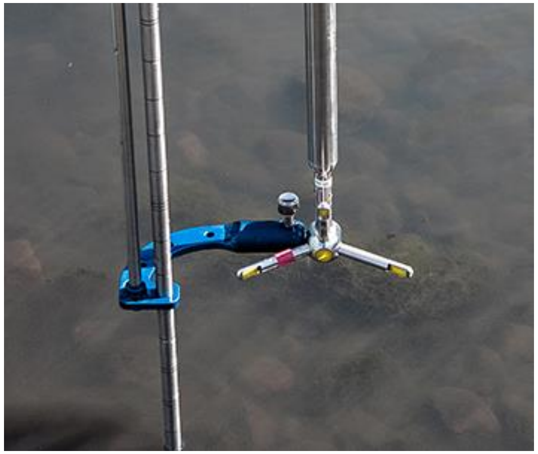
~ use Doppler effect of sonic wave

~ intrusive sensing

~ sampling frequency = 25-50 Hz







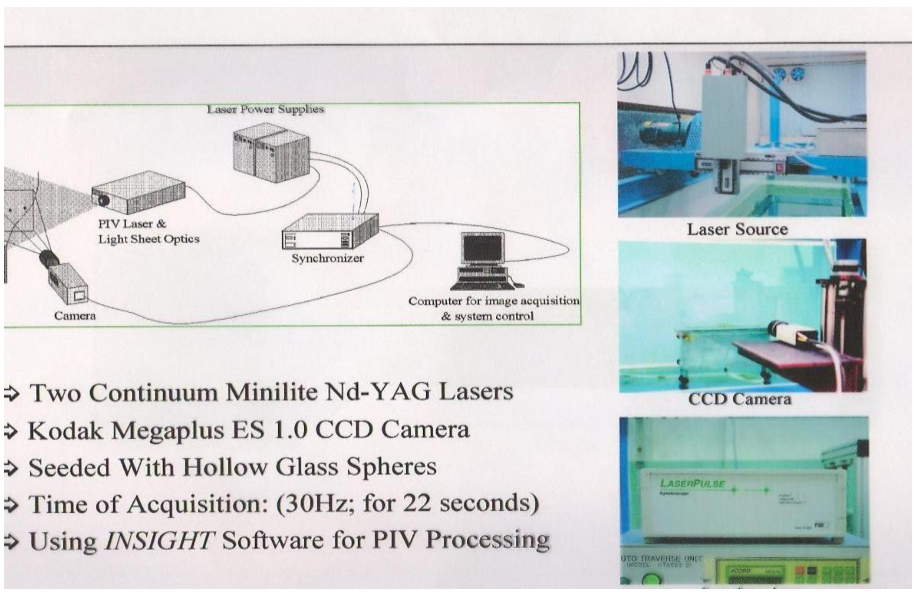
20.1 Velocities and Energies in Turbulence

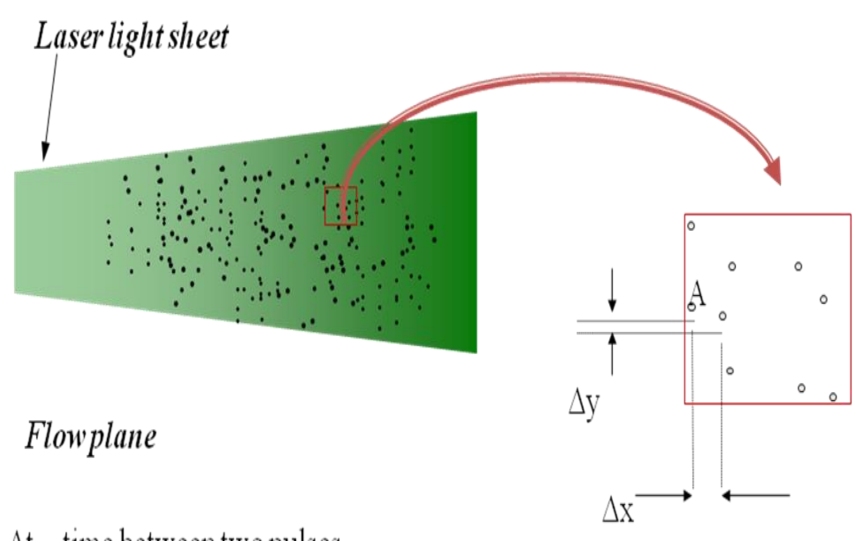
(4) Particle Image Velocimetry (PIV)

