Fundamentals of Geomechanics - Introduction to Geomechanics (Week1, 5 Sept)

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Outline

- Introduction to Rock Mechanics/Geomechanics
  - Terminology
  - Area of Applications
  - Nature of Rock Mechanics/Geomechanics
- Applications of Rock Mechanics/Geomechanics
- Methodology to solve Rock Mechanics/Geomechanics problems

Terminology
Rock Mechanics/Geomechanics

- Rock mechanics: discipline concerned with the stressing, deformation and failure of rock
- Geomechanics: Rock mechanics + Soil Mechanics← becoming more popular in energy industry
- Rock Engineering: Rock mechanics + application to engineering
- Geotechnical Engineering: (Rock mechanics + soil Mechanics) + application to engineering ← used more by civil engineering industry
- Specialized Rock Mechanics/Geomechanics: Mining ---, Petroleum ---, Reservoir ---, Borehole ---,

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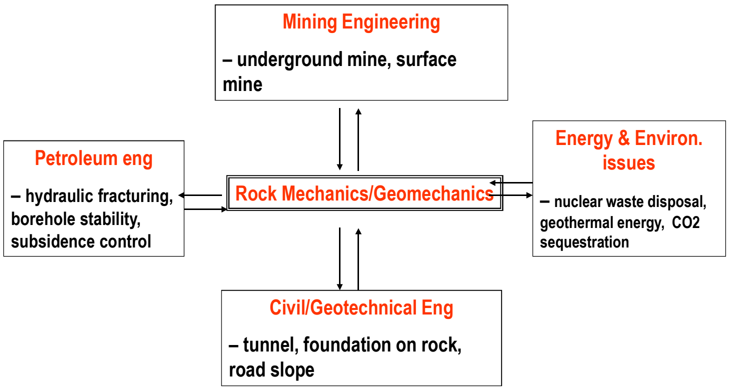
Area of Applications



Mining Engineering

– underground mine, surface mine

Petroleum eng

– hydraulic fracturing, borehole stability, subsidence control

Rock Mechanics/Geomechanics

Energy & Environ. issues

– nuclear waste disposal, geothermal energy, CO2 sequestration

Civil/Geotechnical Eng

– tunnel, foundation on rock, road slope

Nature of problem
Features of Geomechanics

- Limited access to geological data
  - Data limited problem.
  - Heterogeneous and anisotropic
- Effect of fractures
  - Solid understanding on continuum mechanics + consideration on discontinuity needed needed
- Effect of scales
  - What you see is not all and try to see bigger picture
- Uncertain boundary condition
  - In situ stress estimation is the key component
- The mode of loading
  - Removal of rock is the key
- Coupled problem
  - Thermal, Hydraulic and Chemical processes interact each other

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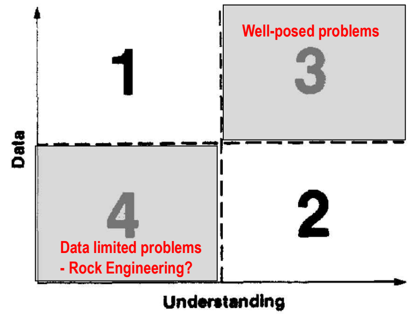
Nature of problem
Data limited problem



Recited from Starfield and Cundall (1988)

Nature of problem
Data limited problem

Rock cutting from Pohang EGS site. ~few mm

↑

REALITY

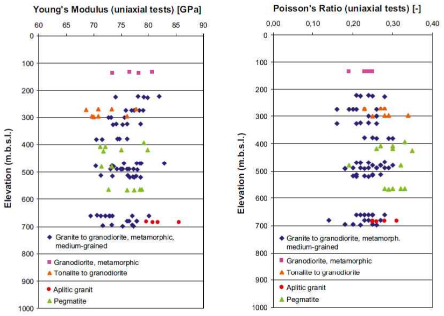
DREAM

↓

One of the biggest rock core in the world at AECL URL in Canada (2002). ~ 1m

Nature of problem
Data limited problem - heterogeneous

- Intact rock – Elastic modulus and Poisson's ratio
  - Forsmark, Sweden



Glamheden, R., A. Fredriksson, K. Röshoff, J. Karlsson, H. Hakami and R. Christiansson, 2007, Rock Mechanics Forsmark. Site descriptive modelling Forsmark stage 2.2, SKB R-07-31, Swedish Nuclear Fuel and Waste Management Co (SKB)

Nature of problem
Data limited problem

- Geological Repository for Nuclear Waste

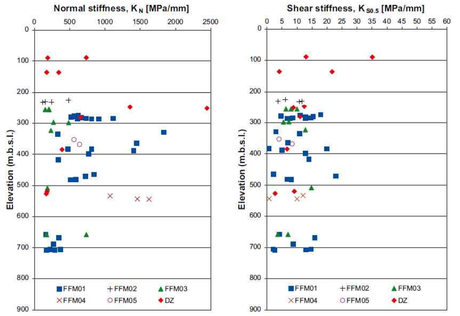
Rock core collection (Forsmark, Oct 2004)

- 25 core-drilled boreholes up to 1,000 m depth.
- 17.8 km core length in total

Core Drilling site (Forsmark, June 2003)

Nature of problem
Data limited problem - heterogeneous

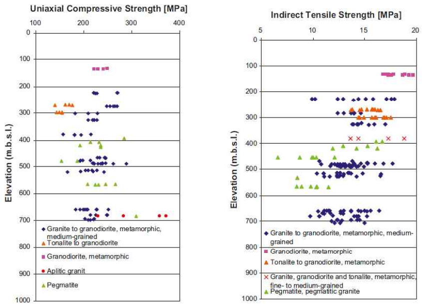
- Fractures – Fracture Normal and Shear Stiffness
  - Forsmark, Sweden



Glamheden, R., A. Fredriksson, K. Röshoff, J. Karlsson, H. Hakami and R. Christiansson, 2007, Rock Mechanics Forsmark. Site descriptive modelling Forsmark stage 2.2, SKB R-07-31, Swedish Nuclear Fuel and Waste Management Co (SKB)

Nature of problem
Data limited problem - heterogeneous

- Intact rock – uniaxial compressive strength and tensile strength
  - Forsmark, Sweden



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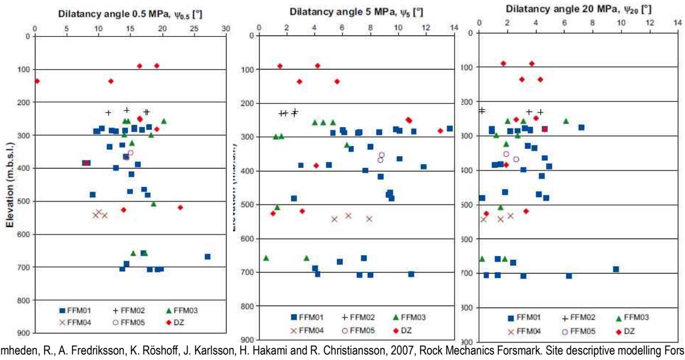
Nature of problem
Data limited problem - heterogeneous

- Fractures – Fracture Dilation behavior
  - Forsmark, Sweden



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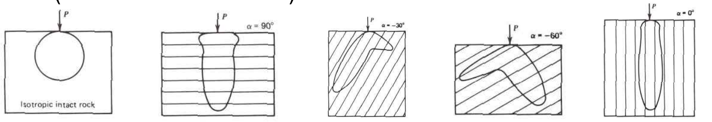
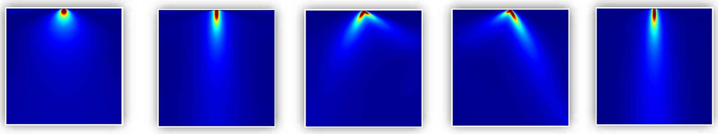
Nature of problem
Data limited problem – Anisotropic

- Foundation under line load on transversely isotropic rock (radial stress is shown)



(Goodman, 1989)



FEM modeling

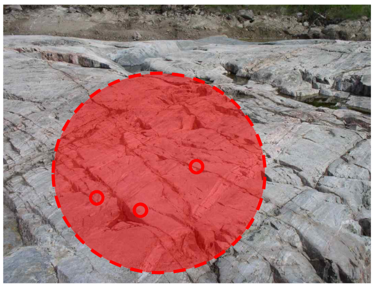
(Park and Min, 2015)

Goodman R, Introduction to rock mechanics, 1989, 2nd ed., Wiley
Park, B. and Min, K.B., Discrete element modeling of transversely isotropic rock applied to foundation and borehole problems, 13rd ISRM Congress, 2015, Vancouver, Canada

Nature of problem
Effect of fractures & Scale

- Effect of fractures has to be properly considered
  - Fractures are main conduit for fluid flow, and more deformable than intact rock
- Scale also matters

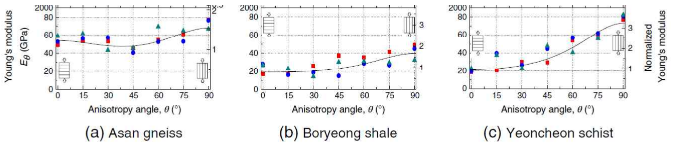
DaeGu Subway 1-10, Korea



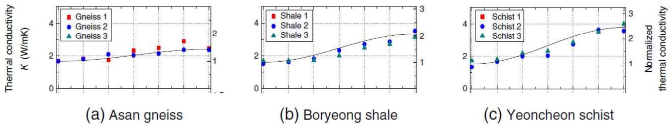
Forsmark, Sweden

Nature of problem
Data limited problem - Anisotropic

Variation of Elastic Modulus (Cho et al., 2012) $S'_{ijkl} = \beta_{im}\beta_{jn}\beta_{kp}\beta_{lp}S_{mnpq}$

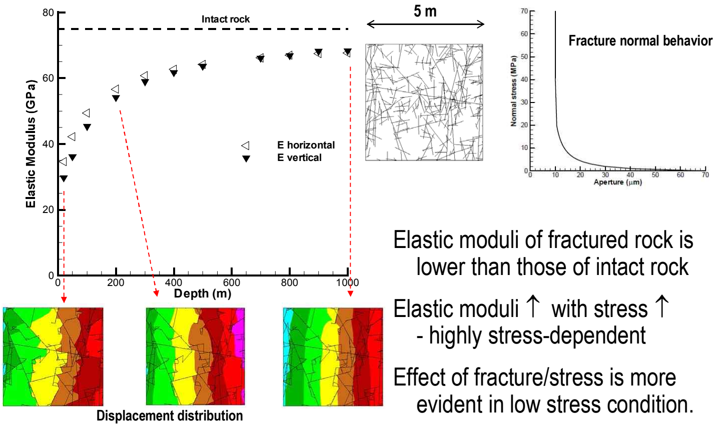


Variation of Thermal Conductivity (Kim et al., 2012) $k^{T'}_{ij} = \beta_{im}\beta_{jn}k^{T}_{mn}$



Cho JW, Kim H, Jeon S, Min KB, Deformation and strength anisotropy of Asan gneiss Boryeong shale, and Yeoncheon schist, IJRMMS, 2012;50:158-169.
Kim H, Cho JW, Song I, Min KB, Anisotropy of elastic moduli, P-wave velocities, and thermal conductivities of Asan Gneiss, Boryeong Shale, and Yeoncheon Schist in Korea, Eng Geol, 2012;147-148:66-77

Nature of problem
Effect of fractures & Scale (example from Forsmark)



Displacement distribution

Elastic moduli of fractured rock is lower than those of intact rock

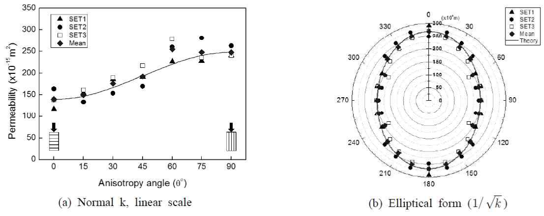
Elastic moduli ↑ with stress ↑
- highly stress-dependent

Effect of fracture/stress is more evident in low stress condition.

Min KB, Stephansson O, The DFN-DEM Approach Applied to Investigate the Effects of Stress on Mechanical and Hydraulic Rock Mass Properties at Forsmark, Sweden, *Tunnel & Underground Space: Journal of Korean Society for Rock Mechanics*, 2011;21(2):117-127

Nature of problem
Data limited problem – Anisotropic

Variation of Permeability (Yang et al., 2013) $k'_{ij} = \beta_{im}\beta_{jn}k_{mn}$



(a) Normal k, linear scale

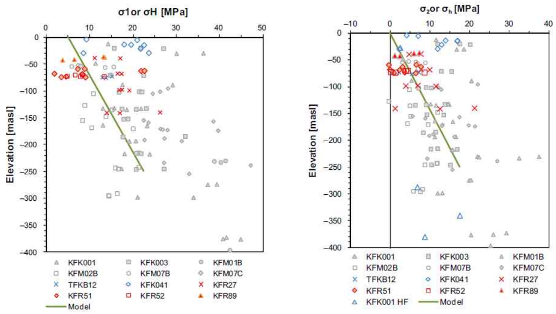
(b) Elliptical form ($1/\sqrt{k}$)

Berea Sandstone ~ 20% porosity

HY Yang, H Kim, K Kim, KY Kim, KB Min, A Study of Locally Changing Pore Characteristics and Hydraulic Anisotropy due to Bedding of Porous Sandstone, J Korean Soc Rock Mech, 2013 23(3):280-240

Nature of problem
Uncertain boundary condition – In situ stress

- In situ stress estimation
  - Forsmark, Sweden

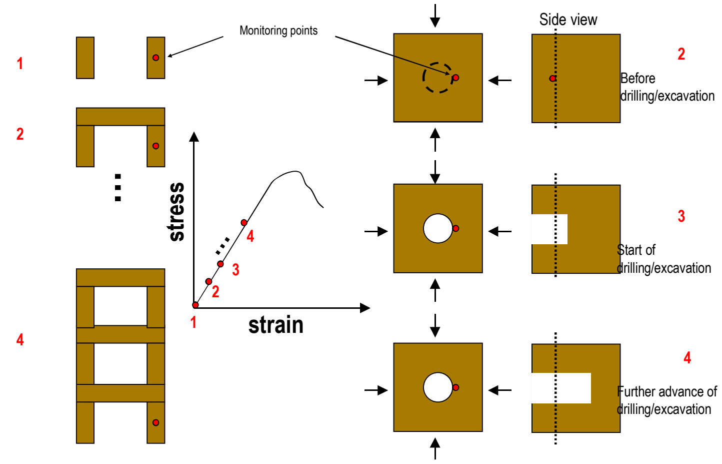


SKB, 2013, Site description of the SFR area at Forsmark at completion of the site investigation phase, SDM-PSU Forsmark, SKB TR-11-04, Swedish Nuclear Fuel and Waste Management Co (SKB).

Nature of Underground Geomechanics

Civil structural problems:
Mechanics of "Addition"

Underground Geomechanics problems:
Mechanics of "Removal"



Nature of problem
Structural problem/Rock Mechanics

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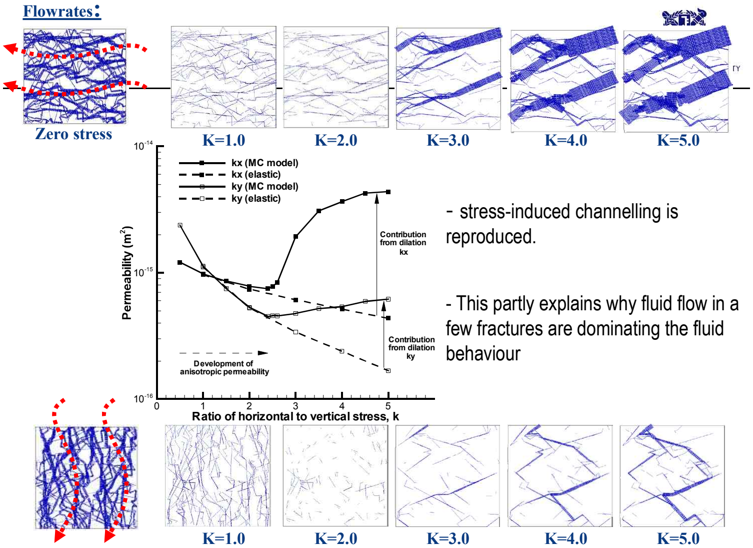
	건축/토목 구조문제 Civil Structural Problem	암석역학 Geomechanics
재료 및 물성 Material & nature of its properties	철강 혹은 콘크리트 Steel, Concrete - 인공물질 (Man-made material) - 균질(Homogeneous) - 연속체(Continuum)	암석 및 토질 (Rock & Soil) - 자연물질 (Natural material) - 불균질 (Heterogeneous) - 불연속체 (Discontinuum) (절리를 함유, contain joints)
경계조건 Boundary condition (loading condition)	자중 + 서비스 하중 (Weight + service load) - 불확실성 적음 (low uncertainty)	현지응력 (In situ stress) - 불확실성 큼 (great uncertainty)
하중재하의 경로 Stress Concentration source	재료의 추가 (상재) (Addition of material)	재료를 없앰 (굴착, removal of material: excavation or drilling)
지하수의 영향 Groundwater	-	매우 중요함 very important
크기 효과 Size effect	-	매우 중요함 Very important

Flowrates:



- stress-induced channelling is reproduced.

- This partly explains why fluid flow in a few fractures are dominating the fluid behaviour

Min KB, Rutqvist J, Tsang CF, Jing L. Jing, Stress-dependent permeability of fractured rock masses: a numerical study, IJRMMS; 2004;41(7):1191-121

Applications
Mining Engineering (1) – Surface Mine

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Prominent Hill, Australia, 2008

Pasir Mine, Indonesia, 2010

Nature of problem
Coupled Process

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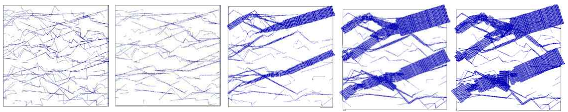
CO2 Geosequestration → Increase of injected CO2 pressure

Nuclear waste repository → Generation of thermal stress
THERMOSHEARING



Geothermal Energy – Enhanced Geothermal System → Increased hydraulic pressure → hydraulic stimulation
HYDROSHEARING

Applications
Mining Engineering (2) – Underground Mine

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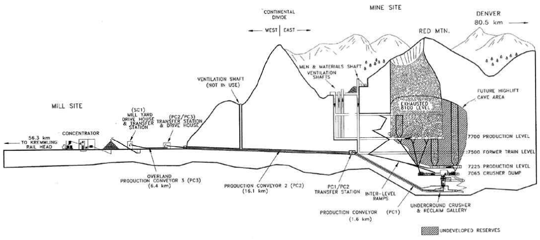
- 헨데센 광산 (Hendersen Mine), 콜로라도, 미국
  - 1976 년 운영시작 (시작전 10년간 약 $500 million 투자)
  - 세계최대의 몰리브덴 광산
  - 1000 미터 하부에 광체, 최대심도 1,600 미터



Hustrulid & Bullock, 2001

Applications
Mining Engineering (2) – Underground Mine

- Drawpoints

Defected steel rib 휘어진 강지보

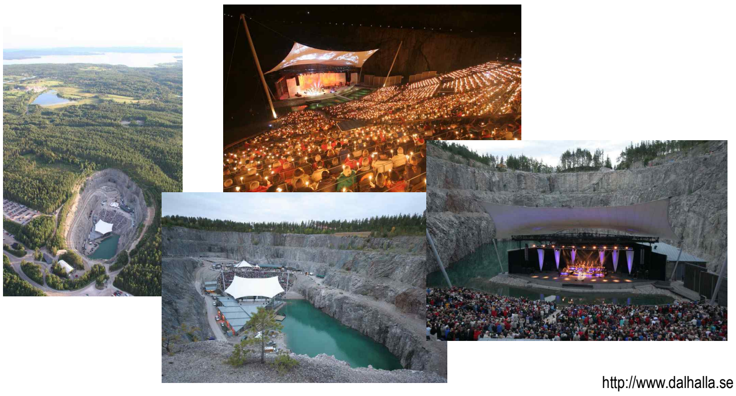
Applications
Mining Engineering (2) – Underground Mine

Relatively large ore size and intact concrete lining

Slabbing at the side of opening (production level)

Applications
Mining Engineering (3) – Quarry

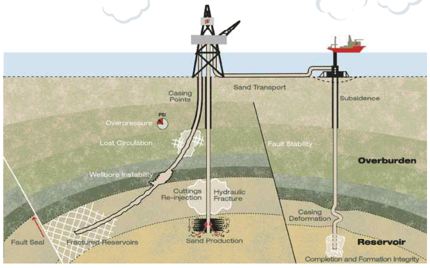
- Dalhalla Concert hall in Sweden – abandoned limestone quarry



http://www.dalhalla.se

Applications
Petroleum Engineering (1)

- Areas of Reservoir Geomechanics
  - Hydraulic Fracturing
  - Borehole Stability
  - Fault reactivation
  - Subsidence
  - Sand Production



http://www.helix-rds.com/EnergyServices/HelixRDS/Capabilities/Geomechanics/tabid/178/Default.aspx

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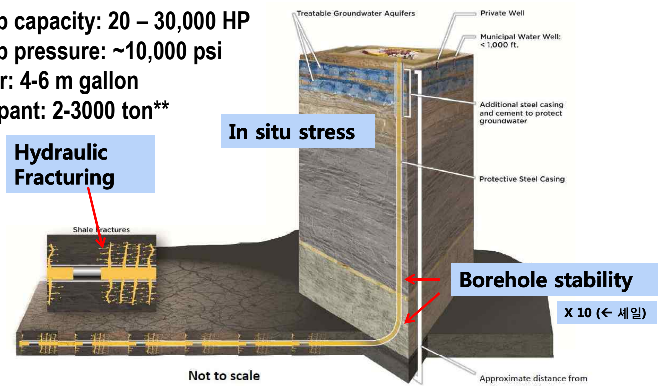
Applications
Petroleum Engineering (2) – Shale Gas production

Pump capacity: 20 – 30,000 HP
Pump pressure: ~10,000 psi
Water: 4-6 m gallon
proppant: 2-3000 ton**

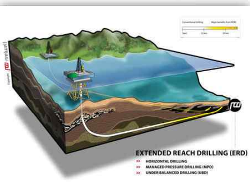


Chesapeake Energy, 2011, Hydraulic Fracturing Fact Sheet, April
** O'Sullivan, 2012, GHGT-2012, Kyoto, Japan

Applications
Petroleum Engineering (3) – wellbore stability

Oseberg in North Sea (Norway)

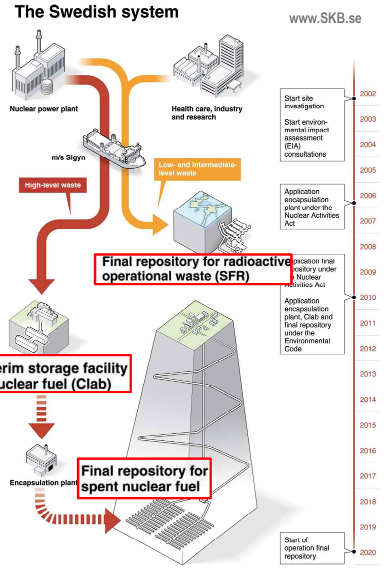
- **Extended Reach Drilling (ERD) has been employed for increasing oil recovery.**
- **Total Depth = 9,327 m**
- **Since 1979, total depth for wells has increased steadily.**



Okland & Cook, SPE, 1998

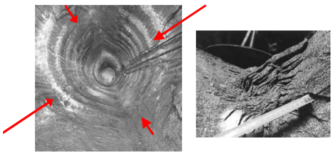
Applications
Geo-Environmental Engineering (1) – Geological repository for nuclear waste

- 지하처분시설:
  - SFR (중저준위)
  - CLAB (고준위 임시저장)
  - 고준위 처분장 (예정)
- 지하연구시설
  - Stripa Mine (1980-1992)
  - Äspö HRL (1995 -)

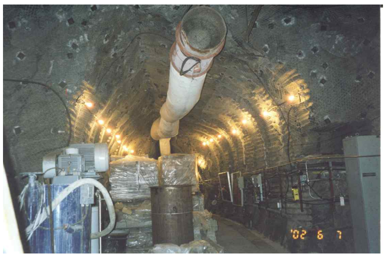


Applications
Geo-Environmental Engineering (1) – Geological repository for nuclear waste

- Underground Research Laboratory in Winnipeg, Canada - Similar observation can be found in underground construction/mining

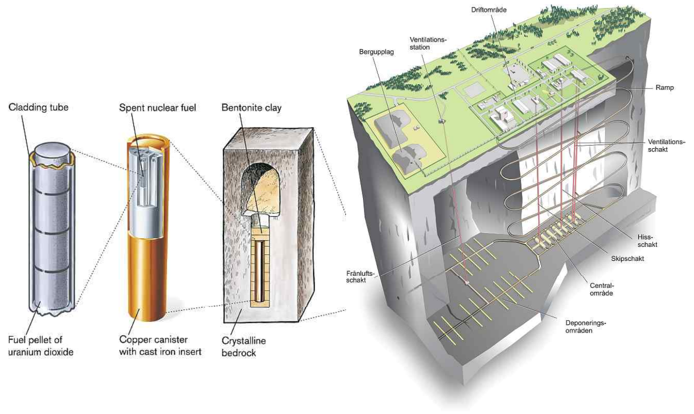


V notched failure due to high in situ stress (400 m, Winnipeg, Canada, Chandler, 2004)



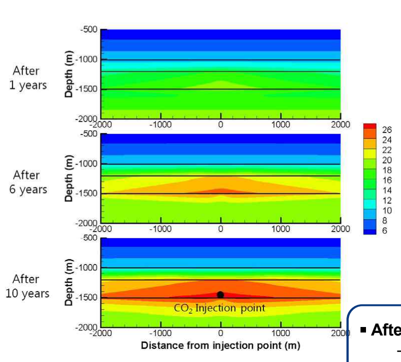
Winnipeg, Canada (Min, 2002)

Applications
Geo-Environmental Engineering (1) – Geological repository for nuclear waste

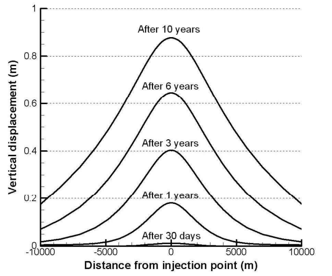


www.skb.se

Applications
Geo-Environmental Engineering (2) – CO2 Geosequestration



Pressure change with time near the injection point (Units: MPa)



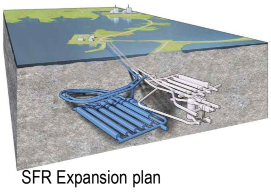
Vertical displacement profile

- After 10 years
  - the pore pressure : about 12 MPa
  - the vertical displacement : about 0.87 m

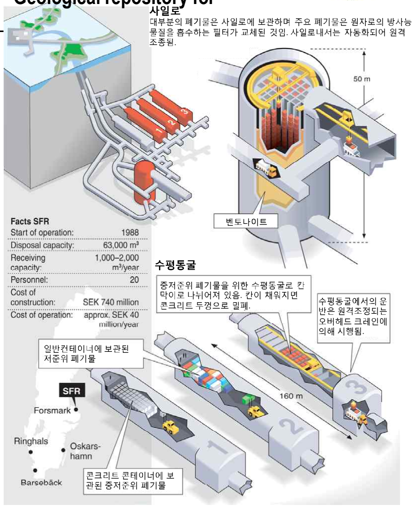
Lee, Min, Rutqvist (2012), RMRE

Applications
Geo-Environmental Engineering (1) – Geological repository for nuclear waste

- 스웨덴 SFR
  - 심도: 60 m
  - 운영시작: 1988년
  - 저장용량: 63,000m^3
  - 30 m x 70 m



SFR Expansion plan



사일로

대부분의 폐기물은 사일로에 보관하며 주요 폐기물은 원자로의 방사능 물질을 흡수하는 필터가 교체된 것임. 사일로내서는 자동화되어 원격 조종됨.

수평동굴

중저준위 폐기물을 위한 수평동굴로 칸막이로 나뉘어져 있음. 칸이 채워지면 콘크리트 두껑으로 밀폐.

수평동굴에서의 운반은 원격조정되는 오버헤드 크레인에 의해 시행됨.

일반컨테이너에 보관된 저준위 폐기물

콘크리트 콘테이너에 보관된 중저준위 폐기물

Facts SFR

Start of operation:	1988
Disposal capacity:	63,000 m^3
Receiving capacity:	1,000–2,000 m^3/year
Personnel:	20
Cost of construction:	SEK 740 million
Cost of operation:	approx. SEK 40 million/year

www.skb.se의 Mats Jerndahl 에서 한글 번역 추가

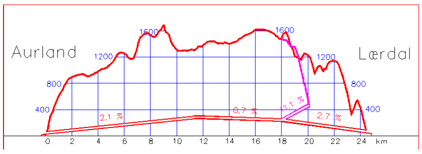
Applications
Civil/Infrastructure (1) – Tunnels

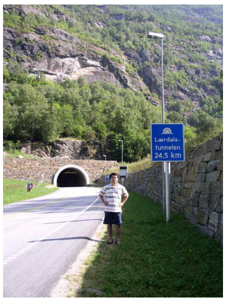
- Civil/Infrastructure
  - Tunnel
  - Slope
  - Dam
  - Oil/Gas Storage Cavern
  - Foundation

T-centralen, Stockholm subway (Per Olof Ultvedt 1975)

Applications
Civil/Infrastructure (1) – Tunnels

- 24.5 km long, 10m wide
- Three 30 m wide mountain hall
- Over 1 km overburden





Applications
Civil/Infrastructure (2) – Slopes

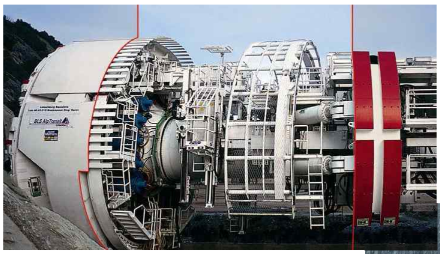
Reinforcement: Rock Anchor

Artificial tunnel

Chunchon, Korea (1999)

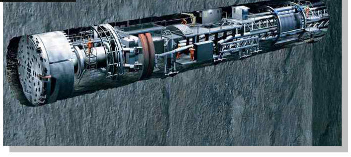
Inje, Korea (1998)

Applications
Civil/Infrastructure (1) – Tunnels



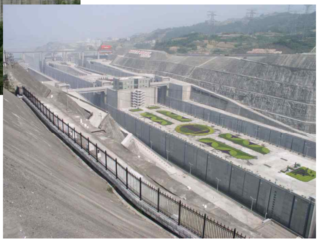
✓ **Tunnel Boring Machine (TBM)**

스위스
Gottard Base Tunnel 에 사용



Applications
Civil/Infrastructure (3) – Dams

Three Gorges Dam (Christoph Filnkößl)



Ship locks for river traffic

Applications
Civil/Infrastructure (2) – Slopes

Slopes to be scaled

Youngyang, Korea (1999)

Goksong, Korea (1999)

Applications
Civil/Infrastructure (4) – Oil/Gas Storage Cavern

발 주 처

SK 가스 주식회사

위 치

한국, 울산시

사업 기간

1985. 11 ~ 1988. 11

사업 개요

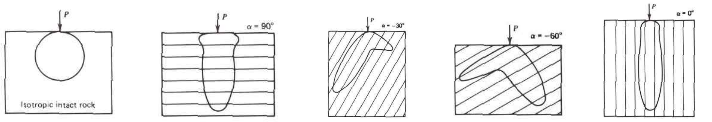
- 저장 유종 : Propane, Butane
- 저장 용량
  - Propane : 275,000 m³ Butane : 225,000 m³
- Propane 저장동굴 : 4 개
  - 저장동굴단면 : 307.24 m²
  - 저장동굴연장 : 835 m
- Butane 저장동굴 : 3 개
  - 저장동굴단면 : 341.45 m²
  - 저장동굴연장 : 616 m

사업 범위

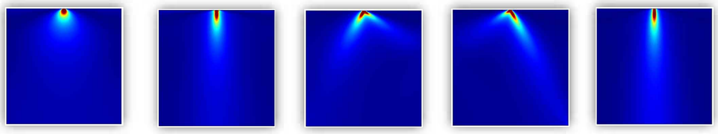
- 전체 사업관리
- 지하 및 지상저장시설 시공
  (현재 운영중)

Applications
Civil/Infrastructure (5) – Foundations

Foundation under line load on transversely isotropic rock (radial stress is shown)



(Goodman, 1989)



FEM modeling

(Park and Min, 2015)

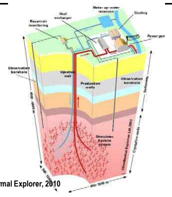
Goodman R, Introduction to rock mechanics, 1989, 2nd ed., Wiley
Park, B. and Min, K.B., Discrete element modeling of transversely isotropic rock applied to foundation and borehole problems, 13rd ISRM Congress, 2015, Vancouver, Canada

---

Aitik Mine, Sweden, Min, 2012

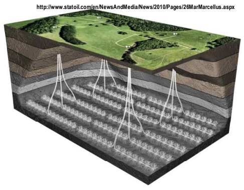
Mining Engineering
Depth : ~ 2.5 km

Enhanced Geothermal System

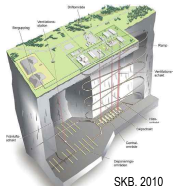


Geothermal Explorer, 2010

http://www.statoil.com/en/NewsAndMedia/News/2010/Pages/26MarMarcellus.aspx



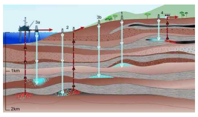
Shale gas production & oil/gas
depth: ~ 3.0 km



SKB, 2010

Underground repository for nuclear waste
depth: 0.5 ~ 5.0 km

THINK BIG!
GO DEEP!!

