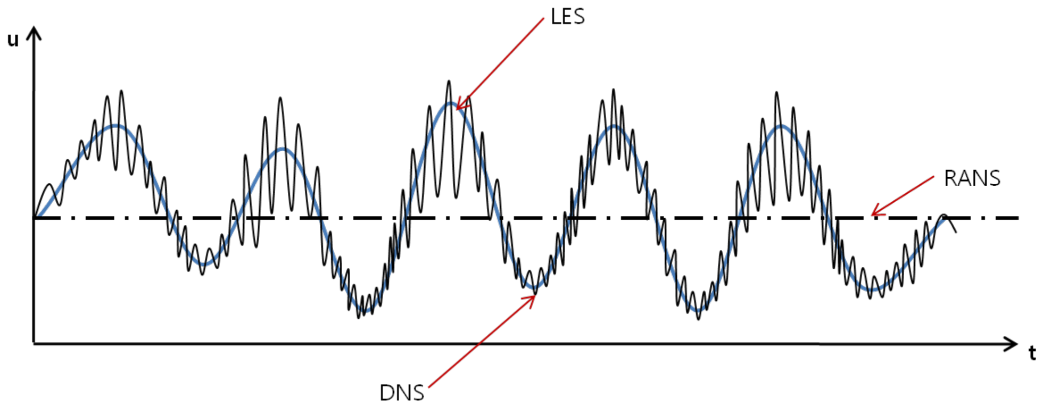
Chapter 10

Turbulence Models and Their Applications



Chapter 10 Turbulence Models and Their Applications

Contents

Objectives

- What is turbulence modeling?
- To derive mean flow equation and specialized equations of motion in natural water bodies
- To study equations of turbulence models

10.4 Turbulence-Closure Models

10.4.4 One-equation models

- This model accounts for transport or history effects (time-rate change) of turbulence quantities by solving differential transport equations.

- One-equation models determine the fluctuating velocity scale from a transport equation rather than the direct link between this scale and the mean velocity gradients.

$$\hat{V} = u = l_m \frac{\partial U}{\partial z}$$

(1) k-equation Model

Velocity fluctuations are to be characterized by $\sqrt{k}$ where k is the **turbulent kinetic energy per unit mass** defined as

$$\hat{V} \propto \sqrt{k}\,, \qquad k = \frac{1}{2}\left(\overline{u^2} + \overline{v^2} + \overline{w^2}\right)$$

10.4 Turbulence-Closure Models

- Eddy viscosity

Length scale ~ empirical

$$\nu_t \propto \hat{V} L$$

$$\nu_t = c_\mu{}' \sqrt{k} L$$ → Kolmogorov-Prandtl equation (1942, 1945)(10.35)

in which $c_\mu{}'$ = empirical constant.

- Turbulent Kinetic Energy (TKE) equation

~ Exact form can be derived from the Navier-Stokes equation.

~ Exact equation contains certain higher-order correlations which must be approximated by models in order to achieve a closure of the equations.

10.4 Turbulence-Closure Models

For high Reynolds number, this equation reads

new unknown correlations

$$\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = -\frac{\partial}{\partial x_i}\left[\overline{u_i\left(\frac{u_j u_j}{2} + \frac{p}{\rho}\right)}\right] - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j}$$

rate of change of k

advective transport due to mean motion

diffusive transport due to velocity and pressure fluctuations

production by turbulent shear stress = P

Mean velocity

$$-\beta g_i \overline{u_i \phi} - \nu \overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}} \qquad (10.36)$$

buoyant production / destruction = G

viscous dissipation into heat = $\varepsilon \, (m^2/s^3)$

10.4 Turbulence-Closure Models

P = transfer of kinetic energy from the mean motion to the turbulent motion (large scale eddies)

G = exchange between the turbulent kinetic energy k and potential energy

~ negative for stable stratification (k is reduced, turbulence is damped while potential energy of the system increases)

~ positive for unstable stratification (k is produced at the expense of the potential energy)

ε = transfers kinetic energy into internal energy of the fluid ~ negative (sink)

10.4 Turbulence-Closure Models

• Concepts used for *k*-equation model

① Energy cascade

~ Kinetic energy extracted from mean motion is first fed into large scale turbulent motion.