~ use Laser and CCD camera

~ <u>measure flow field at once</u>

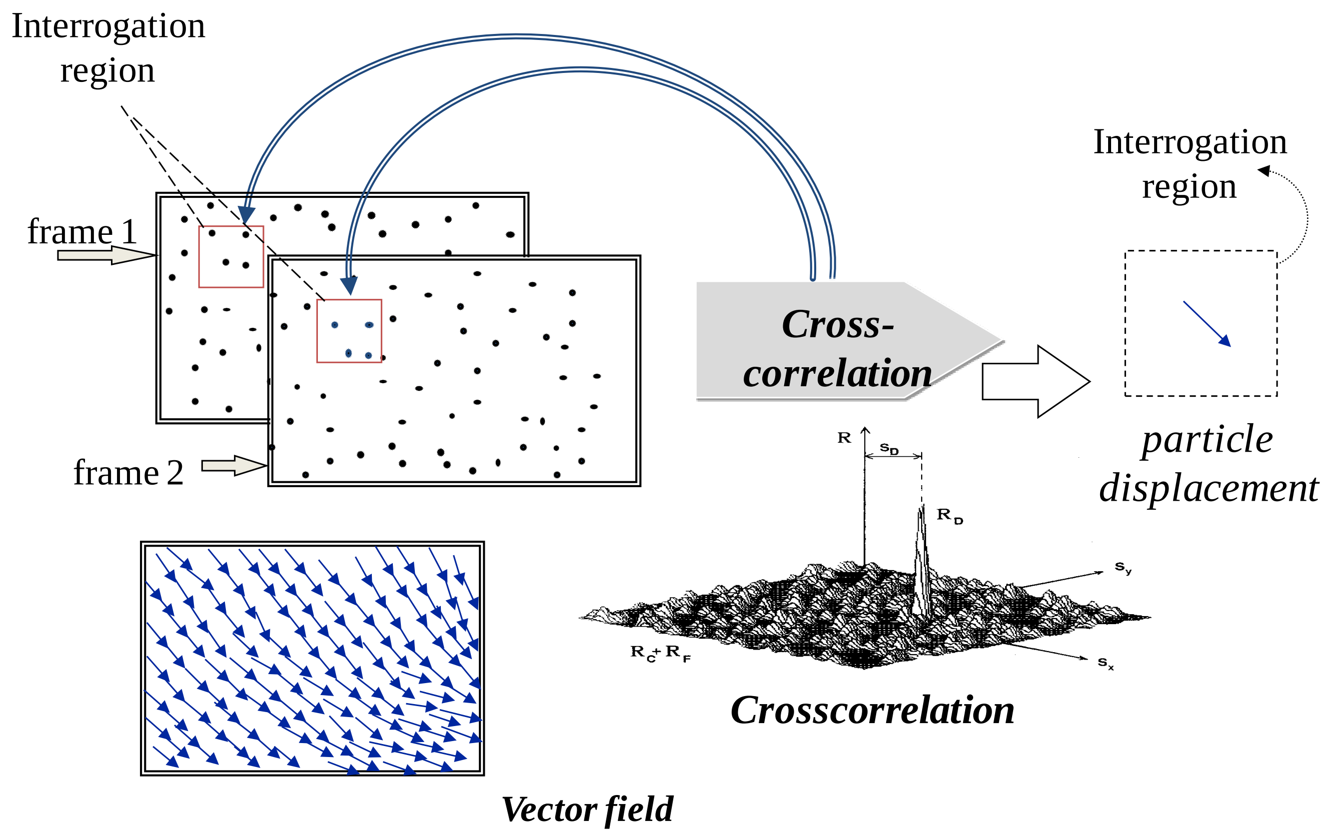
~ sampling frequency = 30 Hz





20.1 Velocities and Energies in Turbulence

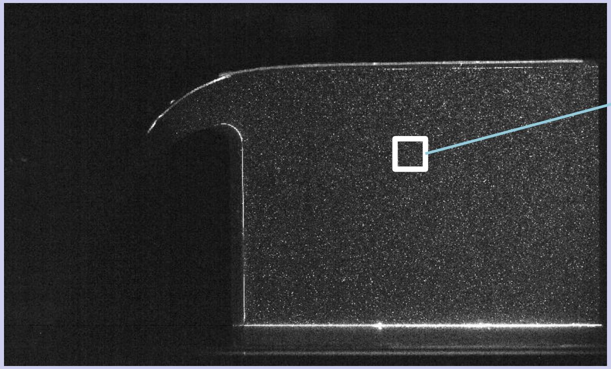
PIV system



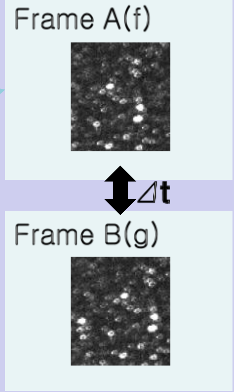
20.1 Velocities and Energies in Turbulence