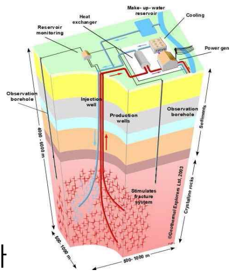


IPCC, 2005

CO_2 sequestration
depth: ~ 2.5 km

Applications
Geothermal Energy

- EGS (Enhanced Geothermal System, 인공저류층 지열시스템): 투수율이나 공극률이 낮은 암반이 경제적인 지열 생산을 가능하도록 투수율을 높힌 인공저류층을 대상으로 한 지열에너지 개발시스템
- EGS 의 핵심기술
  - 심부 시추 (3 ~ 5 km)
  - 인공저류층 형성(수리자극)
  - 저류층 특성화
  - 저류층 모니터링(미소진동 관리)
- 심부지열발전 핵심기술은 석유 가스 등의 자원개발에 필요한 탐사, 개발, 생산 기술과 매우 유사함



Geothermal Explorer, 2010

Fundamentals of Geomechanics - Introduction of the course(Week1, 5 Sept)

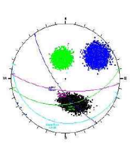
Ki-Bok Min, PhD

Associate Professor
Department of Energy Resources Engineering
Seoul National University

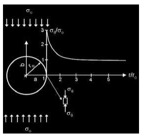
SEOUL NATIONAL UNIVERSITY

Methods for Rock Mechanics/Geomechanics Analysis

- 해석적 방법 (Analytical method)
  - 알려져 있는 수학적 해를 이용하여 응력과 변위를 계산
  - 커쉬해 (Kirsch solution) 등이 원형공동주위의 응력상태를 알려주는 대표적인 수학적 해임.
- 경험적 방법 (Empirical method)
  - 축적된 경험을 이용하여 여러 범주에 점수를 부여하여 해석
  - 암반분류법이 대표적인 예 (RMR (Rock Mass Rating), Q-system)
- 수치해석적 방법 (Numerical Method)
  - 주어진 경계조건과 형상에서 컴퓨터 시뮬레이션을 이용하여 응력과 변위를 계산 (편미분방정식을 푸는 것임)
  - 복잡한 형상에서 효과적임
  - 유한요소법 (Finite Element Method, FEM), 유한차분법 (Finite Difference Method, FDM), 개별요소법 (Discrete Element Method, DEM)



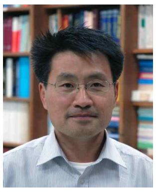
Example of streographic projection method



Stress distribution around a circular opening

Introduction
Schedules, Room and Instructors

- Lectures (3 credits)
  - Mon: 15:30 – 16:20
- Lecture Room: 38-323
- Instructor and Teaching Assistant
  - Ki-Bok Min, Room:38-303, kbmin@snu.ac.kr
  - Kwang-Il Kim, Sehyeok Park, Saeha Kwon



Introduction
Objectives of the course

- Objective;
  - Understand the fundamental concepts of geomechanics
  - Focus on both;
    - classical development of rock mechanics principles
    - State-of-the-art application of the disciplines
  - Topics includes;
    - Rock Failure Criteria
    - In situ stress estimation
    - Stress distribution around an opening
    - Hydromechanics of Rock
    - Rock Anisotropy

Introduction
References

- References (specialized Geomechanics)
  - Paterson MS, Wong T-f, 2005, Experimental rock deformation – the brittle field, 2nd ed., Springer
  - Gueguen Y and Bouteca M (eds), Mechanics of Fluid-Saturated Rocks, Elsevier, 2004 (should be a good publication, haven't checked in detail)
  - Zimmerman RW, Compressibility of Sandstones, Elsevier, 1991 (Prof Zimmerman's PhD thesis)
  - Coussy O, Poromechanics, 2nd Ed., Wiley, 2004 (highly theoretical)
  - Bai M and Elsworth D, Coupled Processes in Subsurface Deformation, Flow and Transport, ASCE Press, 2000 (not particularly reader-friendly)
  - Fjaer E et al., Petroleum-Related Rock Mechanics, Elsevier, 2nd Ed., 2008 (publication by a diligent group in Norway, a lot of experience, application to petroleum engineering)
  - Zoback MD, Reservoir Geomechanics, Cambridge University Press, 2007 (timely publication for petroleum/geothermal applications, mostly Zoback's group's work)
  - Jing L and Stephansson O, Fundamentals of Discrete Element Methods in Rock Engineering, Elsevier, 2007 (good summary for DEM)
  - Amadei B and Stephansson O, Rock Stress and Its Measurement, Chapman & Hall, 1997 (comprehensive and expensive)
  - Stephansson O and Zang Arno, Stress Field of the Earth's Crust, Springer, 2010 (thin with useful animations)
  - Aadnoy BS and Looyeh R, Petroleum rock mechanics – Drilling operations and well design, Elsevier, 2010 (focus on wellbore rock mechanics)

Introduction
Contents of the course

W1- 5 Sept	Introduction to the course/Elasticity
W2 - 12 Sept	Elasticity
W3 - 19 Sept	Rock Failure Criteria
W4 - 26 Sept	Rock Failure Criteria
W5 - 3 Oct	Anisotropic Rock Mechanics
W6 - 10 Oct	Rock Mass Properties
W7 - 17 Oct	- No Lecture (ARMS9 Symposium)
W8 - 24 Oct	In situ stress and its estimation
W9 - 31 Oct	In situ stress and its estimation
W10 – 7 Nov	Stress distribution around an underground opening
W11 - 14 Nov	Hydromechanics of Rock
W12 - 21 Nov	Hydromechanics of Rock
W13 - 28 Nov	Numerical methods in Geomechanics
W14 – 5 Dec	Students Conference

Introduction
Assessment

- Assessment
  - Home Assignment : 50 % ~10 home assignments
  - Final Exam : 20 %
  - Term paper : 20 %
  - Participation : 10 %

Introduction
References

- References (general Geomechanics)
  - Jaeger JG, Cook NGW and Zimmerman RW, Fundamentals of Rock Mechanics, 2007, 4th edition, Blackwell Publishing (highly recommended, many typos).
  - Hoek E and Brown ET, Underground Excavations in Rock, Inst Mining & Metallurgy, 1980 (a classic)
  - Goodman RE, Introduction to Rock Mechanics, 2nd ed. John Wiley & Sons, 1989 (a good and tested book)
  - Hoek E, Practical Rock Engineering, http://www.rocscience.com/hoek/PracticalRockEngineering.asp (practical and concise)
  - Brady BHG and Brown ET, Rock Mechanics for Underground Mining, 4th ed., Kluwer Academic Publishers, 2004 (excellent for mining application)
  - Cornet FH, Elements of Crustal Geomechanics, Cambridge Univ Press, 2015 (new and filling the gap in the engineering geoscience)
  - Hudson JA and Harrison JP, Engineering Rock Mechanics I, Pergamon, 1997 (provides a good perspective but full of typos)
  - Harrison JP and Hudson JA, Engineering Rock Mechanics Part II, Pergamon, 2000 (fun to read)
  - Hoek E, Kaiser PK and Bawden WF, Support of Underground Excavations in Hard Rock, Taylor & Francis, 2000 (practical and useful)
  - Obert L and Duvall WI, Rock Mechanics and the Design of Structures in Rock, John Wiley & Sons, 1967 (many typos)
  - Pusch R, Rock Mechanics on a Geological Base, Elsevier, 1995 (emphasis on geology)

Introduction
Home Assignments (50%)

- Review of classical/recent papers/book chapters (2 pages in both .doc in .ppt)
  - Summary (objective, data, methodology, conclusion, implication)
  - Critical review (limitation/strength, further work)
- Should give 2-3 minutes presentation during the class
- Expected to complete ~2 papers/week, ~20 papers/course
- Submission to eTL (09:00 Monday; Late submission NOT accepted)

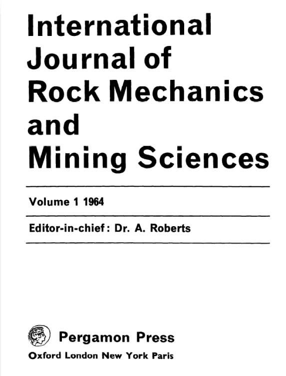
Introduction
Term Paper (20%)

- Select Geomechanics problem of your interest.
- A thorough literature review with your own integration, analysis, criticism and insight
- Coverage of 5-15 papers may be relevant
- You are welcome to choose the topic you are working on or completely different ones

- Timeline
  - ঌ 24 Oct Proposal (1 page)
  - ঌ 5 Dec Final Term Paper Submission (must be < 10 pages) & Presentation (15 m)

Introduction
classical papers

- International Journal of Rock Mechanics and Mining Sciences, Volume 1 1964

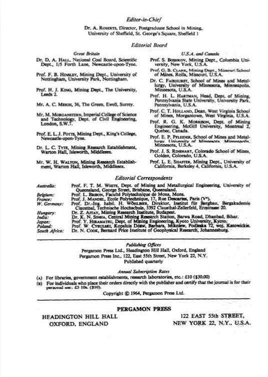


International Journal of Rock Mechanics and Mining Sciences

Volume 1 1964

Editor-in-chief: Dr. A. Roberts

Pergamon Press
Oxford London New York Paris



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Publishing Offices

Pergamon Press Ltd., Headington Hill Hall, Oxford, England
Pergamon Press Inc., 122, East 55th Street, New York 22, N.Y.
Published quarterly

Annual Subscription Rates

(A) For libraries, government establishments, research laboratories, etc.: £10 ($30.00)
(B) For individuals who place their orders directly with the publisher and certify that the journal is for their personal use: £3 10s. ($10).

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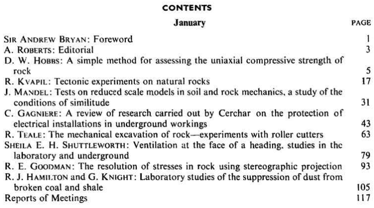
HEADINGTON HILL HALL, OXFORD, ENGLAND
122 EAST 55th STREET, NEW YORK 22, N.Y., U.S.A.

Introduction
Term Paper

- Presentation
  - Presentation is an extremely important part of your professional life. Therefore, you have a good reason to be serious about this.
  - 12 minutes + 3 min (questions)
  - Ask questions

Introduction
classical papers

- Fifty years of Rock Mechanics/Geomechanics
- List of the papers shows the advances and longstanding difficulties in the geomechanics



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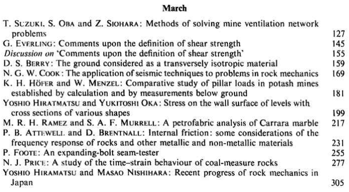

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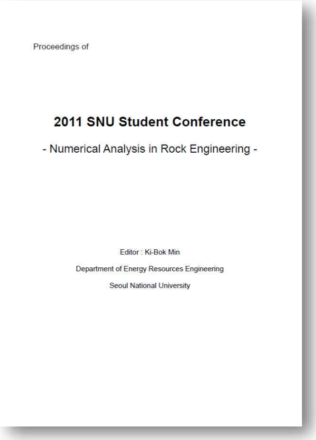
Introduction
Term Paper

- Your term papers will be published as proceedings.
- Your term papers may be used in the introduction of journal papers in the future.



Proceedings of

2011 SNU Student Conference

- Numerical Analysis in Rock Engineering -

Editor : Ki-Bok Min
Department of Energy Resources Engineering
Seoul National University

Home Assignment #1

- Review one of 35 papers from IJRMMS Vol.1 (1964)
  - You need to provide not only summary but also your own insight and criticism based on the selected papers.
  - If necessary, you will have to conduct your own analysis and refer to other papers.
- Marking will be
  - A (100): Excellent
  - B (80): Good
  - C (60): Fair
  - D (40): Poor (You don't seem to know what you are talking about.)

Fundamentals of Geomechanics – Elasticity with Geomechanics focus (Week2, 12 Sept)

Ki-Bok Min, PhD

Associate Professor
Department of Energy Resources Engineering
Seoul National University

Geomechanics
comparison with fluid flow - a convenient truth

Porous media fluid flow (다공성매질의 유체유동)	Geomechanics(암반역학)
Darcy's Law	Hooke's Law
Fluid Flux	Stress (응력)
Pressure gradient	strain (변형율)
Permeability	Elastic modulus (Young's Modulus) & Poisson's ratio
Time dependent	Not time dependent (elastic) time dependent→ creep
Conservation of mass	Equilibrium Equation

$$q = -\frac{k}{\mu}\frac{dP}{dl}$$

$$\sigma = E\varepsilon = E\frac{du}{dx}$$

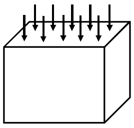
Elasticity with Geomechanics focus
Main References

- Jaeger JG, Cook NGW and Zimmerman RW, 2007, Fundamentals of Rock Mechanics, 4th edition, Blackwell Publishing
  - Chapter 2. Analysis of Stress and Strain (p.9-64)
  - Chapter 5. Linear Elasticity (p.106-144)
- Other Elasticity or Continuum mechanics textbooks (Timoshenko, 1970; Fung, 1994; Malvern, 1969)

Stress (응력)
Definition

- Stress
  - : a force acting over a given area, F/A ← simple definition
  - the internal distribution of force per unit area that balances and reacts to external loads applied to a body← exact definition
- Normal stress: Normal force/Area

$$\sigma = \frac{F_n}{A}$$



- Shear stress: Shear force/Area

$$\tau = \frac{F_s}{A}$$

- Unit: N/m^2=Pa, 10^{6}Pa=MPa, 10^{9}Pa=GPa
  145 psi = 1 MPa = 10 bar = 10 kg중/cm^2

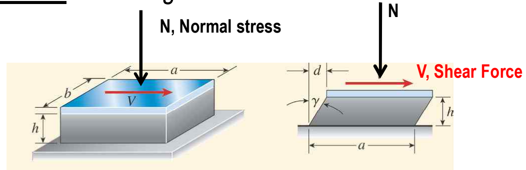
THMC Processes
Physical variables for THMC problems

Physical problem	Conservation Principle $\nabla \cdot q = 0$	State Variable u	Flux σ	Material properties k	Source f	Constitutive equation $\sigma = ku'$
Elasticity	Conservation of linear momentum (equilibrium)	Displacement u	Stress σ	Young's modulus & Poisson's ratio	Body forces	Hooke's law
Heat conduction	Conservation of energy	Temperature T	Heat flux Q	Thermal conductivity k	Heat sources	Fourier's law
Porous media flow	Conservation of mass	Hydraulic head h	Flow rate Q	Permeability k	Fluid source	Darcy's law
Mass transport	Conservation of mass	Concentration C	Diffusive flux q	Diffusion coefficient D	Chemical source	Fick's law

Structure of state variables and fluxes are mathematically similar – ***a convenient truth!***

Stress
Normal stress & Shear stress

- **Stress**: *average force per unit area*
- **Normal stress**: *act in perpendicular to cut surface*
- **Shear stress**: acts *tangential* to the surface of the material



$$\sigma = \frac{P}{A}$$

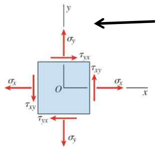
$$\tau = \frac{V}{A}$$

P: Axial Force (N)
V: Shear Force
A: cross sectional area (a x b)

- Unit: N/m^2 = MPa

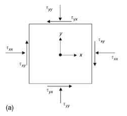
Stress
Definition

- Sign convention
  - Normal stress:
    typical mechanics: tension (+), compression (-)
    rock/geomechanics: tension (-), compression (+)
  - Shear stress:
    - acts on a positive face of an element in the positive direction of an axis (+) : plus-plus or minus-minus
    - acts on a positive face of an element in the negative direction of an axis (-): plus-minus or minus-plus



← Positive normal & shear stresses in other mechanics

in rock mechanics →



Stress (응력) & Force(힘)
stress in 3D

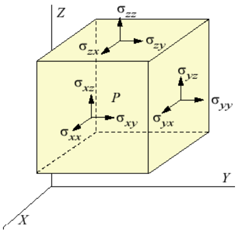
- Stress in 3D

$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

텐서형식
(Tensor form)

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{pmatrix}$$

행렬형식
(matrix form)



9 components → $\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$ ⟶ **6 independent components**

Stress (응력) & Force(힘)
stress in 2D

- Stress in 2D

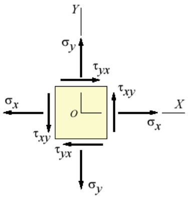
2D: $\begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix}$

τ_{xy}

Direction of surface normal upon which the stress acts

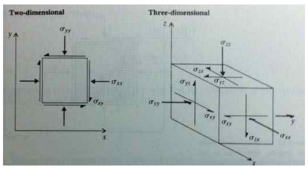
Direction of the stress component

- Normal stress: acting perpendicular to the plane
- Shear stress: acting tangent to the plane
- Stress is a 2nd order tensor
  - Force is 1st order tensor (=vector)
  - Can be defined according to the reference axis
  - Principal stresses are defined

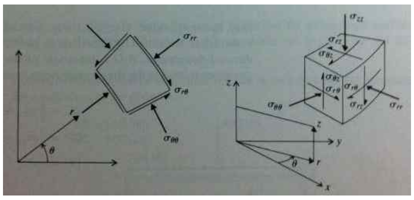


Stress
Definition in 2D and 3D

- 2D & 3D Cartesian Coordinates



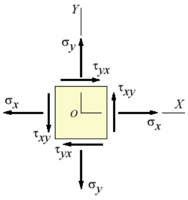
- Polar & Cylindrical coordinates



Stress (응력) & Force(힘)
stress in 2D

- Stress component in 2D $\begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix}$

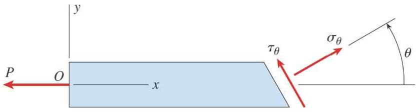
4성분 ⟶ $\tau_{xy} = \tau_{yx}$ ⟶ **3 component is independent**



Stress (응력) & Force(힘)
Stress Transformation

- Uniaxial stress example

$$\begin{pmatrix} \sigma_{x1} & \tau_{x1y1} \\ \tau_{x1y1} & \sigma_{y1} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma_x & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^T$$



$$\sigma_\theta = \frac{N}{A_1} = \frac{P}{A}\cos^2\theta \qquad \tau_\theta = -\frac{V}{A_1} = -\frac{P}{A}\sin\theta\cos\theta$$

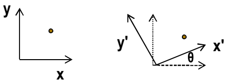
$$\sigma_\theta = \sigma_x \cos^2\theta = \frac{1}{2}\sigma_x(1+\cos 2\theta) \qquad \tau_\theta = -\sigma_x \sin\theta\cos\theta = -\frac{\sigma_x}{2}\sin 2\theta$$

- Stress transformation is conducted by multiplying the direction cosine twice (응력의 변환식은 방향 코사인이 두 번 곱해져서 구해짐).

Stress (응력) & Force(힘)
Stresses on inclined sections (Transformation)

- A different way of obtaining transformed stresses
  - For vector

$$\begin{pmatrix}F_{x1}\\F_{y1}\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}F_x\\F_y\end{pmatrix}$$



  - For tensor (stress)

$$\begin{pmatrix}\sigma_{x1} & \tau_{x1y1}\\ \tau_{x1y1} & \sigma_{y1}\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\sigma_x & \tau_{xy}\\ \tau_{xy} & \sigma_y\end{pmatrix}\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}^T$$

=

$$\sigma_{x_1}=\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos 2\theta+\tau_{xy}\sin 2\theta$$

$$\tau_{x_1y_1}=-\frac{\sigma_x-\sigma_y}{2}\sin 2\theta+\tau_{xy}\cos 2\theta$$

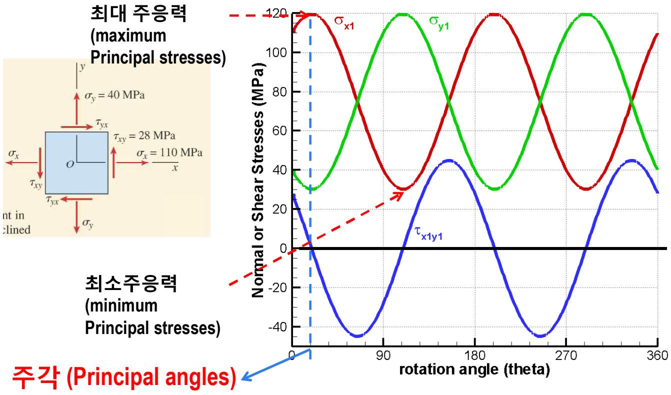
Stress (응력) & Force(힘)
Stress Transformation

- 예)

Cauchy's Formula

- Knowing the component of stress, we can write down at once the traction vector (stress vector) acting on any surface with unit outer normal vector whose components are (n_x, n_y, n_z).
- …assures us that the nine components of stress are necessary and sufficient to define the traction across any surface element in a body. Hence, the stress state in a body is characterized completely by a set of quantities, $\boldsymbol{\tau}$.

Stress (응력) & Force(힘)
Stress Transformation



최대 주응력 (maximum Principal stresses)

최소주응력 (minimum Principal stresses)

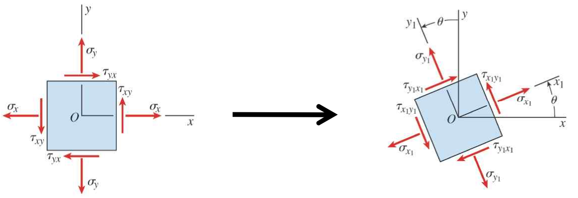
주각 (Principal angles)

Stress (응력) & Force(힘)
Stresses on inclined sections (Transformation)

- Stresses acting on inclined sections assuming that σ_x, σ_y, τ_{xy} are known.
  - x_1y_1 axes are rotated counterclockwise through an angle θ

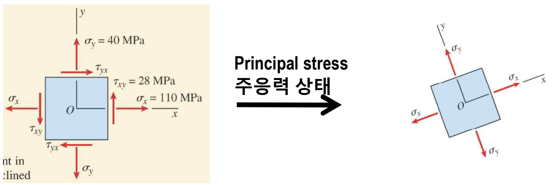
$$\sigma_{x_1}=\frac{\sigma_x+\sigma_y}{2}+\frac{\sigma_x-\sigma_y}{2}\cos 2\theta+\tau_{xy}\sin 2\theta$$

$$\tau_{x_1y_1}=-\frac{\sigma_x-\sigma_y}{2}\sin 2\theta+\tau_{xy}\cos 2\theta$$



Stress (응력) & Force(힘)
Stress Transformation

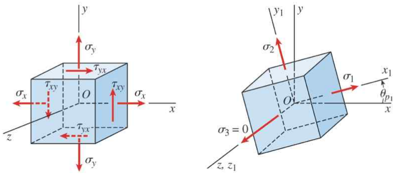
- Principal stress (주응력)
  - The largest (or smallest) normal stress and shear stress at that plane is 0 (가장 큰 수직응력이며 전단응력이 0 임)
  - Vertical stress in the earth is usually principal stress and the other two principal stress is in horizontal direction (통상 지각의 수직방향이 주응력이며, 수평방향으로도 두 개의 주응력 정의 가능)
  - Usually denoted as σ_1, σ_2, σ_3 (크기 순으로 σ_1, σ_2, σ_3 으로 표시)



Principal stress
주응력 상태

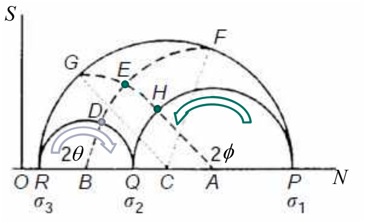
Stress (응력) & Force(힘)
Principal Stresses and Maximum Shear Stresses

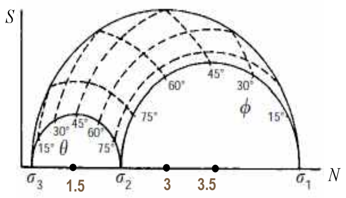
- Stress element is three dimensional
  - Three principal stresses (σ_1 ,σ_2 and σ_3) on three mutually perpendicular planes



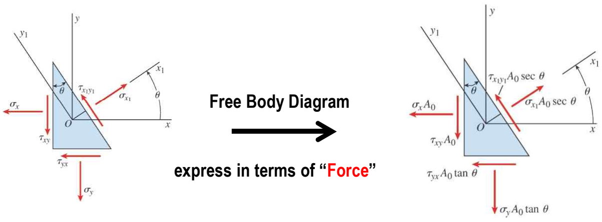
Stress (응력) & Force(힘)
Mohr's Circle

- Mohr's Circles for 3D





Stress
Stresses on inclined sections



Free Body Diagram

express in terms of "Force"

- Force Equilibrium Equations in x_1 and y_1 directions

$$\sum F_{x_1} = \sigma_{x_1} A_0 \sec\theta - \sigma_x A_0 \cos\theta - \tau_{xy} A_0 \sin\theta$$
$$- \sigma_y A_0 \tan\theta \sin\theta - \tau_{yx} A_0 \tan\theta \cos\theta = 0$$

$$\sum F_{y_1} = \tau_{x_1y_1} A_0 \sec\theta + \sigma_x A_0 \sin\theta - \tau_{xy} A_0 \cos\theta$$
$$- \sigma_y A_0 \tan\theta \cos\theta + \tau_{yx} A_0 \tan\theta \sin\theta = 0$$

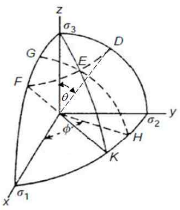
Mohr's Circle
3D - example

For example,
consider the following stress state acting on a point:

$$\sigma_{ij} = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Question: Calculate the normal and shear stress on the plane with normal vector:

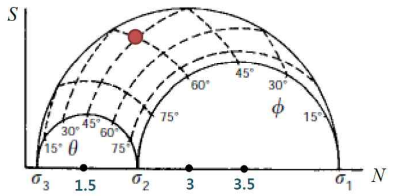
$$n = \left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2} \right)$$



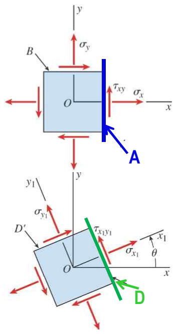
- **Direction cosines**

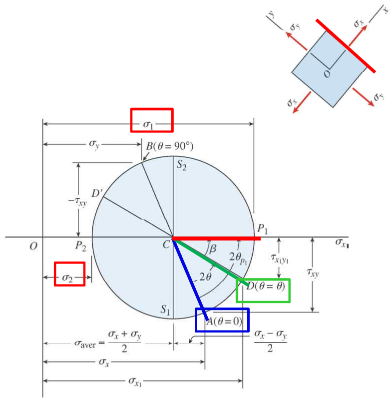
$$n_1 = \cos\phi = \frac{1}{2} \qquad \phi = 60°$$

$$n_3 = \cos\theta = \frac{\sqrt{2}}{2} \qquad \theta = 45°$$



Stress (응력) & Force(힘)
Mohr's Circle





Stress
deviatoric stress/stress invariant

- Deviatoric stress

$$s_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3}\delta_{ij},$$

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} - \begin{bmatrix} \pi & 0 & 0 \\ 0 & \pi & 0 \\ 0 & 0 & \pi \end{bmatrix}$$
$$= \begin{bmatrix} \sigma_{11} - \pi & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} - \pi & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} - \pi \end{bmatrix}.$$

- Stress invariant

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$
$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2$$
$$I_3 = \sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2$$

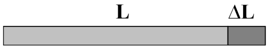
$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$
$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$$
$$I_3 = \sigma_1\sigma_2\sigma_3$$

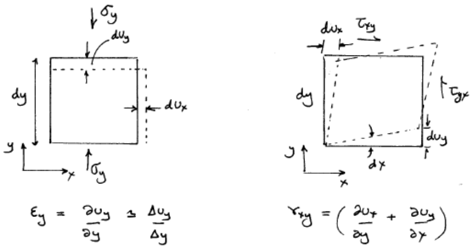
$$\begin{aligned} J_1 &= s_{kk} = 0, \\ J_2 &= \tfrac{1}{2}s_{ij}s_{ji} = \tfrac{1}{2}\mathrm{tr}(\boldsymbol{s}^2) \\ &= \tfrac{1}{2}(s_1^2 + s_2^2 + s_3^2) \\ &= \tfrac{1}{6}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2\right] + \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2 \\ &= \tfrac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \\ &= \tfrac{1}{3}I_1^2 - I_2 = \frac{1}{2}\left[\mathrm{tr}(\boldsymbol{\sigma}^2) - \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma})^2\right], \\ J_3 &= \det(s_{ij}) \\ &= \tfrac{1}{3}s_{ij}s_{jk}s_{ki} = \tfrac{1}{3}\mathrm{tr}(\boldsymbol{s}^3) \\ &= s_1s_2s_3 \\ &= \tfrac{2}{27}I_1^3 - \tfrac{1}{3}I_1I_2 + I_3 = \tfrac{1}{3}\left[\mathrm{tr}(\boldsymbol{\sigma}^3) - \mathrm{tr}(\boldsymbol{\sigma}^2)\mathrm{tr}(\boldsymbol{\sigma}) + \tfrac{2}{9}\mathrm{tr}(\boldsymbol{\sigma})^3\right]. \end{aligned}$$

Strain (변형율) & Displacement (변위)
1D & 2D

- Geometric expression of deformation caused by stress (dimensionless)

1D $\varepsilon = \frac{\Delta L}{L} = \frac{du}{dx}$





$\varepsilon_y = \frac{\partial u_y}{\partial y} \simeq \frac{\Delta u_y}{\Delta y}$

$\gamma_{xy} = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)$

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \varepsilon_{yy} = \frac{\partial u_y}{\partial y},$$

$$\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

Strain and displacement
Transformation equation for plane strain

$$\Delta d = \varepsilon_x dx \cos\theta + \varepsilon_y dy \sin\theta + \gamma_{xy} dy \cos\theta$$

$$\varepsilon_{x1} = \frac{\Delta d}{ds} = \varepsilon_x \frac{dx}{ds}\cos\theta + \varepsilon_y \frac{ds}{ds}\sin\theta + \gamma_{xy} \frac{dy}{ds}\cos\theta$$

$$\varepsilon_{x1} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \cos\theta \sin\theta$$

2D & 3D elasticity
Strain – 2D & 3D

$$\varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \varepsilon_{yy} = \frac{\partial u_y}{\partial y}, \varepsilon_{zz} = \frac{\partial u_z}{\partial z},$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)$$

$$\varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} \right)$$

$$\varepsilon_{xz} = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right)$$

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}$$

Tensor form

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{xy} \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix}$$

matrix form **Engineering strain**

Strain is also a 2nd order tensor and symmetric by definition.

