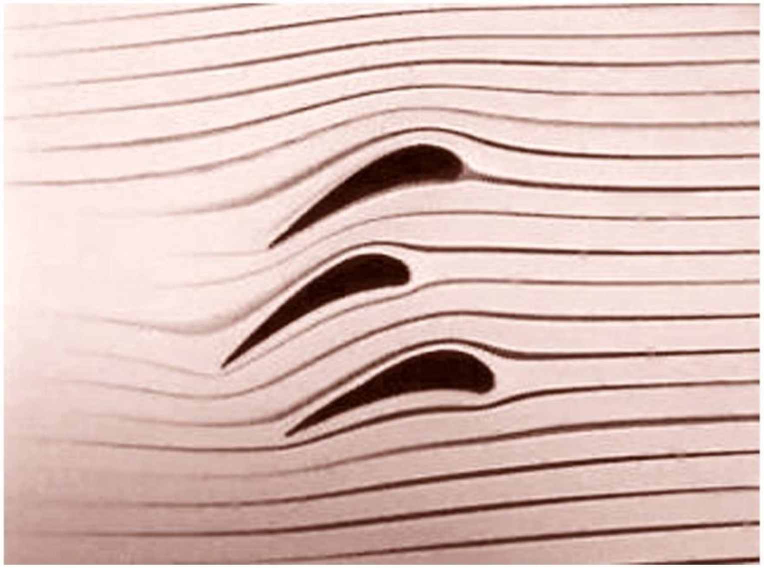
Lecture 2

Kinematics (1)



Lecture 2 Kinematics (1)

Contents

Objectives

- Define methods of flow description
- Study velocity and acceleration field
- Classify fluid motions

Chapter 3 Kinematics of Fluid Motion

- 유체역학(Fluid mechanics): 유체의 운동과 그 운동을 일으키는 힘을 다루는 학문

  - 유체정역학(Fluid statics): 유체의 상대적인 운동이 없는 경우를 다루는 학문; 전단력이 작용하지 않고 수직력만 작용
  - 유체동역학(Fluid dynamics): $\sum \vec{F} = \frac{d}{dt}\left(m\vec{V}\right) = m\frac{d\vec{V}}{dt} + \vec{V}\frac{dm}{dt} = m\vec{a}$
  - 유체운동학(Fluid kinematics): 유체의 운동을 일으키는 힘을 제외하고 운동(변위, 유속, 가속도 등)만을 다루는 다루는 학문
  - 운동역학(Kinetics): 운동과 힘의 관계를 다루는 학문

2.1 The Velocity Field

velocity, acceleration ~ vector quantities

$$\vec{q}(\vec{V}) \qquad \vec{a}$$

Cartesian coordinates

$$x \qquad y \qquad z$$

$$u \qquad v \qquad w$$

$$a_x \qquad a_y \qquad a_z$$

2.1 The Velocity Field

2.1.1 Lagrangian approach

- follow a particular particle through the flow field → *path line*
- fluid properties associated with this particle change as a function of time
- coordinates of moving particles are represented as function of time

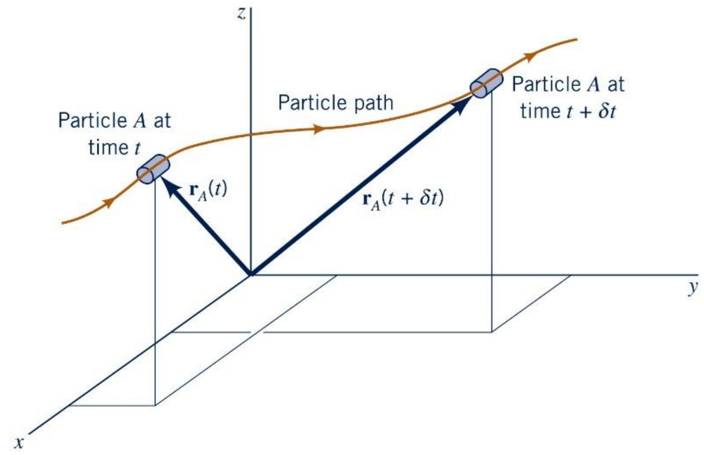
At $t = t_0$ coordinates (position) of a particle $A(a, b, c)$

At $t = t$ position of a particle (x, y, z)

