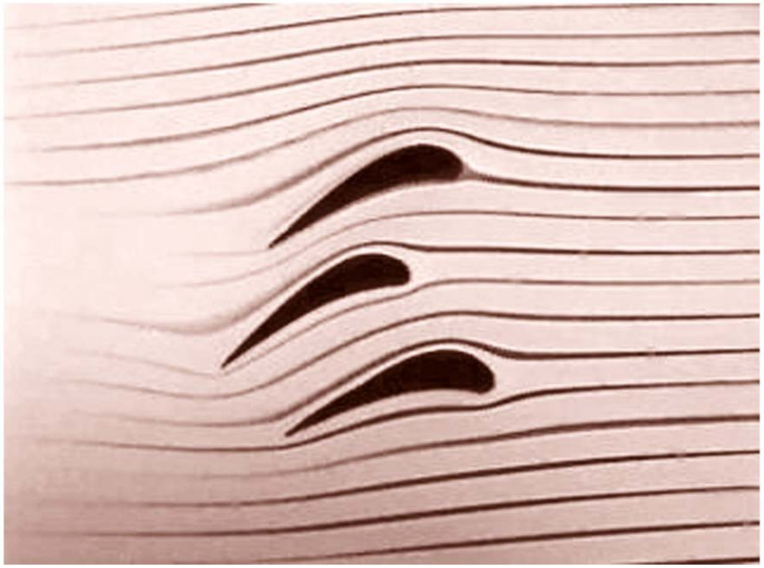
Chapter 2

Kinematics



Chapter 2 Kinematics

Contents

Objectives

- Define methods of flow description
- Classify fluid motions
- Study kinematics of fluid

$$\sum \vec{F} = m\vec{a}$$

2.1 The Velocity Field

velocity, acceleration ~ vector quantities

$\vec{q}$ $\vec{a}$

Cartesian coordinates

x y z

u v w

a_x a_y a_z

2.1 The Velocity Field

2.1.1 Lagrangian approach

~ coordinates of moving particles are represented as function of time

~ follow a particular particle through the flow field → *path line*

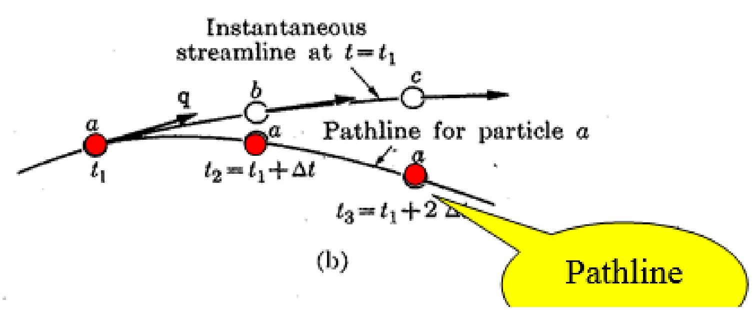
At $t = t_0$ coordinates (position) of a particle (a, b, c)

At $t = t$ position of a particle (x, y, z)

$$x = f_1(a, b, c, t)$$

$$y = f_2(a, b, c, t)$$

$$z = f_3(a, b, c, t)$$



2.1 The Velocity Field

$$u = \frac{\partial x}{\partial t} \qquad a_x = \frac{\partial u}{\partial t} = \frac{\partial^2 x}{\partial t^2}$$

$$v = \frac{\partial y}{\partial t} \qquad a_y = \frac{\partial v}{\partial t} = \frac{\partial^2 y}{\partial t^2}$$

$$w = \frac{\partial z}{\partial t} \qquad a_z = \frac{\partial w}{\partial t} = \frac{\partial^2 z}{\partial t^2}$$

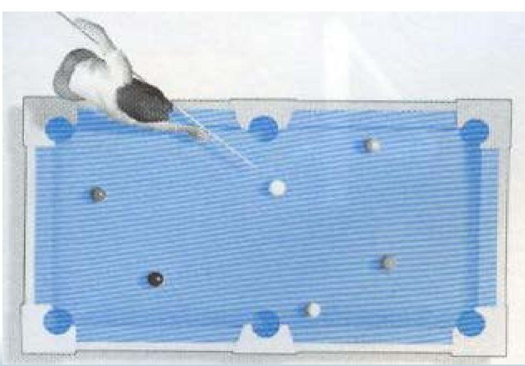
2.1 The Velocity Field

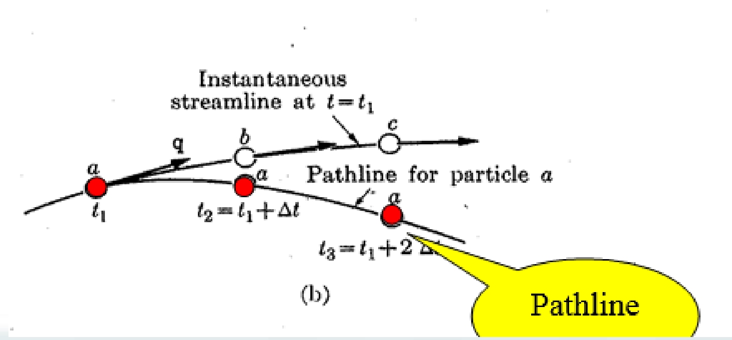
~ commonly used in the solid dynamics

~ convenient to identify a discrete particle, e.g. center of mass of spring - mass system