Strain and displacement
Transformation equation for plane strain

- Shear strain γ_{x1y1}:
  - Decrease in angle between lines that were initially along the x1 and y1 axes.

$$\gamma_{x1y1} = \alpha + \beta$$

$$\frac{\gamma_{x_1y_1}}{2} = -(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta + \frac{\gamma_{xy}}{2}(\cos^2\theta - \sin^2\theta)$$

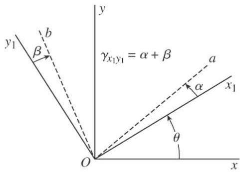


FIG. 7-34 Shear strain $\gamma_{x_1y_1}$ associated with the x_1y_1 axes

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Strain and displacement
Transformation equation for plane strain

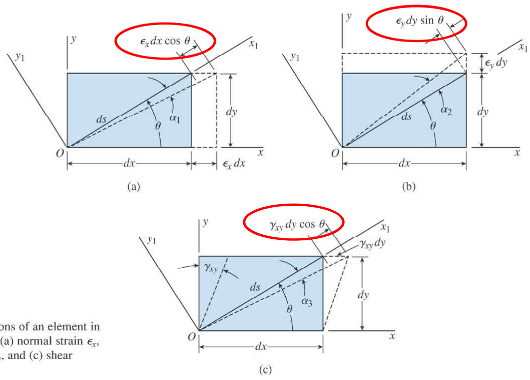


FIG. 7-33 Deformations of an element in plane strain due to (a) normal strain ϵ_x, (b) normal strain ϵ_y, and (c) shear strain γ_{xy}

Strain and displacement
Transformation equation for plane strain

- Transformation equations for plane strain

$$\varepsilon_{x1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x1y1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

compare $\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

compare $\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

$$\varepsilon_{x1} + \varepsilon_{y1} = \varepsilon_x + \varepsilon_y$$

- Similar to the transformation of plane stress

TABLE 7-1 CORRESPONDING VARIABLES IN THE TRANSFORMATION EQUATIONS FOR PLANE STRESS (EQS. 7-4a AND b) AND PLANE STRAIN (EQS. 7-71a AND b)

Stresses	Strains
σ_x	ϵ_x
σ_y	ϵ_y
τ_{xy}	$\gamma_{xy}/2$
σ_{x_1}	ϵ_{x_1}
$\tau_{x_1y_1}$	$\gamma_{x_1y_1}/2$

Strain and displacement
Transformation equation for plane strain

- Principal Angles

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

- Principal Strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{(\frac{\varepsilon_x - \varepsilon_y}{2})^2 + (\frac{\gamma_{xy}}{2})^2}$$

- Maximum Shear Strain (and normal strains for the maximum shear)

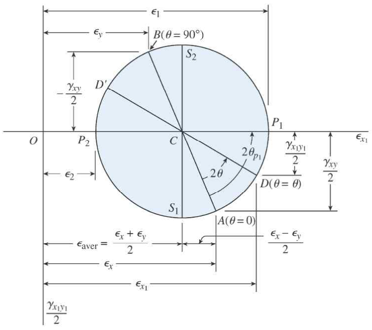
$$\frac{\gamma_{\max}}{2} = \sqrt{(\frac{\varepsilon_x - \varepsilon_y}{2})^2 + (\frac{\gamma_{xy}}{2})^2} \qquad \varepsilon_{aver} = \frac{\varepsilon_x + \varepsilon_y}{2}$$

Strain and displacement
Mohr's Circle

- Mohr's Circle for plane strain ← same as plane stress

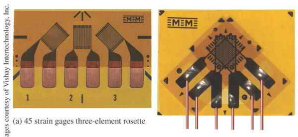
TABLE 7-1 CORRESPONDING VARIABLES IN THE TRANSFORMATION EQUATIONS FOR PLANE STRESS (EQS. 7-4a AND b) AND PLANE STRAIN (EQS. 7-71a AND b)

Stresses	Strains
σ_x	ϵ_x
σ_y	ϵ_y
τ_{xy}	$\gamma_{xy}/2$
σ_{x_1}	ϵ_{x_1}
$\tau_{x_1y_1}$	$\gamma_{x_1y_1}/2$



Strain and displacement
Strain Measurements

- Strain gages
  - A device for measuring normal strains on the surface of a stressed object (e.g., rock)
  - Electrical resistance of the wire is altered when it stretches or shortens → converted to strain
  - Sensitive: can measure 1x10^{-6}
  - Three measurement → strains in any direction
- Strain rosette
  - A group of three gages arranged in a particular direction



Images courtesy of Vishay Intertechnology, Inc.

(a) 45 strain gages three-element rosette

(b) Three-element strain-gage rosettes prewired

Constitutive Equation
Hooke's Law

- Hooke's law

$$\sigma = E\varepsilon = E\frac{du}{dx} \longleftarrow \quad q''_x = -k\frac{dT}{dx} \qquad q_x = -K_x\frac{\partial h}{\partial x}$$

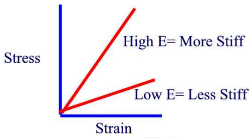
- Elastic (Young's) modulus, E (N/m^2=Pa)

$$E = \frac{\sigma_y}{\varepsilon_y}$$

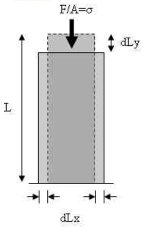
- Poisson's ratio, ν (dimensionless)

$$\nu = -\frac{lateral\ strain}{axial\ strain} = -\frac{\varepsilon_x}{\varepsilon_y}$$

- Typical range of properties
  - Concrete σ_c = 20-50 MPa E ~ 25 GPa
  - Granite σ_c = 100-200 MPa E ~ 60 GPa
  - Alloy steel σ_c = >500 MPa E ~ 200 GPa



$$\varepsilon = \frac{\Delta L}{L}$$



Constitutive Equation
Hooke's Law

- Hooke's Law

1D $\sigma = E\varepsilon$

- Shear modulus G

$$\tau_{xy} = G\gamma_{xy}$$

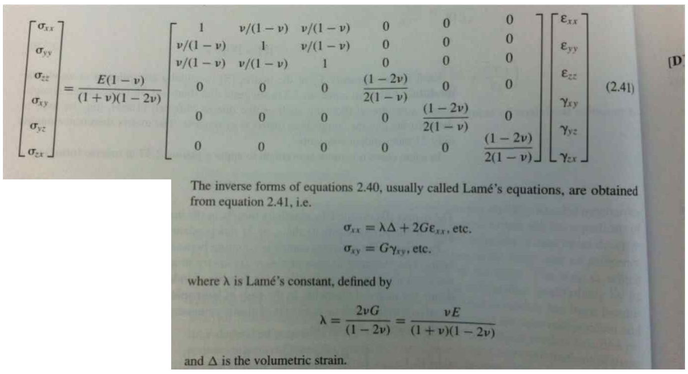
- Generalized Hooke's law (isotropy)

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{pmatrix} \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{pmatrix}$$

- 2 independent parameters (E, ν) for isotropic material

$$G = \frac{E}{2(1+\nu)}$$

Constitutive Equation
Hooke's Law (in inverse form)



$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \nu/(1-\nu) & \nu/(1-\nu) & 0 & 0 & 0 \\ \nu/(1-\nu) & 1 & \nu/(1-\nu) & 0 & 0 & 0 \\ \nu/(1-\nu) & \nu/(1-\nu) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\nu)}{2(1-\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2(1-\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2(1-\nu)} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} \quad (2.41)$$

The inverse forms of equations 2.40, usually called Lamé's equations, are obtained from equation 2.41, i.e.

$$\sigma_{xx} = \lambda\Delta + 2G\varepsilon_{xx}, \text{ etc.}$$
$$\sigma_{xy} = G\gamma_{xy}, \text{ etc.}$$

where λ is Lamé's constant, defined by

$$\lambda = \frac{2\nu G}{(1-2\nu)} = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

and Δ is the volumetric strain.

Constitutive Equation
Hooke's Law

- Normal strain in plane stress (평명응력하에서의 수직변형율)

$$\textbf{Normal strain, } \varepsilon_x = \frac{1}{E}\sigma_x + -\frac{\nu}{E}\sigma_y$$

  - Similarly

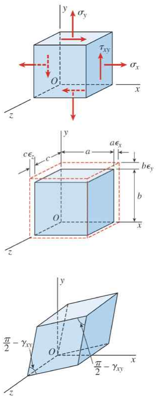
$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$$

- Normal strain in plane stress(평면응력하에서의 전단변형율)
  - Shear strain is a change of angle(전단변형율은 각도의 변화)
  - No influence from σ_x & σ_y (수직응력 σ_x 과 σ_y 은 영향이 없다.)

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right) \qquad \varepsilon_z = -\frac{\nu}{E}\left(\sigma_x + \sigma_y\right)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

G: 전단계수



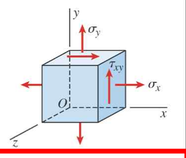
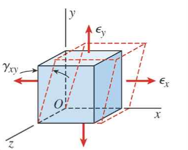
Hooke's Law
General Perspective - Anisotropy

- The most general case
  - Stress and Strain are linearly related

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16}\\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26}\\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36}\\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46}\\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56}\\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66}\end{pmatrix} \begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

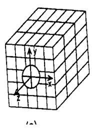
- Compliance matrix has 21 independent parameters
  (By the symmetry of stress tensor, strain tensor and consideration of strain energy)

Plane Strain (평면변형율)
Plane strain versus plane stress

	Plane stress	Plane strain
		
Stresses	$\sigma_z = 0$ $\tau_{xz} = 0$ $\tau_{yz} = 0$ σ_x, σ_y, and τ_{xy} may have nonzero values	$\tau_{xz} = 0$ $\tau_{yz} = 0$ σ_x, σ_y, σ_z, and τ_{xy} may have nonzero values
Strains	$\gamma_{xz} = 0$ $\gamma_{yz} = 0$ ϵ_x, ϵ_y, ϵ_z, and γ_{xy} may have nonzero values	$\epsilon_z = 0$ $\gamma_{xz} = 0$ $\gamma_{yz} = 0$ ϵ_x, ϵ_y, and γ_{xy} may have nonzero values

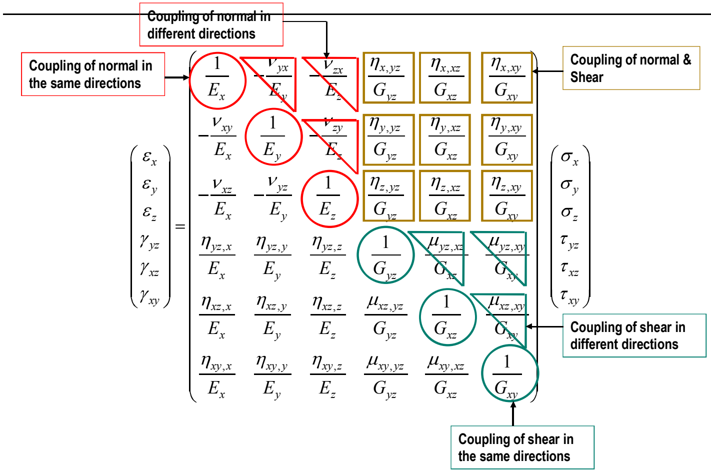
Orthotropic
Three orthogonal planes of elastic symmetry

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix} = \begin{pmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0\\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0\\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{xz}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}}\end{pmatrix} \begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$



- Three orthogonal planes elastic symmetry
- 9 independent constants

Hooke's Law
General Perspective - Anisotropy



Coupling of normal in different directions

Coupling of normal in the same directions

Coupling of normal & Shear

Coupling of shear in different directions

Coupling of shear in the same directions

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix} = \begin{pmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & \frac{\eta_{x,yz}}{G_{yz}} & \frac{\eta_{x,xz}}{G_{xz}} & \frac{\eta_{x,xy}}{G_{xy}}\\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & \frac{\eta_{y,yz}}{G_{yz}} & \frac{\eta_{y,xz}}{G_{xz}} & \frac{\eta_{y,xy}}{G_{xy}}\\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & \frac{\eta_{z,yz}}{G_{yz}} & \frac{\eta_{z,xz}}{G_{xz}} & \frac{\eta_{z,xy}}{G_{xy}}\\ \frac{\eta_{yz,x}}{E_x} & \frac{\eta_{yz,y}}{E_y} & \frac{\eta_{yz,z}}{E_z} & \frac{1}{G_{yz}} & \frac{\mu_{yz,xz}}{G_{xz}} & \frac{\mu_{yz,xy}}{G_{xy}}\\ \frac{\eta_{xz,x}}{E_x} & \frac{\eta_{xz,y}}{E_y} & \frac{\eta_{xz,z}}{E_z} & \frac{\mu_{xz,yz}}{G_{yz}} & \frac{1}{G_{xz}} & \frac{\mu_{xz,xy}}{G_{xy}}\\ \frac{\eta_{xy,x}}{E_x} & \frac{\eta_{xy,y}}{E_y} & \frac{\eta_{xy,z}}{E_z} & \frac{\mu_{xy,yz}}{G_{yz}} & \frac{\mu_{xy,xz}}{G_{xz}} & \frac{1}{G_{xy}}\end{pmatrix} \begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

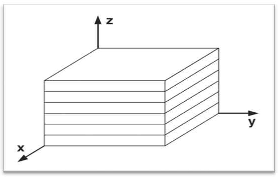
Lekhnitskii(1963) & Hudson (1997)

Transversely Isotropic
One axis of symmetry

- Transversely isotropic – 5 independent parameters

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu'}{E'} & -\frac{\nu}{E} & 0 & 0 & 0\\ -\frac{\nu'}{E'} & \frac{1}{E'} & -\frac{\nu'}{E'} & 0 & 0 & 0\\ -\frac{\nu}{E} & -\frac{\nu'}{E'} & \frac{1}{E} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G'} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G'}\end{bmatrix} \begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix}$$

$$E_x = E_z = E$$
$$E_y = E'$$
$$\nu_{xz} = \nu_{zx} = \nu$$
$$\nu_{yx} = \nu_{yz} = \nu'$$
$$G_{xy} = G_{yz} = G'$$



(H. Gercek, 2006)

Plane Strain (평면변형율)
Plane strain versus plane stress

- Stress and strain in different dimensions are coupled. Therefore, we need a special consideration –plane strain and plane stress
- Plane strain
  - 3rd dimensional strain goes zero
  - Stresses around drill hole or 2D tunnel

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{1}{E} & \enclose{circle}{-\frac{\nu}{E}} & \enclose{circle}{-\frac{\nu}{E}} & 0 & 0 & 0\\ \enclose{circle}{-\frac{\nu}{E}} & \frac{1}{E} & \enclose{circle}{-\frac{\nu}{E}} & 0 & 0 & 0\\ \enclose{circle}{-\frac{\nu}{E}} & \enclose{circle}{-\frac{\nu}{E}} & \frac{1}{E} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

$$\begin{pmatrix}\varepsilon_{xx} & \varepsilon_{xy} & 0\\ \varepsilon_{yx} & \varepsilon_{yy} & 0\\ 0 & 0 & 0\end{pmatrix}\rightarrow\begin{pmatrix}\sigma_{xx} & \sigma_{xy} & 0\\ \sigma_{yx} & \sigma_{yy} & 0\\ 0 & 0 & \sigma_{zz}\end{pmatrix}$$

$$\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{(1-\nu^2)}{E} & -\frac{\nu(1+\nu)}{E} & 0\\ -\frac{\nu(1+\nu)}{E} & \frac{(1-\nu^2)}{E} & 0\\ 0 & 0 & \frac{2(1+\nu)}{E}\end{pmatrix}\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{pmatrix}$$

Governing Equation
1D

- Strain-displacement relationship
- Stress-strain relationship
- Static Equillibrium Equation

$$\varepsilon=\frac{du}{dx}$$

$$\varepsilon=\frac{1}{E}\sigma$$

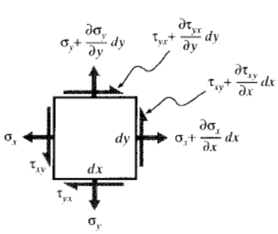
$$\frac{\partial\sigma_{xx}}{\partial x}+\rho b_x=0$$

- Final equation for elasticity

$$E\frac{\partial^2 u_x}{\partial x^2}+\rho b_x=0$$

Conservation Equation
Equilibrium equation

- Sum of traction, body forces (and moment) are zero (static case)



$$\sum F_i=m\frac{\partial^2 u_i}{\partial t}\xrightarrow{\text{Very slow loading}}\sum F_i=0$$

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}+\rho b_x=0$$

$$\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\tau_{zy}}{\partial z}+\rho b_y=0$$

$$\frac{\partial\tau_{xz}}{\partial x}+\frac{\partial\tau_{yz}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}+\rho b_z=0$$

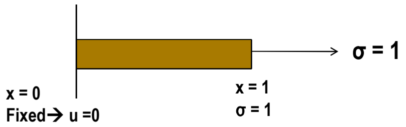
- b_x, b_y, b_z are components of acceleration due to gravity.

$$\sum M_i=0\longrightarrow\tau_{xy}=\tau_{yx}$$

Governing Equation
1D - example

- With no body force, and static case

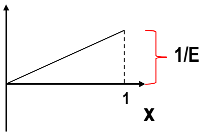
$$E\frac{\partial^2 u_x}{\partial x^2}+\cancel{\rho b_x}=\rho\cancel{\frac{\partial^2 u_x}{\partial t^2}}\qquad E\frac{\partial^2 u_x}{\partial x^2}=0$$



$E\frac{\partial u_x}{\partial x}=C_1$ → **From σ = 1, C_1=1**

$Eu_x=x+C_2$ → **From u = 0 at x=0, C_2=0**

$$u_x=\frac{x}{E}$$



Governing Equation
3D – Navier’s Equationv

- Strain-displacement relationship (6)
- Stress-strain relationship (6)
- Equation of motion (3)

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i})$$

$$\sigma_{ij}=C_{ijkl}\varepsilon_{kl}$$

$$\sigma_{ji,j}+\rho b_i=\rho\frac{\partial^2 u_i}{\partial t^2}$$

- Navier’s equation

$$Gu_{i,jj}+(\lambda+G)u_{j,ji}+\rho b_i=0$$

$$G\nabla^2\mathbf{u}+(\lambda+G)\nabla\nabla\cdot\mathbf{u}+\rho\mathbf{b}=0$$

$$G\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+(\lambda+G)\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_y}{\partial x\partial y}+\frac{\partial^2 u_z}{\partial x\partial z}\right)+\rho b_x=0$$

$$G\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)+(\lambda+G)\left(\frac{\partial^2 u_x}{\partial x\partial y}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_z}{\partial y\partial z}\right)+\rho b_y=0$$

$$G\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+(\lambda+G)\left(\frac{\partial^2 u_x}{\partial x\partial z}+\frac{\partial^2 u_y}{\partial y\partial z}+\frac{\partial^2 u_z}{\partial z^2}\right)+\rho b_z=0$$

  - Three governing equations for three displacement components

$$\lambda=\frac{E\nu}{(1+\nu)(1-2\nu)}$$

2D & 3D elasticity
Comparison with diffusion equation

- Diffusion equation

$$A\frac{\partial c}{\partial t}+\nabla\cdot(-D\nabla c)=R$$

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=\rho c\frac{\partial T}{\partial t}$$

  - Time-dependent
  - One parameter k is necessary for steady state behaviour

- Navier’s equation

$$G\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+(\lambda+G)\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_y}{\partial x\partial y}+\frac{\partial^2 u_z}{\partial x\partial z}\right)+\rho b_x=0$$

$$G\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)+(\lambda+G)\left(\frac{\partial^2 u_x}{\partial x\partial y}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_z}{\partial y\partial z}\right)+\rho b_y=0$$

$$G\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+(\lambda+G)\left(\frac{\partial^2 u_x}{\partial x\partial z}+\frac{\partial^2 u_y}{\partial y\partial z}+\frac{\partial^2 u_z}{\partial z^2}\right)+\rho b_z=0$$

  - Not time-dependent
  - Three coupled equations
  - Two parameters (isotropy)

1D & steady state

1D

$$-\frac{d}{dx}\left(\alpha(x)\frac{dU(x)}{dx}\right)=f(x)$$

Fundamentals of Geomechanics – Deformation and Failure of Rock (Week 3 & 4, 19 & 26 Sept)

Ki-Bok Min, PhD

Associate Professor
Department of Energy Resources Engineering
Seoul National University

Rock Failure
Classical References

- Test on rock failure
  - Cook, N. G. W. (1965). "The failure of rock." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 2(4): 389-403.
  - Hudson, J. A., et al. (1972). "SOFT, STIFF AND SERVO-CONTROLLED TESTING MACHINES - REVIEW WITH REFERENCE TO ROCK FAILURE." Engineering Geology 6(3): 155-189.
  - Hoek, E. and Franklin. JA (1968). "A simple triaxial cell for field and laboratory testing of rock." Trans. Instn Min. Metall. 77: A22–A26.
  - Hudson, J. A. (1971). "EFFECT OF TIME ON MECHANICAL BEHAVIOUR OF FAILED ROCK." Nature 232(5307): 185-186.
- Rock mass failure criteria
  - Hoek, E. and E. T. Brown (1980). "EMPIRICAL STRENGTH CRITERION FOR ROCK MASSES." Journal of the Geotechnical Engineering Division-Asce 106(9): 1013-1035.
  - Hoek, E. (1983). "23RD RANKINE-LECTURE - STRENGTH OF JOINTED ROCK MASSES." Geotechnique 33(3): 185-223.
  - Hoek, E. and E. T. Brown (1997). "Practical estimates of rock mass strength." International Journal of Rock Mechanics and Mining Sciences 34(8): 1165-1186.
- Failure criteria
  - Coulomb, 1773,
  - Mohr, 1900,
  - Wiebols, G. A. and N. G. W. Cook (1968). "AN ENERGY CRITERION FOR STRENGTH OF ROCK IN POLYAXIAL COMPRESSION." International Journal of Rock Mechanics and Mining Sciences 5(6): 529-&.

Introduction
Objectives of the course

- Objective;
  - Understand the fundamental concept of Rock failure
  - Topics includes;
    - Failure criteria
    - Brittle vs. Ductile
    - Post peak response
    - True triaxial failure criteria
    - Anisotropic failure criteria
    - Outstanding issues

Rock Failure
Classical References

- Friction on rock surfaces
  - Byerlee, J. (1978). "Friction of rocks." Pure and Applied Geophysics 116(4): 615-626. (citation > 2,000)
  - Recent one? Compare with Barton's equation? Criticism? Relevance?
- Effect of pore pressure
  - Nur, A. and J. D. Byerlee (1971). "Exact Effective Stress Law for Elastic Deformation of Rock with Fluids." Journal of Geophysical Research 76(26): 6414-6419. (citation >400)
  - Effect of pore pressure on strength?
- Point load test
  - Broch, E. and J. A. Franklin (1972). "POINT-LOAD STRENGTH TEST." International Journal of Rock Mechanics and Mining Sciences 9(6): 669-697. (citation >200)
  - Recent one?
- Brazilian Tensile Strength test (effect of anisotropy)
  - Li, D. Y. and L. N. Y. Wong (2013). "The Brazilian Disc Test for Rock Mechanics Applications: Review and New Insights." Rock Mechanics and Rock Engineering 46(2): 269-287.
  - Claesson, J. and B. Bohloli (2002). "Brazilian test: stress field and tensile strength of anisotropic rocks using an analytical solution." International Journal of Rock Mechanics and Mining Sciences 39(8): 991-1004.
- Earthquake and fracture slip
  - Brace, W. F. and J. D. Byerlee (1966). "STICK-SLIP AS A MECHANISM FOR EARTHQUAKES." Science 153(3739): 990-992. (citation >380)

Rock Failure
Main References

- Jaeger JG, Cook NGW and Zimmerman RW, 2007, Fundamentals of Rock Mechanics, 4th edition, Blackwell Publishing
  - Chapter 3. Friction on Rock Surfaces (p.65-p.79)
  - Chapter 4. Deformation and Failure of Rock (p.80-p.105)
- Brady BHG and Brown ET, 2004, Rock Mechanics for Underground Mining, 4th ed., Kluwer Academic Publishers
  - Chapter 4. Rock Strength and Deformability (p.85-141)
- Hoek E and Brown ET, 1980, Underground Excavations in Rock, Inst Mining & Metallurgy
  - Chapter 6. Strength of Rock and Rock Mass (p.131-182)
- R Ulusay (ed.), 2015, The ISRM Suggested Methods for Rock Characterization, Testing and Monitoring:2007-2014,
  - Part IV. Failure Criteria (p.223-p.262)

Rock Failure
Classical References

- Fundamentals of failure
  - Brace, W. F., et al. (1966). "DILATANCY IN FRACTURE OF CRYSTALLINE ROCKS." Journal of Geophysical Research 71(16): 3939-&.
  - Wong, T. F. (1982). "MICROMECHANICS OF FAULTING IN WESTERLY GRANITE." International Journal of Rock Mechanics and Mining Sciences 19(2): 49-64.
  - Fredrich, J. T., et al. (1990). "EFFECT OF GRAIN-SIZE ON BRITTLE AND SEMIBRITTLE STRENGTH - IMPLICATIONS FOR MICROMECHANICAL MODELING OF FAILURE IN COMPRESSION." Journal of Geophysical Research-Solid Earth and Planets 95(B7): 10907-10920.
- True Triaxial Test
  - Mogi, K. (1971). "EFFECT OF TRIAXIAL STRESS SYSTEM ON FAILURE OF DOLOMITE AND LIMESTONE." Tectonophysics 11(2): 111-&.
  - Mogi, K. (1971). "FRACTURE AND FLOW OF ROCKS UNDER HIGH TRIAXIAL COMPRESSION." Journal of Geophysical Research 76(5): 1255-&.
  - Haimson, B. and C. Chang (2000). "A new true triaxial cell for testing mechanical properties of rock, and its use to determine rock strength and deformability of Westerly granite." International Journal of Rock Mechanics and Mining Sciences 37(1-2): 285-296.
  - Chang, C. and B. Haimson (2000). "True triaxial strength and deformability of the German Continental Deep Drilling Program (KTB) deep hole amphibolite." Journal of Geophysical Research-Solid Earth 105(B8): 18999-19013.
  - Chang, C. and B. Haimson (2005). "Non-dilatant deformation and failure mechanism in two Long Valley Caldera rocks under true triaxial compression." International Journal of Rock Mechanics and Mining Sciences 42(3): 402-414.
- Anisotropy
  - Donath, F. A. (1961). "EXPERIMENTAL STUDY OF SHEAR FAILURE IN ANISOTROPIC ROCKS." Geological Society of America Bulletin 72(6): 985-989.

Home Assignment #2

- Review papers in rock failure
  - You need to provide not only summary but also your own insight and criticism based on the selected papers.
  - If necessary, you will have to conduct your own analysis and refer to other papers.
  - One classical paper (try to refer to recent papers on the subject)
    - 09:00 19 Sept through eTL
  - One classical paper (try to refer to recent papers on the subject)
    - 09:00 26 Sept through eTL

Outline
Deformation and Failure of rock

- Introduction
- The stress-strain curve
- Effects of confining stress and temperature
- Types of fracture
- Coulomb Failure criterion
- Mohr’s hypothesis
- Effects of pore fluids
- Failure under true-triaxial conditions
- The effect of anisotropy on strength

Outline
Deformation and Failure of Rock

- Jaeger JG, Cook NGW and Zimmerman RW, 2007, Fundamentals of Rock Mechanics, 4th edition, Blackwell Publishing
  - Chapter 3. Friction on Rock Surfaces (p.65-p.79)
  - Chapter 4. Deformation and Failure of Rock (p.80-p.105)

Friction on Rock Surfaces
Introduction

- Phenomenon by which a tangential shearing force is required in order to displace two contacting surfaces along a direction parallel to their nominal contact plane
- Importance:
  - Microscopic scale: minute Griffith cracks
  - Somewhat larger scale: friction between grains,
  - Macroscale (~m^2): fracture and fault

Outline
Friction on Rock Surfaces

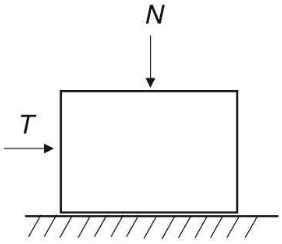
- Introduction
- Amonton’s law
- Friction on rock surfaces
- Stick-slip oscillations
- Sliding on a plane of weakness
- Effects of time and velocity

Friction on Rock Surfaces
Amonton’s law

- Amonton’s law (1699)

$$T = \mu N$$

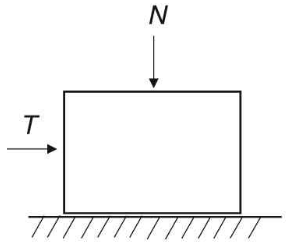
μ : coefficient of friction



Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Friction on Rock Surfaces
Friction on rock surface - Friction coefficient

- Friction
  - Phenomenon by which a tangential shearing force is required in order to displace two contacting surfaces along a direction parallel to their nominal contact plane
  - Importance: friction between grains, fracture and fault



$\tau = \mu\sigma$

τ : shear stress

σ : normal stress

μ : coefficient of friction

Also called 'friction angle'. Why?

$\tau = \mu_d \sigma$

μ_d : coefficient of dynamic friction

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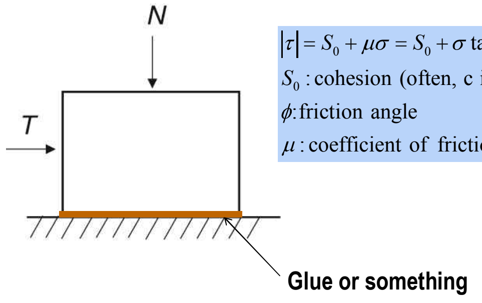
Friction on Rock Surfaces
Friction on rock surface - cohesion

- Coulomb failure criterion (on fractures)



Glue or something

$|\tau| = S_0 + \mu\sigma = S_0 + \sigma \tan\phi$

S_0 : cohesion (often, c is used), or 'shear strength'

ϕ: friction angle

μ : coefficient of friction angle

Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

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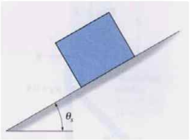
Friction on Rock Surfaces
Friction on rock surfaces - Friction coefficient

- Friction angle

$\mu = \tan\phi$

ϕ = friction angle

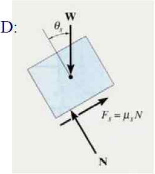
DETERMING μ_s EXPERIMENTALLY



A block with weight **W** is placed on an inclined plane. The plane is slowly tilted until the block just begins to slip.

The inclination, θ_s, is noted. Analysis of the block just before it begins to move gives (using $F_s = \mu_s N$):

FBD:



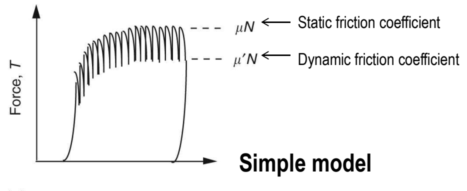
$\nwarrow + \ \Sigma F_y = N - W\cos\theta_s = 0$

$\nearrow + \ \Sigma F_X = \mu_S N - W\sin\theta_s = 0$

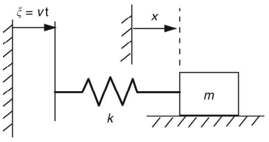
Using these two equations, we get μ_s = $(\mathbf{W}\sin\theta_s) / (\mathbf{W}\cos\theta_s) = \tan\theta_s$
This simple experiment allows us to find the μ_S between two materials in contact.