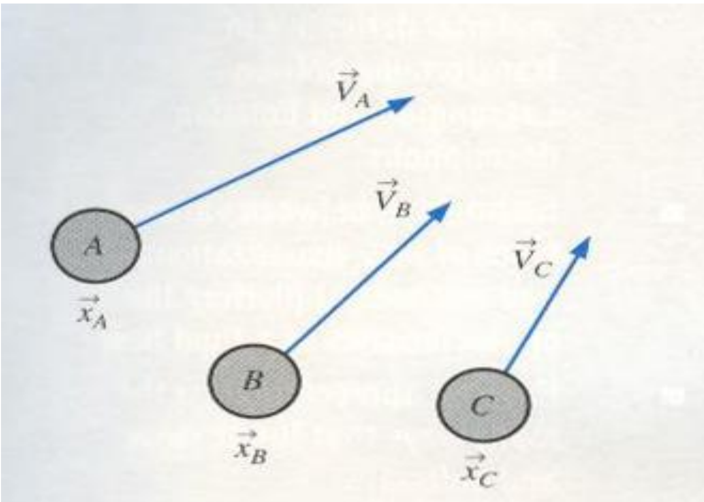
$$x = f_1(a, b, c, t) \quad (2.1a)$$

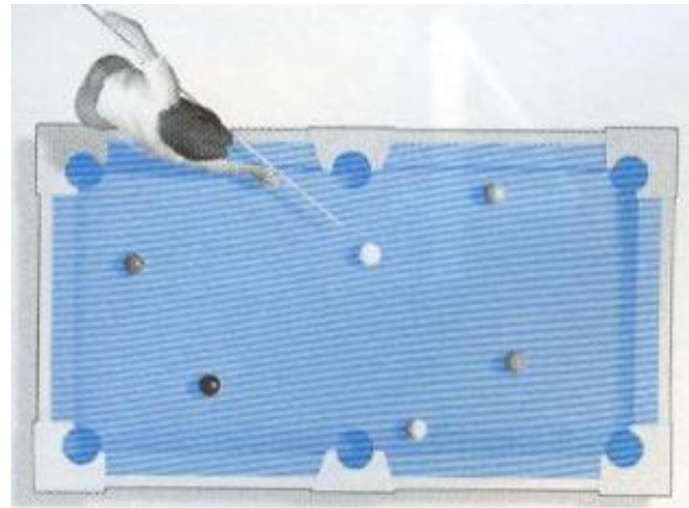
$$y = f_2(a, b, c, t) \quad (2.1b)$$

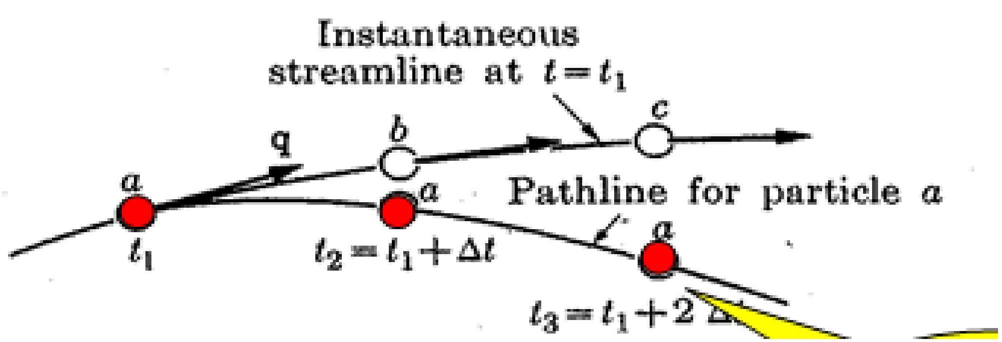
$$z = f_3(a, b, c, t) \quad (2.1c)$$



2.1 The Velocity Field







2.1 The Velocity Field

- Path line (유적선)

~ the position is plotted as a function of time

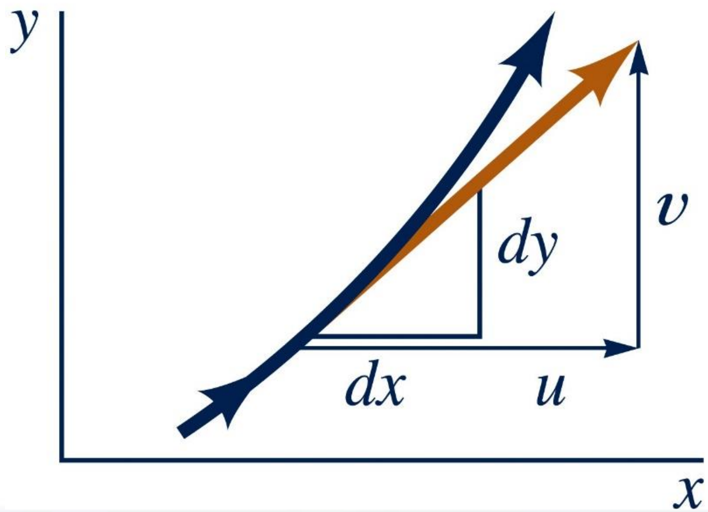
= trajectory of the particle → path line