→ This energy is then passed on to smaller and smaller eddies by vortex stretching (vortex trail, vortex street) until viscous force become active and dissipate the energy.

② Anisotropy vs. Isotropy

~ Large-scale turbulences are anisotropic, whereas small-scale turbulences are isotropic.

Because of interaction between large-scale turbulent motion and mean flow, the large-scale turbulent motion depends strongly on the boundary conditions.

10.4 Turbulence-Closure Models

During the energy cascade process, energy is passed on to smaller eddies by vortex stretching. → The direction sensitivity is diminished, and small-scale turbulence tend to be isotropic

- **Modeled form of the k-equation**

~ The exact k-equation contains new unknown correlations.

→ To obtain a closed set of equations, model assumptions must be introduced for these terms.

i) Diffusion term

~ In analogy to the diffusion expression for the scalar quantity ϕ, the diffusion flux of k is assumed proportional to the gradient k.

10.4 Turbulence-Closure Models

$$-\overline{u_i\left(\frac{u_j u_j}{2}+\frac{p}{\rho}\right)}=\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i}$$

in which σ_k = empirical diffusion constant.

ii) Reynolds stress

$$-\overline{u_i u_j}=\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)$$

iii) heat (mass) flux

$$-\overline{u_i\phi}=\frac{\nu_t}{\sigma_t}\frac{\partial \Phi}{\partial x_i}$$

in which σ_t = turbulent Prandtl or Schmidt number

10.4 Turbulence-Closure Models

iv) viscous dissipation

$$\varepsilon = c_D \frac{k^{3/2}}{L}$$

in which c_D= empirical constant.

Substituting i) ~ iv) into exact k-equation yields

$$\frac{\partial k}{\partial t} + U_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \nu_t\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} + \beta g_i \frac{\nu_t}{\sigma_t}\frac{\partial \Phi}{\partial x_i} - c_D \frac{k^{3/2}}{L} \quad (10.37)$$

~ This model is restricted to high Reynolds number flows; $c_\mu{}' c_D \approx 0.08 \ and \ \sigma_k \approx 1$

~ For low Reynolds number flows, a viscous diffusion term should be accounted for and empirical constants are functions of the turbulent Reynolds number, $\mathrm{Re}_f = \sqrt{k}L / \nu$

10.4 Turbulence-Closure Models

- Special case of local equilibrium

~ Turbulence is said to be in local equilibrium when the rate of change, advection, and diffusion terms in Eq. (10.37) are zero.

→ Then, production of *k* is equal to dissipation.

$$\frac{\partial k}{\partial t}+U_i\frac{\partial k}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right)+\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}+\beta g_i\frac{\nu_t}{\sigma_t}\frac{\partial \Phi}{\partial x_i}-c_D\frac{k^{3/2}}{L}$$

For non-buoyant shear layers,

$$\nu_t\left(\frac{\partial U}{\partial z}\right)^2=c_D\frac{k^{3/2}}{L}$$

$$k^{1/2}=\left[\nu_t\left(\frac{\partial U}{\partial z}\right)^2\frac{L}{c_D}\right]^{1/3} \qquad (1)$$

10.4 Turbulence-Closure Models

Substitute (1) into Kolmogorov-Prandtl expression [$\nu_t = c_u{}' \sqrt{k} L$]

$$\nu_t = \left(\frac{c_\mu{}'^3}{c_D} \right)^{\frac{1}{2}} L^2 \left| \frac{\partial U}{\partial z} \right|$$

Set l_m = mixing length = $\left(\frac{c' \mu^3}{c_D} \right)^{\frac{1}{4}} L$

Then we have the mixing-length model

$$\nu_t = l_m{}^2 \left| \frac{\partial U}{\partial z} \right|$$

Thus, the mixing-length model is suitable only for flows where the turbulence is in local equilibrium.