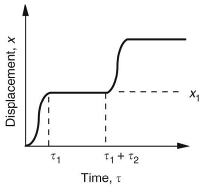
Friction on Rock Surfaces
Stick-slip oscillations

- Stick-slip oscillation
  - May provide a mechanism for earthquakes



Simple model





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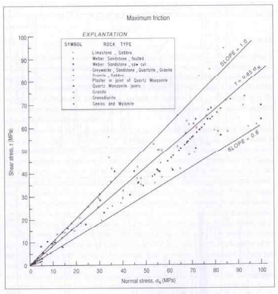
Friction on Rock Surfaces
Friction on rock surfaces - Friction coefficient

- Typical Range of Friction coefficient (Byerlee, 1978)
  - 0.6 ~ 1.0
  - Wider variability in low normal stress

$\tau = 0.85\sigma \qquad \sigma < 200\text{MPa}$

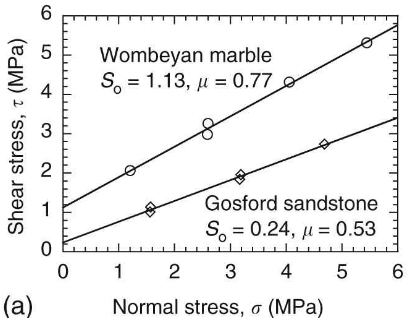
$\tau = 50\text{MPa} + 0.6\sigma \qquad 200\text{MPa} < \sigma < 1700\text{MPa}$



Byerlee, J. (1978). "Friction of rocks." Pure and Applied Geophysics 116(4): 615-626.

Friction on Rock Surfaces
Sliding on a plane of weakness

- Example



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Friction on Rock Surfaces
Sliding on a plane of weakness

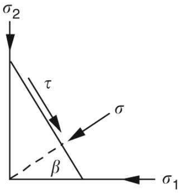
- Coulomb Failure criteria of a fracture (plane of weakness)

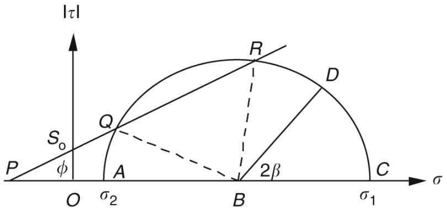
$$|\tau| = S_0 + \mu\sigma = S_0 + \sigma\tan\phi$$

S_0 : cohesion (often, c is used), or 'shear strength'

ϕ : friction angle

μ : coefficient of friction





Friction on Rock Surfaces
Sliding on a plane of weakness

- The stress difference that is required to cause a slip with a given β and σ_2

$$\sigma_1 - \sigma_2 = \frac{2(S_0 + \mu\sigma_2)}{(1-\mu\cot\beta)\sin\beta} \qquad \sigma_1 = \frac{2S_0\cos\phi}{(1-k)\sin(2\beta-\phi)-(1+k)\sin\phi}$$

$$\sigma_{1,\min} = \sigma_2 + 2(S_0 + \mu\sigma_2)\left(\sqrt{\mu^2+1}+\mu\right)$$

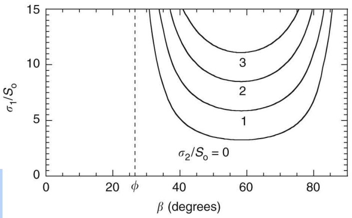
- Solution exists only for

$$\phi < \beta$$

- Range of β (for a given stress state)

$$\beta_1 < \beta < \beta_2$$

$$2\beta_1 = \phi + \sin^{-1}\left[\{(\sigma_m + S_0\cot\phi)/\tau_m\}\sin\phi\right]$$
$$2\beta_2 = \pi + \phi - \sin^{-1}\left[\{(\sigma_m + S_0\cot\phi)/\tau_m\}\sin\phi\right]$$



Variation of σ_1 needed to cause sliding on a fracture for μ=0.5

Deformation and Failure of Rock
Introduction

- Various loading conditions (and specimen)
  - Test on intact rock is important
  - In real conditions, stress can be in situ stress or induced stress

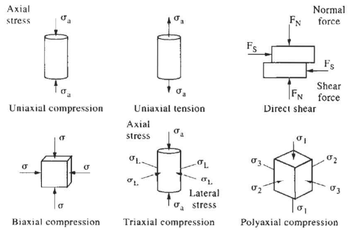
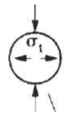
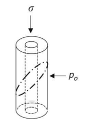


Figure 6.13 Specimen loading conditi ons in general laboratory use.

True triaxial compression



Brazilian Tensile test



Hollow cylinder test

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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Deformation and Failure of Rock
Introduction – Laboratory experiment

- Geological Repository for Nuclear Waste

Rock core collection (Forsmark, Oct 2004)

- 25 core-drilled boreholes up to 1,000 m depth.
- 17.8 km core length in total

Core Drilling site (Forsmark, June 2003)

Deformation and Failure of Rock
Introduction – Laboratory experiment

- Pohang EGS project
  - 3.6 m long 4 inch (~10 cm) core at 4.2 km depth

Deformation and Failure of Rock
Introduction – Laboratory experiment

Deformation and Failure of Rock
Introduction – Laboratory experiment

UCS & Triaxial: 1 inch cores
BTS: 1.5 inch cores
Divided-bar (thermal conductivity measurement): NX (54mm) disks

Deformation and Failure of Rock
The stress-strain curve

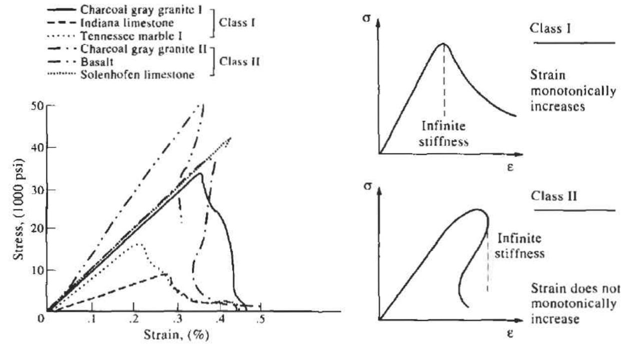

Figure 6.8 Examples of complete stress–strain curves for different rocks (from Wawersik and Fairhurst, 1970).

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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Deformation and Failure of Rock
The stress-strain curve

- Initial behavior: concave upward
- Acoustic emission at 50% level
- Post peak behavior may not be important in civil engineering but it is important (encouraged) for some applications. e.g., block caving

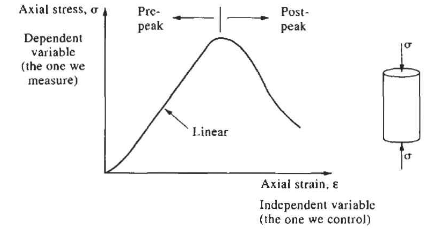

Figure 6.1 The complete stress–strain curve.

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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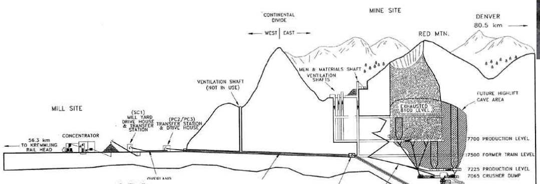
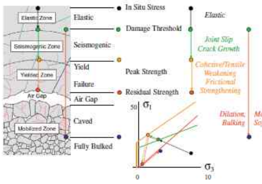
Deformation and Failure of Rock
The stress-strain curve

- 헨데센 광산 (Hendersen Mine), 콜로라도, 미국
  - 1976 년 운영시작 (세계최대의 몰리브덴 광산)
  - 1000 미터 하부에 광체, 최대심도 1,600 미터

Min, 2006



Hustrulid & Bullock, 2001



Caving mechanics

Mas Ivars et al., 2011

Deformation and Failure of Rock
The stress-strain curve

- Full stress-strain curve by servo-controlled testing
  - Stress controlled test will generate uncontrolled failure

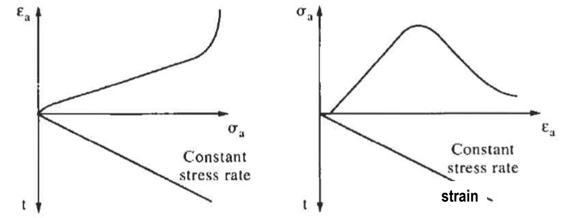


Figure 6.5 Stress- and strain-controlled stress–strain curves.

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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Deformation and Failure of Rock
The stress-strain curve – Brittle vs. Ductile

- Brittle vs ductile
  - Ductile: rock support an increasing load as it deforms
  - Brittle: load decreases as the strain increases
- Brittle-ductile transition
  - Rock becomes more ductile with increasing confining pressure

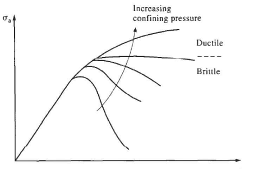

Figure 6.15 The effect of confining pressure in the triaxial test and the brittle–ductile transition.

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

Deformation and Failure of Rock
The stress-strain curve – Brittle vs. Ductile

- Quantitative description of brittleness is still an open question
  - There are many definitions for brittleness index

$$B_1 = \frac{\varepsilon_{el}}{\varepsilon_{tot}} = \frac{\varepsilon_{el}}{\varepsilon_{el} + \varepsilon_{pl}}$$

$$B_2 = \frac{W_{el}}{W_{tot}}$$

$$B_3 = \frac{C_0 - T_0}{C_0 + T_0}$$

$$B_4 = \sin\varphi$$

$$B_5 = \frac{\tau_{max} - \tau_{res}}{\tau_{max}}$$

$$B_6 = \left|\frac{\varepsilon_f^p - \varepsilon_c^p}{\varepsilon_c^p}\right|$$

$$B_7 = OCR^b$$

$$B_8 = \frac{1}{2}\cdot\left(\frac{E_{dyn}[Mpsi](0.8-\phi)-1}{8-1} + \frac{\upsilon_{dyn}-0.4}{0.15-0.4}\right)\cdot 100$$

Holt R. M. et al (2011)

Deformation and Failure of Rock
Other effects - time

- Creep:
  - Continued deformation when the applied stress is held constant
- Relaxation:
  - Decrease in stress when applied strain is held constant
- Fatigue
  - Increase in strain due to cyclic loading

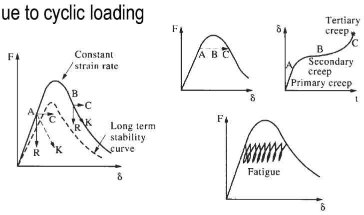

Figure 6.16 Time-dependent effects and the complete stress–strain curve.

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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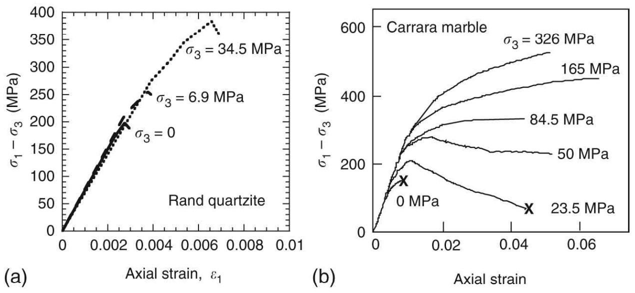
Deformation and Failure of Rock
Other effects - confining stress

- With increasing confining pressure
  - Strength increases
  - Becomes more ductile



Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

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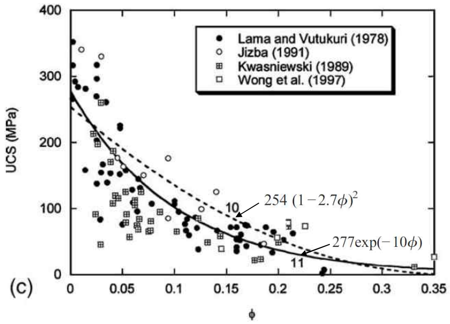
Deformation and Failure of Rock
Other effects - porosity

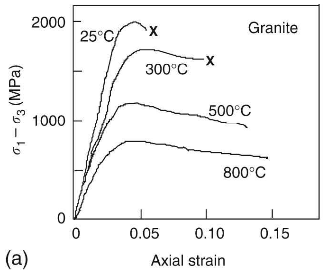
- Relationship with other parameters
  - UCS tends to decrease with porosity



Chang, Chandong, Mark D. Zoback, and Abbas Khaksar. "Empirical Relations between Rock Strength and Physical Properties in Sedimentary Rocks." *Journal of Petroleum Science and Engineering* 51, no. 3–4 (2006): 223-37.

Deformation and Failure of Rock
Other effects - temperature

- Increase in temperature tends to:
  - Reduces elastic modulus & compressive strength
  - Increases the ductility



At confining pressure of 500 MPa

Implication to EGS?

Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

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Deformation and Failure of Rock
Other effects - Size effect

- Size effect: properties varies with size
  - Elastic modulus: relatively less affected
  - Strength: tends to decreases with increase of size. Why?
  - One could choose "representative elementary volume (REV)" to overcome this.

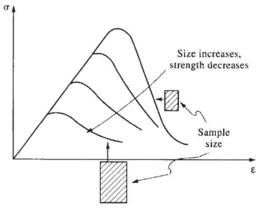


Figure 6.11 The size effect in the uniaxial complete stress–strain curve.

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

Deformation and Failure of Rock
Other effects - Size effect

- Representative Elementary Volume: Volume after which a property does not vary

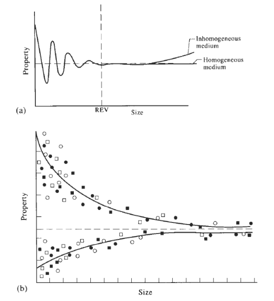
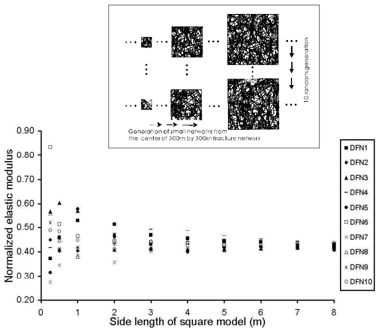


Figure 4.12 Variability in measured values with respect to sample volume, illustrating the REV. (a) General concept. (b) Example data scatter.



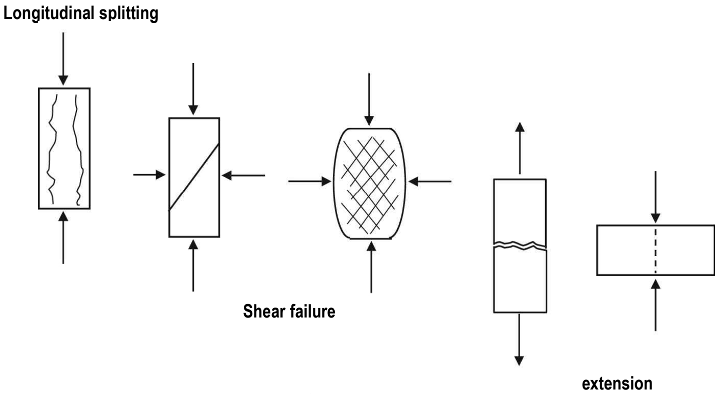
Normalized elastic moduli of fractured rock in various scales

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

Min KB, Jing L, 2003, Numerical determination of the equivalent elastic compliance tensor for fractured rock masses using the distinct element method, *International Journal of Rock Mechanics & Mining Sciences* 2003;40(6):795-816.

Deformation and Failure of Rock
Types of fracture

- Failure patterns vs. loading conditions



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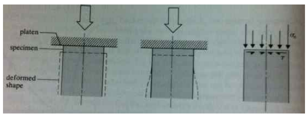
Deformation and Failure of Rock
Other effects - Shape effect

- Aspect ratio matters
  - Low ratio tends to have larger strength. Why?
  - Solution?
    - use large enough ratio >2.0-2.5
    - Improve testing procedure



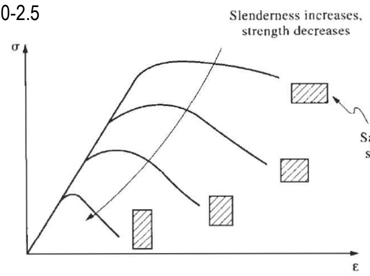


Figure 6.12 The shape effect in uniaxial compression.

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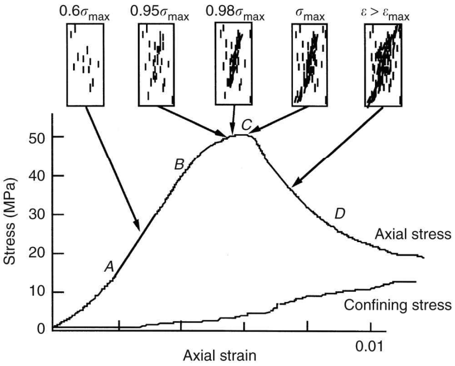
Deformation and Failure of Rock
Types of fracture

- Microcracking with the increase of axial and lateral stress



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Deformation and Failure of Rock
Other effects - Shape effect

- Other factors for lab test and in situ behavior
  - Moisture content
  - Desiccation – especially for clay
  - Slaking
  - Swelling - bentonite
  - Pore pressure – via effective stress
  - Groundwater chemistry – dissolution (chalk, limestone)
  - Free-thaw mechanism – cold region

Deformation and Failure of Rock
Coulomb failure criterion

- Coulomb Failure Criterion (on a rock) (or Mohr-Coulomb Failure Criterion)

$$|\tau| = S_0 + \mu\sigma = S_0 + \sigma\tan\phi$$

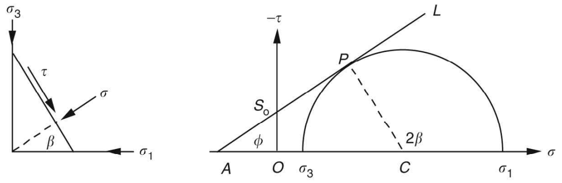
S_0 : cohesion (often, c is used), or 'shear strength'

ϕ:internal friction angle

μ : coefficient of internal friction angle

Same equation with different notation

$$|\tau| = c + \mu\sigma$$



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Deformation and Failure of Rock
Coulomb failure criterion

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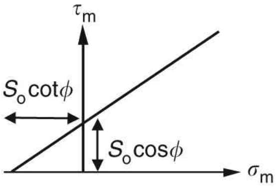
- Different expression (1)

$\sigma_m = \frac{1}{2}(\sigma_1 + \sigma_3)$ mean normal, $\tau_m = \frac{1}{2}(\sigma_1 - \sigma_3)$ maximum shear

$$\tau_m = S_0 \cos\phi + \sigma_m \sin\phi$$

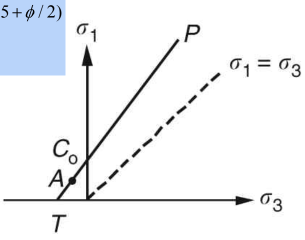


- Different expression (2)

$$\sigma_1 = 2S_0 \tan\beta + \sigma_3 \tan^2\beta = C_0 + \sigma_3 \tan^2\beta = C_0 + \sigma_3 \tan^2(45 + \phi/2)$$
$$= 2S_0\left[\left(1+\mu^2\right)^{1/2} + \mu\right] + \sigma_3\left[\left(1+\mu^2\right)^{1/2} + \mu\right]^2$$

$$C_0 = 2S_0 \tan\beta = 2S_0\left[\left(1+\mu^2\right)^{1/2} + \mu\right]$$

C_0:uniaxial compressive strength



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Deformation and Failure of Rock
Coulomb failure criterion

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- Limitations
  - Prediction of too high tensile strength
    - Tension cut-off needed

$$\frac{\sigma_c}{\sigma_t} = \tan^2\beta = \left[\left(1+\mu^2\right)^{1/2} + \mu\right]^2$$

  - Actual σ-τ is not linear
    - Angle β decreases with higher confining pressure
  - Does not consider intermediate principal stress
    - Additional consideration is needed

Deformation and Failure of Rock
Coulomb failure criterion

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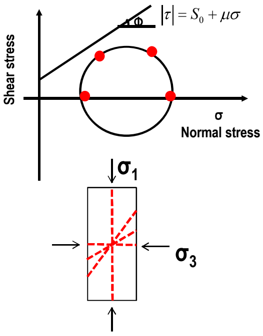
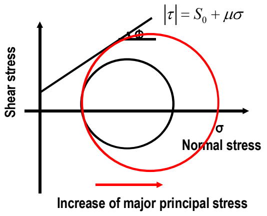
- Conditions for failure
  - A set of normal and shear stress within a rock must satisfy failure criterion





Deformation and Failure of Rock
Coulomb failure criterion

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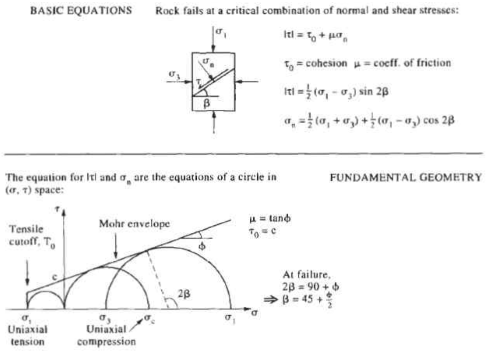
- Coulomb Failure Criterion for intact rock



Rock fails at a critical combination of normal and shear stresses:

$|\tau| = \tau_0 + \mu\sigma_n$

τ_0 = cohesion μ = coeff. of friction

$|\tau| = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta$

$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\beta$

The equation for $|\tau|$ and σ_n are the equations of a circle in (σ, τ) space:

$\mu = \tan\phi$
$\tau_0 = c$

At failure,
$2\beta = 90 + \phi$
$\Rightarrow \beta = 45 + \frac{\phi}{2}$

Figure 6.18 The Mohr–Coulomb failure criterion.

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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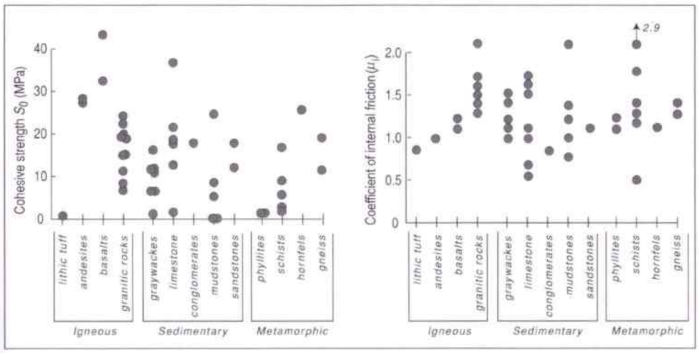
Deformation and Failure of Rock
Coulomb failure criterion

SEOUL NATIONAL UNIVERSITY

- Examples of measured cohesive strength (cohesion) and coefficient of internal friction



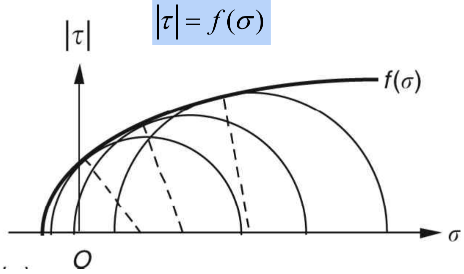
Zoback, 2007

Deformation and Failure of Rock
Mohr's hypothesis

SEOUL NATIONAL UNIVERSITY

- Mohr's nonlinear failure criterion
  - Experiment shows that σ_1 increase at a rate less than linear rate with σ_3
  - Failure angle (β) decrease with increasing confining stress.

$$|\tau| = f(\sigma)$$



Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Deformation and Failure of Rock
Hoek-Brown Failure Criterion

- Advantage
  - Non-linear form fits better with experimental data over a range of confining pressure
  - Developed through extensive lab tests on a wide range of rock type
  - Straightforwardly used

$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$

σ_1 : maximum principal stress at failure

σ_3 : minimum principal stress at failure

σ_c : uniaxial compressive strength

m: Hoek-Brown material constants ($0 \le$ m)

s: Hoek-Brown material constants ($0 \le$ s ≤ 1)

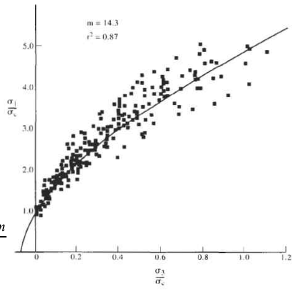


Figure 6.20 The Hoek–Brown empirical failure criterion.

  - More realistic tensile strength $\frac{\sigma_c}{\sigma_t} = \frac{\sqrt{m^2+4s}+m}{2s}$

More general form: $\sigma_1 = \sigma_3 + \left(m\sigma_c\sigma_3 + s\sigma_c^2\right)^a$

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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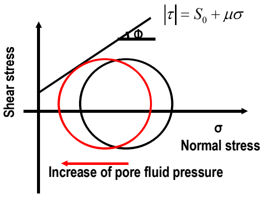
Deformation and Failure of Rock
Effects of pore fluids

- Required pore pressure to induce fracture with a given stress condition;

$$p_w = \sigma_3 - \frac{(\sigma_1-\sigma_3)-C_0}{\tan^2(45+\frac{\phi}{2})-1}$$



Deformation and Failure of Rock
Hoek-Brown Failure Criterion

- Values of the constant *m* for intact rock, by rock group (Note that values in parenthesis are estimates)

Rock type	Class	Group	Texture			
			Coarse	Medium	Fine	Very fine
SEDIMENTARY	Clastic		Conglomerates* (21 ± 3) Breccias (19 ± 5)	Sandstones 17 ± 4	Siltstones 7 ± 2 Greywackes (18 ± 3)	Claystones 4 ± 2 Shales (6 ± 2) Marls (7 ± 2)
	Non-Clastic	Carbonates	Crystalline Limestone (12 ± 3)	Sparitic Limestones (10 ± 2)	Micritic Limestones (9 ± 2)	Dolomites (9 ± 3)
		Evaporites		Gypsum 8 ± 2	Anhydrite 12 ± 2	
		Organic				Chalk 7 ± 2
METAMORPHIC	Non Foliated		Marble 9 ± 3	Hornfels (19 ± 4) Metasandstone (19 ± 3)	Quartzites 20 ± 3	
	Slightly foliated		Migmatite (29 ± 3)	Amphibolites 26 ± 6		
	Foliated**		Gneiss 28 ± 5	Schists 12 ± 3	Phyllites (7 ± 3)	Slates 7 ± 4
IGNEOUS	Plutonic	Light	Granite 32 ± 3 Granodiorite (29 ± 3)	Diorite 25 ± 5		
		Dark	Gabbro 27 ± 3 Norite 20 ± 5	Dolerite (16 ± 5)		
	Hypabyssal		Porphyries (20 ± 5)		Diabase (15 ± 5)	Peridotite (25 ± 5)
	Volcanic	Lava		Rhyolite (25 ± 5) Andesite 25 ± 5	Dacite (25 ± 3) Basalt (25 ± 5)	Obsidian (19 ± 3)
		Pyroclastic	Agglomerate (19 ± 3)	Breccia (19 ± 5)	Tuff (13 ± 5)	

* Conglomerates and breccias may present a wide range of m_i values depending on the nature of the cementing material and the degree of cementation, so they may range from values similar to sandstone to values used for fine grained sediments.

* *These values are for intact rock specimens tested normal to bedding or foliation. The value of m_i will be significantly different if failure occurs along a weakness plane.

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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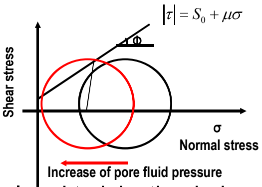
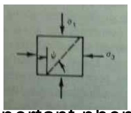
Deformation and Failure of Rock
Effects of pore fluids

- Required pore pressure to induce sliding of a given fracture with a specific orientation under a specific stress condition;

$$p_w = \frac{S_0}{\tan\phi} + \sigma_3 + (\sigma_1-\sigma_3)\left(\sin^2\theta - \frac{\sin\theta\cos\theta}{\tan\phi}\right)$$





- Extremely important phenomenon related to injection induced microearthquake

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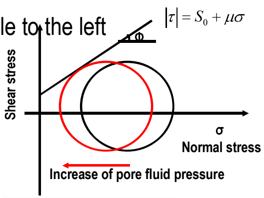
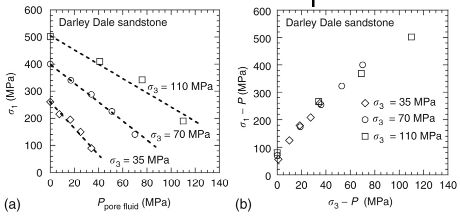
Deformation and Failure of Rock
Effects of pore fluids

- Mechanical effect
  - Pore pressure translate the Mohr's circle to the left



- Chemical interactions
  - between rock and the fluid



Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

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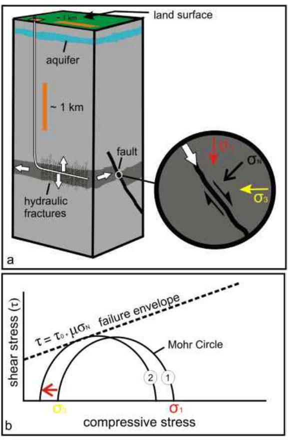
Deformation and Failure of Rock
Effects of pore fluids



(Richard Davies, 2013)

미소 진동 발생 원인

(a) 공극압 증가: 유효 응력 감소

(b) 지하 암반의 온도 변화

(c) 유체 생산/주입

(d) 화학 물질 첨가: 마찰계수 변화

⬇

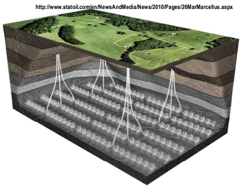
지하 암반 내 유효 응력의 감소/증가

스웨덴 Aitik 광산, 민기복, 2012

광물자원개발

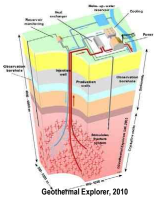
다양한 지중 에너지 관련
응용분야에서 미소진동
발생 가능

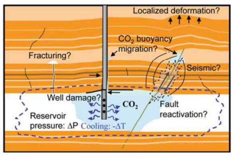
http://www.statoil.com/en/NewsAndMedia/News/2010/Pages/26MarMarcellus.aspx



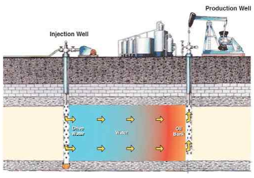
셰일가스 등 석유가스자원개발

EGS 지열발전



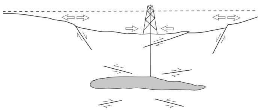


CO_2 지중저장



NRC, 2013

Enhanced Oil Recovery



석유 가스 생산 (Segall, 1989)

Deformation and Failure of Rock
Failure under true-triaxial conditions

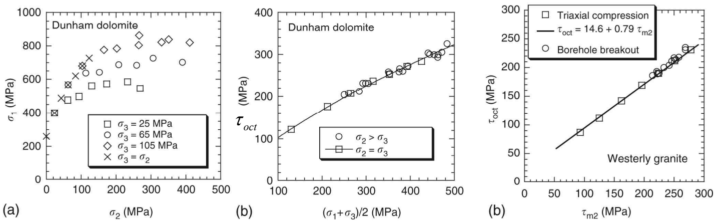
- It is (generally) known that intermediate principal stress also affect the failure.
  - Failure criterion under true triaxial stress conditions is of the form;

$$\tau_{oct} = f(\tau_{m2})$$

$$|\tau_{oct}| = \frac{1}{3}\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right\}^{1/2} = \frac{\sqrt{2}}{3}\left\{I_1^2+3I_2\right\}^{1/2} = \sqrt{\frac{2}{3}J_2}$$

$$\tau_{oct} = a + b\tau_{m2}$$

$$\tau_{m2} = \frac{(\sigma_1+\sigma_3)}{2}$$



Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

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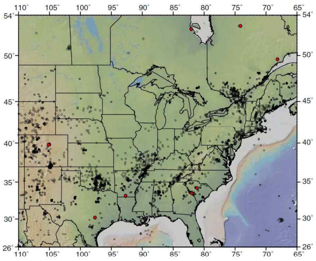
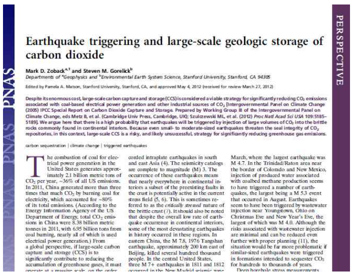
Deformation and Failure of Rock
Effects of pore fluids