~ since path line is tangent to the instantaneous velocity at each point along the path, changes in the particle location over an infinitesimally small time are given by

$$dx = u dt; \; dy = v dt; \; dz = w dt$$

$$u = \frac{dx}{dt}; \; v = \frac{dy}{dt} \; w = \frac{dz}{dt} \qquad (2.2a)$$

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} = \frac{dt}{1} \qquad (2.2b)$$



2.1 The Velocity Field

$$u = \frac{\partial x}{\partial t} \qquad v = \frac{\partial y}{\partial t} \qquad w = \frac{\partial z}{\partial t} \tag{2.2a}$$

$$a_x = \frac{\partial u}{\partial t} = \frac{\partial^2 x}{\partial t^2} \tag{2.3a}$$

$$a_y = \frac{\partial v}{\partial t} = \frac{\partial^2 y}{\partial t^2} \tag{2.3b}$$

$$a_z = \frac{\partial w}{\partial t} = \frac{\partial^2 z}{\partial t^2} \tag{2.3c}$$

2.1 The Velocity Field

Lagrangian description is commonly used in the solid dynamics because it is convenient to identify a discrete particle, e.g. center of mass of spring - mass system.

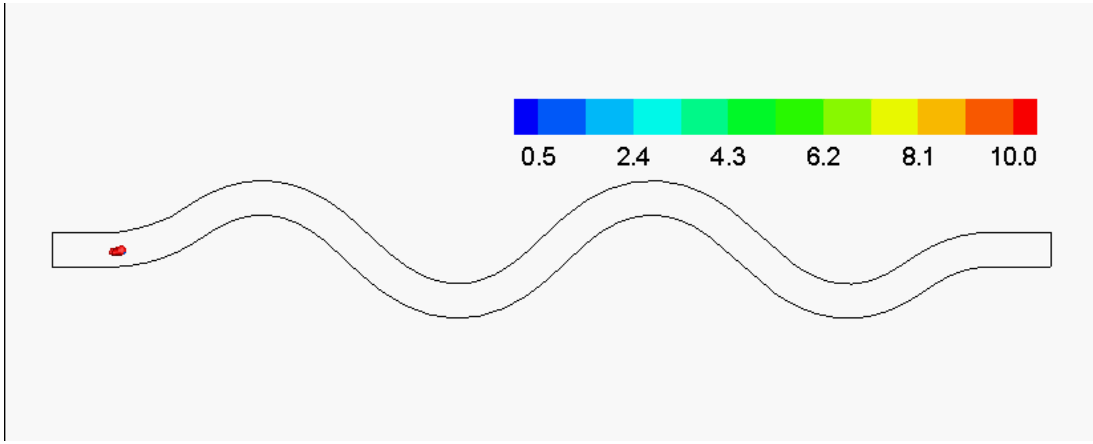
However, it is cumbersome when dealing with a fluid as a continuum of particles due to deformation of fluid.

~ We are not usually concerned with the detailed history of an individual particle, but rather with interrelation of flow properties at individual points in the flow field. → Eulerian description

2.1 The Velocity Field

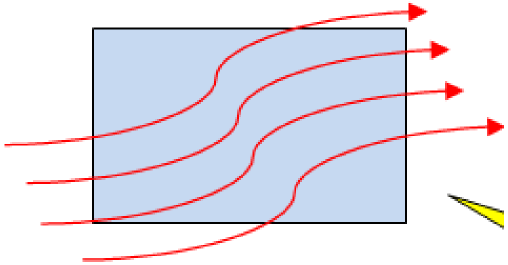
[Re] Examples of Lagrangian description in fluid mechanics

- Numerical fluid mechanics simulations using LPTM (Lagrangian Particle Tracking Model)
- Tagging of individual fluid particles in the experiments or field survey