10.4 Turbulence-Closure Models

▪ Length-scale determination

~ Because the length scale L appears both in Kolmogorov-Prandtl equation and in dissipation term of the k-equation, this must be specified empirically.

~ In most models, L is determined from simple empirical relations similar to those for the mixing length, l_m.

→ Launder and Spalding (1972) for estuary; Smith and Takhar (1977) for open-channel

▪ Bobyleva et al. (1965) 's length scale formula

~ similar to von Kaman's formula

$$L = \kappa \frac{\Psi}{\frac{\partial \Psi}{\partial z}}$$

10.4 Turbulence-Closure Models

where κ = von Karman's const.

$$\Psi = \frac{k^{\frac{1}{2}}}{L} = turbulence\ \ parameter$$

~ applicable to flows where turbulent transport is mainly in vertical direction

- When the turbulence is in local equilibrium in the shear layer,

$$\frac{k^{\frac{1}{2}}}{L} \propto \frac{\partial U}{\partial z}$$

$$L = l_m = \kappa \left| \frac{\partial U / \partial z}{\partial^2 U / \partial z^2} \right|$$ → von Karman's formula

10.4 Turbulence-Closure Models

(2) Bradshaw et al.'s Model

→ $\overline{uv}$ -equation model for wall boundary layers

~ This model does not employ the eddy viscosity concept.

~ It solves a transport equation for the shear stress $\overline{uv}$.

For 2D wall boundary layers, relation between k (normal stress) and $\overline{uv}$ (shear stress) is given as

$$\frac{\overline{uv}}{k} = a_1 \approx const \approx 0.3$$ (experiment)

Convert k-equation to $\overline{uv}$-equation for steady flows

10.4 Turbulence-Closure Models

$$\underbrace{U\frac{\partial \frac{\overline{uv}}{a_1}}{\partial x}+V\frac{\partial \frac{\overline{uv}}{a_1}}{\partial y}}_{\text{advection}} = \underbrace{-\frac{\partial}{\partial y}\left[G\overline{uv}(\overline{uv}_{\max})^{\frac{1}{2}}\right]}_{\text{diffusion}} - \underbrace{\overline{uv}\frac{\partial U}{\partial y}}_{\text{production}} - \underbrace{\frac{\overline{uv}^{3/2}}{L}}_{\text{dissipation}} \qquad (10.38)$$

in which $G=\left(\frac{\overline{uv}_{\max}}{U_\infty^2}\right)^{\frac{1}{2}} f_1\left(\frac{y}{\delta}\right)$

$L=f_2\left(\frac{y}{\delta}\right)\delta$ → empirical

→ For the diffusion flux of $\overline{uv}$, the gradient-diffusion concept is not employed. Instead the diffusion flux is assumed to proportional to a bulk velocity, $(\overline{uv}_{\max})^{\frac{1}{2}}$.

10.4 Turbulence-Closure Models

- Modeling transport of heat and mass by one-equation model

i) Find eddy viscosity (ν_t) or the shear stress ($\overline{u_i u_j}$) using one-equation model.

$$\Gamma_t = \frac{\nu_t}{\sigma_t}$$

ii) Use gradient-diffusion concept to calculate heat and mass transfer by turbulence

$$-\overline{u_i \phi} = \Gamma_t \frac{\partial \Phi}{\partial x_i}$$

iii) Solve scalar transport equation

10.4 Turbulence-Closure Models

• Advantage of one-equation model

① One-equation models can account for advective and diffusive transport and for history effects on the turbulent velocity scale.

→ One-equation model is superior to the mixing-length model when these effects are important

Examples: non-equilibrium shear layers with rapidly changing free stream conditions, abrupt changes in the boundary conditions, shear layers in estuary with velocity reversal, heat and mass exchange in area with vanishing velocity gradients

② Buoyancy term appears automatically in the k-equation model.