- 이산화탄소 주입으로 인한 미소진동 및 이로 인한 누출로 인해 대규모 CCS는 성공할 가능성이 낮다* (스탠포드대학 Zoback교수의견)





PNAS

PERSPECTIVE

Earthquake triggering and large-scale geologic storage of carbon dioxide

Mark D. Zoback and Steven M. Gorelick

규모 3 이상의 지진현황. • 는 인공지진

Zoback MD & Gorelick SM, Earthquake triggering and large-scale geologic storage of carbon dioxide, Proc National Academy of Science of the USA (PNAS), June 2012

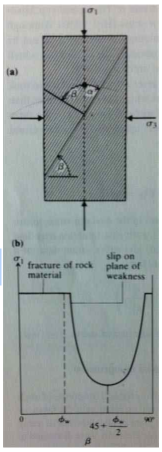
Deformation and Failure of Rock
The effects of anisotropy on strength

- Strength of anisotropic rock
  - can be estimated assuming a failure through predominant layers (which could be assumed to behave similar to fractures)

$$\sigma_1 = \sigma_3 + \frac{2(S_w + \mu_w\sigma_3)}{(1-\mu_w\cot\beta)\sin 2\beta}$$

  - Minimum strength when $\tan 2\beta_w = -\frac{1}{\mu_w}$ in other words, $\beta_w = 45 + \frac{\phi}{2}$

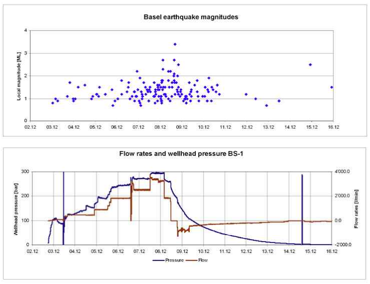
$$\sigma_1^{\min} = \sigma_3 + 2(S_w + \mu_w\sigma_3)\left[\sqrt{\mu_w^2+1} + \mu_w\right]$$



Deformation and Failure of Rock
Effects of pore fluids

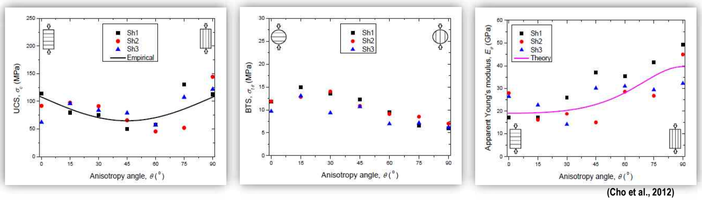
3 MW 발전 + 20 MW 난방
5 km 시추, 수리자극 @4.6 km
6일간 11,500 m^3 주입
유량 ~50 liter/s, 유압 ~30 MPa
최대 M_{Lmax}: 3.4
(1356년 규모 6.6 지진 기록)
→ 프로젝트 중단

주입압/유량 과 관측된 미소진동



Deformation and Failure of Rock
The effects of anisotropy on strength

- 각도에 따른 강도 및 탄성계수의 변화 (보령셰일)



(a) 일축압축강도 **(b) 인장강도** **(c) 탄성계수**

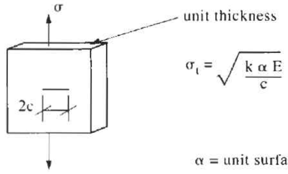
(Cho et al., 2012)

보령셰일의 역학적, 탄성파적, 열전도도적 이방성비				
압축강도 이방성비 (UCS/UCS′)	인장강도 이방성비 (BTS/BTS′)	탄성 계수 이방성비 (E/E′)	P파 속도 이방성비 ($V_{P(90°)}/V_{P(0°)}$′)	열전도도 이방성비 ($K_{(90°)}/K_{(0°)}$′)
2.6	2.2	2.1	1.5	2.1

(Cho, Kim, Min and Jeon, 2012)

Deformation and Failure of Rock
Griffith Failure Criterion

Material fractures when sufficient strain energy is released to enable cracks to propagate



$\sigma_t = \sqrt{\frac{k\,\alpha\,E}{c}}$ $k = \frac{2}{\pi}$ for plane stress
$= \frac{2}{\pi}(1 - \nu^2)$ for plane strain

α = unit surface energy of the crack

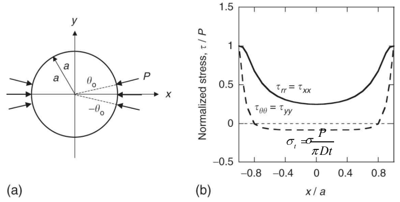
In compression:
$(\sigma_1 - \sigma_3)^2 = 8T_0(\sigma_1 + \sigma_3)$ when $\sigma_1 + 3\sigma_3 > 0$
$\sigma_3 = -T_0$ when $\sigma_1 + 3\sigma_3 < 0$
Note: compression positive, T_0 positive $(-T_0 = \sigma_t)$

Figure 6.19 The plane Griffith failure criterion.

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

Deformation and Failure of Rock
Other strength test – Brazilian strength

- The reason why Brazilian Test Works…



When $2\theta_0$=15°

  - Stress distribution along the x-axis $\rho = r / a$

$$\tau_{rr}(\theta = 0) = \frac{2P}{\pi}\left\{\frac{(1-\rho^2)\sin 2\theta_o}{(1-2\rho^2\cos 2\theta_o + \rho^4)} + \arctan\left[\frac{(1+\rho^2)}{(1-\rho^2)}\tan\theta_o\right]\right\}, \quad (8.161)$$

$$\tau_{\theta\theta}(\theta = 0) = -\frac{2P}{\pi}\left\{\frac{(1-\rho^2)\sin 2\theta_o}{(1-2\rho^2\cos 2\theta_o + \rho^4)} - \arctan\left[\frac{(1+\rho^2)}{(1-\rho^2)}\tan\theta_o\right]\right\}, \quad (8.162)$$

$$\tau_{rr}(\theta = \pi/2) = -\frac{2P}{\pi}\left\{\frac{(1-\rho^2)\sin 2\theta_o}{(1+2\rho^2\cos 2\theta_o + \rho^4)} - \arctan\left[\frac{(1-\rho^2)}{(1+\rho^2)}\tan\theta_o\right]\right\}, \quad (8.163)$$

$$\tau_{\theta\theta}(\theta = \pi/2) = \frac{2P}{\pi}\left\{\frac{(1-\rho^2)\sin 2\theta_o}{(1+2\rho^2\cos 2\theta_o + \rho^4)} + \arctan\left[\frac{(1-\rho^2)}{(1+\rho^2)}\tan\theta_o\right]\right\}. \quad (8.164)$$

$$\tau_{rr}(\theta = 0) = \frac{W(3+\rho^2)}{\pi a(1-\rho^2)}, \quad \tau_{\theta\theta}(\theta = 0) = -\frac{W}{\pi a},$$

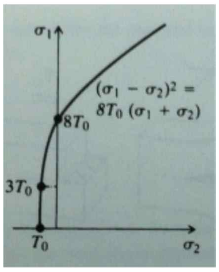
$$\sigma_t = \frac{W}{\pi a} = \frac{2P}{\pi Dt}$$

W: line load per unit length

Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Deformation and Failure of Rock
Griffith Failure Criterion

$$(\sigma_1 - \sigma_2)^2 = 8T_0(\sigma_1 + \sigma_2), \quad \sigma_1 + 3\sigma_2 > 0$$
$$\sigma_2 = -T_0, \quad \sigma_1 + 3\sigma_2 < 0$$



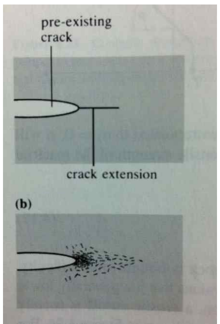


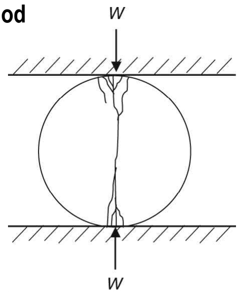
Figure 4.25 Extension of a preexisting crack, (a) Griffith's hypothesis, (b) the actual case for rock.

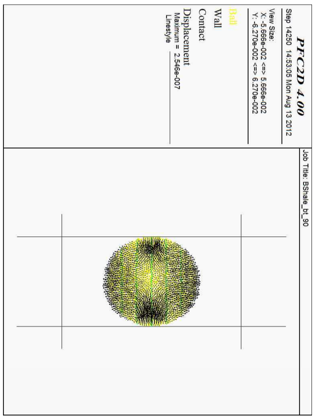
Hypothesis (model)

reality

Deformation and Failure of Rock
Other strength test – Brazilian strength

Example of Brazilian Tensile Strength Test by a numerical simulation using Discrete Element Method



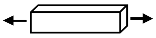


Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Deformation and Failure of Rock
Other strength test – Brazilian strength

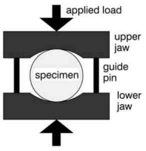
- Tensile strength : Maximum sustainable stress under tensile condition

$$\sigma_t = \frac{T_{\max}}{A}$$



Tensile loading

- <u>Tensile strength is 1/10 ~ 1/20 of UCS</u>
- Tensile strength is measured by Brazilian Test = 2P/(πdt)



Deformation and Failure of Rock
Other strength test – tensile strength

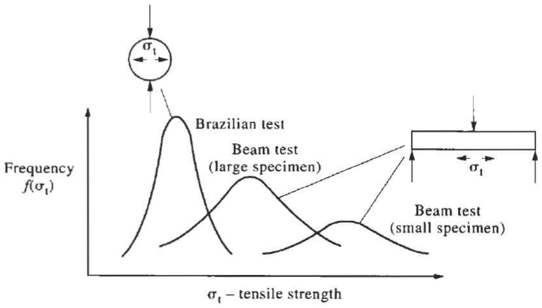


Figure 6.14 Tensile strength variation as a function of specimen volume and type of test.

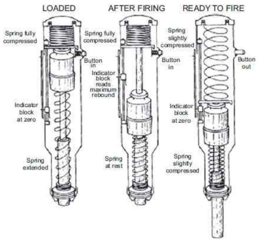
- How about direct tensile test?

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

Deformation and Failure of Rock
Other strength test – Schmidt Hammer Rebound Hardness Test

- Spring-driven cylindrical hammer rebounds off the rock surface
- The rebound distance is a measure of rock quality (e.g., strength)
- Often used on rock fracture surface
- Condition of rock surface has significant effect on the results

http://rammedearth.blogspot.kr/2006/06/hammer-time.html



Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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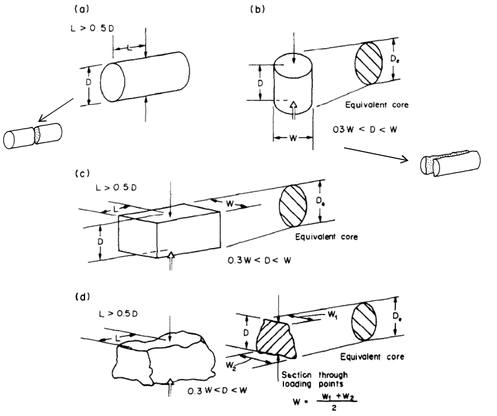
Deformation and Failure of Rock
Other strength test – Point Load Test



. Specimen shape requirements for (a) the diametral test, (b) the axial test, (c) the block test, and (d) the irregular lump test

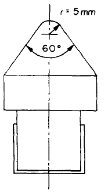


Fig. 2. Platen shape and tip radius.

Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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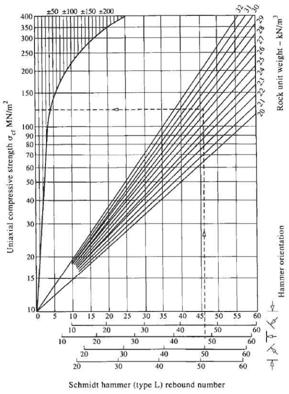
Deformation and Failure of Rock
Other strength test – Schmidt Hammer Rebound Hardness Test

- Use chart relating the rebound number and UCS



Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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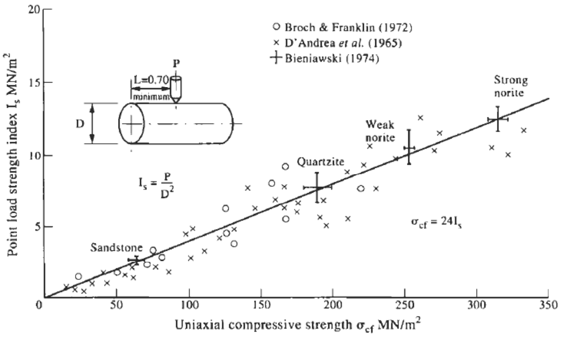
Deformation and Failure of Rock
Other strength test – Point Load Test



Hudson & Harrison, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon

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Deformation and Failure of Rock
Other strength test –Point Load Test

- Index test used mainly to predict the uniaxial compressive strength of rock
- Measures the 'Point Load Strength Index' $I_{s(50)}$
- Rock specimens in the form of either core, cut blocks, or irregular lumps are broken by application of concentrated load through a pair of spherically truncated, conical platens.
- Little or no specimen preparation is needed.

$$I_{s(50)} = \frac{P}{D^2}$$

$I_{s(50)}$: Point Load Strength Index (50 mm)

P: Peak load

D: Distance between the two platen contacts

$$UCS = (20 \sim 25) * I_{s(50)}$$

http://www.controls-group.com/eng/rock-mechanics-testing-equipment/rock-strength-index-apparatus.php

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Fundamentals of Geomechanics – Rock Anisotropy (Week 24 & 31 Oct)

Ki-Bok Min, PhD

Associate Professor
Department of Energy Resources Engineering
Seoul National University

Introduction
Objectives of the course

- Objective;
  - Understand the importance of rock anisotropy
  - Mechanical behavior;
    - Constitutive equation &Transformation of compliance matrix
    - Elastic material properties and their bounds
    - Anisotropic Strength
  - Hydraulic, thermal, and seismic properties

Rock Failure
Classical References

- Model for Anisotropic Rock Mass Properties
  - Gerrard, C. M. (1982). "ELASTIC MODELS OF ROCK MASSES HAVING 1, 2 AND 3 SETS OF JOINTS." International Journal of Rock Mechanics and Mining Sciences 19(1): 15-23.
  - Hornby, B. E., et al. (1994). "ANISOTROPIC EFFECTIVE-MEDIUM MODELING OF THE ELASTIC PROPERTIES OF SHALES." Geophysics 59(10): 1570-1583.
  - Amadei B. and Goodman RE. A 3-D constitutive relation for fractured rock masses, In Selvadurai APS (Ed), Proc. International Symposium on the mechanical behavior of structured media, Ottawa, Part B, 1981;249-268
  - Fossum AF. Effective elastic properties for a randomly jointed rock mass, Int J Rock Mech Min Sci & Geomech Abstr, 1985; 22(6):467-470
- Drilling and borehole stability
  - Brown, E. T., et al. (1981). "THE INFLUENCE OF ROCK ANISOTROPY ON HOLE DEVIATION IN ROTARY DRILLING - A REVIEW." International Journal of Rock Mechanics and Mining Sciences 18(5): 387-401.
  - Karfakis, M. G. and J. F. Evers (1987). "EFFECTS OF ROCK LAMINATION ANISOTROPY ON DRILLING PENETRATION AND DEVIATION." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 24(6): 371-374.
  - Cuisiat, E. D. E. and J. A. Hudson (1993). "THE INFLUENCE OF ROCK ANISOTROPY ON BOREHOLE BREAKOUTS - A MICROSTATISTICAL APPROACH." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 30(7): 1077-1083.
  - Ong, S. H. and J. C. Roegiers (1993). "Influence of anisotropies in borehole stability." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 30(7): 1069-1075.
- Anisotropic POROELASTICITY
  - Cheng, A. H. D. (1997). "Material coefficients of anisotropic poroelasticity." International Journal of Rock Mechanics and Mining Sciences 34(2): 199-205.
  - Kanj, M., et al. (2003). "Poromechanics of anisotropic hollow cylinders." Journal of Engineering Mechanics-Asce 129(11): 1277-1287.

Rock Anisotropy
Main References

- Jaeger JG, Cook NGW and Zimmerman RW, 2007, Fundamentals of Rock Mechanics, 4th edition, Blackwell Publishing
  - Chapter 5.10. Stress strain relatins for anisotropic materials (p.137-144)
- Brady BHG and Brown ET, 2004, Rock Mechanics for Underground Mining, 4th ed., Kluwer Academic Publishers
  - Chapter 4.6 Strength of anisotropic rock material in triaxial compression (p.117-119)
- Lekhnitskii, S. G., 1963. Theory of elasticity of an anisotropic body. San Francisco, Holden-Day.
  - Chapter 1. General equations of the theory of elasticity of an anisotropic body (p.1-73)
- Ting, T. C. T., 1996, Anisotropic Elasticity. New Yrok, Oxford University Press.
  - Chapter 1. Matrix Algebra (p.1-31)
  - Chapter 2. Linear anisotropic elastic materials (p.32-64)
- Min KB, Park B, Kim H, Cho JW, Jing L, Experimental and Numerical Anisotropic Rock Mechanics, Feng XT (ed), Rock Mechanics and Rock Engineering, Chapter 4 (p.109-138?)

Rock Failure
Classical References

- Testing Method
  - Chen, C. S., et al. (1998). "Determination of deformability and tensile strength of anisotropic rock using Brazilian tests." International Journal of Rock Mechanics and Mining Sciences 35(1): 43-61.
  - Liao, J. J., et al. (1997). "Determination of dynamic elastic constants of transversely isotropic rocks using a single cylindrical specimen." International Journal of Rock Mechanics and Mining Sciences 34(7): 1045-1054.
  - Talesnick, M. L. and E. A. Bloch-Friedman (1999). "Compatibility of different methodologies for the determination of elastic parameters of intact anisotropic rocks." International Journal of Rock Mechanics and Mining Sciences 36(7): 919-940.
  - Talesnick, M. L. and M. Ringel (1999). "Completing the hollow cylinder methodology for testing of transversely isotropic rocks: torsion testing." International Journal of Rock Mechanics and Mining Sciences 36(5): 627-639.
  - Li, C. (2001). "A method for graphically presenting the deformation modulus of jointed rock masses." Rock Mechanics and Rock Engineering 34(1): 67-75.
  - Nunes, A. (2002). "A new method for determination of transverse isotropic orientation and the associated elastic parameters for intact rock." International Journal of Rock Mechanics and Mining Sciences 39(2): 257-273.
  - Cho, J. W., et al. (2012). "Deformation and strength anisotropy of Asan gneiss, Boryeong shale, and Yeoncheon schist." International Journal of Rock Mechanics and Mining Sciences 50: 158-169.
  - Young, R. P. and D. A. Hutchins (1987). "MEASURING ANISOTROPY IN ROCKS USING LASER-GENERATED ULTRASOUND." Geophysical Journal of the Royal Astronomical Society 91(2): 501-516.
- Anisotropy in thermal and seismic properties
  - Kim, H., et al. (2012). "Anisotropy of elastic moduli, P-wave velocities, and thermal conductivities of Asan Gneiss, Boryeong Shale, and Yeoncheon Schist in Korea." Engineering Geology 147: 68-77.
  - Wang, Z. J. (2002). "Seismic anisotropy in sedimentary rocks, part 2: Laboratory data." Geophysics 67(5): 1423-1440.
  - Crampin, S. (1989). "SUGGESTIONS FOR A CONSISTENT TERMINOLOGY FOR SEISMIC ANISOTROPY." Geophysical Prospecting 37(7): 753-770.
  - Young, R. P. and D. A. Hutchins (1987). "MEASURING ANISOTROPY IN ROCKS USING LASER-GENERATED ULTRASOUND." Geophysical Journal of the Royal Astronomical Society 91(2): 501-516.

Rock Failure
Classical References

- Early works - Anisotropic Strength and Failure Criteria, etc. (1st ISRM Congress, and other works at similar periods)
  - Arguelles, H., et al. (1966). Analysis of Heterotropic And Anisotropic Properties of Rock Masses. 1st ISRM congress, International Society for Rock Mechanics.
  - Pinto, J. L. (1966). Stresses And Strains In an Anisotropic-orthotropic Body. 1st ISRM Congress, International Society for Rock Mechanics: 625-635.
  - Rodrigues, F. P. (1966). Anisotropy of Granites. Modulus of Elasticity And Ultimate Strength Ellipsoids, Joint Systems, Slope Attitudes, And Their Correlations. 1st ISRM congress, International Society for Rock Mechanics: 721-731.
  - Ruiz, M. D. (1966). Anisotropy of Rock Masses In Various Underground Projects In Brazil. 1st ISRM congress, International Society for Rock Mechanics: 263-267.
  - And other works.
- Application of anisotropic analysis
  - Exadaktylos, G. E. and K. N. Kaklis (2001). "Applications of an explicit solution for the transversely isotropic circular disc compressed diametrically." International Journal of Rock Mechanics and Mining Sciences 38(2): 227-243.
  - Wang, C. D. and J. J. Liao (1998). "Stress influence charts for transversely isotropic rocks." International Journal of Rock Mechanics and Mining Sciences 35(6): 771-785.
- In situ stress
  - Amadei, B. (1996). "Importance of anisotropy when estimating and measuring in situ stresses in rock." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 33(3): 293-325.

Home Assignment #4

- Review papers in anisotropy
  - You need to provide not only summary but also your own insight and criticism based on the selected papers.
  - If necessary, you will have to conduct your own analysis and refer to other papers.
  - One classical paper (try to refer to recent papers on the subject)
    - 09:00 24 Oct through eTL

Outline

- Introduction
- Anisotropic elasticity
  - Constitutive equation
  - Bounds of elastic constants/Transformation of compliance matrix
- Some insight into the anisotropic behaviour
- Experiments
  - Determination of anisotropic elastic constants
- Numerical approach
  - UDEC modeling of transversely isotropic rock

Introduction
Anisotropy vs. Isotropy

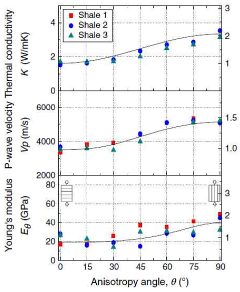
- Convenient Truth about Anisotropy
  - Normal (NOT abnormal)
  - General (NOT special)
  - Predictable (NOT unpredictable)

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}S_{11}&S_{12}&S_{13}&S_{14}&S_{15}&S_{16}\\ S_{21}&S_{22}&S_{23}&S_{24}&S_{25}&S_{26}\\ S_{31}&S_{32}&S_{33}&S_{34}&S_{35}&S_{36}\\ S_{41}&S_{42}&S_{43}&S_{44}&S_{45}&S_{46}\\ S_{51}&S_{52}&S_{53}&S_{54}&S_{55}&S_{56}\\ S_{61}&S_{62}&S_{63}&S_{64}&S_{65}&S_{66}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

Complete Anisotropy

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{1}{E}&-\frac{\nu}{E}&-\frac{\nu}{E}&0&0&0\\ -\frac{\nu}{E}&\frac{1}{E}&-\frac{\nu}{E}&0&0&0\\ -\frac{\nu}{E}&-\frac{\nu}{E}&\frac{1}{E}&0&0&0\\ 0&0&0&\frac{1}{G}&0&0\\ 0&0&0&0&\frac{1}{G}&0\\ 0&0&0&0&0&\frac{1}{G}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

Isotropy



Boryeong Shale, Korea (Cho et al., 2012)

$$S'_{ijkl}=\beta_{im}\beta_{jn}\beta_{kp}\beta_{lp}S_{mnpq}$$

Components of compliance tensor can be calculated from 4th order tensor transformation

Cho JW, Kim H, Jeon S, Min KB, Deformation and strength anisotropy of Asan gneiss Boryeong shale, and Yeoncheon schist, IJRMMS, 2012;50:158-169.

Anisotropic Rock Mechanics
Outline

- Introduction
- Anisotropic elasticity
  - Constitutive equation
  - Some insight into the anisotropic behavior
- Experimental Anisotropic Rock Mechanics
- Numerical Anisotropic Rock Mechanics
  - Blocky DEM (UDEC) modeling for fractured rock mass
  - Particulate DEM (PFC) modeling for transversely isotropic rock
- Concluding Remarks & Acknowledgement
- Prof Min's Research Group

Outline

- Introduction
- Anisotropic elasticity
  - Constitutive equation
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- Experimental Anisotropic Rock Mechanics
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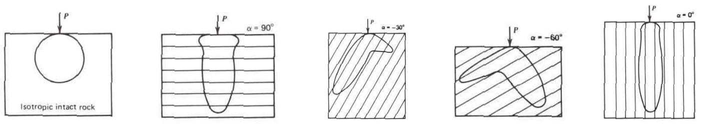
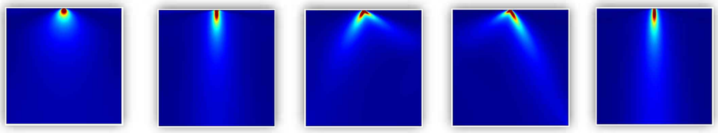
Introduction
Anisotropy vs. Isotropy

Foundation under line load on transversely isotropic rock



(Goodman, 1989)

FEM modeling



(Park and Min, 2015)

Radial stress distribution

Goodman R, Introduction to rock mechanics, 1989, 2nd ed., Wiley
Park, B. and Min, K.B., 2015, Discrete element modeling of transversely isotropic rock applied to foundation and borehole problems, 13rd ISRM Congress, Vancouver, Canada

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Anisotropic Elasticity
Constitutive Equation

$$\varepsilon_{ij}=S_{ijkl}\sigma_{kl}$$

Contracted form

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}S_{11}&S_{12}&S_{13}&S_{14}&S_{15}&S_{16}\\ S_{21}&S_{22}&S_{23}&S_{24}&S_{25}&S_{26}\\ S_{31}&S_{32}&S_{33}&S_{34}&S_{35}&S_{36}\\ S_{41}&S_{42}&S_{43}&S_{44}&S_{45}&S_{46}\\ S_{51}&S_{52}&S_{53}&S_{54}&S_{55}&S_{56}\\ S_{61}&S_{62}&S_{63}&S_{64}&S_{65}&S_{66}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

- Compliance matrix has 21 independent parameters
  (By the symmetry of stress tensor, strain tensor and consideration of strain energy)

Anisotropic Elasticity
Constitutive Equation

Coupling of normal in different directions

Coupling of normal in the same directions

Coupling of normal & Shear

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & \frac{\eta_{x,yz}}{G_{yz}} & \frac{\eta_{x,xz}}{G_{xz}} & \frac{\eta_{x,xy}}{G_{xy}}\\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & \frac{\eta_{y,yz}}{G_{yz}} & \frac{\eta_{y,xz}}{G_{xz}} & \frac{\eta_{y,xy}}{G_{xy}}\\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & \frac{\eta_{z,yz}}{G_{yz}} & \frac{\eta_{z,xz}}{G_{xz}} & \frac{\eta_{z,xy}}{G_{xy}}\\ \frac{\eta_{yz,x}}{E_x} & \frac{\eta_{yz,y}}{E_y} & \frac{\eta_{yz,z}}{E_z} & \frac{1}{G_{yz}} & \frac{\mu_{yz,xz}}{G_{xz}} & \frac{\mu_{yz,xy}}{G_{xy}}\\ \frac{\eta_{xz,x}}{E_x} & \frac{\eta_{xz,y}}{E_y} & \frac{\eta_{xz,z}}{E_z} & \frac{\mu_{xz,yz}}{G_{yz}} & \frac{1}{G_{xz}} & \frac{\mu_{xz,xy}}{G_{xy}}\\ \frac{\eta_{xy,x}}{E_x} & \frac{\eta_{xy,y}}{E_y} & \frac{\eta_{xy,z}}{E_z} & \frac{\mu_{xy,yz}}{G_{yz}} & \frac{\mu_{xy,xz}}{G_{xz}} & \frac{1}{G_{xy}}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

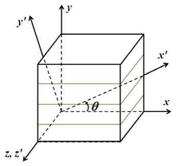
Coupling of shear in different directions

Coupling of shear in the same directions

Adapted from Hudson & Harrison (1997)

Constitutive Equation
Transversely Isotropic - One axis of elastic symmetry of rotation

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}=\begin{bmatrix}\frac{1}{E} & -\frac{\nu'}{E'} & -\frac{\nu}{E} & 0 & 0 & 0\\ -\frac{\nu'}{E'} & \frac{1}{E'} & -\frac{\nu'}{E'} & 0 & 0 & 0\\ -\frac{\nu}{E} & -\frac{\nu'}{E'} & \frac{1}{E} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G'} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G'}\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix}$$



$E_x = E_z = E$

$E_y = E'$

$\nu_{xz} = \nu_{zx} = \nu$

$\nu_{yx} = \nu_{yz} = \nu'$

$G_{xy} = G_{yz} = G'$

$$\nu_{xy} = \nu_{zy} = \nu'\frac{E}{E'}$$

- 5 independent constants

Constitutive Equation
Monoclinic – One plane of elastic symmetry

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & \frac{\eta_{x,xy}}{G_{xy}}\\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & \frac{\eta_{y,xy}}{G_{xy}}\\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & \frac{\eta_{z,xy}}{G_{xy}}\\ 0 & 0 & 0 & \frac{1}{G_{yz}} & \frac{\mu_{yz,xz}}{G_{xz}} & 0\\ 0 & 0 & 0 & \frac{\mu_{xz,yz}}{G_{yz}} & \frac{1}{G_{xz}} & 0\\ \frac{\eta_{xy,x}}{E_x} & \frac{\eta_{xy,y}}{E_y} & \frac{\eta_{xy,z}}{E_z} & 0 & 0 & \frac{1}{G_{xy}}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

- With a plane of symmetry normal to z-axis
- 13 independent constants

Constitutive Equation
Isotropic - Complete symmetry

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0\\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0\\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

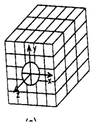
$E' = E$

$\nu' = \nu$

$G' = G$

Constitutive Equation
Orthotropic - Three orthogonal planes of elastic symmetry

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0\\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0\\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{xz}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

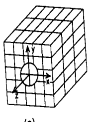


- Three orthogonal planes elastic symmetry
- 9 independent constants

Anisotropic Elasticity
Constitutive Equation

Orthotropic

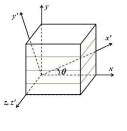
$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0\\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0\\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G_{xz}} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$



9 independent constants

Transversely Isotropic

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{1}{E} & -\frac{\nu'}{E'} & -\frac{\nu}{E} & 0 & 0 & 0\\ -\frac{\nu'}{E'} & \frac{1}{E'} & -\frac{\nu'}{E'} & 0 & 0 & 0\\ -\frac{\nu}{E} & -\frac{\nu'}{E'} & \frac{1}{E} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G'} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G'}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$



$E_x = E_z = E$

$E_y = E'$

$\nu_{xz} = \nu_{zx} = \nu$

$\nu_{yx} = \nu_{yz} = \nu'$

$G_{xy} = G_{yz} = G'$

5 independent constants

isotropic

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0\\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0\\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

$E' = E$

$\nu' = \nu$

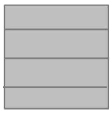
$G' = G$

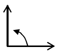
2 independent constants

Anisotropic Elasticity
Insight into its behavior – uniaxial compression

$$\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}=\begin{bmatrix}\frac{1}{E} & -\frac{\nu'}{E'} & -\frac{\nu}{E} & 0 & 0 & 0\\ -\frac{\nu'}{E'} & \frac{1}{E'} & -\frac{\nu'}{E'} & 0 & 0 & 0\\ -\frac{\nu}{E} & -\frac{\nu'}{E'} & \frac{1}{E} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G'} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{2(1+\nu)}{E} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G'}\end{bmatrix}\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix}$$

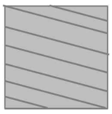
$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}S'_{11} & S'_{12} & S'_{13} & 0 & 0 & S'_{16}\\ S'_{21} & S'_{22} & S'_{23} & 0 & 0 & S'_{26}\\ S'_{31} & S'_{32} & S'_{33} & 0 & 0 & S'_{36}\\ 0 & 0 & 0 & S'_{44} & S'_{45} & 0\\ 0 & 0 & 0 & S'_{54} & S'_{55} & 0\\ S'_{61} & S'_{62} & S'_{63} & 0 & 0 & S'_{66}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$





$$\xrightarrow{\qquad\qquad\qquad}$$

$$S'_{ijkl}=\beta_{im}\beta_{jn}\beta_{kp}\beta_{lp}S_{mnpq}$$



$$S'_{11}=\frac{\cos^4\theta}{E}+\frac{\sin^4\theta}{E'}+\frac{\sin^2 2\theta}{4}\left(-\frac{2\nu'}{E'}+\frac{1}{G'}\right),$$
$$S'_{12}=\frac{\sin^2 2\theta}{4}\left(\frac{1}{E}+\frac{1}{E'}-\frac{1}{G'}\right)-\frac{\nu'}{E'}\left(\cos^4\theta+\sin^4\theta\right),$$
$$S'_{13}=-\cos^2\theta\frac{\nu}{E}-\sin^2\theta\frac{\nu'}{E'},$$
$$S'_{22}=\frac{\sin^4\theta}{E}+\frac{\cos^4\theta}{E'}+\frac{\sin^2 2\theta}{4}\left(-\frac{2\nu'}{E'}+\frac{1}{G'}\right),$$
$$S'_{32}=-\sin^2\theta\frac{\nu}{E}-\cos^2\theta\frac{\nu'}{E'}$$
$$\ldots$$

No shear strain under uniaxial stress

Shear strain even under uniaxial stress

Constitutive Equations
Bounds of elastic constants

- Example) compliance matrix of elastic material
  - Positive strains energy requires the Positive definiteness of matrix.
    → constraints of elastic parameters (elastic modulus and Poisson's ratio)

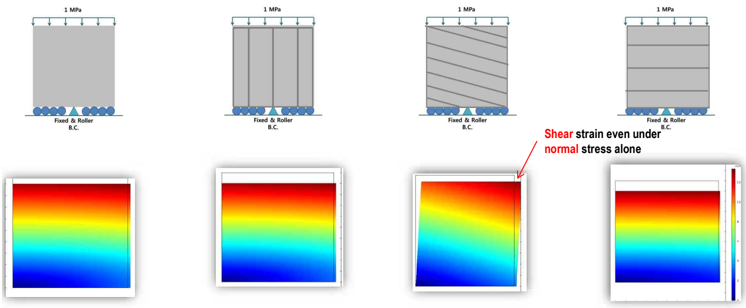
$$W=\frac{1}{2}\sigma^T S\sigma$$

W: strain energy intensity
S: compliance matrix

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix}=\begin{pmatrix}\frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0\\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0\\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}\longrightarrow\quad\begin{matrix}E>0\\ -1<\nu<\frac{1}{2}\end{matrix}$$

Anisotropic Elasticity
Insight into its behavior – uniaxial compression

FEM modeling of Uniaxial loading on transversely isotropic rock



Shear strain even under normal stress alone

Total displacement distribution

Constitutive Equations
Bounds of elastic constants

- Definiteness
  - $Q(x)=\mathbf{x}^T\mathbf{A}\mathbf{x}$ and symmetric matrix A are called
  - Positive definite if Q(x)>0 for all x≠0
  - Negative definite if Q(x)<0 for all x≠0
  - Indefinite if Q(x)>0 and Q(x)<0 for all x≠0
- Positive Definiteness
  - All the principal minors are positive

$$a_{11}>0\qquad\begin{vmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}>0\qquad\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}>0\qquad\det\mathbf{A}>0$$

$$\begin{bmatrix}a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \cdot & \cdot & \cdots & \cdot\\ a_{n1} & a_{n2} & \cdots & a_{nn}\end{bmatrix}$$

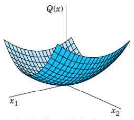
Constitutive Equations
Bounds of elastic constants

$$W=\frac{1}{2}\sigma_{ij}\varepsilon_{ij}\qquad W=\frac{1}{2}\sigma^T S\sigma$$

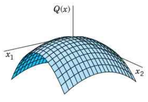
- the 6×6 matrices of elastic constants must be positive definite (Ting, 1996)
- A necessary and sufficient condition for the quadratic form to be positive definite is that all principal minors of matrix (that is all minor determinants in the matrix having diagonal elements coincident with the principal diagonal of the matrix) are positive (Amadei et al 1987).