Velocity measurement using PIV system

Mono 10bit PIV particle video acquisition

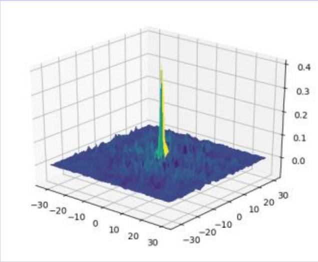


Video frame extraction

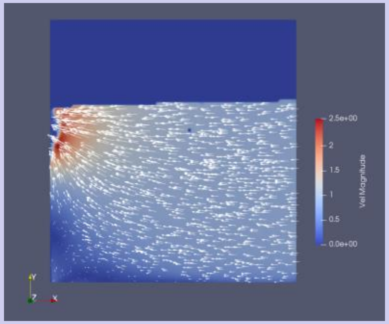


Correlation analysis using FFT

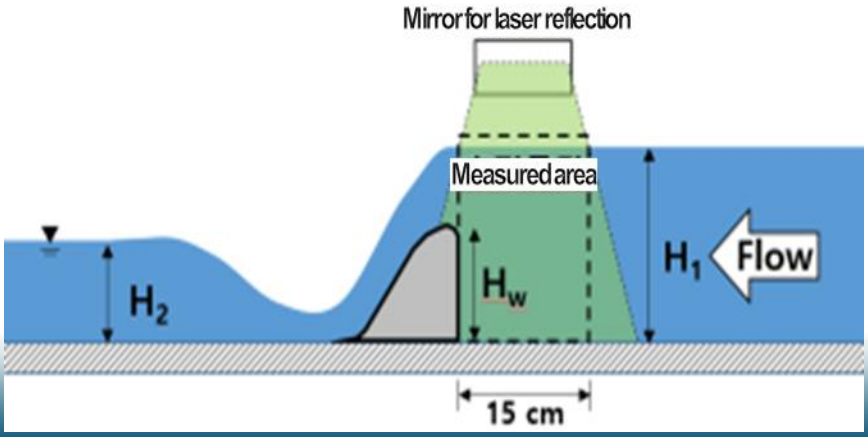
$$R(s,t)=F^{-1}\left(F^{*}(f)F(g)\right)$$



