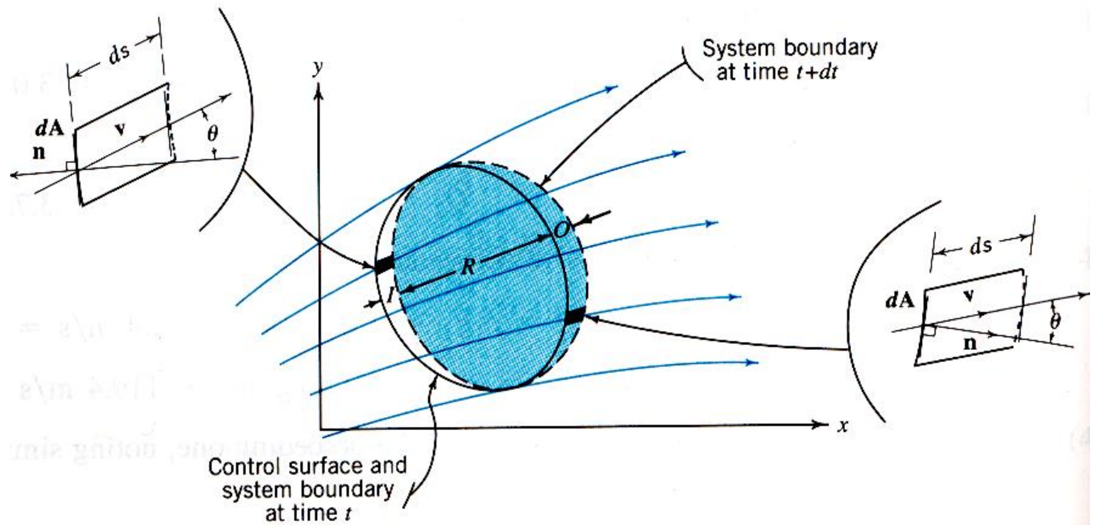
Lecture 6

Continuity, Energy, and Momentum Equations (2)



Lecture 6 Continuity, Energy, and Momentum Equations (2)

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6.1 The General Energy Equation

6.2 Simplified Energy Equation

Objectives

- Derive the energy equation based on the 1st law of thermodynamics and using Reynolds Transport Theorem
- Derive the simplified equations for the energy equation
- Derive Bernoulli equation for ideal fluid

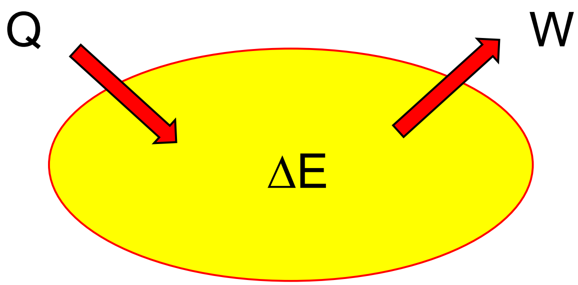
6.1 The General Energy Equation

6.1.1 The 1st law of thermodynamics

- The 1st law of thermodynamics:

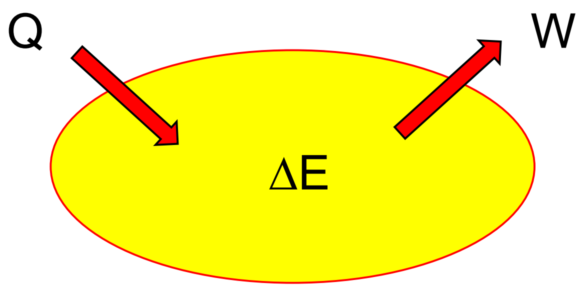
The difference between the heat added to a system of masses and the work done by the system depends only on the initial and final states of the system (→ change in energy).