Constitutive Equations
Bounds of elastic constants

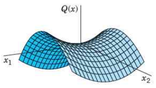
- Positive definite if Q(x)>0 for all x≠0
- Negative definite if Q(x)<0 for all x≠0
- Indefinite if Q(x)>0 and Q(x)<0 for all x≠0



(a) Positive definite form



(b) Negative definite form



(c) Indefinite form

Fig. 162. Quadratic forms in two variables (Problem 24)

Constitutive Equations
Bounds of elastic constants - Orthogonal

$$E_x, E_y, E_z, G_x, G_y, G_z > 0$$

$$\begin{vmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} \end{vmatrix} = \frac{1}{E_xE_y} - \frac{\nu_{yx}\nu_{xy}}{E_xE_y} \rangle 0 \xrightarrow{\qquad} 1-\nu_{yx}\nu_{xy} \rangle 0$$

$$\begin{vmatrix} \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} \\ -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} \end{vmatrix} = \frac{1}{E_yE_z} - \frac{\nu_{zy}\nu_{yz}}{E_yE_z} \rangle 0 \xrightarrow{\qquad} 1-\nu_{zy}\nu_{yz} \rangle 0$$

$$\begin{vmatrix} \frac{1}{E_x} & -\frac{\nu_{zx}}{E_z} \\ -\frac{\nu_{xz}}{E_x} & \frac{1}{E_z} \end{vmatrix} = \frac{1}{E_xE_z} - \frac{\nu_{zx}\nu_{xz}}{E_xE_z} \rangle 0 \xrightarrow{\qquad} 1-\nu_{zx}\nu_{xz} \rangle 0$$

$$\begin{vmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} \end{vmatrix} = \frac{1}{E_xE_yE_z} - \frac{\nu_{zy}\nu_{yz}}{E_xE_yE_z} - \frac{\nu_{yx}\nu_{xy}}{E_xE_yE_z} - \frac{\nu_{xz}\nu_{zx}}{E_xE_yE_z} - \frac{\nu_{yx}\nu_{xz}\nu_{zy}}{E_xE_yE_z} - \frac{\nu_{zx}\nu_{xy}\nu_{yz}}{E_xE_yE_z} \rangle 0 \xrightarrow{\qquad} 1-\nu_{zy}\nu_{yz}-\nu_{yx}\nu_{xy}-\nu_{xz}\nu_{zx}-\nu_{yx}\nu_{xz}\nu_{zy}-\nu_{zx}\nu_{xy}\nu_{yz} \rangle 0$$

Constitutive Equations
Bounds of elastic constants - Isotropic

$$E > 0, G > 0$$

$$-1 < \nu < 1 \xrightarrow{\qquad} \begin{matrix} -1 < \nu < \frac{1}{2} \\ E > 0 \end{matrix}$$

$$-\sqrt{\frac{(1-\nu)}{2}} < \nu < \sqrt{\frac{(1-\nu)}{2}}$$

Constitutive Equations
Bounds of elastic constants - Orthogonal

$$E_x, E_y, E_z, G_x, G_y, G_z > 0$$

$$-\sqrt{\frac{E_x}{E_y}} \langle \nu_{xy} \langle \sqrt{\frac{E_x}{E_y}}$$

$$-\sqrt{\frac{E_y}{E_z}} \langle \nu_{yz} \langle \sqrt{\frac{E_y}{E_z}}$$

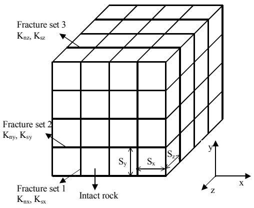
$$-\sqrt{\frac{E_x}{E_z}} \langle \nu_{xz} \langle \sqrt{\frac{E_x}{E_z}}$$

$$1-\frac{E_z}{E_y}\nu_{yz}{}^2 - \frac{E_y}{E_x}\nu_{xy}{}^2 - \frac{E_z}{E_x}\nu_{xz}{}^2 - 2\frac{E_z}{E_x}\nu_{xy}\nu_{xz}\nu_{yz} \rangle 0$$

Application to fractured rock masses
- Amadei (1981)

Rock masses with three perpendicular fracture sets can modelled as orthogonally isotropic rock

$$\begin{pmatrix} \frac{1}{E_x}+\frac{1}{K_{nx}S_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{yx}}{E_y} & \frac{1}{E_y}+\frac{1}{K_{ny}S_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{zx}}{E_z} & -\frac{\nu_{zy}}{E_z} & \frac{1}{E_z}+\frac{1}{K_{nz}S_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{yz}}+\frac{1}{K_{sy}S_y}+\frac{1}{K_{sz}S_z} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{xz}}+\frac{1}{K_{sx}S_x}+\frac{1}{K_{sz}S_z} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}}+\frac{1}{K_{sx}S_x}+\frac{1}{K_{sy}S_y} \end{pmatrix}$$



Constitutive Equations
Bounds of elastic constants - Transversely Isotropic

$$E, E', G' > 0$$

$$-1 < \nu < 1$$

$$-\sqrt{\frac{E'}{E}\frac{(1-\nu)}{2}} < \nu' < \sqrt{\frac{E'}{E}\frac{(1-\nu)}{2}}$$

Compliance matrix
Transformation

- 0th order tensor (scalar) : no need to transform, independent of coordinate
- 1th order tensor (vector) : $x_i' = \beta_{ij}x_j$
- 2nd order tensor : $\sigma'_{ij} = \beta_{im}\beta_{jn}\sigma_{mn}$
  - i.e. stress, strain, permeability
- 4th order tensor : $S'_{ijkl} = \beta_{im}\beta_{jn}\beta_{kp}\beta_{lp}S_{mnpq}$
  - Compliance tensor

$$\beta_{ij} = \begin{pmatrix} \cos(x',x) & \cos(x',y) & \cos(x',z) \\ \cos(y',x) & \cos(y',y) & \cos(y',z) \\ \cos(z',x) & \cos(z',y) & \cos(z',z) \end{pmatrix} \qquad \beta_{ij} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

General transformation **Rotation**

Compliance matrix
Transformation

$$S'_{ijkl} = \beta_{im}\beta_{jn}\beta_{kp}\beta_{lp}S_{mnpq}$$

$$\downarrow$$

$$S'_{ij} = S_{mn}q_{mi}q_{nj}$$

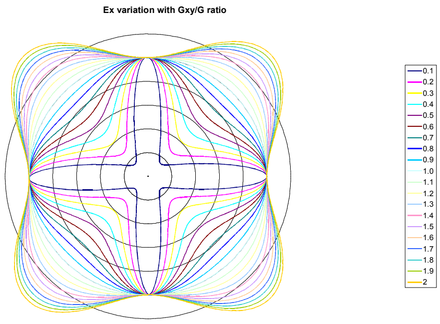
$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16}\\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26}\\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36}\\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46}\\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56}\\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$

	X	Y	z
X'	α_1	β_1	γ_1
Y'	α_2	β_2	γ_2
Z'	α_3	β_3	γ_3

	1	2	3	4	5	6
1	α_1^2	α_2^2	α_3^2	$2\alpha_2\alpha_3$	$2\alpha_3\alpha_1$	$2\alpha_1\alpha_2$
2	β_1^2	β_2^2	β_3^2	$2\beta_2\beta_3$	$2\beta_3\beta_1$	$2\beta_1\beta_2$
3	γ_1^2	γ_2^2	γ_3^2	$2\gamma_2\gamma_3$	$2\gamma_3\gamma_1$	$2\gamma_1\gamma_2$
4	$\beta_1\gamma_1$	$\beta_2\gamma_2$	$\beta_3\gamma_3$	$\beta_2\gamma_3+\beta_3\gamma_2$	$\beta_1\gamma_3+\beta_3\gamma_1$	$\beta_1\gamma_2+\beta_2\gamma_1$
5	$\gamma_1\alpha_1$	$\gamma_2\alpha_2$	$\gamma_3\alpha_3$	$\gamma_2\alpha_3+\gamma_3\alpha_2$	$\gamma_1\alpha_3+\gamma_3\alpha_1$	$\gamma_1\alpha_2+\gamma_2\alpha_1$
6	$\alpha_1\beta_1$	$\alpha_2\beta_2$	$\alpha_3\beta_3$	$\alpha_2\beta_3+\alpha_3\beta_2$	$\alpha_1\beta_3+\alpha_3\beta_1$	$\alpha_1\beta_2+\alpha_2\beta_1$

Transformation of compliance tensor
Elastic modulus

Ex=Ey
Nu = 0.25
Gxy/G varies



- Orthotropic rock

Compliance matrix
Transformation

$$S'_{11} = \frac{\cos^4\theta}{E} + \frac{\sin^4\theta}{E'} + \frac{\sin^2 2\theta}{4}\left(-\frac{2\nu'}{E'} + \frac{1}{G'}\right),$$

$$S'_{12} = \frac{\sin^2 2\theta}{4}\left(\frac{1}{E} + \frac{1}{E'} - \frac{1}{G'}\right) - \frac{\nu'}{E'}\left(\cos^4\theta + \sin^4\theta\right),$$

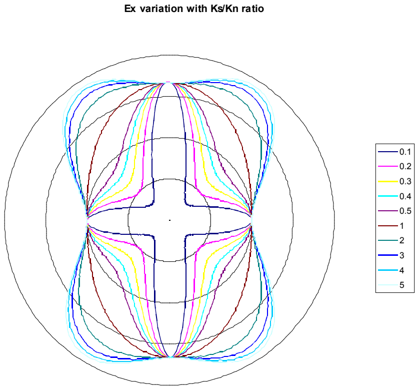
$$S'_{13} = -\cos^2\theta\frac{\nu}{E} - \sin^2\theta\frac{\nu'}{E'},$$

$$S'_{22} = \frac{\sin^4\theta}{E} + \frac{\cos^4\theta}{E'} + \frac{\sin^2 2\theta}{4}\left(-\frac{2\nu'}{E'} + \frac{1}{G'}\right),$$

$$S'_{32} = -\sin^2\theta\frac{\nu}{E} - \cos^2\theta\frac{\nu'}{E'}$$

…

Transformation of compliance tensor
Elastic modulus

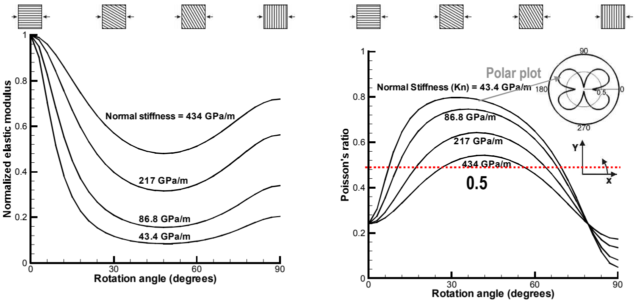


K_n for set 1

= 2* K_n for set 2

Fractured Rock Masses

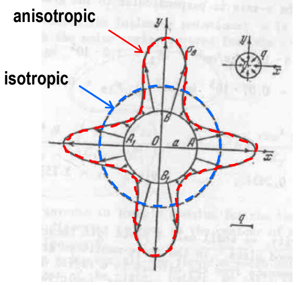
Transformation of compliance tensor
Elastic modulus and Poisson's ratio (Min & Jing, 2004)



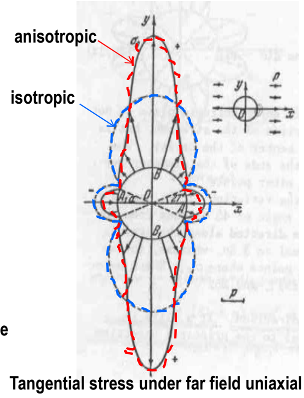
Transversely Isotropic rock

Anisotropic Elasticity
Insight into its behavior – circular opening

Some formulae considering anisotropy (internal pressure, uniaxial stress, Lekhnitskii, 1963)



Tangential stress under internal pressure

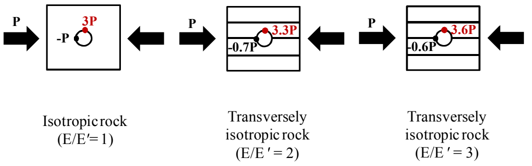


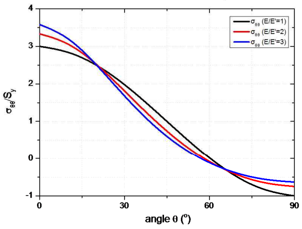
Tangential stress under far field uniaxial tension

Lekhnitskii, Theory of elasticity of an anisotropic elastic body, 1963

Anisotropic Elasticity
Insight into its behavior – circular opening

Stress concentration around a circular opening (Kim, 2012)

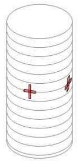




In anisotropic case, tangential stress varies depending upon the anisotropy ratio of elastic constants.

Kim H, 2012, MS thesis SNU

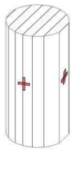
Experimental Anisotropic Rock Mechanics
Laboratory observation – sample



(a) θ =0°

$$\frac{\varepsilon_x}{\sigma_y} = -\frac{\nu'}{E'}$$

$$\frac{\varepsilon_y}{\sigma_y} = \frac{1}{E'}$$

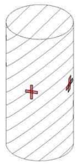


(b) θ =90°

$$\frac{\varepsilon_y}{\sigma_y} = \frac{1}{E}$$

$$\frac{\varepsilon_x}{\sigma_x} = -\frac{\nu'}{E'}$$

$$\frac{\varepsilon_z}{\sigma_y} = -\frac{\nu}{E}$$



(c) 0°<θ<90°

$$\varepsilon'_x/\sigma'_y = \frac{\sin^2 2\theta}{4}\left(\frac{1}{E}+\frac{1}{E'}-\frac{1}{G'}\right)-\frac{\nu'}{E'}\left(\cos^4\theta+\sin^4\theta\right)$$

$$\varepsilon'_y/\sigma'_y = \frac{\sin^4\theta}{E}+\frac{\cos^4\theta}{E'}+\frac{\sin^2 2\theta}{4}\left(-\frac{2\nu'}{E'}+\frac{1}{G'}\right)$$

$$\varepsilon'_z/\sigma'_y = -\sin^2\theta\frac{\nu}{E}-\cos^2\theta\frac{\nu'}{E'}$$

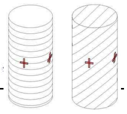
Experimental Anisotropic Rock Mechanics
Laboratory observation - sample

Asan Gneiss

Boryeong Shale

Yeoncheon Schist

Experimental Anisotropic Rock Mechanics
Laboratory observation – parameter determination



- **Combinations of 2 specimens - Least Square Method**

$$\begin{bmatrix} \varepsilon_{a(x1)}/\sigma_a \\ \varepsilon_{a(y1)}/\sigma_a \\ \varepsilon_{a(z1)}/\sigma_a \\ \varepsilon_{a(y2)}/\sigma_a \\ \varepsilon_{c(x1\theta)}/\sigma_c \\ \varepsilon_{c(y1\theta)}/\sigma_c \\ \varepsilon_{c(z1\theta)}/\sigma_c \\ \varepsilon_{c(y2\theta)}/\sigma_c \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \sin^2 2\theta/4 & \sin^2 2\theta/4 & 0 & -\cos^4\theta/4-\sin^4\theta/4 & \sin^2 2\theta/4 \\ \sin^4\theta & \cos^4\theta & 0 & -2\sin^2 2\theta/4 & \sin^2 2\theta/4 \\ 0 & 0 & -\sin^2\theta & -\cos^2\theta & 0 \\ \sin^4\theta & \cos^4\theta & 0 & -2\sin^2 2\theta/4 & \sin^2 2\theta/4 \end{bmatrix} \times \begin{bmatrix} \frac{1}{E} \\ \frac{1}{E'} \\ \frac{\nu}{E} \\ \frac{\nu'}{E'} \\ \frac{1}{G'} \end{bmatrix}$$

→ **5 independent equations**

► **5 constants (*E, E’, v, v’* and *G’*) can be determined.**

- Minimum number of specimens is **two**, where one of the specimens is inclined to the isotropic plane

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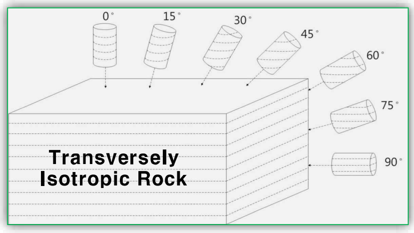
Experimental Anisotropic Rock Mechanics
Laboratory observation

- Laboratory experiments for the validation of tensor transformation (Cho et al., 2012)

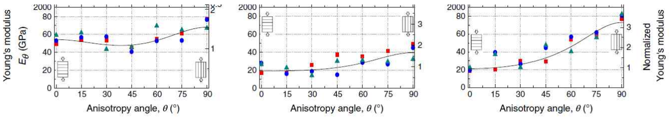
Directional Coring System



Cho JW, Kim H, Jeon S, Min KB, Deformation and strength anisotropy of Asan gneiss Boryeong shale, and Yeoncheon schist, IJRMMS, 2012;50:158-169.

Experimental Anisotropic Rock Mechanics
Laboratory observation

Variation of Elastic Modulus (Cho et al., 2012) $S'_{ijkl} = \beta_{im}\beta_{jn}\beta_{kp}\beta_{lp}S_{mnpq}$



(a) Asan gneiss (b) Boryeong shale (c) Yeoncheon schist

$$MPE = \frac{1}{N}\sum_{i=0}^{90}\left|\frac{y_{\theta(ex)}-y_{\theta(th)}}{y_{\theta(th)}}\right|$$

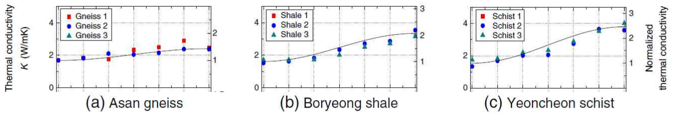
The relative difference between the theoretical prediction and measurement (mean prediction error) ~20%

Kim H, Cho JW, Song I, Min KB, Anisotropy of elastic moduli, P-wave velocities, and thermal conductivities of Asan Gneiss, Boryeong Shale, and Yeoncheon Schist in Korea, Eng Geol, 2012;147-148:66-77

Experimental Anisotropic Rock Mechanics
Laboratory observation

Variation of Thermal Conductivity (Kim et al., 2012)

$$k^{T'}_{ij} = \beta_{im}\beta_{jn}k^{T}_{mn}$$



(a) Asan gneiss (b) Boryeong shale (c) Yeoncheon schist

$$MPE = \frac{1}{N}\sum_{i=0}^{90}\left|\frac{y_{\theta(ex)} - y_{\theta(th)}}{y_{\theta(th)}}\right|$$

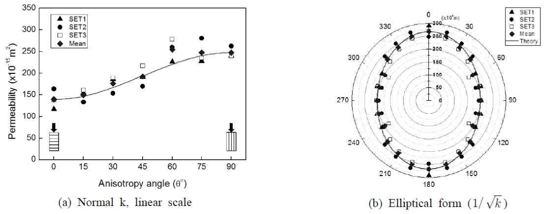
The relative difference between the theoretical prediction and measurement (mean prediction error) ~20%

Kim H, Cho JW, Song I, Min KB, Anisotropy of elastic moduli, P-wave velocities, and thermal conductivities of Asan Gneiss, Boryeong Shale, and Yeoncheon Schist in Korea, Eng Geol, 2012;147-148:66-77

Experimental Anisotropic Rock Mechanics
Laboratory observation

Variation of Permeability (Yang et al., 2013)

$$k'_{ij} = \beta_{im}\beta_{jn}k_{mn}$$



(a) Normal k, linear scale (b) Elliptical form $(1/\sqrt{k})$

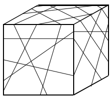
Berea Sandstone ~ 20% porosity

HY Yang, H Kim, K Kim, KY Kim, KB Min, A Study of Locally Changing Pore Characteristics and Hydraulic Anisotropy due to Bedding of Porous Sandstone, J Korean Soc Rock Mech, 2013 23(3):280-240

DEM for Anisotropic Rock Mechanics
Methodology for numerical experiments

$$\varepsilon_{ij} = S_{ijkl}\sigma_{kl}$$

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{pmatrix} = \begin{pmatrix}S_{11}&S_{12}&S_{13}&S_{14}&S_{15}&S_{16}\\ S_{21}&S_{22}&S_{23}&S_{24}&S_{25}&S_{26}\\ S_{31}&S_{32}&S_{33}&S_{34}&S_{35}&S_{36}\\ S_{41}&S_{42}&S_{43}&S_{44}&S_{45}&S_{46}\\ S_{51}&S_{52}&S_{53}&S_{54}&S_{55}&S_{56}\\ S_{61}&S_{62}&S_{63}&S_{64}&S_{65}&S_{66}\end{pmatrix}\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{pmatrix}$$



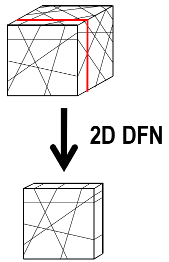
6 Boundary conditions

$$\begin{pmatrix}\varepsilon^{(1)}_{xx}&\varepsilon^{(2)}_{xx}&\varepsilon^{(3)}_{xx}&\varepsilon^{(4)}_{xx}&\varepsilon^{(5)}_{xx}&\varepsilon^{(6)}_{xx}\\ \varepsilon^{(1)}_{yy}&\varepsilon^{(2)}_{yy}&\varepsilon^{(3)}_{yy}&\varepsilon^{(4)}_{yy}&\varepsilon^{(5)}_{yy}&\varepsilon^{(6)}_{yy}\\ \varepsilon^{(1)}_{zz}&\varepsilon^{(2)}_{zz}&\varepsilon^{(3)}_{zz}&\varepsilon^{(4)}_{zz}&\varepsilon^{(5)}_{zz}&\varepsilon^{(6)}_{zz}\\ \gamma^{(1)}_{yz}&\gamma^{(2)}_{yz}&\gamma^{(3)}_{yz}&\gamma^{(4)}_{yz}&\gamma^{(5)}_{yz}&\gamma^{(6)}_{yz}\\ \gamma^{(1)}_{zx}&\gamma^{(2)}_{zx}&\gamma^{(3)}_{zx}&\gamma^{(4)}_{zx}&\gamma^{(5)}_{zx}&\gamma^{(6)}_{zx}\\ \gamma^{(1)}_{xy}&\gamma^{(2)}_{xy}&\gamma^{(3)}_{xy}&\gamma^{(4)}_{xy}&\gamma^{(5)}_{xy}&\gamma^{(6)}_{xy}\end{pmatrix} = \begin{pmatrix}S_{11}&S_{12}&S_{13}&S_{14}&S_{15}&S_{16}\\ S_{21}&S_{22}&S_{23}&S_{24}&S_{25}&S_{26}\\ S_{31}&S_{32}&S_{33}&S_{34}&S_{35}&S_{36}\\ S_{41}&S_{42}&S_{43}&S_{44}&S_{45}&S_{46}\\ S_{51}&S_{52}&S_{53}&S_{54}&S_{55}&S_{56}\\ S_{61}&S_{62}&S_{63}&S_{64}&S_{65}&S_{66}\end{pmatrix}\begin{pmatrix}\sigma^{(1)}_{xx}&\sigma^{(2)}_{xx}&\sigma^{(3)}_{xx}&\sigma^{(4)}_{xx}&\sigma^{(5)}_{xx}&\sigma^{(6)}_{xx}\\ \sigma^{(1)}_{yy}&\sigma^{(2)}_{yy}&\sigma^{(3)}_{yy}&\sigma^{(4)}_{yy}&\sigma^{(5)}_{yy}&\sigma^{(6)}_{yy}\\ \sigma^{(1)}_{zz}&\sigma^{(2)}_{zz}&\sigma^{(3)}_{zz}&\sigma^{(4)}_{zz}&\sigma^{(5)}_{zz}&\sigma^{(6)}_{zz}\\ \sigma^{(1)}_{yz}&\sigma^{(2)}_{yz}&\sigma^{(3)}_{yz}&\sigma^{(4)}_{yz}&\sigma^{(5)}_{yz}&\sigma^{(6)}_{yz}\\ \sigma^{(1)}_{zx}&\sigma^{(2)}_{zx}&\sigma^{(3)}_{zx}&\sigma^{(4)}_{zx}&\sigma^{(5)}_{zx}&\sigma^{(6)}_{zx}\\ \sigma^{(1)}_{xy}&\sigma^{(2)}_{xy}&\sigma^{(3)}_{xy}&\sigma^{(4)}_{xy}&\sigma^{(5)}_{xy}&\sigma^{(6)}_{xy}\end{pmatrix}$$

- 6 linearly independent B.C. – 3D
- 3 linearly independent B.C. – 2D

DEM for Anisotropic Rock Mechanics
Methodology for numerical experiments

$$\begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\end{pmatrix} = \begin{pmatrix}S_{11}&S_{12}&S_{13}&S_{16}\\ S_{21}&S_{22}&S_{23}&S_{26}\\ S_{31}&S_{32}&S_{33}&S_{36}\\ S_{61}&S_{62}&S_{63}&S_{66}\end{pmatrix}\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\end{pmatrix} \qquad \begin{pmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\end{pmatrix} = \begin{pmatrix}S_{11}&S_{12}&S_{16}\\ S_{21}&S_{22}&S_{26}\\ S_{61}&S_{62}&S_{66}\end{pmatrix}\begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{pmatrix} + \begin{pmatrix}S_{13}\\ S_{23}\\ S_{63}\end{pmatrix}(\sigma_{zz})$$



3 Boundary conditions

$$\begin{pmatrix}\varepsilon^{(1)}_{xx}&\varepsilon^{(2)}_{xx}&\varepsilon^{(3)}_{xx}\\ \varepsilon^{(1)}_{yy}&\varepsilon^{(2)}_{yy}&\varepsilon^{(3)}_{yy}\\ \gamma^{(1)}_{xy}&\gamma^{(2)}_{xy}&\gamma^{(3)}_{xy}\end{pmatrix} - \begin{pmatrix}S_{13}\\ S_{23}\\ S_{63}\end{pmatrix}\begin{pmatrix}\sigma^{(1)}_{zz}&\sigma^{(2)}_{zz}&\sigma^{(2)}_{zz}\end{pmatrix} = \begin{pmatrix}S_{11}&S_{12}&S_{16}\\ S_{21}&S_{22}&S_{26}\\ S_{61}&S_{62}&S_{66}\end{pmatrix}\begin{pmatrix}\sigma^{(1)}_{xx}&\sigma^{(2)}_{xx}&\sigma^{(3)}_{xx}\\ \sigma^{(1)}_{yy}&\sigma^{(2)}_{yy}&\sigma^{(3)}_{yy}\\ \sigma^{(1)}_{xy}&\sigma^{(2)}_{xy}&\sigma^{(3)}_{xy}\end{pmatrix}$$

$$[\varepsilon]-[S_z][\sigma_z]=[S][\sigma]$$

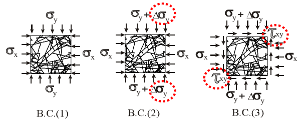
$$[\varepsilon][\sigma]^{-1}-[S_z][\sigma_z][\sigma]^{-1}=[S]$$

σ_{ij} : BC

ε_{ij} : measured

S_{ij} : pre-determined

S_{ij} : ?



- In 2D plane strain condition, 6 elastic constants are determined.

