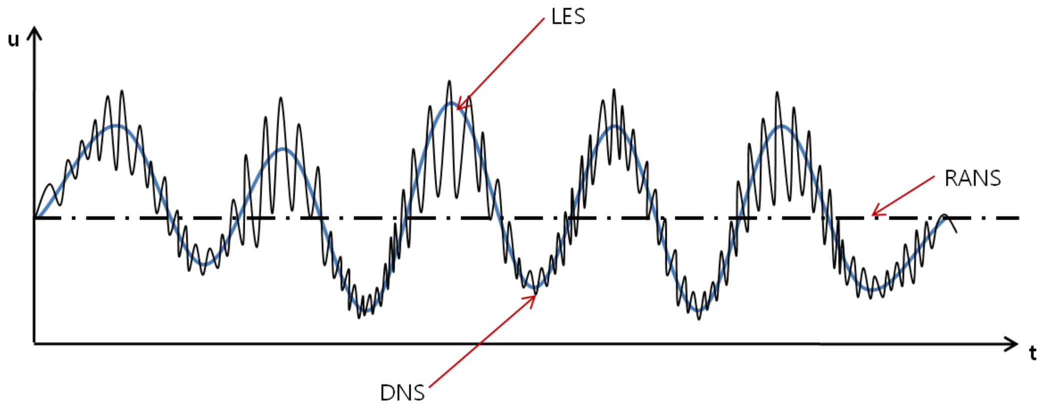
Lecture 26

Turbulence Models (1)



Lecture 26 Turbulence Models (1)

Contents

Objectives

- What is turbulence modeling?
- Derive the mean flow equation

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- References

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26.1 Introduction

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Turbulence is a <u>dangerous topic</u> which is at the <u>origin of serious fights</u> in scientific meetings since it represents extremely different points of view, all of which have in common their complexity, as well as an inability to solve the problem. It is even difficult to agree on <u>what exactly is the problem to be solved</u>.

26.1 Introduction

26.1.1 The Role of turbulence models

- **Why we need turbulence models?**

▪ Turbulent flows of practical relevance

→ highly random, unsteady, three-dimensional

→ Turbulent motion (velocity distribution), heat and mass transfer processes are extremely difficult to describe and to predict theoretically.

▪ **Solution for turbulent flows**

(1) Navier-Stokes equation (DNS)

- Exact equations describing the turbulent motion are known.
- Numerical procedures are available to solve N-S eqs.

26.1 Introduction

- Size of numerical grids used must be small enough to resolve the smallest significant eddy scale present in the flow, and the simulation must be carried out for a significantly long time that initial conditions have died out and significant features of the flow have evolved.

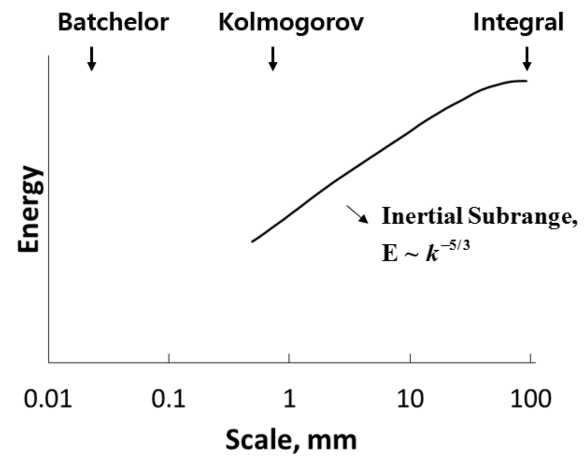
→ Storage capacity and speed of present-day computers are still not sufficient to allow a solution for any practically relevant turbulent flows.

- Thus, computer simulations of the full N-S equation are usually limited to flows where periodicity of the flow can be assumed and the boundaries are simple, usually rectangular.