2.1 The Velocity Field

2.1.2 Eulerian method

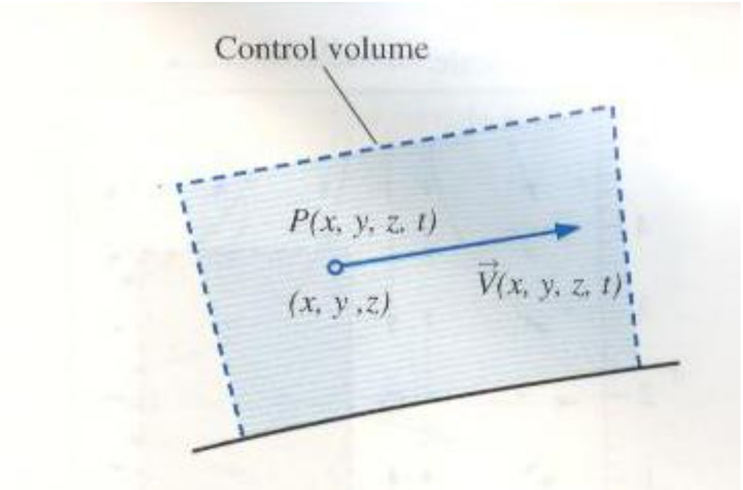


- use the field concept
- observer fixes attention at discrete points
- notes flow characteristics in the vicinity of a fixed point as particles pass by
- focus on the fluid which passes through a control volume that is fixed in space
- familiar framework in which most fluid problems are solved
- instantaneous picture of the velocities and accelerations of every particle

  → streamline
- Velocities (pressure, density) at various points are given as function of time