Blocky DEM for Anisotropic Rock Mechanics
Results

- Complete 2D compliance tensor was obtained in the fractured rock mass (Min & Jing, 2013)

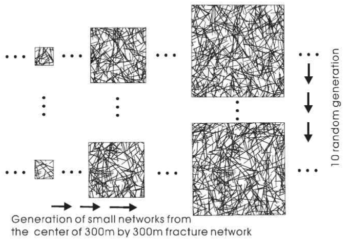
$$\begin{pmatrix}S_{11}&S_{12}&S_{13}&S_{16}\\ S_{21}&S_{22}&S_{23}&S_{26}\\ S_{31}&S_{32}&S_{33}&S_{36}\\ S_{61}&S_{62}&S_{63}&S_{66}\end{pmatrix} = \begin{pmatrix}2.7709&-0.2933&-0.2800&-0.0445\\ -0.2933&2.3934&-0.2814&-0.0405\\ -0.2800&-0.2814&1.1820&-0.0051\\ -0.0591&-0.0653&-0.0051&5.5066\end{pmatrix} \times E-11\ (1/\mathrm{Pa})$$

- This method has been applied to a series of problems.
  - Sellafield DFN data (essentially a generic nature)
  - Forsmark (Swedish repository site)

Min KB, Jing L, Numerical determination of the equivalent elastic compliance tensor for fractured rock masses using the distinct element method, IJRMMS 2003;40(6):795-816.

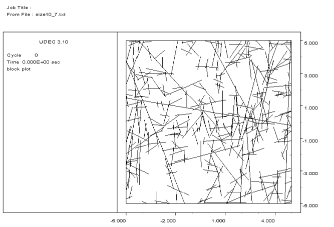
Blocky DEM for Anisotropic Rock Mechanics
Results – case 1) scale dependency (Min & Jing, 2003)

- Complete 2D compliance tensor was obtained in the fractured rock mass (Min & Jing, 2013)

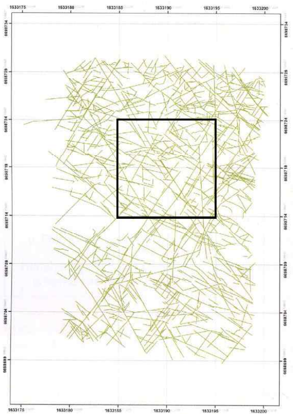


$$\begin{pmatrix}S_{11}&S_{12}&S_{13}&S_{16}\\ S_{21}&S_{22}&S_{23}&S_{26}\\ S_{31}&S_{32}&S_{33}&S_{36}\\ S_{61}&S_{62}&S_{63}&S_{66}\end{pmatrix} = \begin{pmatrix}2.7709&-0.2933&-0.2800&-0.0445\\ -0.2933&2.3934&-0.2814&-0.0405\\ -0.2800&-0.2814&1.1820&-0.0051\\ -0.0591&-0.0653&-0.0051&5.5066\end{pmatrix} \times E-11\ (1/\mathrm{Pa})$$

Blocky DEM for Anisotropic Rock Mechanics
Results – case 3) stress dependent E (Forsmark)



Generated DFN model (10 m X 10 m scale)

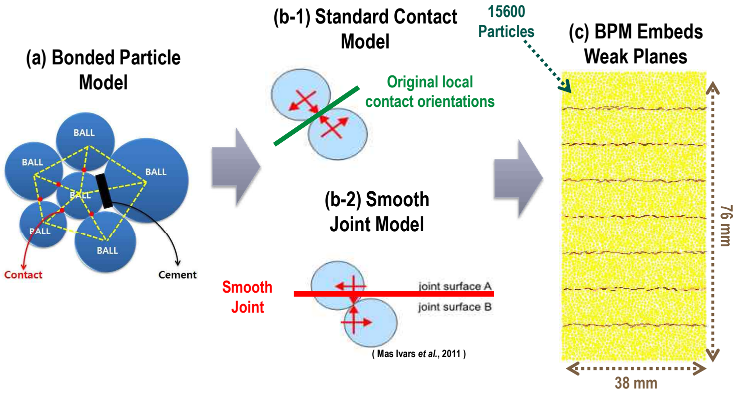


Trace map at Forsmark drillsite 2 (version 1.1)

Square is 10 m X 10 m scale.

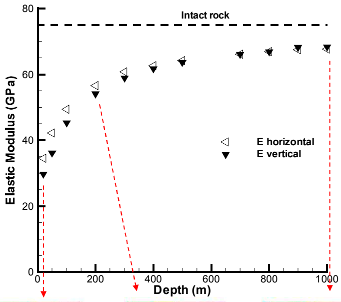
DEM for Anisotropic Rock Mechanics
Methodology

PFC^{2D} with Smooth Joint Model



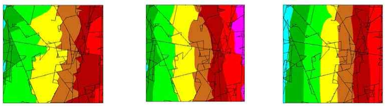
(Mas Ivars *et al.*, 2011)

Blocky DEM for Anisotropic Rock Mechanics
Results – case 3) stress dependent E (Forsmark)



Elastic moduli ↑ with stress ↑
- highly stress-dependent

Stress induced anisotropy
- E_h 20% higher than E_v in shallow depth



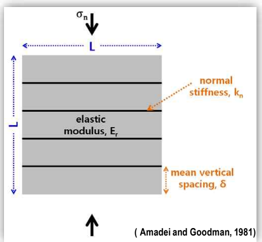
Effect of stress is more evident in low stress condition.

Displacement distribution

Min KB, Stephansson O, Jing L, Effect of stress on mechanical and hydraulic rock mass properties – application of DFN-DEM approach on the data from Site Investigation at Forsmark, Sweden, EUROCK 2005, Brno, Czech Republic, 2005, pp.389-395

DEM for Anisotropic Rock Mechanics
Verification – Elastic modulus & Strength

Equivalent Continuum Model



(Amadei and Goodman, 1981)

$$\frac{1}{E_e}=\left(\frac{1}{E_r}+\frac{1}{k_n\cdot\delta}\right),\qquad \frac{1}{G_e}=\left(\frac{1}{G_r}+\frac{1}{k_s\cdot\delta}\right)$$

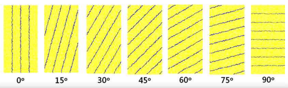
E_e , G_e : equivalent elastic & shear modulus,
E_r, G_r : intact rock elastic & shear modulus,
k_n , k_s : normal & shear stiffness
on weak planes,
δ : mean vertical spacing

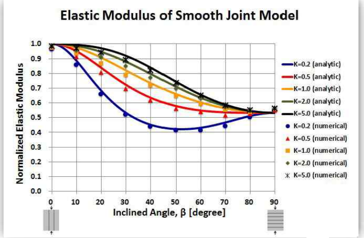
K is stiffness ratio

Tensor Transformation

$$\frac{1}{E_\beta}=\frac{1}{E_r}+\cos^2\beta\cdot\left(\frac{\cos^2\beta}{k_n\cdot\delta}+\frac{sin^2\beta}{k_s\cdot\delta}\right)$$

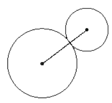
(E_β varies with respect to inclination, *β*)





Particulate DEM – Bonded Particulate system
Bonding logic

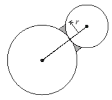
contact bond



2 microproperties:
normal & shear strengths

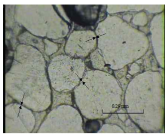
models adhesion over vanishingly small area of contact point
(does not resist moment)
breaks if normal or shear force exceeds bond strength

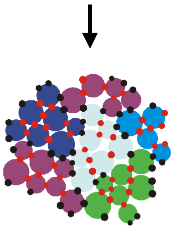
parallel bond



5 microproperties:
bond radius
normal & shear stiffnesses
normal & shear strengths

models additional material deposited after balls are in contact
(does resist moment)
breaks if normal or shear stress exceeds bond strength





from www.HCItasca.com

DEM for Anisotropic Rock Mechanics
Sliding on a plane of weakness

- The stress difference that is required to cause a slip with a given β and σ_2

$$\sigma_1-\sigma_2=\frac{2\left(S_0+\mu\sigma_2\right)}{\left(1-\mu\cot\beta\right)\sin\beta}\qquad \sigma_1=\frac{2S_0\cos\phi}{(1-k)\sin(2\beta-\phi)-(1+k)\sin\phi}$$

$$\sigma_{1,\min}=\sigma_2+2\left(S_0+\mu\sigma_2\right)\left(\sqrt{\mu^2+1}+\mu\right)$$

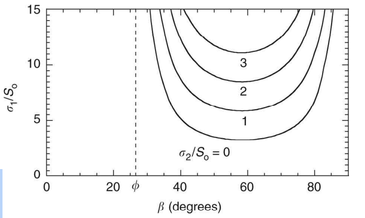
- Solution exists only for

$$\phi<\beta$$

- Range of β (for a given stress state)

$$\beta_1<\beta<\beta_2$$

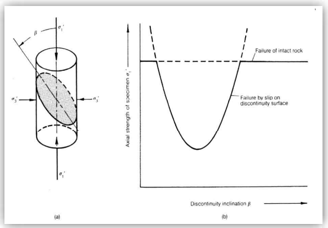
$$2\beta_1=\phi+\sin^{-1}\left[\left\{\left(\sigma_m+S_0\cot\phi\right)/\tau_m\right\}\sin\phi\right]$$
$$2\beta_2=\pi+\phi-\sin^{-1}\left[\left\{\left(\sigma_m+S_0\cot\phi\right)/\tau_m\right\}\sin\phi\right]$$



Variation of σ_1 needed to cause sliding on a fracture for μ=0.5

DEM for Anisotropic Rock Mechanics
Verification – Elastic modulus & Strength

Strength Anisotropy with respect to Weak Planes



(Jaeger and Cook, 2007)

✓ Strength of Fractured Rock

$$\sigma_1 = \sigma_3 + \frac{2\cdot(C+\sigma_3\cdot\tan\varphi)}{(1-\tan\varphi\cdot\tan\beta)\cdot\sin 2\beta}$$

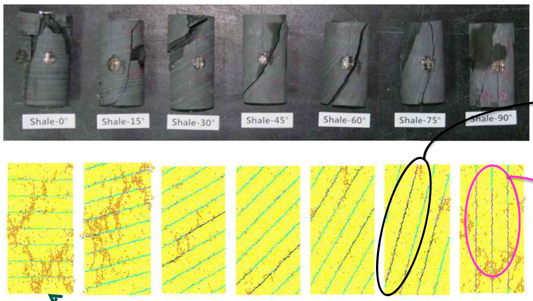
σ_1 : axial strength, σ_3 : confining stress,
C : cohesion, φ : friction angle,
β : inclination

✓ Fixed Cohesion (C) : 10 MPa

✓ Various Friction Angle (φ) : 0° – 30°

✓ Mechanical Behaviors
-> Smoothly Change

DEM for Anisotropic Rock Mechanics
Investigation into the failure mechanism - compression



Shear Failure along the Weak Planes

Tensile Failure along the Weak Planes

15600 Particles

Red: Tensile Cracks on Contact/Parallel Bond
Blue: Shear Cracks on Contact/Parallel Bond
Magenta: Tensile Cracks on Smooth Joint
Black: Shear Cracks on Smooth Joint

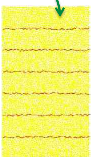
DEM for Anisotropic Rock Mechanics
Validation against laboratory measurements

(a) Step 1 (Intact Rock Part)

Microproperties of Bonded Particle Model

Grain	Cement
Elastic modulus = 38 Gpa	Elastic modulus = 38 Gpa
Stiffness ratio = 3.5	Stiffness Ratio = 3.5
Friction coefficient = 0.839	Tensile stress = 75 Mpa

BPM



SJM

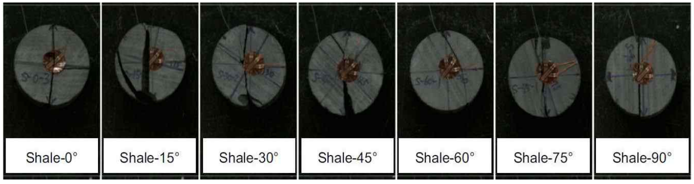
(b) Step 2 (Weak Plane Part)

Microproperties of Smooth Joint Model

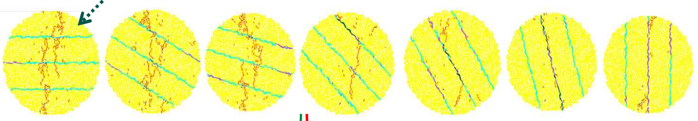
Normal stiffness = 33700 Gpa/m	Dilation angle = 0°
Shear stiffness = 960 Gpa/m	Tensile strength = 3 Mpa
Friction coefficient = 0.364 (20°)	Cohesion = 15

DEM for Anisotropic Rock Mechanics
Investigation into the failure mechanism - tensile

(Cho *et al.*, 2012)



6158 Particles



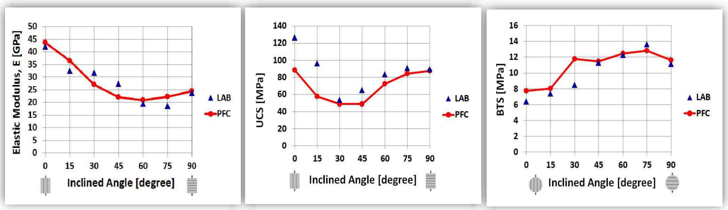
Dominated by Intact Rock (BPM)

Dominated by Layers (Smooth Joint Model)

DEM for Anisotropic Rock Mechanics
Validation against laboratory measurements

Boryeong Shale vs. Numerical Model

(a) Elastic Modulus (b) UCS (c) BTS



(Cho *et al.*, 2012)

▲ : Lab Experiments / ●: Numerical Results

Capture the overall trend of anisotropic mechanical behaviors

Park, B. and Min, K.B., 2015, Bonded-Particle Discrete Element Modeling of Mechanical Behavior of Transversely Isotropic Rock, IJRMMS, under review

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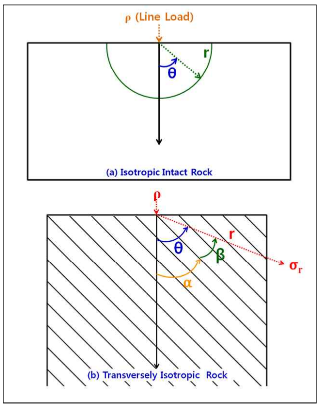
DEM for Anisotropic Rock Mechanics
Upscaling – transversely isotropic rock under line load

Boussinesq-Flamant's Problem



✓ Stress distribution in (an)isotropic, homogeneous, and infinite medium when line load is applied

✓ Radial stress as a function of distance 'r' and the angle 'θ'

$$\sigma_r = \frac{2P\cos\theta}{\pi r} \quad \text{(Isotropic)}$$

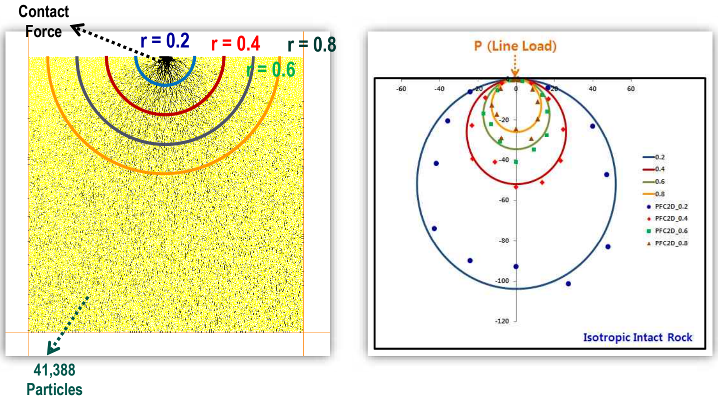
$$\sigma_r = \frac{h}{\pi r}\left(\frac{X\cos\beta + Yg\sin\beta}{(\cos^2\beta - g\sin^2\beta)^2 + h^2\sin^2\beta\cos^2\beta}\right)$$

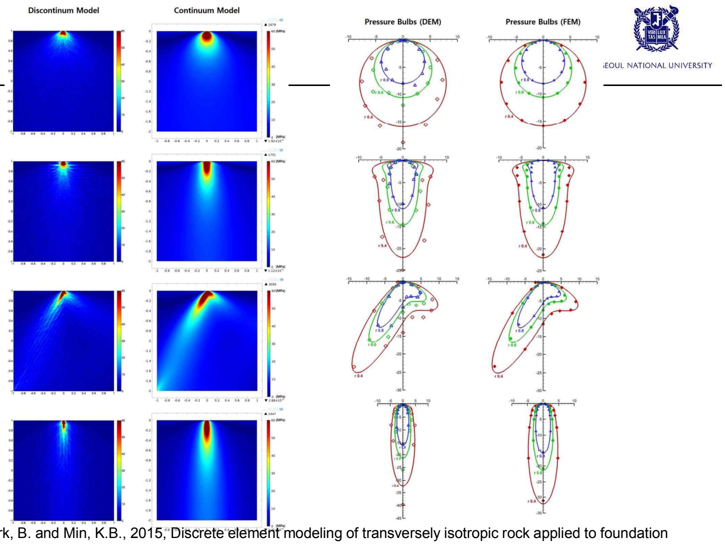
Transversely isotropic (Bray, 1977)

P : line load, r : radial distance,
θ : radial stress direction from line load
h, g : material constants
X : x-direction load / Y : y-direction load

DEM for Anisotropic Rock Mechanics
Upscaling – transversely isotropic rock under line load

Analytic Solution vs. PFC^{2D}





Park, B. and Min, K.B., 2015, Discrete element modeling of transversely isotropic rock applied to foundation and borehole problems, 13rd ISRM Congress, Vancouver, Canada

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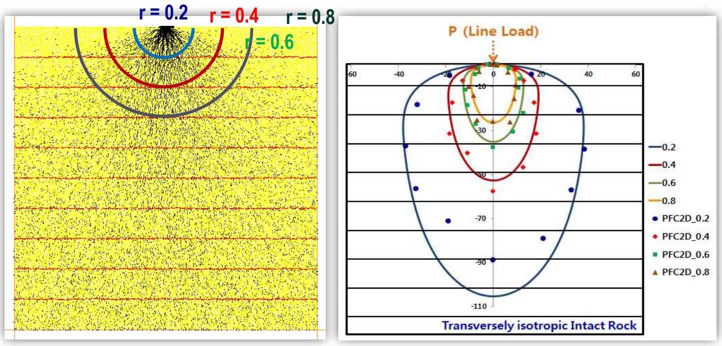
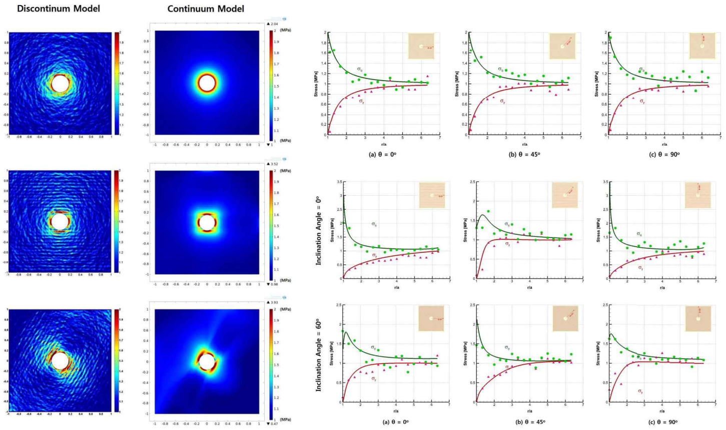
DEM for Anisotropic Rock Mechanics
Upscaling – transversely isotropic rock under line load

Analytic Solution vs. PFC^{2D}



DEM for Anisotropic Rock Mechanics
Upscaling – circular hole under biaxial load



Park, B. and Min, K.B., 2015, Discrete element modeling of transversely isotropic rock applied to foundation and borehole problems, 13rd ISRM Congress, Vancouver, Canada

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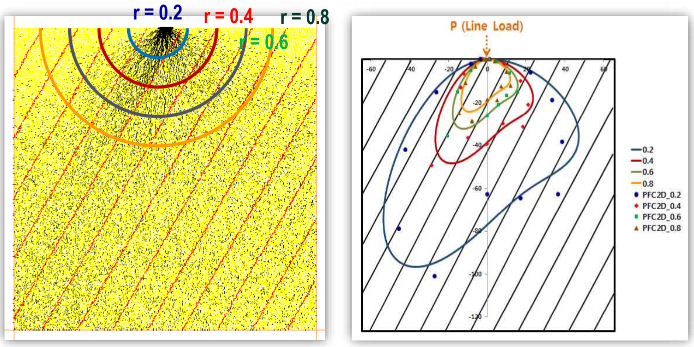
DEM for Anisotropic Rock Mechanics
Upscaling – transversely isotropic rock under line load

Analytic Solution vs. PFC^{2D}



✓ Stress distribution matches with analytical solution

Fundamentals of Geomechanics – Rock Mass Properties (Week 7 Nov)

Ki-Bok Min, PhD

Associate Professor
Department of Energy Resources Engineering
Seoul National University

Introduction
Objectives of the course

- Objective;
  - Understand the importance of rock mass properties determination
  - Mechanical behavior;
    - Empirical approach
    - Analytical approach
    - Numerical approach
    - In situ testing
  - Hydraulic properties
    - Upscaling approach
    - In situ testing

Rock Mass Properties
Main References

- Brady BHG and Brown ET, 2004, Rock Mechanics for Underground Mining, 4th ed., Kluwer Academic Publishers
  - Chapter 3 Rock Mass Structure and Characterization (p.46-84)
  - Chapter 4.9 Behaviour of discontinuous rock masses (p.133-139)
- Goodman RE, 1989, Introduction to Rock Mechancis. 2nd ed., John Wiley & Sons
  - Chapter 6. Deformability of Rocks (p.179-220)
- Hudson JA & Harrison JP, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon
  - Chapter 8. Rock Masses (p.141-148)
  - Chapter 9. Permeability (p.149-162)

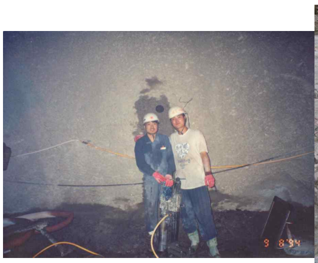
Rock Mass Properties
Classical References

- Rock Mass properties – empirical approach
  - Bieniawski, Z. T. (1978). "DETERMINING ROCK MASS DEFORMABILITY - EXPERIENCE FROM CASE HISTORIES." International Journal of Rock Mechanics and Mining Sciences 15(5): 237-247.
- Rock Mass properties – in situ testing
  - Unal, E. (1997). "Determination of in situ deformation modulus: New approaches for plate-loading tests." International Journal of Rock Mechanics and Mining Sciences 34(6): 897-915.
- Rock Mass properties – analytical approach
  - Oda M. Fabric tensor for discontinuous geological materials. Soils Fdns, 1982;22(4):96-108
  - Oda, M., et al. (1993). "ELASTIC STRESS AND STRAIN IN JOINTED ROCK MASSES BY MEANS OF CRACK TENSOR ANALYSIS." Rock Mechanics and Rock Engineering 26(2): 89-112.
- Rock Mass properties – numerical approach
  - Min, K. B. and L. R. Jing (2003). "Numerical determination of the equivalent elastic compliance tensor for fractured rock masses using the distinct element method." International Journal of Rock Mechanics and Mining Sciences 40(6): 795-816.
  - Min, K. B. and L. Jing (2004). "Stress-dependent mechanical properties and bounds of Poisson's ratio for fractured rock masses investigated by a DFN-DEM technique." International Journal of Rock Mechanics and Mining Sciences 41(3): 431-432.

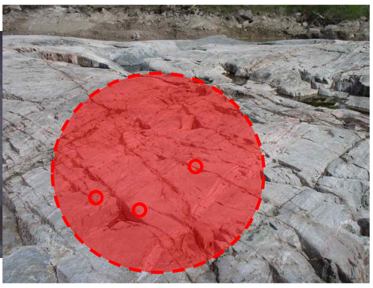
Rock Mass Properties
Classical References

- Stress dependent rock properties
  - Santarelli, F. J., et al. (1986). "ANALYSIS OF BOREHOLE STRESSES USING PRESSURE-DEPENDENT, LINEAR ELASTICITY." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 23(6): 445-449.
  - Brown, E. T., et al. (1989). "INFLUENCE OF STRESS-DEPENDENT ELASTIC-MODULI ON STRESSES AND STRAINS AROUND AXISYMMETRIC BOREHOLES." Rock Mechanics and Rock Engineering 22(3): 189-203.
- In situ characterization of Permeability/transmissivity
  - Theis, C. V., 1935, The lowering of peizometer surface and the rate of discharge of a well using groundwater storage, Trans. Am. Geophys. Union, 16: 519-524
  - Neuman, S. P. and P. A. Witherspoon (1969). "THEORY OF FLOW IN A CONFINED 2 AQUIFER SYSTEM." Water Resources Research 5(4): 803-+.
  - A paper dealing with fractured rock hydraulic properties
  - Fokker, P. A., et al. (2012). "Estimating reservoir heterogeneities from pulse testing." Journal of Petroleum Science and Engineering 86-87: 15-26.
  - Long J.C.S., Remer J.S., Wilson C.R., Witherspoon P.A., 1982, Porous media equivalents for Networks of discontinuous fractures, Water Resources Research, vol.18, No.3, pp.645-658

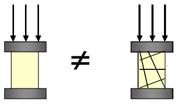
Introduction
Rock vs Rock Mass



DaeGu Subway 1-10, Korea



Forsmark, Sweden



- Determining rock mass properties is a critical issue for mechanical, hydraulic and thermal behavior – especially fractured (jointed) rock mass

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Introduction
Rock vs Rock Mass – Representative Elementary Volume (REV)

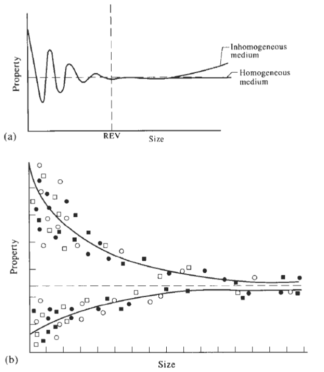

Figure 4.12 Variability in measured values with respect to sample volume, illustrating the REV. (a) General concept. (b) Example data scatter.

Hudson JA, Harrison JP, Engineering Rock Mechanics, 1997, Pergamon

Fractures
Geometry

- Fractures (Discontinuities)

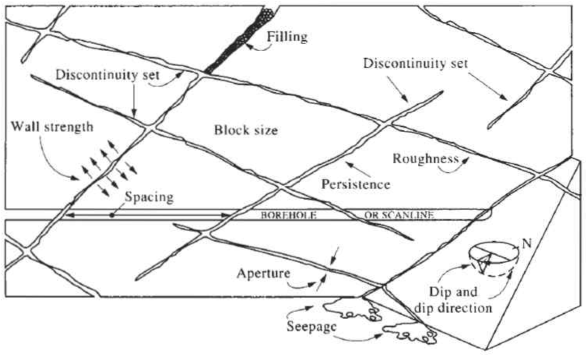

Figure 7.3 Schematic of the primary geometrical properties of discontinuities in rock (from Hudson, 1989).

Hudson JA, Harrison JP, Engineering Rock Mechanics, 1997, Pergamon

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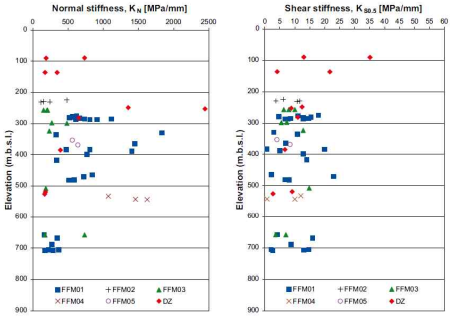
Fractures
Mechanical properties – stiffness

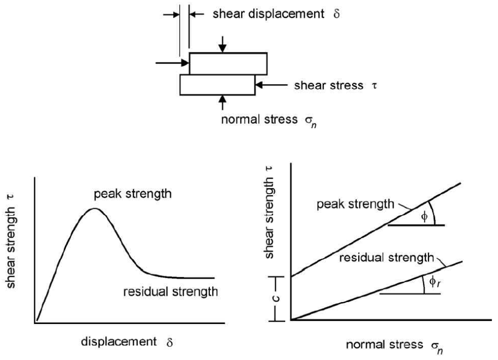


Introduction
Approaches

- Analytical approach
  - Oda (1982, 1993), Amadei (1981)
- Experimental approach (in situ test)
  - Plate loading, flat jack, dilatometer, goodman jack,
  - Injection (production) test
- Empirical approach
  - Rock mass classification (RMR, Q, GSI)
- Numerical approach
  - Numerical experiments (Min's work, 2003; Pouya, 2001; Stietel et al., 1996)

Fractures
Mechanical properties – shear test

- Shear testing of fractures



Hoek E, 2007, Practical Rock Engineering, ,https://www.rocscience.com/learning/hoek-s-corner/books

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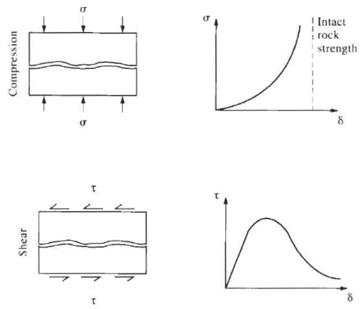
Fractures
Mechanical properties - Stiffness

- Normal stiffness
  - Unit: Stress/length (MPa/m)
  - Linear model $\sigma_n = K_n \delta_n$
  - Non-linear model

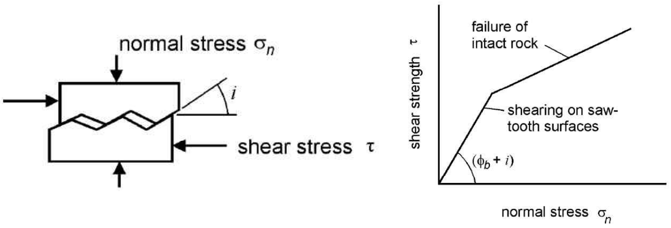
$$\delta_n = \frac{\sigma_n}{c + d\sigma_n}$$

- Shear stiffness
  - Unit: Stress/length (MPa/m)
  - Linear model $\sigma_s = K_s \delta_s$
  - Non-linear model: e.g., Barton's equation (strength + full path)



Fractures
Mechanical properties – shear strength

- Patton's law (1966) for fracture friction angle



$$\tau = \sigma_n \tan(\phi_b + i)$$

where ϕ_b is the basic friction angle of the surface and i is the angle of the saw-tooth face.

Hoek E, 2007, Practical Rock Engineering, ,https://www.rocscience.com/learning/hoek-s-corner/books

Fractures
Mechanical properties – shear strength

- Barton's equation (1977)

$$\tau = \sigma_n \tan\left[JRC \log_{10}(JCS/\sigma_n) + \phi_r\right]$$

τ: Shear Strength of a fracture

JRC: Joint Roughness Coefficient

JCS : Joint Wall Compressive Strength

ϕ_r : residual friction angle

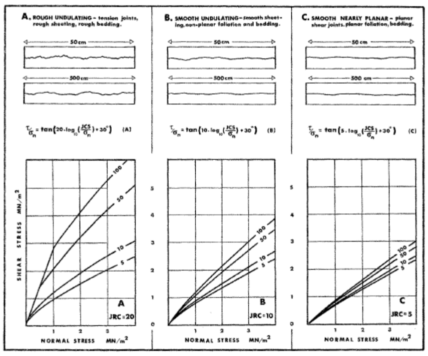


Fig. 1. Empirical law of friction in graphical form. Each curve is numbered with the appropriate *JCS* value (units of MN/m²). The roughness profiles are intended as an approximate guide to the appropriate *JRC* values 20, 10 and 5. Completely smooth planar joints have *JRC* = 0

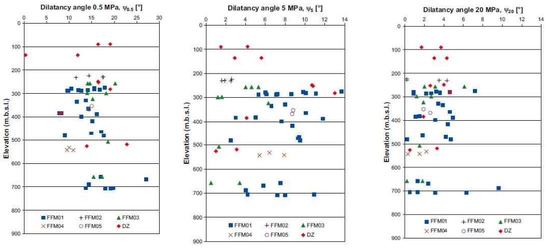
Barton & Choubey, 1977, The shear strength of rock joints in theory and practice, *Rock Mech Rock Eng*, 10(1-2):1-54

Fractures
Mechanical properties – dilation

– Aperture change due to shear dilation (SKB, 2007)

- 3.2° dilation angle under 20 MPa
- Dilation angles from direct shear tests for the FFM01 fracture domain

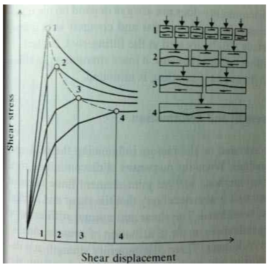
Normal load (MPa)	Mean (°)	Std. dev. (°)	Minimum (°)	Maximum (°)	Uncertainty of mean (%)
0.5	14.6	4.1	7.8	27.1	±10.2
5	7.7	2.7	2.5	13.7	±12.8
20	3.2	2.1	0.2	9.6	±23.9



SKB R-7-31

Fractures
Mechanical properties – shear strength

- Size dependent behavior of a fracture



$$JRC_n = JRC_o \left(\frac{L_n}{L_o}\right)^{-0.02JRC_o} \qquad (8)$$

where JRC_o, and L_o (length) refer to 100 mm laboratory scale samples and JRC_n, and L_n refer to in situ block sizes.