26.1 Introduction

- **Kolmogorov microscale**

- Dissipation length scale of small eddy is



$$\eta \propto \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}} \qquad (26.1)$$

$$\varepsilon \propto \tilde{u}^2 \cdot \frac{\tilde{u}}{l} \propto \frac{\tilde{u}^3}{l} \qquad (26.2)$$

- l is <u>integral length scale</u> which is the same as turbulent velocity field.
- Integral scale divided by dissipation length scale is

$$\frac{l}{\eta} = \left(\frac{\tilde{u}l}{\nu}\right)^{3/4} = \mathrm{Re}^{3/4} \qquad (26.3)$$

- Thus, number of computational grid is proportional $\mathrm{Re}^{3/4}$

$[Ex]\ \mathrm{Re} = 10^5 \rightarrow \text{No. of grid points in 3D} \sim 10^{45/4}$

26.1 Introduction

(2) Reynolds equation (RANS)

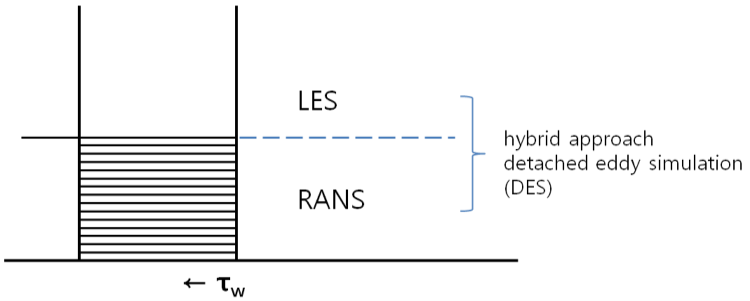
- Average N-S equations to remove turbulent fluctuations completely

-Describe the complete effect of turbulence on the average motion by using turbulence model

- Model the small eddies vis turbulence modeling

(3) LES

- numerical resolution of only the large eddies
- small-scale motion is filtered out
- In case of high Re, wall model is needed.



26.1 Introduction

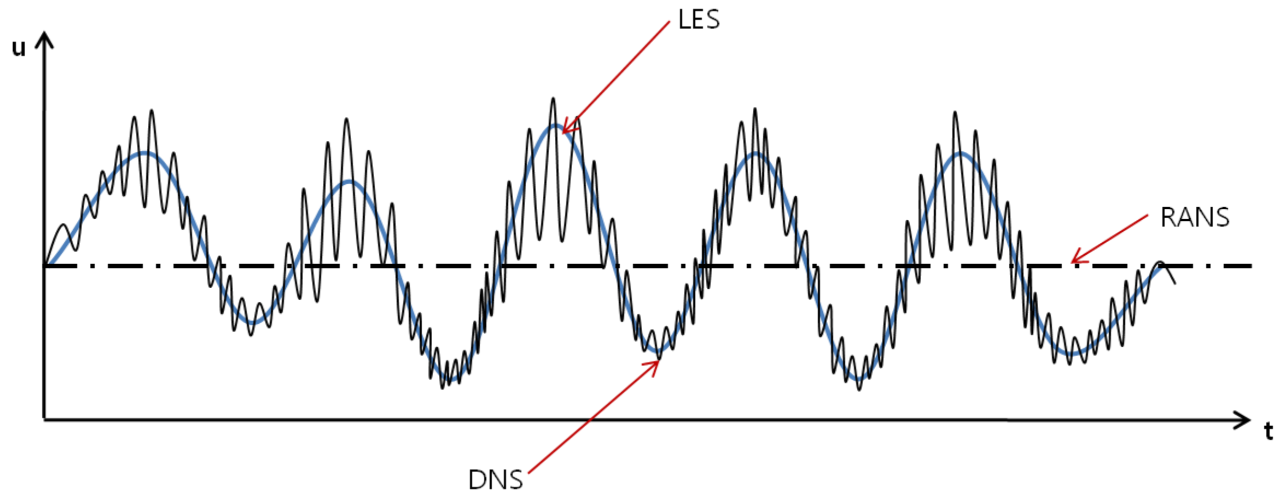
- **Solution for turbulent flows**

DNS: direct numerical simulation of N-S eq.

LES: numerical resolution of only the large eddies (periodic vortex)

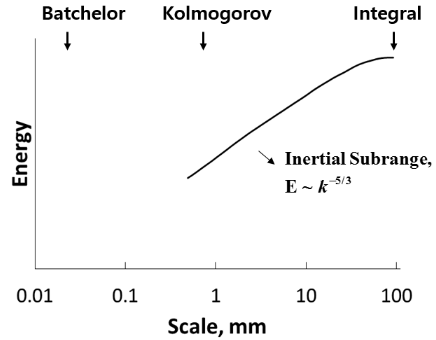
RANS: solution of Reynolds-Averaged N-S eq.