→ Conservation of energy



6.1 The General Energy Equation

$$\delta Q - \delta W = dE \qquad (6.1)$$



where δQ = heat added to the system from surroundings

δW = work done by the system on its surroundings

δE = increase in energy of the system

6.1 The General Energy Equation

[Re]

- Property of a system: position, velocity, pressure, temperature, mass, volume
- State of a system: condition as identified through properties of the system

Consider time rate of change

$$\frac{\delta Q}{dt} - \frac{\delta W}{dt} = \frac{dE}{dt} \qquad (6.2)$$

6.1 The General Energy Equation

- **Work**

$W_{pressure}$ = work of normal stresses acting on the system boundary

W_{shear} = work of tangential stresses done at the system boundary on adjacent external fluid in motion

W_{shaft} = shaft work done on a rotating element in the system

- **Energy**

Consider e = energy per unit mass = $E/mass$

e_u = internal energy associated with fluid temperature = u

e_p = potential energy per unit mass = gh

where h = local elevation of the fluid

e_q = kinetic energy per unit mass = $\frac{q^2}{2}$

6.1 The General Energy Equation

$$u + \frac{p}{\rho} = \text{enthalpy}$$

$$e = e_u + e_p + e_q = u + gh + \frac{q^2}{2} \quad (6.3)$$

- Internal energy

  = activity of the molecules comprising the substance

  = force existing between the molecules

  ~ depend on temperature and change in phase

6.1 The General Energy Equation

6.1.2 General energy equation

$$\frac{\delta Q}{dt} - \frac{\delta W}{dt} = \frac{dE}{dt} \qquad (6.4)$$

Consider work done

$$\frac{\delta W}{dt} = \frac{\delta W_{pressure}}{dt} + \frac{\delta W_{shaft}}{dt} + \frac{\delta W_{shear}}{dt} \qquad (6.4a)$$

$\frac{\delta W_{pressure}}{dt}$ = net rate at which work of pressure is done by the fluid on the surroundings (유체 압력이 외부에 한 일)

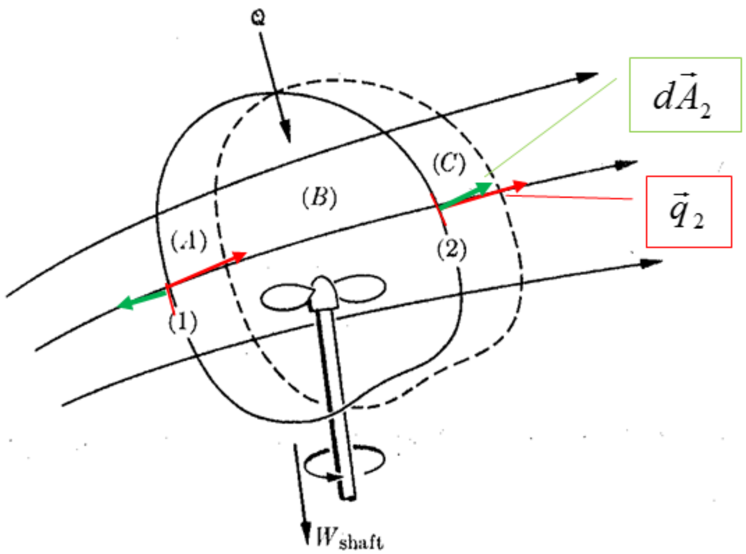
= work flux$_{out}$ - work flux$_{in}$

$$= \oint_{CS} p\left(\vec{q}\cdot d\vec{A}\right)$$

6.1 The General Energy Equation

[Re] Reynolds Transport Theorem (RTT)

$$\frac{dE}{dt} = \oint_{CS} e\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV}\left(e\rho\, dV\right)$$



For mass conservation: $\frac{dE}{dt} = 0; e = 1$

$$\oint_{CS} \rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV}\left(\rho\, dV\right) = 0$$

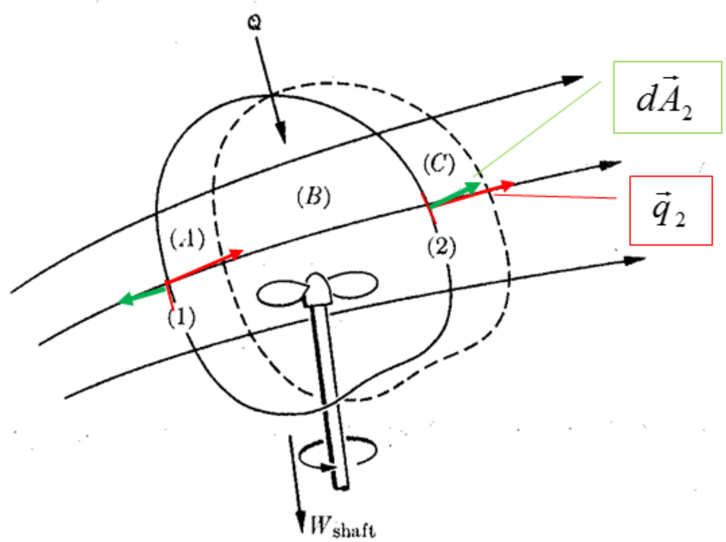
$-\oint_{CS} \rho\vec{q}\cdot d\vec{A}$ = <u>net flux of matter </u>through the control surface