2.1 The Velocity Field

- Velocity field



$$\vec{q} = \vec{i}u + \vec{j}v + \vec{k}w \qquad (2.4)$$

where $$u = f_1(x, y, z, t) \qquad (2.5a)$$

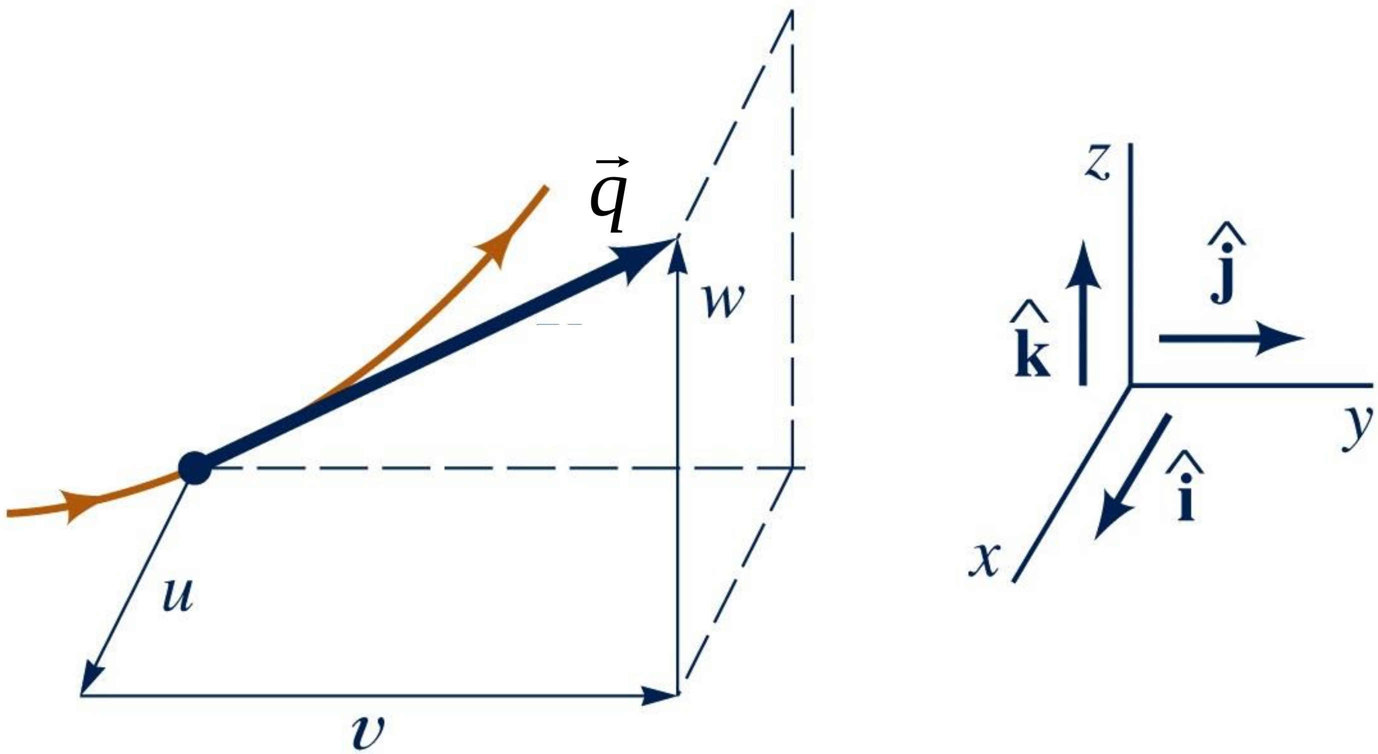
$$v = f_2(x, y, z, t) \qquad (2.5b)$$

$$w = f_3(x, y, z, t) \qquad (2.5c)$$

$x,\ y,\ z,\ t$ = independent variables

$\vec{i}, \vec{j}, \vec{k}$ = unit vectors

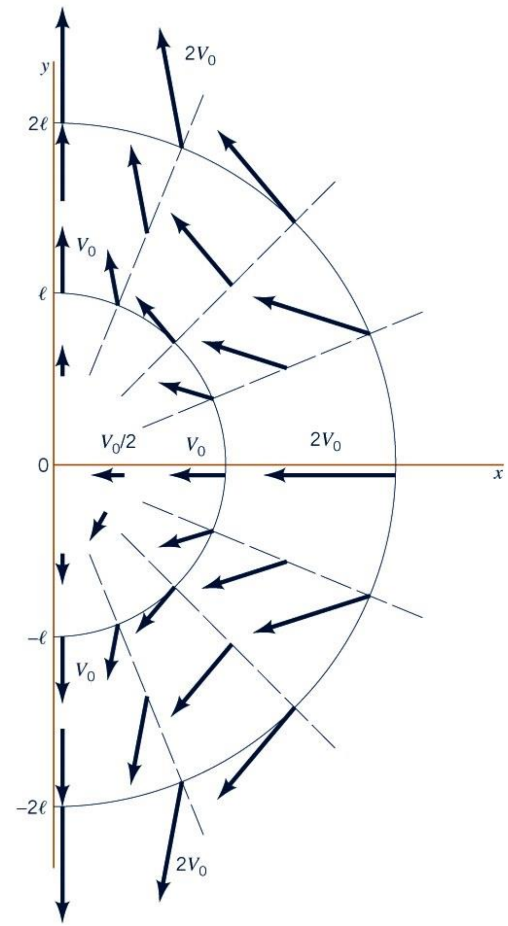
2.1 The Velocity Field



2.1 The Velocity Field

- Speed

$$q = |\vec{q}| = \left(u^2 + v^2 + w^2\right)^{1/2} \qquad (2.6)$$



A change in velocity results in an acceleration.

The acceleration may be due to a change in speed and/or direction.