~ cumbersome when dealing with a fluid as a continuum of particles

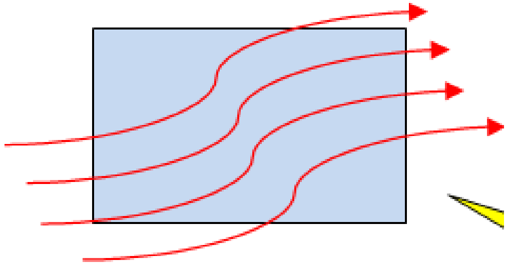
→ Due to deformation of fluid, we are not usually concerned with the detailed history of an individual particle, but rather with interrelation of flow properties at individual points in the flow field.





2.1 The Velocity Field

2.1.2 Eulerian method



~ observer fixes attention at discrete points

~ notes flow characteristics in the vicinity of a fixed point as particles pass by

~ focused on the fluid which passes through a control volume that is fixed in space

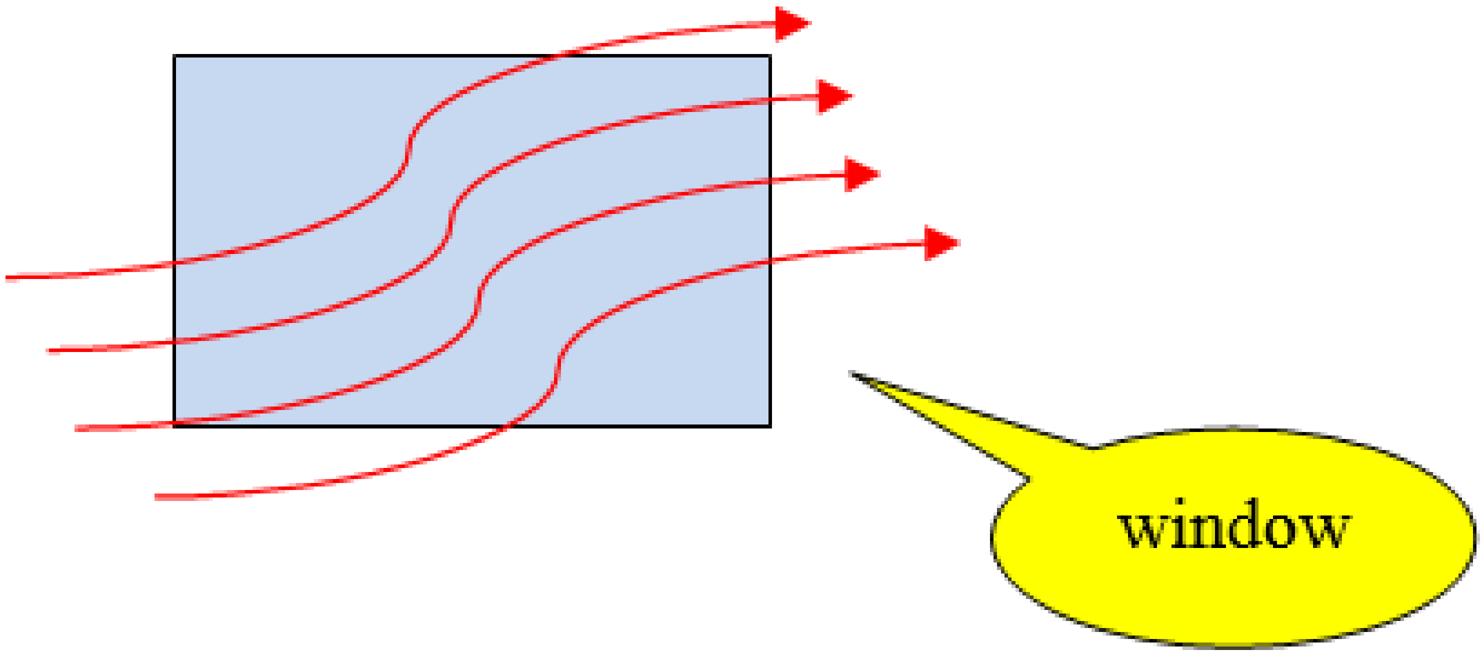
~ familiar framework in which most fluid problems are solved