(외부에서 유입된 질량 -유출된 질량)

= flux in – flux out

6.1 The General Energy Equation

$$\vec{q}\cdot d\vec{A} = \begin{cases} \text{positive for outflow into CV} \\ \\ \text{negative for inflow} \end{cases}$$



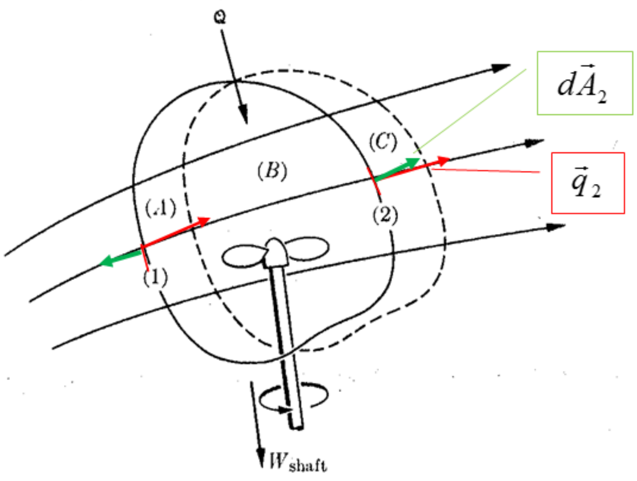
$$\vec{q}\cdot d\vec{A} = Q = L^3/t$$

$$p\left(\vec{q}\cdot d\vec{A}\right) = \frac{F}{L^2}\frac{L^3}{t} = FL/t = E/t$$

Thus, (6.4a) becomes

$$\frac{\delta W}{dt} = \oint_{CS} p\left(\vec{q}\cdot d\vec{A}\right) + \frac{\delta W_{shaft}}{dt} + \frac{\delta W_{shear}}{dt} \qquad (6.4b)$$

6.1 The General Energy Equation



Now, consider energy change term

$$\frac{dE}{dt}$$ = total rate change of stored energy

= net rate of energy flux through C.V.

+ time rate of change inside C.V.

$$\frac{dE}{dt} = \oint_{CS} e\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV}\left(e\rho\, dV\right)$$

(6.4c)

Reynolds Transport Theorem

$$e = E / mass;\ \rho\left(\vec{q}\cdot d\vec{A}\right) = mass / time$$

$$e\rho\left(\vec{q}\cdot d\vec{A}\right) = E / t$$

6.1 The General Energy Equation

Substituting (6.4b) and (6.4c) into Eq. (6.4) yields

$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} - \frac{\delta W_{shear}}{dt} - \oint_{CS} p\left(\vec{q}\cdot d\vec{A}\right)$$

$$= \oint_{CS} e\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV}\left(e\rho\, dV\right)$$

$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} - \frac{\delta W_{shear}}{dt} \tag{6.5}$$

$$= \oint_{CS}\left(\frac{p}{\rho} + e\right)\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV}\left(e\rho\, dV\right) \tag{6.6}$$

6.1 The General Energy Equation

Assume potential energy $e_p = gh$ (due to gravitational field of the earth)

Then $e = u + gh + \frac{q^2}{2}$

Then, Eq. (6.7) becomes

$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} - \frac{\delta W_{shear}}{dt} = \oint_{CS} \left(\frac{p}{\rho} + u + gh + \frac{q^2}{2} \right) \rho \left(\vec{q} \cdot d\vec{A} \right) + \frac{\partial}{\partial t} \int_{CV} e \rho \, dV \qquad (6.7)$$