Acquisition of velocity vector using displacement analysis

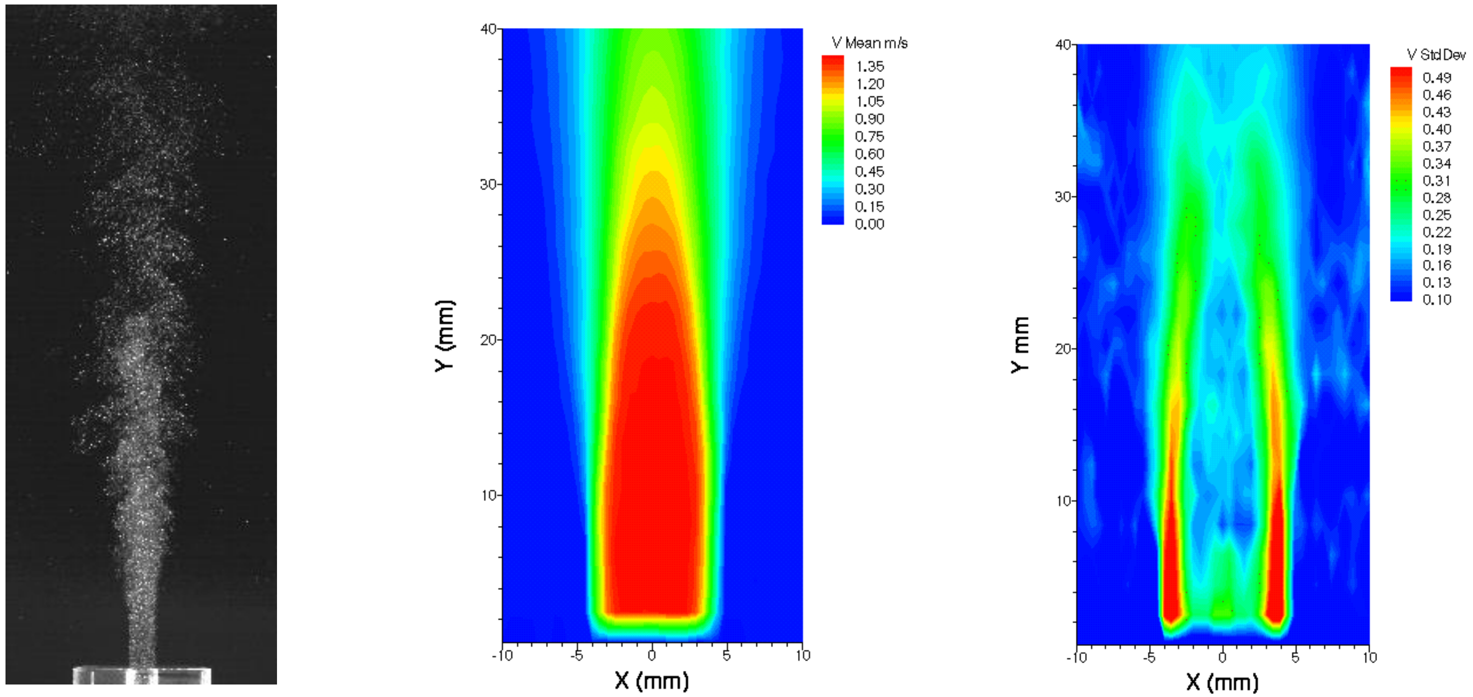


Velocity fields near ogee weir



PIV measurement

20.1 Velocities and Energies in Turbulence

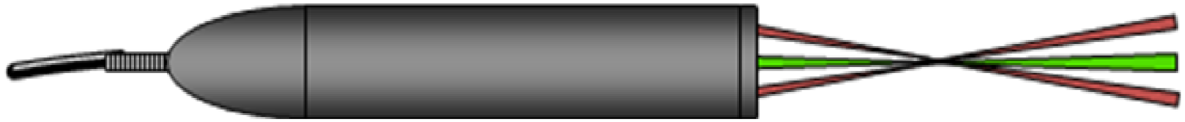


a) Image b)Velocity c)Turbulence Intensity

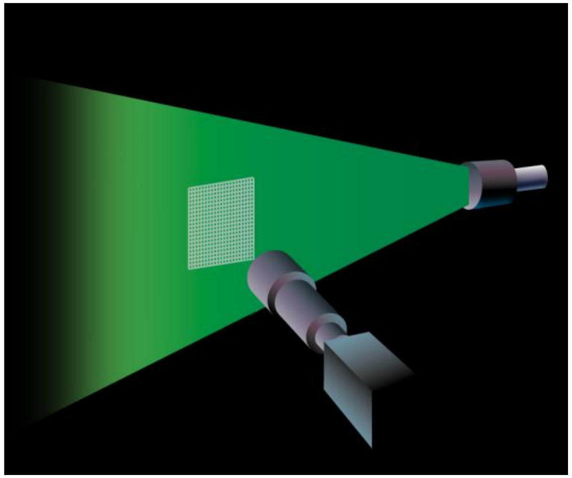
b) Fig. 1 Jet Characteristics Measured by PIV (Seo et al., 2002)