10.4 Turbulence-Closure Models

• Disadvantage

① The application is restricted to shear-layer situation not applicable to more complex flows.

②The empirical formulas for calculating length scale in general flows so far been tested insufficiently.

→ For calculating general flows, the trend has been to move on to two-equation models which determine the length scale from a transport equation.

10.4 Turbulence-Closure Models

10.4.5 Two-equation models

(1) Types of two-equation models

Length scale L characterizing the size of the large, energy-containing eddies is also subject to transport processes in a similar manner to the kinetic energy k.

- For example, eddies generated by a grid are advected downstream so that their size at any station depends on their initial size. → history effect
- Dissipation destroys the small eddies and thus effectively increases the eddy size.
- Vortex stretching connected with the energy cascade reduces the eddy size.

→ The balance of all these processes can be expressed in a transport model for L.

10.4 Turbulence-Closure Models

- Length scale equations

(i) The general form

$$Z = k^m l^n$$

Length scale transport equation of which the exact form can be derived from Navier-Stokes eq. is given as

$$\underbrace{\frac{\partial Z}{\partial t}}_{\text{rate of change}} + \underbrace{U_i \frac{\partial Z}{\partial x_i}}_{\text{advection}} = \underbrace{\frac{\partial}{\partial x_i}\left(\frac{\sqrt{k}L}{\sigma_z}\frac{\partial Z}{\partial x_i}\right)}_{\text{diffusion}} + \underbrace{c_{z1}\frac{Z}{k}P}_{\text{production}} - \underbrace{c_{z2} Z \frac{\sqrt{k}}{L}}_{\text{destruction}} + S \qquad (10.39)$$

where σ_z , c_{z1} , c_{z2} = empirical constants

10.4 Turbulence-Closure Models

P = production of kinetic energy $\left(= -\overline{u_i u_j} \dfrac{\partial u_i}{\partial x_j}\right)$

S = secondary source term which is important near walls

(ii) Other length scales

Dissipation rate: kL	Rotta (1968)
Frequency: $\dfrac{k^{\frac{1}{2}}}{L}$	Kolmogorov (1941)
Turbulence vorticity: $\omega = k / L^2$	Spalding (1971), Saffman (1970

10.4 Turbulence-Closure Models

(iii) Energy dissipation rate

Use energy dissipation rate as a combination of length scale and energy

Chou (1945), Davidov (1961), Jones & Launder (1972)

$$\varepsilon \propto \frac{k^{3/2}}{L}$$

- The use of the ε–equation has been criticized because the process of dissipation is associated with the small-scale turbulence while it is the length scale L characterizing the large-scale, energy-containing eddies that needs to be determined.

- However, the amount of energy dissipated is controlled by the energy fed from the large-scale motion through the spectrum to the small-scale motion.

10.4 Turbulence-Closure Models

(2) Standard k - ε model

- At high Reynolds numbers where local isotropy prevails, turbulence is characterized by the two parameters k and ε.

- ε model works better near walls than other equations.

- The ε-equation does not require a near-wall correction term *S*.

- The model employs the eddy viscosity/diffusivity concept.

$$\nu_t = c_\mu \frac{k^2}{\varepsilon} \qquad (10.40)$$

$$\Gamma_t = \frac{\nu_t}{\sigma_t} \qquad (10.41)$$

10.4 Turbulence-Closure Models

[Re] Derivation

$$\nu_t = c_\mu ' \sqrt{k} L \qquad \text{(a)}$$

$$\varepsilon = c_D \frac{k^{3/2}}{L} \rightarrow L = c_D \frac{k^{3/2}}{\varepsilon} \qquad \text{(b)}$$

Substitute (b) into (a)

$$\nu_t = c_\mu ' \sqrt{k} \frac{c_D k^{3/2}}{\varepsilon} = c_\mu ' c_D \frac{k^2}{\varepsilon}$$

Set $c_\mu = c_\mu ' c_D$

Then

$$\nu_t = c_\mu \frac{k^2}{\varepsilon}$$

10.4 Turbulence-Closure Models

Exact ε - equation can be derived from N-S equations for fluctuating vorticity.