6.1 The General Energy Equation

- Application: generalized apparatus

At boundaries normal to flow lines → no shear

$$\rightarrow W_{shear} = 0 \qquad (6.8)$$

$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} = \oint_{CS} \left(\frac{p}{\rho} + u + gh + \frac{q^2}{2} \right) \rho \left(\vec{q} \cdot d\vec{A} \right) + \frac{\partial}{\partial t} \int_{CV} e\rho \, dV \qquad (6.9)$$

For steady motion,

$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} = \oint_{CS} \left(\frac{p}{\rho} + u + gh + \frac{q^2}{2} \right) \rho \left(\vec{q} \cdot d\vec{A} \right) \qquad (6.10)$$

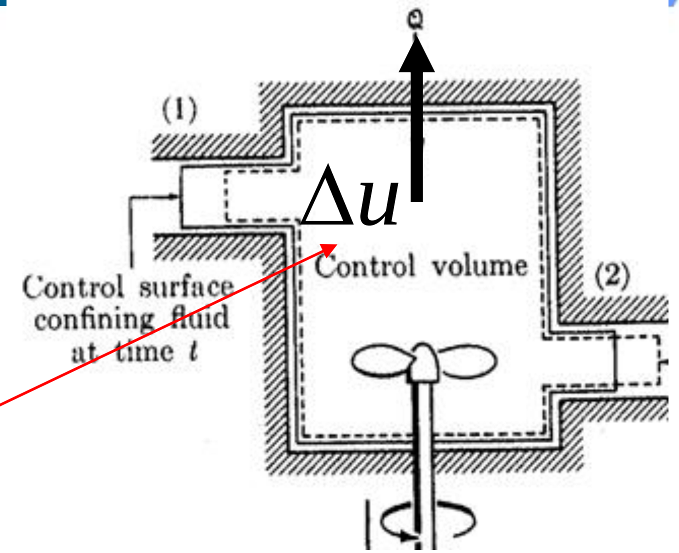
6.1 The General Energy Equation

❖ Effect of friction

- This effect is accounted for implicitly.
- This results in a degradation of mechanical energy into heat which may be transferred away (Q, heat transfer), or may cause a temperature change

→ modification of internal energy

- Thus, Eq. (6.10) can be applied to both viscous fluids and non-viscous fluids (ideal frictionless processes).

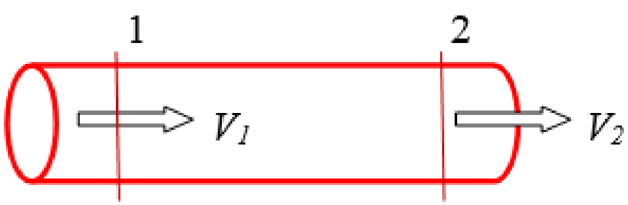


6.2 Simplified Energy Equation

6.2.1 1 D Steady flow equations

For flow through conduits, properties are uniform normal to the flow direction.

→ one-dimensional steady flow



$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} = \oint_{CS}\left(u + \frac{p}{\rho} + gh + \frac{q^2}{2}\right)\rho\left(\vec{q}\cdot d\vec{A}\right)$$

Integrated form of Eq. (6.10) = ② − ①

$$\frac{heat}{dt} - \frac{\delta W_{shaft}}{dt} = \left[u + \frac{p}{\rho} + gh + \frac{V^2}{2}\right]_{②} \rho Q - \left[u + \frac{p}{\rho} + gh + \frac{V^2}{2}\right]_{①} \rho Q$$

6.2 Simplified Energy Equation

where $\dfrac{V^2}{2}$ = average kinetic energy per unit mass

$$Section\,1: \int_1 \rho\left(\vec{q}\cdot d\vec{A}\right) = -\rho Q = \text{mass flow rate into CV}$$

$$Section\ 2: \int_2 \rho\left(\vec{q}\cdot d\vec{A}\right) = \rho Q = \text{mass flow from CV}$$

Divide by ρQ (mass/time)

$$M = \rho Q\ dt$$

$$\frac{heat\ transfer}{mass} - \frac{W_{shaft}}{mass} = \left[u + \frac{p}{\rho} + gh + \frac{V^2}{2}\right]_{②} - \left[u + \frac{p}{\rho} + gh + \frac{V^2}{2}\right]_{①}$$