20.1 Velocities and Energies in Turbulence

LDV: single point measurement



PIV: field measurement



20.1 Velocities and Energies in Turbulence

[Re] Reynolds rules of averages: Schlichting (1979) Boundary-Layer Theory

Let *f* and *g* are two dependent variables whose time mean values are to be found. *s* is any one of the independent variables *x, y, z, t*.

$$\bar{\bar{f}} = \bar{f}$$

$$\overline{f + g} = \bar{f} + \bar{g}$$

$$\overline{f \cdot \bar{g}} = \bar{f} \cdot \bar{g}$$

$$\overline{\frac{\partial f}{\partial s}} = \frac{\partial \bar{f}}{\partial s} \rightarrow \left(\begin{array}{l} \text{since time averaging is carried out by integrating over a long} \\ \text{period of time, which commutes with differentiation with respect} \\ \text{to another independant variable} \end{array} \right.$$

$$\overline{\int f \, ds} = \int \bar{f} \, ds$$

20.2 Continuity for Turbulent Motion

20.2.1 Continuity equation for turbulent motion

Continuity equation for incompressible fluid

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad \text{(A)}$$

Substitute velocity decomposition into (A)

$$\frac{\partial(\bar{u}+u')}{\partial x}+\frac{\partial(\bar{v}+v')}{\partial y}+\frac{\partial(\bar{w}+w')}{\partial z}=0 \qquad (20.14)$$

$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial \bar{v}}{\partial y}+\frac{\partial \bar{w}}{\partial z}+\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0 \qquad \text{(B)}$$

20.2 Continuity for Turbulent Motion

Take time-averages of each term of (B)

$$\overline{\frac{\partial \bar{u}}{\partial x}} + \overline{\frac{\partial \bar{v}}{\partial y}} + \overline{\frac{\partial \bar{w}}{\partial z}} + \cancel{\overline{\frac{\partial u'}{\partial x}}} + \cancel{\overline{\frac{\partial v'}{\partial y}}} + \cancel{\overline{\frac{\partial w'}{\partial z}}} = 0$$

$$\left(\because \overline{\frac{\partial u'}{\partial x}} = \frac{\partial (\overline{u'})}{\partial x} = 0 \right)$$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \qquad (20.15)$$

Substitute (20.15) into (B)

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \qquad (20.16)$$

→ Both mean-motion components and the superposed turbulent-motion components must satisfy the continuity equation.

→ Continuity must be satisfied for both turbulent and laminar motions.

20.2 Continuity for Turbulent Motion

[Re] Continuity Eq. for compressible fluid

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

$$\frac{\partial (\bar{\rho} + \rho')}{\partial t} + \frac{\partial \left\{ (\bar{\rho} + \rho')(\bar{u}_i + u_i') \right\}}{\partial x_i} = 0$$

Time averaging yields

$$\overline{\frac{\partial (\bar{\rho} + \rho')}{\partial t}} + \overline{\frac{\partial \{ (\bar{\rho} + \rho')(\bar{u}_i + u_i') \}}{\partial x_i}} = 0$$

$$\frac{\partial \bar{\bar{\rho}}}{\partial t} + \overline{\frac{\partial \rho'}{\partial t}} + \frac{\partial}{\partial x_i} \overline{(\bar{\rho}\bar{u}_i + \rho' \bar{u}_i + \bar{\rho} u_i' + \rho' u_i')} = 0$$

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_i} (\bar{\rho}\bar{u}_i + \overline{\rho' u_i'}) = 0$$

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho}\bar{u}}{\partial x} + \frac{\partial \bar{\rho}\bar{v}}{\partial y} + \frac{\partial \bar{\rho}\bar{w}}{\partial z} + \frac{\partial}{\partial x}(\overline{\rho' u'}) + \frac{\partial}{\partial y}(\overline{\rho' v'}) + \frac{\partial}{\partial z}(\overline{\rho' w'}) = 0 \qquad (20.17)$$