→ effects of all turbulent motions are accounted for by the turbulence model



26.1 Introduction

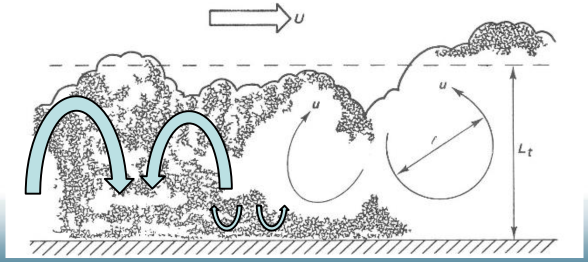
▪ Scale of turbulence



- eddying motion with a wide spectrum of eddy sizes and a corresponding spectrum of fluctuation frequencies

i) Large-scale eddies:

- contain much of the kinetic energy and little of the vorticity
- Large eddies tend to be anisotropic.
- The forms of the largest eddies (low-frequency fluctuations) are determined by the boundary conditions (size of the flow domain).
- These large eddies gradually break down into smaller eddies.

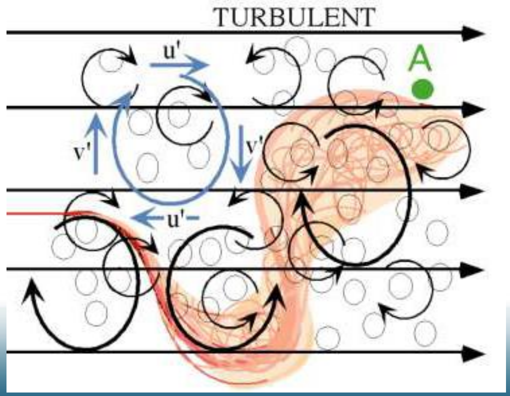


26.1 Introduction

ii) Small eddies:

- have little kinetic energy but much vorticity
- The small eddies tend to be isotropic.
- The forms of the smallest eddies (highest-frequency fluctuations) are determined by the viscous forces.
- several orders of magnitude smaller

→ In numerical solution, to resolve the small-scale turbulent motion, 10^9 to 10^{12} grid points would be necessary to cover the flow domain in three dimensions.



26.1 Introduction

- Classification of turbulence

i) Anisotropic turbulence: general turbulence; it varies in intensity in direction

ii) Isotropic turbulence: smallest turbulence; independent of direction (orientation)

$$\overline{u_i u_j} = \begin{cases} 0, \; i \neq j \\ const. \;, i = j \end{cases}$$

iii) Nonhomogeneous turbulence

iv) Homogeneous turbulence: statistically independent of the location

$$\overline{u_{i_a}^{\;2}} = \overline{u_{i_b}^{\;2}}$$

$$\overline{u_i u_j}_{\;a} = \overline{u_i u_j}_{\;b}$$

26.1 Introduction

26.1.2 Turbulence models

Turbulence models: a set of equations (algebraic or differential) which determine the turbulent transport terms in the mean-flow equations and thus close the system of equations

1) Time-averaging approaches (models)

Model	No. of turbulent transport eqs.	Turbulence quantities transported
Zero equation model	0	None
One equation model	1	*k* (turbulent kinetic energy)
Two equation model	2	*k*-ε (turbulent energy, dissipation rate), *k*-*l*
Stress/flux model	6	$\overline{u_i u_j}$ components (stress terms)
Algebraic stress model	2	*k*, ε used to calculate

26.1 Introduction

2) Space-averaged approaches

→ Large Eddy Simulation (LES)

- simulate the larger and more easily-resolvable scales of the motions while accepting the smaller scales will not be properly represented

26.2 Mean Flow Equation and Closure Problem

26.2.1 Reynolds averaged basic equation

▪ Navier-Stokes eq.

~ Eq. of motion for turbulent motion

~ describes all the details of the turbulent fluctuating motion

~ These details cannot presently be resolved by a numerical calculation procedure.

~ Engineers are not interested in obtaining these details but interested in average quantities.

26.2 Mean Flow Equation and Closure Problem

- Definition of mean quantities by Reynolds

$$U_i = \frac{1}{T}\int_0^T \tilde{u}_i \, dt \qquad (26.4a)$$

$$\Phi = \frac{1}{T}\int_0^T \tilde{\phi} \, dt \qquad (26.4b)$$

where T = averaging time; $\tilde{\phi}$ = scalar quantity (temperature, concentration)

- Averaging time should be long compared with the time scale of the turbulent motion but small compared with that of the mean flow in transient (unsteady) problems.