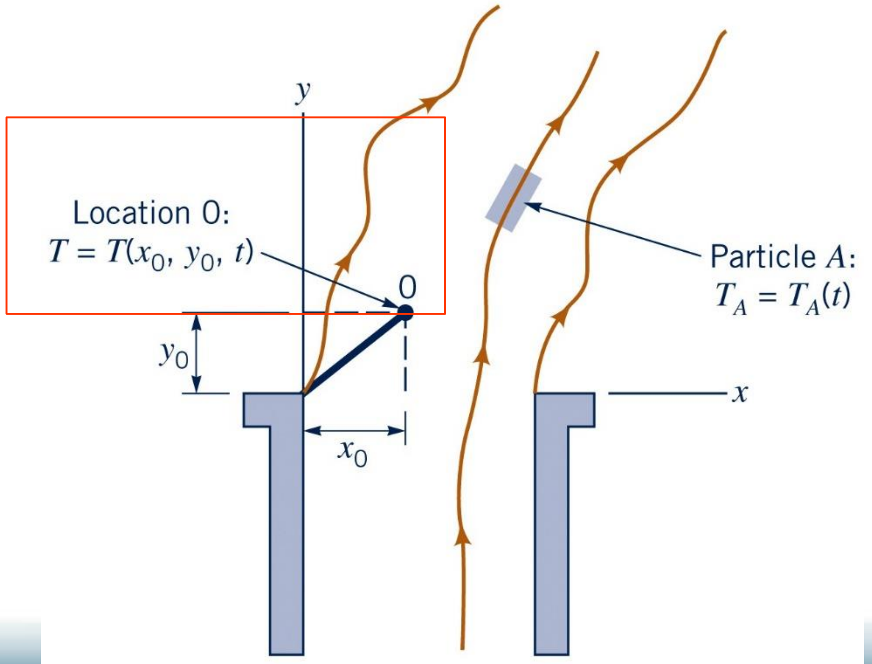
2.1 The Velocity Field

[Re] Two views

Eulerian method: record the temperature at the fixed point *0*

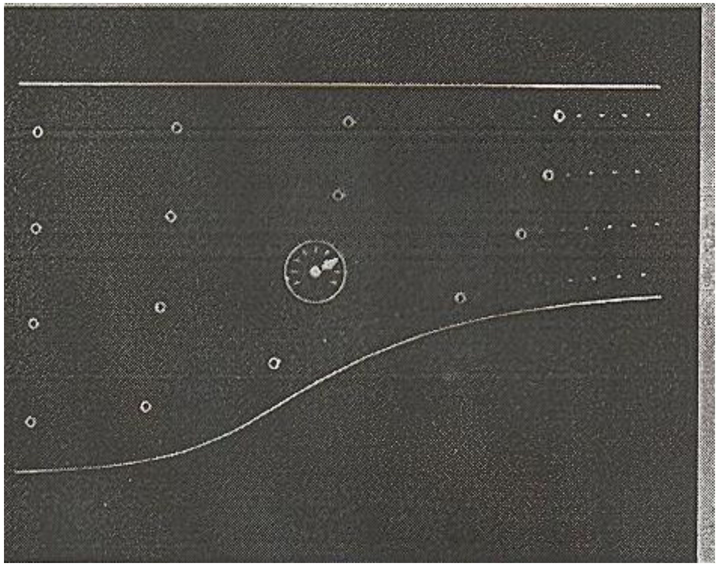
Lagrangian method: follow particle *A*

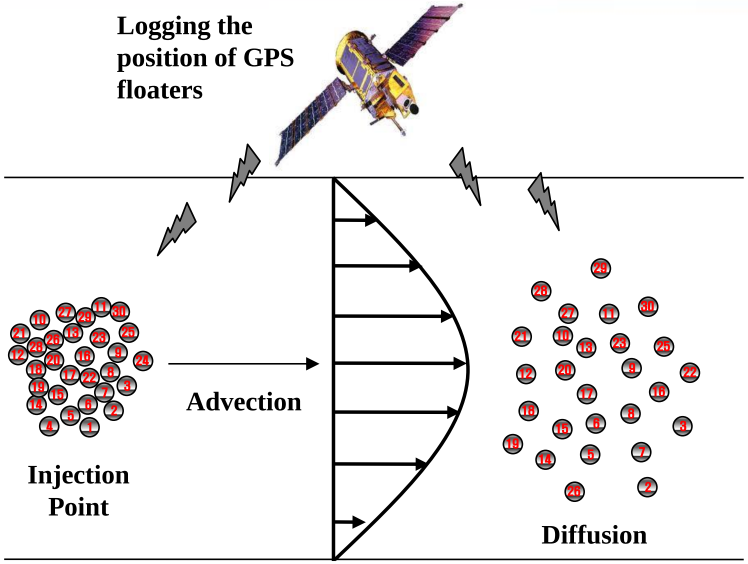
If enough information in Eulerian form is available, Lagrangian information can be derived from the Eulerian data, and vice versa.