6.2 Simplified Energy Equation

Divide by g

$$\frac{heat\ transfer}{weight} - \frac{W_{shaft}}{weight} = \left[\frac{u}{g} + \frac{p}{\gamma} + h + \frac{V^2}{2g}\right]_{②} - \left[\frac{u}{g} + \frac{p}{\gamma} + h + \frac{V^2}{2g}\right]_{①}$$

(6.11)

- Energy Equation for 1-D steady flow: Eq. (6.11)
  - use average values for p, γ, h, u, and V at each flow section
  - use K_e (energy correction coeff.) to account for non-uniform velocity distribution over flow cross section

6.2 Simplified Energy Equation

$$K_e \frac{\rho}{2} V^2 Q = \int \frac{\rho}{2} q^2 dQ$$ ---- kinetic energy/time $= \frac{1}{2}\frac{mV^2}{t}$

$$K_e = \frac{\int \frac{\rho}{2} q^2 dQ}{\frac{\rho}{2} V^2 Q} \geq 1 \qquad (6.12)$$

$$\frac{heat\ transfer}{weight} - \frac{W_{shaft}}{weight} = \left[\frac{p}{\gamma} + h + K_e \frac{V^2}{2g}\right]_{②} - \left[\frac{p}{\gamma} + h + K_e \frac{V^2}{2g}\right]_{①} + \frac{u_2 - u_1}{g}$$

(6.13)

$K_e =$
- 2, for laminar flow (parabolic velocity distribution)
- 1.06, for turbulent flow (smooth pipe)

6.2 Simplified Energy Equation

For a fluid of uniform density γ

$$\frac{p_1}{\gamma} + h_1 + K_{e_1}\frac{V_1^2}{2g} = \frac{p_2}{\gamma} + h_2 + K_{e_2}\frac{V_2^2}{2g} + \frac{W_{shaft}}{weight} \boxed{- \frac{heat\ transfer}{weight} + \frac{u_2 - u_1}{g}} \quad (6.14)$$

→ unit: m (energy per unit weight)

For viscous fluid;

$$-\frac{heat\ transfer}{weight} + \frac{u_2 - u_1}{g} = \boxed{H_{L_{1-2}}}$$

→ loss of mechanical energy

~ irreversible in liquid

6.2 Simplified Energy Equation

Then, Eq. (6.14) becomes

$$\frac{p_1}{\gamma} + h_1 + K_{e_1}\frac{V_1^{\,2}}{2g} = \frac{p_2}{\gamma} + h_2 + K_{e_2}\frac{V_2^{\,2}}{2g} + \Delta H_M + \Delta H_{L_{1-2}} \quad (6.14a)$$

where ΔH_M = shaft work transmitted from the system to the outside

$$H_1 = H_2 + \Delta H_M + \Delta H_{L_{1-2}} \quad (6.14b)$$

where H_1 , H_2= weight flow rate average values of total head

6.2 Simplified Energy Equation

6.2.2 Bernoulli Equation

Assume

① ideal fluid → friction losses are negligible

② no shaft work $\rightarrow \Delta H_M = 0$

③ no heat transfer and internal energy is constant $\rightarrow \Delta H_{L_{1-2}} = 0$

$$\frac{p_1}{\gamma} + h_1 + K_{e_1}\frac{V_1^{\,2}}{2g} = \frac{p_2}{\gamma} + h_2 + K_{e_2}\frac{V_2^{\,2}}{2g} \tag{6.15}$$

$$H_1 = H_2$$

6.2 Simplified Energy Equation

If $K_{e1} = K_{e2} = 1$, then Eq. (6.15) reduces to

work

Pressure head

Potential head

Velocity head

$$H = \frac{p_1}{\gamma} + h_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + h_2 + \frac{V_2^2}{2g} \qquad (6.16)$$