26.2 Mean Flow Equation and Closure Problem

- Decomposition of instantaneous values

$$\tilde{u}_i = U_i + u_i \qquad (26.5a)$$

$$\tilde{\phi} = \Phi + \phi \qquad (26.5b)$$

$\downarrow$ mean $\searrow$ fluctuations

Substitute (26.5) into time-dependent equations of continuity and N-S eqs.

and average over time as indicated by (26.4)

→ mean flow equations (RANS)

26.2 Mean Flow Equation and Closure Problem

$\frac{\mu}{\rho}\nabla^2 U_i$ (molecular viscosity) dropped

Continuity:

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}+\frac{\partial W}{\partial z}=0 \tag{26.6}$$

x-momentum:

$$\frac{\partial U}{\partial t}+\frac{\partial U^2}{\partial x}+\frac{\partial (VU)}{\partial y}+\frac{\partial (WU)}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x}+fV-\frac{\partial \overline{u^2}}{\partial x}-\frac{\partial \overline{uv}}{\partial y}-\frac{\partial \overline{uw}}{\partial z} \tag{26.7}$$

y-momentum:

$$\frac{\partial V}{\partial t}+\frac{\partial UV}{\partial x}+\frac{\partial (V^2)}{\partial y}+\frac{\partial (WV)}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y}-fU-\frac{\partial \overline{vu}}{\partial x}-\frac{\partial \overline{v^2}}{\partial y}-\frac{\partial \overline{vw}}{\partial z} \tag{26.8}$$

26.2 Mean Flow Equation and Closure Problem

z-momentum

$$\frac{\partial W}{\partial t}+\frac{\partial (UW)}{\partial x}+\frac{\partial (VW)}{\partial y}+\frac{\partial (W^2)}{\partial z} \qquad (26.9)$$

$$=-\frac{1}{\rho}\frac{\partial P}{\partial z}-g-\frac{\partial \overline{wu}}{\partial x}-\frac{\partial \overline{wv}}{\partial y}-\frac{\partial \overline{w^2}}{\partial z}$$

Scalar transport:

$$\frac{\partial \Phi}{\partial t}+\frac{\partial (U\Phi)}{\partial x}+\frac{\partial (V\Phi)}{\partial y}+\frac{\partial (W\Phi)}{\partial z} \qquad (26.10)$$

$$=S_\Phi-\frac{\partial \overline{u\phi}}{\partial x}-\frac{\partial \overline{v\phi}}{\partial y}-\frac{\partial \overline{w\phi}}{\partial z}$$

$D\nabla^2 U_i$ (molecular diffusion) dropped

in which P = mean static pressure

f = Coriolis parameter

ρ = fluid density

S_Φ = volumetric source/sink term of scalar quantity

26.2 Mean Flow Equation and Closure Problem

Eqs. (26.6)~(26.10) do not form a closed set.

- Non-linearity of the original N-S eq. and scalar transport eq. introduce unknown correlations between fluctuating velocities and between velocity and scalar fluctuations in the averaging processes.

$\rho \overline{u^2}$ $etc.$ = rate of transport of momentum = turbulent Reynolds stresses

$\rho \overline{u\phi}$ $etc.$ = rate of transport of heat or mass = turbulent heat or mass fluxes

26.2 Mean Flow Equation and Closure Problem

▪ In Eqs. (26.6)~(26.10), viscous stresses and molecular heat or mass fluxes are neglected because they are much smaller than their turbulent counterparts except in the viscous sublayer very near walls.

▪ Eqs. (26.6)~(26.10) can be solved for average dependent variables when the turbulence correlation can be determined in some way.

→ task of the **turbulence models**

26.2 Mean Flow Equation and Closure Problem

- Level of a turbulence model

~ depends on the relative importance of the turbulent transport terms

For the turbulent jet motion, simulation of turbulence is important.

For the horizontal motion in large shallow water bodies, refined turbulence modeling is not important because the inertial term in the momentum equations are balanced mainly by the pressure gradient and/or buoyancy terms.

→ The simulation of turbulence in heat and mass transport models is always important because the scalar transport equation does not contain any pressure gradient and/or buoyancy terms.