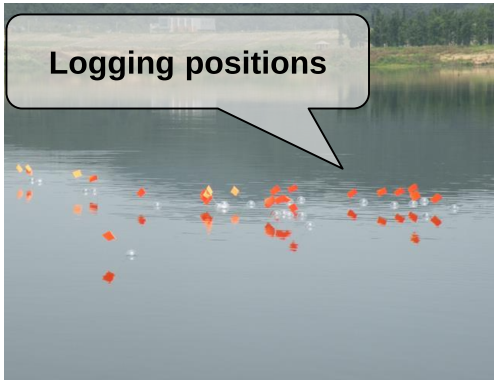


2.1 The Velocity Field

- **Lagrangian measurements**





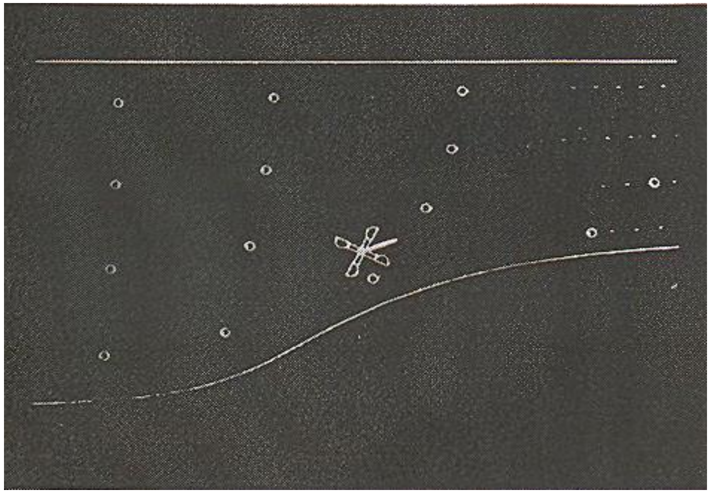


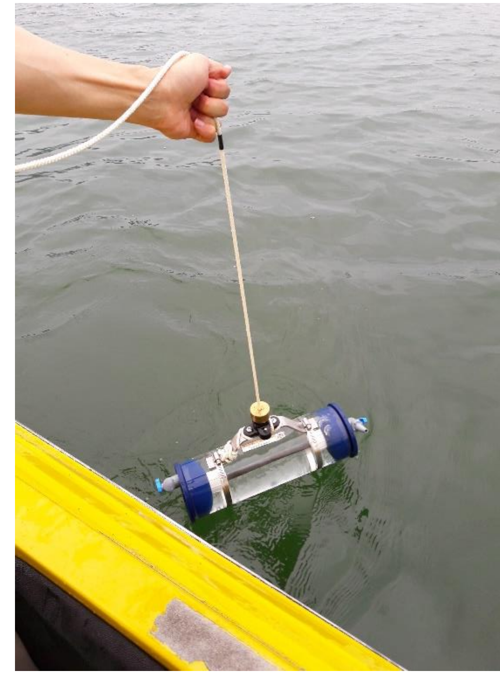
<GPS floater>

Park, I., Seo, I. W., Kim, Y. D., and Han, E. J. (2017). "Turbulent Mixing of Floating Pollutants at the Surface of the River," *Journal of Hydraulic Engineering*.

2.1 The Velocity Field

- Eulerian measurements





Shin, J. H., and Seo, I. W., and Baek, D. (2020). "Longitudinal and Transverse Dispersion Coefficients of 2D Contaminant Transport Model for Mixing Analysis in Open Channels," *Journal of Hydrology*.

2.2 The Acceleration Field

Obtain the acceleration field if the velocity field is known in the Eulerian description.

(1) Total change in velocity (material /substantial derivative)

= sum of partial derivatives of the four independent variables, x, y, z, t

$$x-\text{dir} \;:\; du = \frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

total derivative: $$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

$$= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \qquad (2.6a)$$

local change due to unsteadiness

convective change due to translation

2.2 The Acceleration Field

$$y-\text{dir} \quad : \quad \frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \qquad (2.6b)$$

$$z-\text{dir} \quad : \quad \frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \qquad (2.6c)$$

(2) Total rate of density change of compressible fluid

$$\rho = \rho(x, y, z, t)$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} = \frac{\partial \rho}{\partial t} + u_j\frac{\partial \rho}{\partial x_j} \qquad (2.7)$$

For incompressible fluid, $\frac{d\rho}{dt} = 0$

For steady flow, $\frac{\partial \rho}{\partial t} = 0$

2.2 The Acceleration Field

(3) Acceleration

- time rate of change of velocity

$$\vec{a} = \frac{d\vec{q}}{dt} = \frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\,\vec{q} \qquad (2.8)$$

$$\vec{a} = \vec{i}a_x + \vec{j}a_y + \vec{k}a_z \qquad (2.9)$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial u}{\partial t} + u_j\frac{\partial u}{\partial x_j} \qquad (2.10a)$$

$$a_y = \frac{dv}{dt} = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \frac{\partial v}{\partial t} + u_j\frac{\partial v}{\partial x_j} \qquad (2.10b)$$

$$a_z = \frac{dw}{dt} = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \frac{\partial w}{\partial t} + u_j\frac{\partial w}{\partial x_j} \qquad (2.10c)$$

local acceleration

convective acceleration

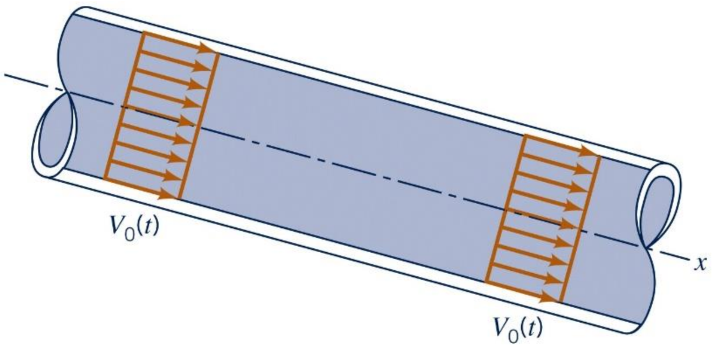
2.3 Steady versus Uniform motion

i) steady motion: no changes with time at fixed point ⟷ <u>unsteady motion</u>

$$\frac{\partial \vec{q}}{\partial t} = 0 \rightarrow \text{local acceleration} = 0$$

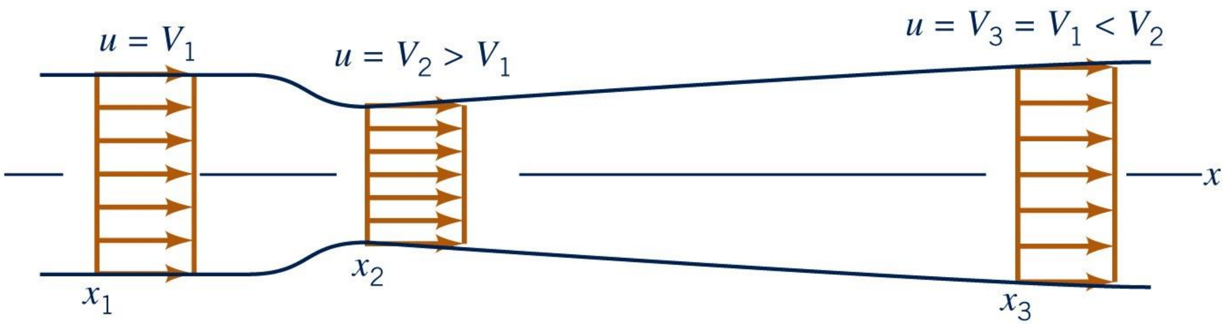
ii) uniform motion: no changes with space ⟷ non-uniform motion