→ rate of change + advection = diffusion + generation of vorticity due to vortex stretching + viscous destruction of vorticity

→ need model assumptions for diffusion, generation, and destruction terms (diffusion is modeled with gradient assumption).

- Modeled ε-equation

$$\underbrace{\frac{\partial \varepsilon}{\partial t}}_{\text{rate of change}} + \underbrace{U_i \frac{\partial \varepsilon}{\partial x_i}}_{\text{advection}} = \underbrace{\frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right)}_{\text{diffusion}} + \underbrace{c_{1\varepsilon}\frac{\varepsilon}{k}\left(P+G\right)\left(1+c_{3\varepsilon}R_f\right) - c_{2\varepsilon}\frac{\varepsilon^2}{k}}_{\text{generation-destruction}} \tag{10.42}$$

10.4 Turbulence-Closure Models

where P = stress production of kinetic energy k;

G = buoyancy production of kinetic energy k

$R_f = \text{Richardson number}$

- Complete k - ε model

$$\frac{\partial k}{\partial t}+U_i\frac{\partial k}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_i}\right)+\nu_t\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}+\beta g_i\frac{\nu_t}{\sigma_t}\frac{\partial \Phi}{\partial x_i}-\varepsilon \qquad (10.43)$$

$$\frac{\partial \varepsilon}{\partial t}+U_i\frac{\partial \varepsilon}{\partial x_i}=\frac{\partial}{\partial x_i}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_i}\right)+c_{1\varepsilon}\frac{\varepsilon}{k}\left(P+G\right)\left(1+c_{3\varepsilon}R_f\right)-c_{2\varepsilon}\frac{\varepsilon^2}{k} \qquad (10.44)$$

Table 10.3 Values of constants in the model

c_μ	$c_{1\varepsilon}$	$c_{2\varepsilon}$	σ_k	σ_ε	$c_{3\varepsilon}$
0.09	1.44	1.92	1.0	1.3	1 (G>0), 0-0.2 (G<0)

10.3 Specialized Model Equations

(3) k - ε model for Depth-averaged 2D model

continuity: $$\frac{\partial \zeta}{\partial t}+\frac{\partial (H\bar{U})}{\partial x}+\frac{\partial (H\bar{V})}{\partial y}=0$$

x-momentum: $$\frac{\partial (H\bar{U})}{\partial t}+\frac{\partial (H\bar{U}^2)}{\partial x}+\frac{\partial (H\bar{V}\bar{U})}{\partial y}=-gH\frac{\partial \zeta}{\partial x}$$

$$+\frac{1}{\rho}\frac{\partial (H\overline{\tau_{xx}})}{\partial x}+\frac{1}{\rho}\frac{\partial (H\overline{\tau_{xy}})}{\partial y}+\frac{\tau_{sx}-\tau_{bx}}{\rho}$$

$$+\frac{1}{\rho}\frac{\partial}{\partial x}\int_{-h}^{\zeta}\rho (U-\bar{U})^2 dz+\frac{1}{\rho}\frac{\partial}{\partial y}\int_{-h}^{\zeta}\rho (U-\bar{U})(V-\bar{V})dz$$

10.4 Turbulence-Closure Models

y-momentum:

$$\frac{\partial(H\bar{V})}{\partial t}+\frac{\partial(H\overline{\bar{U}\bar{V}})}{\partial x}+\frac{\partial(H\bar{V}^2)}{\partial y}=-gH\frac{\partial\zeta}{\partial y}+\frac{1}{\rho}\frac{\partial(H\overline{\tau_{yx}})}{\partial x}+\frac{1}{\rho}\frac{\partial(H\overline{\tau_{yy}})}{\partial y}+\frac{\tau_{sy}-\tau_{by}}{\rho}$$
$$+\frac{1}{\rho}\frac{\partial}{\partial x}\int_{-h}^{\zeta}\rho(U-\bar{U})(V-\bar{V})dz+\frac{1}{\rho}\frac{\partial}{\partial y}\int_{-h}^{\zeta}\rho(V-\bar{V})^2dz \qquad (10.47)$$