~ total head along a conduct is constant

6.2 Simplified Energy Equation

- Grade lines

1) Energy (total head) line (E.L) ~ H above datum

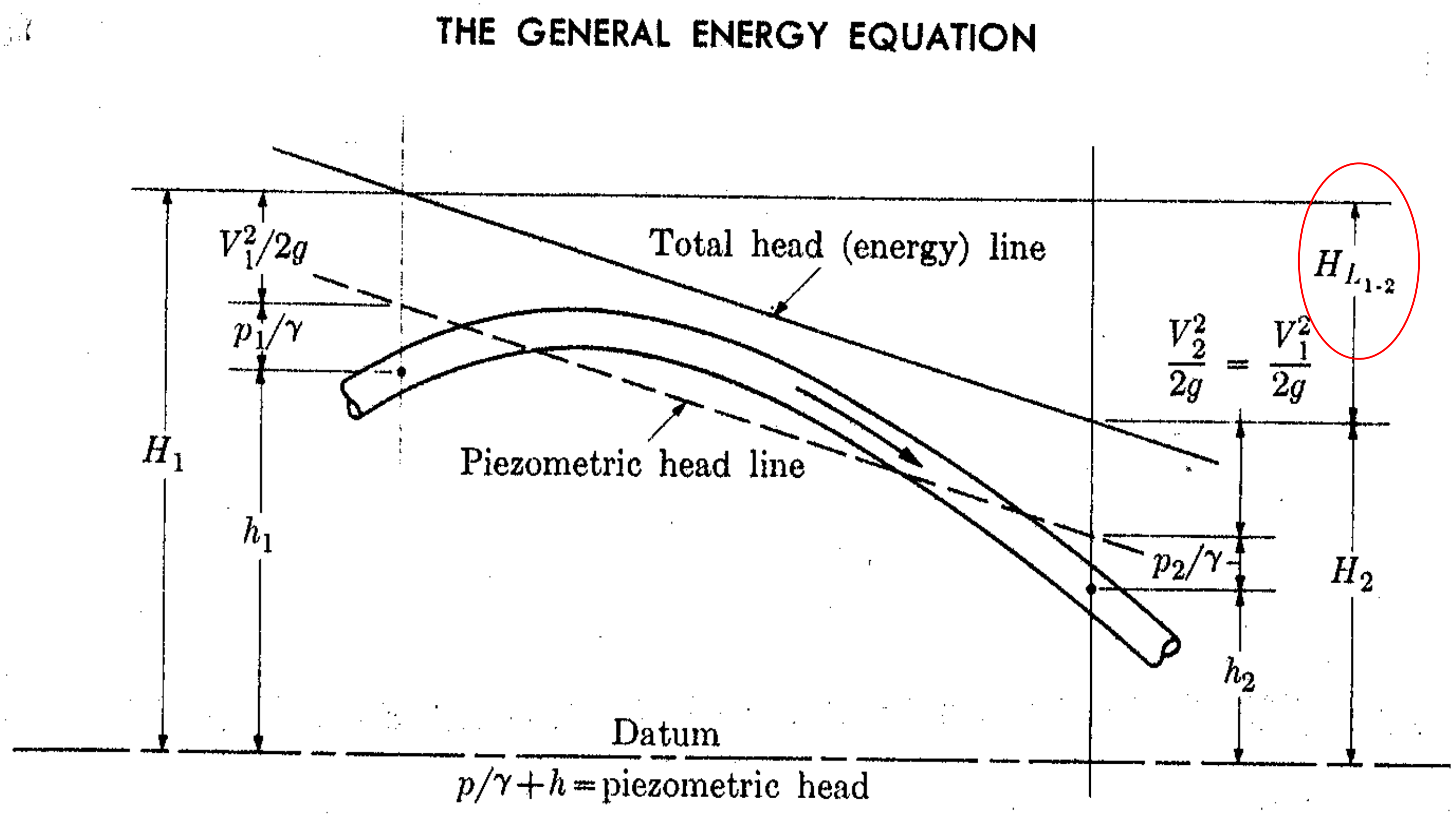
2) Hydraulic (piezometric head) grade line (H.G.L.)

$$= \left(\frac{p}{\gamma} + h\right) \text{above datum}$$

For flow through a pipe with a constant diameter

$$V_1 = V_2 \quad \rightarrow \quad \frac{V_1^{\,2}}{2g} = \frac{V_2^{\,2}}{2g}$$

6.2 Simplified Energy Equation



6.2 Simplified Energy Equation

1) If the fluid is real (viscous fluid) and if no energy is being added, then the energy line may never be horizontal or slope upward in the direction of flow.

2) Vertical drop in energy line represents the head loss by turbine or energy dissipation by hydraulic jump.