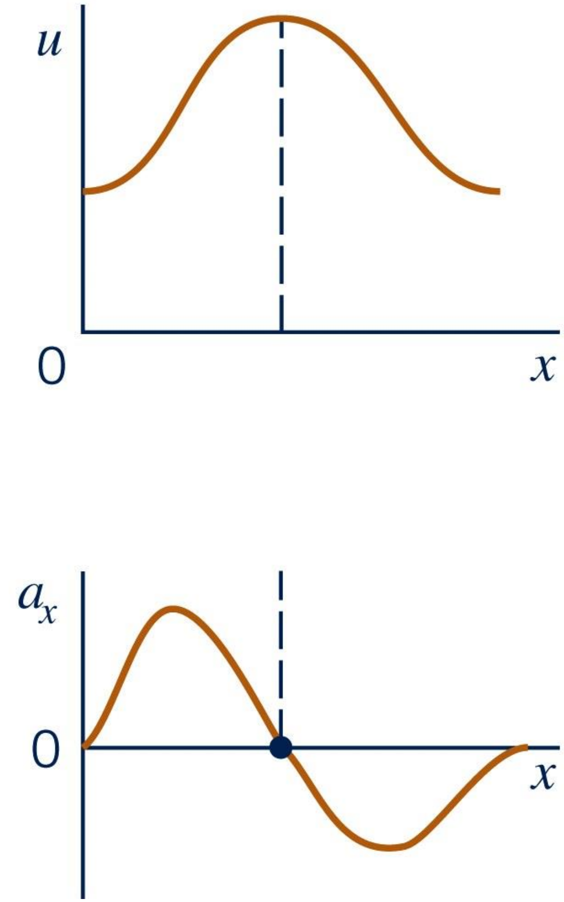
$$(\vec{q} \cdot \nabla)\vec{q} = 0 \rightarrow \text{convective acceleration} = 0$$



2.3 Steady versus Uniform motion

- Non-uniform flow→ convective acceleration





Appendix

(1) Vector differential operators: ∇ → "del" or "nabla"

$$\nabla = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

Gradient: $$\nabla f = grad\ f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

Divergence: $$\nabla \cdot \vec{q} = div\ \vec{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

(2) Vector product

i) dot product → scalar

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\phi$$

Appendix

ϕ = angle between the vectors

$$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1 \qquad (\cos 0^\circ = 1)$$

$$\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{j}\cdot\vec{i} = \vec{k}\cdot\vec{j} = 0 \qquad (\because \cos 90^\circ = 0)$$

ii) cross product $\rightarrow$ vector

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\phi$$

Direction = perpendicular to the plane of $\vec{a}\ and\ \vec{b}$ $\rightarrow$ right-hand rule

$$\vec{q}\cdot\nabla = (\vec{i}u + \vec{j}v + \vec{k}w)\cdot(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z})$$

$$= u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Appendix

$$(\vec{q} \cdot \nabla)\,\vec{q} = \left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \right) (\vec{i}u + \vec{j}v + \vec{k}w)$$

$$= \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \right)\vec{i}$$

$$+ \left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \right)\vec{j}$$

$$+ \left(u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \right)\vec{k}$$

$$\nabla^2 = \nabla \cdot \nabla = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \right) \cdot \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z} \right)$$

$$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Appendix

$$\nabla^2 \phi = 0 \quad \rightarrow \quad \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$ → Laplace Eq.

$$grad\ (u + v) = \nabla (u + v) = \nabla u + \nabla v$$

$$div\ (\vec{u} + \vec{v}) = \nabla \cdot (\vec{u} + \vec{v}) = \nabla \cdot \vec{u} + \nabla \cdot \vec{v}$$

$$grad\ (uv) = \nabla (uv) = v \nabla u + u \nabla v$$

$$div\ (u\vec{v}) = \nabla \cdot (u\vec{v}) = \nabla u \cdot \vec{v} + u \nabla \cdot \vec{v}$$

$$div\ grad\ u = \nabla \cdot \nabla u = \nabla^2 u$$