~ instantaneous picture of the velocities and accelerations of every particle

→ streamline

~ velocities at various points are given as function of time

2.1 The Velocity Field



2.1 The Velocity Field

$$\vec{q} = \vec{i}u + \vec{j}v + \vec{k}w$$

where
$$u = f_1(x, y, z, t)$$

$$v = f_2(x, y, z, t)$$

Independent variables

$$w = f_3(x, y, z, t)$$

x, y, z, t = independent variables

$\vec{i}, \vec{j}, \vec{k}$ = unit vectors

2.1 The Velocity Field

2.1.3 Total Derivative

(1) Total change in velocity

= sum of partial derivatives of the four independent variables, x, y, z, t

$$x-\mathrm{dir} \;:\; du = \frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

total derivative: $$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

$$= \underline{\frac{\partial u}{\partial t}} + \underline{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}$$

local change due to unsteadiness

convective change due to translation

2.1 The Velocity Field

$$y-\mathrm{dir}\ :\ \frac{dv}{dt}=\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}$$

$$z-\mathrm{dir}\ :\ \frac{dw}{dt}=\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}$$

(2) Total rate of density change of <u>compressible</u> fluid

$$\rho=\rho(x,y,z,t)$$

$$\frac{d\rho}{dt}=\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+v\frac{\partial\rho}{\partial y}+w\frac{\partial\rho}{\partial z}=\frac{\partial\rho}{\partial t}+u_j\frac{\partial\rho}{\partial x_j}$$

For incompressible fluid, $\frac{d\rho}{dt}=0$

For steady flow, $\frac{\partial\rho}{\partial t}=0$

2.1 The Velocity Field

Acceleration

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial u}{\partial t} + u_j\frac{\partial u}{\partial x_j}$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \frac{\partial v}{\partial t} + u_j\frac{\partial v}{\partial x_j}$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \frac{\partial w}{\partial t} + u_j\frac{\partial w}{\partial x_j}$$

Vector notation

$$\vec{a} = \vec{i}a_x + \vec{j}a_y + \vec{k}a_z$$

$$\vec{a} = \frac{d\vec{q}}{dt} = \frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\,\vec{q}$$

local acceleration

convective acceleration

2.2 Steady versus Uniform motion

i) steady motion: no changes with time at fixed point $\longleftrightarrow$ unsteady motion

$$\frac{\partial \vec{q}}{\partial t} = 0 \rightarrow \text{local acceleration} = 0$$

ii) uniform motion: no changes with space $\longleftrightarrow$ non-uniform motion

$$(\vec{q} \cdot \nabla)\vec{q} = 0 \rightarrow \text{convective acceleration} = 0$$

2.2 Steady versus Uniform motion

- Vector differential operators: ∇ → "del" or "nabla"

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

Gradient: $$\nabla f = grad\ f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

Divergence: $$\nabla \cdot \vec{q} = div\ \vec{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

[Re] Vector product

i) dot product → scalar

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\phi$$

2.2 Steady versus Uniform motion

ϕ = angle between the vectors

$$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1 \qquad (\cos 0^{\circ} = 1)$$

$$\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{j}\cdot\vec{i} = \vec{k}\cdot\vec{j} = 0 \qquad (\because \cos 90^{\circ} = 0)$$

ii) cross product → vector

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\phi$$

Direction = perpendicular to the plane of $\vec{a}\ and\ \vec{b}$ → right-hand rule

$$\vec{q}\cdot\nabla = (\vec{i}u + \vec{j}v + \vec{k}w)\cdot(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z})$$

$$= u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

2.2 Steady versus Uniform motion

$$(\vec{q}\cdot\nabla)\,\vec{q} = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)(\vec{i}u + \vec{j}v + \vec{k}w)$$

$$= \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)\vec{i}$$

$$+ \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)\vec{j}$$

$$+ \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)\vec{k}$$

$$\nabla^2 = \nabla\cdot\nabla = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)\cdot\left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right)$$

$$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

2.2 Steady versus Uniform motion

$$\nabla^2 \phi = 0 \quad \rightarrow \quad \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$ → Laplace Eq.

$$grad\ (u + v) = \nabla (u + v) = \nabla u + \nabla v$$

$$div\ (\vec{u} + \vec{v}) = \nabla \cdot (\vec{u} + \vec{v}) = \nabla \cdot \vec{u} + \nabla \cdot \vec{v}$$

$$grad\ (uv) = \nabla (uv) = v \nabla u + u \nabla v$$

$$div\ (u\vec{v}) = \nabla \cdot (u\vec{v}) = \nabla u \cdot \vec{v} + u \nabla \cdot \vec{v}$$

$$div\ grad\ u = \nabla \cdot \nabla u = \nabla^2 u$$