Scalar transport:

$$\frac{\partial(H\bar{\Phi})}{\partial t}+\frac{\partial(H\bar{U}\bar{\Phi})}{\partial x}+\frac{\partial(H\bar{V}\bar{\Phi})}{\partial y}=\frac{1}{\rho}\frac{\partial(H\bar{J}_x)}{\partial x}+\frac{1}{\rho}\frac{\partial(H\bar{J}_y)}{\partial y}+\frac{q_s}{\rho}$$
$$+\frac{1}{\rho}\frac{\partial}{\partial x}\int_{-h}^{\zeta}\rho(U-\bar{U})(\Phi-\bar{\Phi})dz+\frac{1}{\rho}\frac{\partial}{\partial y}\int_{-h}^{\zeta}\rho(V-\bar{V})(\Phi-\bar{\Phi})dz \qquad (10.48)$$

10.4 Turbulence-Closure Models

- $k - \varepsilon$ model

Assumptions;

$\overline{\tau}_{ij}$ ~ depth-averaged turbulent stress ($-\rho\overline{uv}$) acting in x_i -direction on a face perpendicular to x_j

$$\frac{\overline{\tau}_{ij}}{\rho} = \tilde{\nu}_t \left(\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_j}{\partial x_i} \right) - \frac{2}{3}\tilde{k}\delta_{ij} \tag{10.49}$$

$\overline{J}_i$ ~ depth-averaged flux of $\Phi(-\rho\overline{u\phi}\ or\ -\rho\overline{v\phi})$ in direction x_i

$$\frac{\overline{J}_i}{\rho} = \tilde{\Gamma}_t \frac{\partial \overline{\Phi}}{\partial x_i} \tag{10.50}$$

10.4 Turbulence-Closure Models

with

$$\tilde{\nu}_t = c_\mu \frac{\tilde{k}^2}{\tilde{\varepsilon}} \qquad (10.51)$$

$$\tilde{\Gamma}_t = \frac{\tilde{\nu}_t}{\sigma_t} \qquad (10.52)$$

where $\tilde{k}$, $\tilde{\varepsilon}$ are depth-averaged values

The variation of $\tilde{k}$, $\tilde{\varepsilon}$ is determined from the following transport equations

$$\bar{U}\frac{\partial \tilde{k}}{\partial x} + \bar{V}\frac{\partial \tilde{k}}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\tilde{\nu}_t}{\sigma_k}\frac{\partial \tilde{k}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\tilde{\nu}_t}{\sigma_k}\frac{\partial \tilde{k}}{\partial y}\right) + P_h + P_{kV} - \tilde{\varepsilon} \qquad (10.53)$$

$$\bar{U}\frac{\partial \tilde{\varepsilon}}{\partial x} + \bar{V}\frac{\partial \tilde{\varepsilon}}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\tilde{\nu}_t}{\sigma_\varepsilon}\frac{\partial \tilde{\varepsilon}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\tilde{\nu}_t}{\sigma_\varepsilon}\frac{\partial \tilde{\varepsilon}}{\partial y}\right) + c_{1\varepsilon}\frac{\tilde{\varepsilon}}{\tilde{k}}P_h + P_{\varepsilon V} - c_{2\varepsilon}\frac{\tilde{\varepsilon}^2}{\tilde{k}} \qquad (10.54)$$

10.4 Turbulence-Closure Models

where

$$P_h = \tilde{\nu}_t \left[2\left(\frac{\partial \bar{U}}{\partial x}\right)^2 + 2\left(\frac{\partial \bar{V}}{\partial y}\right)^2 + \left(\frac{\partial \bar{U}}{\partial y} + \frac{\partial \bar{V}}{\partial x}\right)^2 \right] \quad (10.55)$$

→ production of due to interaction of the turbulent stresses with the horizontal mean velocity gradients

$$P_{kV} = c_k \frac{U_*^3}{h} \quad (10.56)$$

$$P_{\varepsilon V} = c_\varepsilon \frac{U_*^4}{h^2} \quad (10.57)$$

$$U_* = \sqrt{c_f \left(\bar{U}^2 + \bar{V}^2\right)} \quad (10.58)$$

$$c_k = \frac{1}{\sqrt{c_f}} \quad (10.59)$$

$$c_\varepsilon = \frac{1}{\sqrt{e^* \sigma_t}} \frac{c_{2\varepsilon}}{c_f^{3/4}} \sqrt{c_\mu} = 3.6 \frac{c_{2\varepsilon}}{c_f^{3/4}} \sqrt{c_\mu} \quad (10.60)$$

c_f = friction coefficient

10.4 Turbulence-Closure Models

- P_{kV}, $P_{\varepsilon V}$

~ All terms originating from nonuniformity of vertical profiles are assumed to be absorbed by the additional source terms, P_{kV}, $P_{\varepsilon V}$.

~ stems from the significant vertical velocity gradients near the bottom

~ relatively large turbulence shear stresses in the near-bottom region produce turbulence

~ depends strongly on the bottom roughness, via the friction velocity, U_*

- e^*

$$e^* = \frac{\overline{\Gamma}_t}{U_* h}$$

For wide laboratory flume, $e^* = 0.15$

10.4 Turbulence-Closure Models

For real rivers, e^{*} = 0.60 (Fischer et al., 1979)

→ The diffusivity obtained in real rivers accounts not only for turbulent transport but also for dispersive transport due to vertical non-uniformities of scalar quantities and velocity components.

When significant secondary motions in cross-sectional planes are present, relatively small non-uniformities of temperature or concentration may cause relatively large dispersion contribution to the Φ -Eq.

In natural rivers, secondary motions may arise from large-scale irregularities in the river bed and from rivers bends.

σ_t = 0.5

10.4 Turbulence-Closure Models

- Assessment of two-equation models

i) Advantage

① Two-equation models account for the transport of the turbulent velocity scale and the length scale.

② Two-equation models are the simplest ones that promise success for those flows for which the length scale cannot be prescribed empirically in an easy way. Examples: separated flows, complex shear layers

③ With efficient solution procedure, the additional solution of the length scale equation is not computationally expensive.

10.4 Turbulence-Closure Models

④ The k - ε model is one of the most widely tested and successfully applied turbulence models.

⑤ The depth-averaged version of k - ε model has been applied with success in a number of different calculations of the flow field and pollutant transport processes.

→ This model was found to be particularly suitable for situations involving the interaction of turbulence generated both at the river bed and by the shear layers of discharging jets.

10.4 Turbulence-Closure Models

ii) Limitation of model

① k - ε model uses the same eddy viscosity/diffusivity for all Reynolds stress and heat or mass flux components.

- The standard $k-\varepsilon$ model assumes an isotropic eddy viscosity/diffusivity and hence constructs a direct relation to one velocity scale $\sqrt{k}$.

- But in certain flow situations the assumption of an isotropic eddy viscosity is too crude.

10.4 Turbulence-Closure Models

② It cannot produce the turbulence driven secondary motions in straight open channels.

→ It does not allow any directional influences on the stresses and fluxes, for example, those due to buoyancy forces.

③ In order to allow for the non-isotropic nature of the eddy viscosity/diffusivity, the $k-\varepsilon$ model should be refined by introducing a so called algebraic stress/flux model to replace the simple combined relations, Eq. (10.23) and Eq. (10.41).