Mechanical properties
Elastic modulus (deformation modulus)

- Elastic (deformation) modulus of fractured rock mass

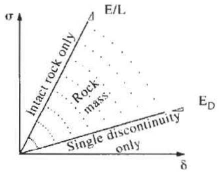
$$\frac{1}{E_m} = \left(\frac{1}{E_i} + \frac{1}{k_n \cdot S}\right)$$

$$\frac{1}{E_m} = \left(\frac{1}{E_i} + \frac{\lambda}{E_D}\right)$$

E_m : Elastic modulus of rock mass

k_n : normal stiffness of a fracture

S : spacing of fractures

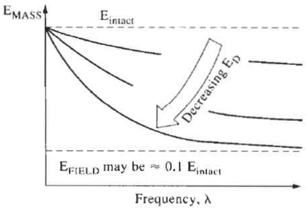


$$\frac{1}{G_m} = \left(\frac{1}{G_i} + \frac{1}{k_s \cdot S}\right)$$

G_m : Shear modulus of rock mass

k_s : shear stiffness of a fracture

S : spacing of fractures



Hudson JA, Harrison JP, Engineering Rock Mechanics, 1997, Pergamon

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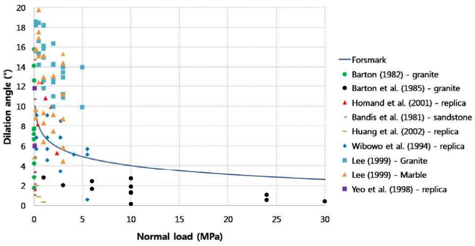
Fractures
Mechanical properties – dilation

- Fracture dilation during fracture shearing
  - Great implication for geo-environmental engineering

$$\phi_{dilation} = \tan^{-1}\left(\frac{\text{normal dilation}}{\text{shear displacement}}\right)$$

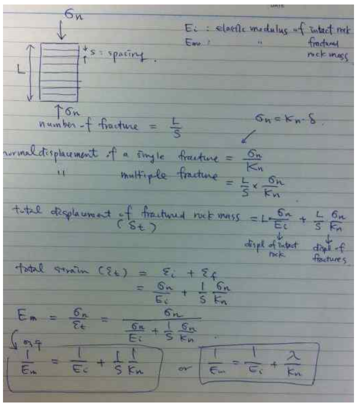
$$\phi_{dilation} = \frac{1}{2} JRC \log\left(\frac{JCS}{\sigma_n}\right)$$




Shear dilation

Mechanical properties
Elastic modulus (deformation modulus)

- Derivation


σn

L

s : spacing.

σn

Ei : elastic modulus of intact rock

Em : " fractured rock mass

number of fracture = L/S

σn = kn·δ

normal displacement of a single fracture = σn/kn

" multiple fracture = L/S × σn/kn

total displacement of fractured rock mass (δt) = L σn/Ei + L/S σn/kn

displ of intact rock

displ of fractures

total strain (εt) = εi + εf

= σn/Ei + 1/S σn/kn

Em = σn/εt = σn / (σn/Ei + 1/S σn/kn)

eq. 9

1/Em = 1/Ei + 1/S 1/kn or 1/Em = 1/Ei + λ/kn

Mechanical properties
Elastic modulus (deformation modulus)

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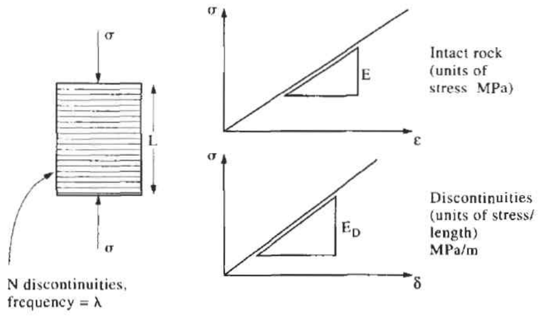

$\delta = 1/\lambda =$ spacing of fractures

Hudson JA, Harrison JP, Engineering Rock Mechanics, 1997, Pergamon

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Mechanical properties
Rock Mass Strength

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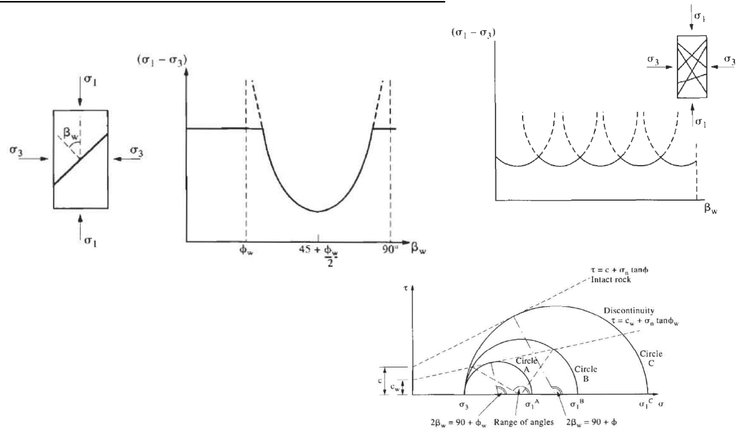

Figure 8.5 Mohr's circle representation of the possible modes of failure for rock containing a single plane of weakness.

Hudson JA, Harrison JP, Engineering Rock Mechanics, 1997, Pergamon

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Hydraulic properties
Permeability of fractured rock mass – cubic law

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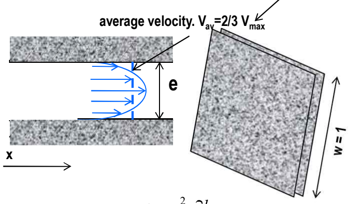
$$v = -\frac{1}{8\mu}\left(e^2 - 4y^2\right)\frac{d(\rho_w gh)}{dx}$$

average velocity. V_{av}=2/3 V_{max}



$$Q = \int_{-\frac{e}{2}}^{\frac{e}{2}} v(wdy) = -\frac{we^3}{12\mu}\frac{d}{dx}(\rho_w gh)$$

$$Q = -\frac{e^3}{12\mu}\frac{d}{dx}(\rho_w gh) = -\frac{\rho_w ge^3}{12\mu}\frac{dh}{dx}$$

Hydraulic conductivity (K) of parallel plate model

$$q = Q/A = Q/ew = -\frac{\rho_w ge^2}{12\mu}\frac{\partial h}{\partial x}$$

$$\boxed{Q = -\frac{\rho_w ge^3}{12\mu}\frac{\partial h}{\partial x}} \qquad \boxed{Q = -\frac{e^3}{12\mu}\frac{\partial p}{\partial x}} \longleftarrow$$

with zero elevation

ρ_w: density of fluid
g: acceleration of gravity
μ: viscosity

- Cubic law: for a given gradient in head and unit width (w), flow rate through a fracture is proportional to the **cube** of the fracture aperture.

Hydraulic properties
Permeability of fractured rock mass

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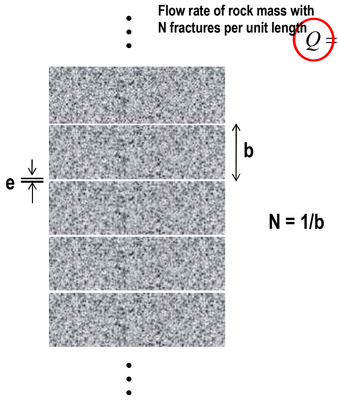

Flow rate of rock mass with N fractures per unit length

Equivalent hydraulic conductivity (K) of multiple parallel plate models

$$Q = -\frac{\rho_w ge^3}{12\mu}\frac{\partial h}{\partial x} \times N = -\frac{\rho_w ge^3}{12\mu}\frac{1}{b}bN\frac{\partial h}{\partial x}$$

$$K = \frac{\rho_w gNe^3}{12\mu} = \frac{\rho_w ge^3}{12\mu b} \qquad k = \frac{Ne^3}{12} = \frac{e^3}{12b}$$

- K: hydraulic conductivity
- k: permeability
- e: aperture
- N: number of fracture per unit distance = frequency, (L^{-1})
- b: spacing (L)

Hydraulic properties
Permeability of fractured rock mass

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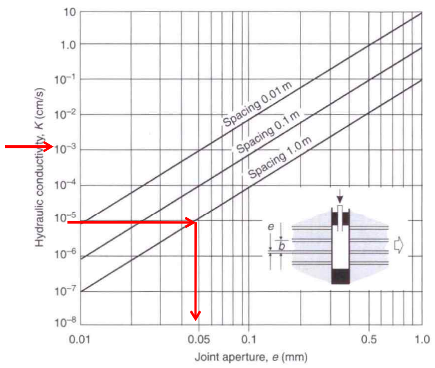
10^{-5} m/s
= 10^{-12} m^2
= 1 D



$$K = \frac{\rho_w ge^3}{12\mu b}$$

A sandstone with K of 10^{-5} cm/s (which is 10^{-7} m/s ~10^{-14} m^2~10^{-2}Darcy~10mD) correspond to aperture 50μm in 1 m interval.

Influence of fracture aperture e and spacing b on hydraulic conductivity K in the direction of a set of smooth parallel fractures in a rock mass (Hoek et al., 2004)

Hydraulic properties
Permeability of fractured rock mass

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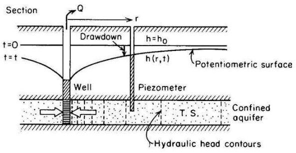
Transient Theis solution - Flow to a well in a confined aquifer



- Based on the work by Theis (1935)
- Used the analogy with heat transfer
- When a steadying pumping is conducted in a well, the head difference at any given radius is expressed as follows.

$$h_0 - h = s = \frac{Q}{4\pi T}\int_{r^2S/4Tt}^{\infty}\frac{e^{-z}}{z}dz = \frac{Q}{4\pi T}W(u) \qquad u = \frac{r^2S}{4Tt} = \frac{r^2S_s}{4Kt}$$

h_0: original head at any distance *r* from a fully penetrating well at time *t* equals zero
h: head at some later time *t*
s: drawdown, difference between h_0 and h
Q: steady pumping rate (m^3/sec)
T: transmissivity (hydraulic conductivity x thickness), m^2/sec
S: storativity (specific storage x thickness), dimensionless

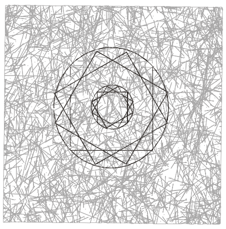
Equivalent Continuum model
Requirements

- Two conditions for application of ECM (Long et al.,1982*)
  - Existence of REV
  - The properties (k_{ij}, S_{ijkl}) should be represented as tensors to be used for continuum mechanics analyses

$$Q_i = A\frac{\boxed{k_{ij}}}{\mu}\frac{\partial P}{\partial x_j}, \qquad \varepsilon_{ij} = \boxed{S_{ijkl}}\sigma_{kl}$$

*Long JCS, Remer JS, Wilson CR, Witherspoon PA. Porous media equivalents for networks of discontinuous fractures, Water Resour Res, 1982;18(3):645-658

Equivalent Continuum model
Requirements



$$k'_{ij} = \beta_{im}\beta_{jn}k_{mn}$$

$$S'_{ijkl} = \beta_{im}\beta_{jn}\beta_{kp}\beta_{lq}S_{mnpq}$$

- Calculated properties with rotated models should be compatible with the prediction made by the tensor transformation

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Case study (1) – Beniawski (1978)
Rock Mass Properties – Elastic modulus

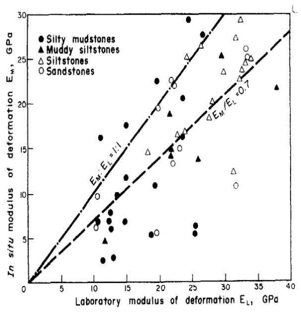


Fig. 2. Comparison of *in-situ* and laboratory moduli of deformation determined at the Orange–Fish Tunnel, after Olivier [27].

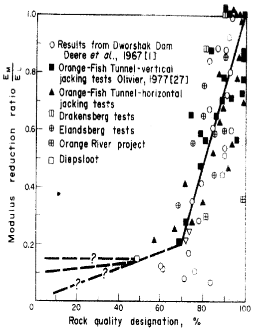


Fig. 6. Comparison of RQD with modulus reduction ratio: E_M, modulus of rock mass to E_L, modulus of rock material.

Bieniawski, Z. T. (1978). "DETERMINING ROCK MASS DEFORMABILITY - EXPERIENCE FROM CASE HISTORIES." International Journal of Rock Mechanics and Mining Sciences 15(5): 237-247.

Case study (1) – Beniawski (1978)
Rock Mass Properties – Elastic modulus

$$E_M = 1.76 \times RMR - 84.3 \qquad E_M = 2 \times RMR - 100.$$

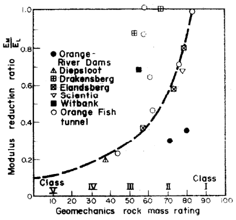


Fig. 7. Relationship between rock mass rating and ratio of deformation modulus of rock mass E_M to that of rock material E_L.

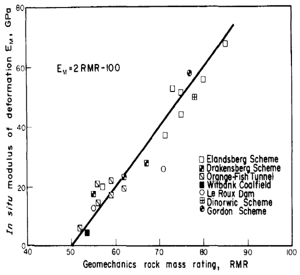
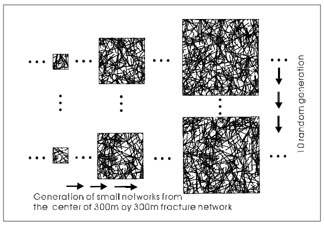


Fig. 8. Relationship between *in-situ* modulus and rock mass rating.

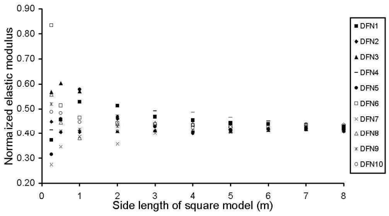
Bieniawski, Z. T. (1978). "DETERMINING ROCK MASS DEFORMABILITY - EXPERIENCE FROM CASE HISTORIES." International Journal of Rock Mechanics and Mining Sciences 15(5): 237-247.

Case study (2) – Min and Jing (2003)
Rock Mass Properties – Compliance Tensor

- Compliance tensor of fractured rock mass
  - Data from Sellafield, UK
  - Obtained from numerical experiment (DFN-DEM approach)



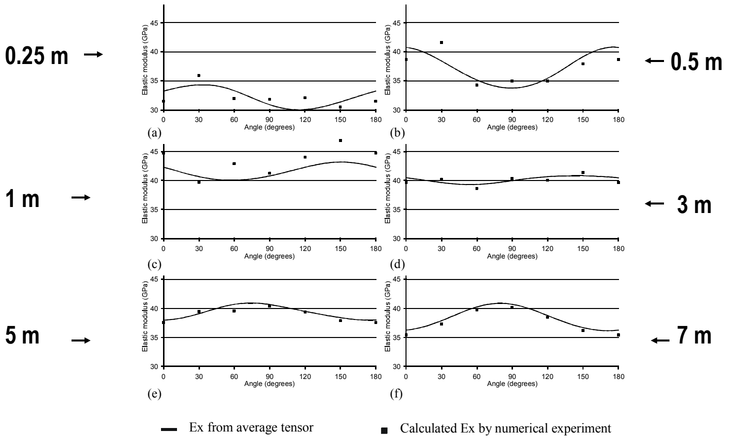
Generation of small networks from the center of 300m by 300m fracture network



Normalized elastic moduli of fractured rock in various scales

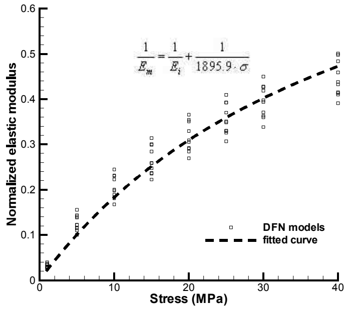
Min KB, Jing L, 2003, Numerical determination of the equivalent elastic compliance tensor for fractured rock masses using the distinct element method, *International Journal of Rock Mechanics & Mining Sciences* 2003;40(6):795-816.

Case study (2) – Min and Jing (2003)
Rock Mass Properties – Compliance Tensor

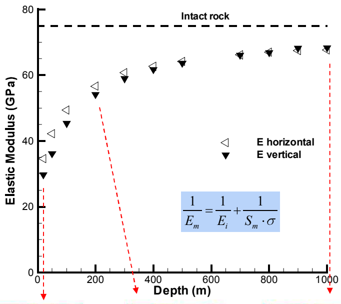


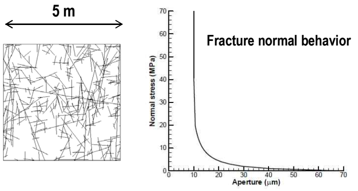
Min KB, Jing L, 2003, Numerical determination of the equivalent elastic compliance tensor for fractured rock masses using the distinct element method, *International Journal of Rock Mechanics & Mining Sciences* 2003;40(6):795-816.

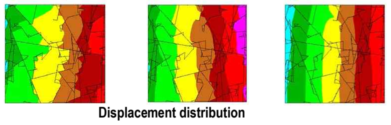
Case study (3) – Min & Jing (2004), Min and Stephansson (2011)
Rock Mass Properties – Stress dependent Elastic modulus



Case study (3) – Min & Jing (2004), Min and Stephansson (2011)
Rock Mass Properties – Stress dependent Elastic modulus







Elastic moduli of fractured rock is lower than those of intact rock

Elastic moduli ↑ with stress ↑ - highly stress-dependent

Effect of fracture/stress is more evident in low stress condition.

Min KB, Stephansson O, The DFN-DEM Approach Applied to Investigate the Effects of Stress on Mechanical and Hydraulic Rock Mass Properties at Forsmark, Sweden, *Tunnel & Underground Space: Journal of Korean Society for Rock Mechanics*, 2011;21(2):117-127

Case study (4) – Long et al. (1982)
Permeability tensor for equivalent continuum medium (ECM)

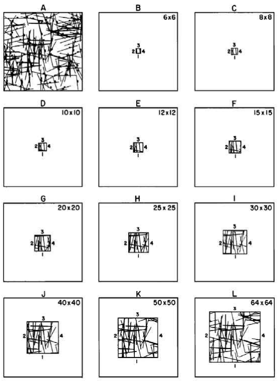


Fig. 11. Fracture samples of increasing size.

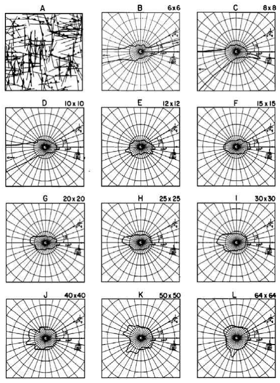
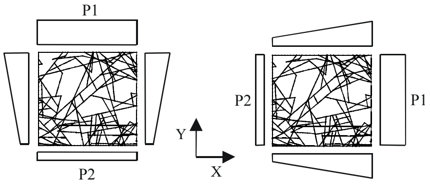


Fig. 12. Permeability plots for fracture samples of increasing size.

Long, J. C. S., et al. (1982). "Porous media equivalents for networks of discontinuous fractures." Water Resources Research 18(3): 645-658.

Case study (5) – Min et al. (2004)
Permeability tensor for equivalent continuum medium (ECM)

- Boundary Condition

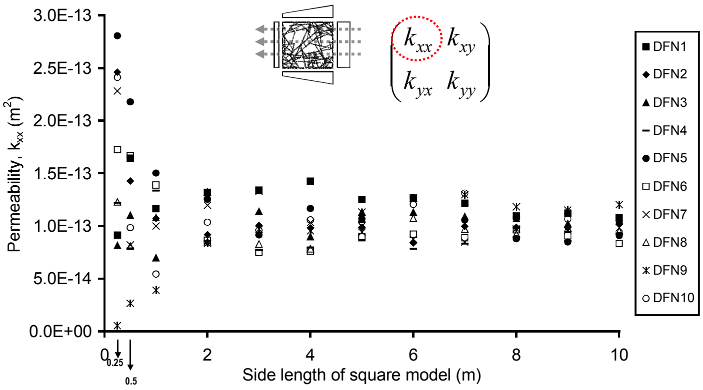


- Constitutive Relation

$$Q_i = A \frac{k_{ij}}{\mu} \frac{\partial P}{\partial x_j}$$

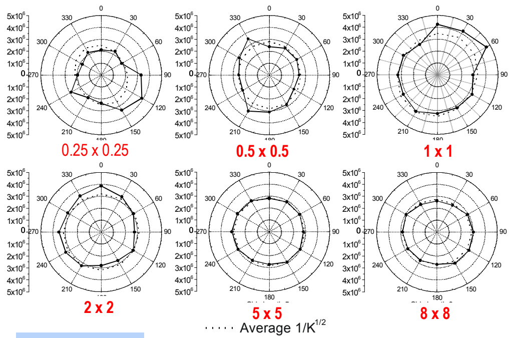
Min, K. B., et al. (2004). "Determining the equivalent permeability tensor for fractured rock masses using a stochastic REV approach: Method and application to the field data from Sellafield, UK." Hydrogeology Journal 12(5): 497-510.

Case study (5) – Min et al. (2004)
Permeability tensor for equivalent continuum medium (ECM)



Min, K. B., et al. (2004). "Determining the equivalent permeability tensor for fractured rock masses using a stochastic REV approach: Method and application to the field data from Sellafield, UK." Hydrogeology Journal 12(5): 497-510.

Case study (5) – Min et al. (2004)
Permeability tensor for equivalent continuum medium (ECM)



$$\frac{x^2}{1/k_x} + \frac{y^2}{1/k_y} = 1$$

Min, K. B., et al. (2004). "Determining the equivalent permeability tensor for fractured rock masses using a stochastic REV approach: Method and application to the field data from Sellafield, UK." Hydrogeology Journal 12(5): 497-510.

Fundamentals of Geomechanics – in situ stress (14, 21 Nov)

Ki-Bok Min, PhD

Associate Professor
Department of Energy Resources Engineering
Seoul National University

SEOUL NATIONAL UNIVERSITY

Rock Mass Properties
Classical References

- In situ stress – importance and overview
  - Kim, K. and J. A. Franklin (1987). "SUGGESTED METHODS FOR ROCK STRESS DETERMINATION." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 24(1): 53-73.
  - Fairhurst, C. (2003). "Stress estimation in rock: a brief history and review." International Journal of Rock Mechanics and Mining Sciences 40(7-8): 957-973.
  - Ljunggren, C., et al. (2003). "An overview of rock stress measurement methods." International Journal of Rock Mechanics and Mining Sciences 40(7-8): 975-989.
- Overcoring method
  - Leeman, E. R. (1968). "DETERMINATION OF COMPLETE STATE OF STRESS IN ROCK IN A SINGLE BOREHOLE-LABORATORY AND UNDERGROUND MEASUREMENTS." International Journal of Rock Mechanics and Mining Sciences 5(1): 31-38.
  - Sjoberg, J., et al. (2003). "ISRM suggested methods for rock stress estimation - Part 2: Overcoring methods." International Journal of Rock Mechanics and Mining Sciences 40(7-8): 999-1010.
- Numerical method
  - Hart, R. (2003). "Enhancing rock stress understanding through numerical analysis." International Journal of Rock Mechanics and Mining Sciences 40(7-8): 1089-1097.
  - Hakami E., H. H., Christiansson R. (2006). Depicting a plausible in situ stress distribution by numerical analysis - examples from two candidate sites in Sweden. In-Situ Rock Stress, Trondheim, Taylor & Francis Group.

Introduction
Objectives of the course

- Objective;
  - Understand the importance of in situ stress for geomechanics
  - In situ stress measurement;
    - Early method (USBM, flat jack method, ..)
    - Overcoring method
    - Hydraulic Fracturing method
    - Indirect method
    - Novel approach
  - Estimation
    - Integrated method
    - World Stress
    - Stress in Korea

Rock Mass Properties
Classical References

- Hydraulic fracturing method
  - Hubbert, M. K. and D. G. Willis (1957). "Mechanics of hydraulic fracturing." Trans. AIME 210(6): 153-163.
  - Haimson, B. C. and Fairhurst, C., Initiation and extension of hydraulic fractures in rock, Soc. Petr. Engrg. J., 7, 310-318, 1967
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  - Rutqvist, J. and O. Stephansson (1996). "A cyclic hydraulic jacking test to determine the in situ stress normal to a fracture." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 33(7): 695-711.
  - Rutqvist, J., et al. (2000). "Uncertainty in the maximum principal stress estimated from hydraulic fracturing measurements due to the presence of the induced fracture." International Journal of Rock Mechanics and Mining Sciences 37(1-2): 107-120.
- Borehole breakout
  - Zoback, M. D., et al. (1985). "WELL BORE BREAKOUTS AND INSITU STRESS." Journal of Geophysical Research-Solid Earth and Planets 90(NB7): 5523-5530 (cited >270).
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  - Schmitt, D. R., et al. (2012). "Crustal stress determination from boreholes and rock cores: Fundamental principles." Tectonophysics 580: 1-26.

Rock Mass Properties
Main References

- Brady BHG and Brown ET, 2004, Rock Mechanics for Underground Mining, 4th ed., Kluwer Academic Publishers
  - Chapter 5 Pre-mining state of stress (p.142-164)
- Goodman RE, 1989, Introduction to Rock Mechancis. 2nd ed., John Wiley & Sons
  - Chapter 4. Initial stresses in rocks and their measurement (p.101-140)
- Hudson JA & Harrison JP, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon
  - Chapter 4. In situ stress (p.41-70)
- Zang A, Stephansson O (2010) Stress field of the Earth's crust. Springer Science and Business Media BV, Dordrecht
- Amadei and Stephansson, 1997, Rock Stress and its measurement,

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Rock Mass Properties
Classical References

- Integrated method
  - Brudy, M., et al. (1997). "Estimation of the complete stress tensor to 8 km depth in the KTB scientific drill holes: Implications for crustal strength." Journal of Geophysical Research-Solid Earth 102(B8): 18453-18475 (cit >227)
  - Zoback, M. D., et al. (2003). "Determination of stress orientation and magnitude in deep wells." International Journal of Rock Mechanics and Mining Sciences 40(7–8): 1049-1076 (cit>172).
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  - Christiansson, R. and J. A. Hudson (2003). "ISRM suggested methods for rock stress estimation - Part 4: Quality control of rock stress estimation." International Journal of Rock Mechanics and Mining Sciences 40(7-8): 1021-1025.
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  - Brown, E. T. and E. Hoek (1978). "TRENDS IN RELATIONSHIPS BETWEEN MEASURED INSITU STRESSES AND DEPTH." International Journal of Rock Mechanics and Mining Sciences 15(4): 211-215.
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  - 임한욱, 이정인, 1980, 공경변화법에 의한 암반내 초기응력 측정에 관한 연구, 대한광산학회지, 17권 1호, p.30-37, 3호 131-137
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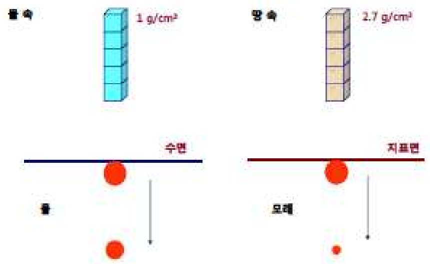
  - 신중호, 박찬, 이병주. "한반도지역의 현지응력장 분포 패턴 및 지질시대별 전이 추이." 터널과 지하공간, 23.6 (2013.12): 457-469.

Home Assignment #5

- Review papers in rock mass properties
  - You need to provide not only summary but also your own insight and criticism based on the selected papers.
  - If necessary, you will have to conduct your own analysis and refer to other papers.
  - One classical paper (try to refer to recent papers on the subject)
    - 09:00 14, 21, Nov through eTL

Introduction

$$p = \rho g z = \text{밀도} \times \text{중력가속도} \times \text{심도}$$



$$p = \rho g z = 0.01 \times z(MPa) \qquad p = \rho g z = 0.027 \times z(MPa)$$

전석원, 2008

In situ Stress in Rock
Outline

- Introduction
- Method of stress determination
  - Direct method
    - Flatjack method
    - Hydraulic fracturing test
    - USBM overcoring method
    - CSIRO (type) overcoring method
  - Indicator method
    - Borehole breakout
    - Other methods
- In situ stress in Korea and worldwide

Introduction
Prediction of in situ stress

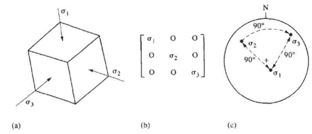
- Heim's rule
  - Assumption: no lateral deformation

Introduction
Importance

- Boundary condition for a engineering problem
  - In situ stress orientation and magnitude is a critical factor for various rock mechanics applications
    - Tunnel/mine/opening design/stability
    - Hydraulic fracturing
    - Borehole stability
    - Earthquake anallysis

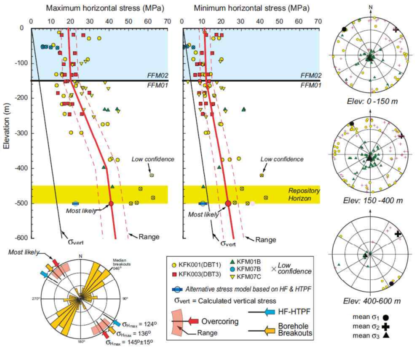
Introduction
Presentation of in situ stress

- Principal stress is presented



$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

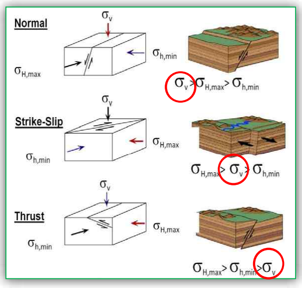
Figure 4.1 (a) Principal stresses acting on a small cube. (b) Principal stresses expressed in matrix form. (c) Principal stress orientations shown on a hemispherical projection.



Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

Introduction
State of Stress

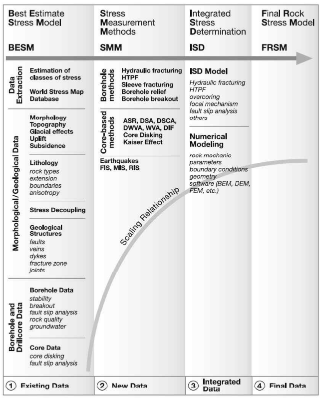
- Three types of stress regime
  - Normal fault stress regime
  - Strike-slip stress regime
  - Thrust fault stress regime



Itasca Consulting Group, 2011)

Introduction
Integrated stress measurement

- Multiple methods are often needed



Zang A, Stephansson O (2010) Stress field of the Earth's crust. Springer Science and Business Media BV, Dordrecht

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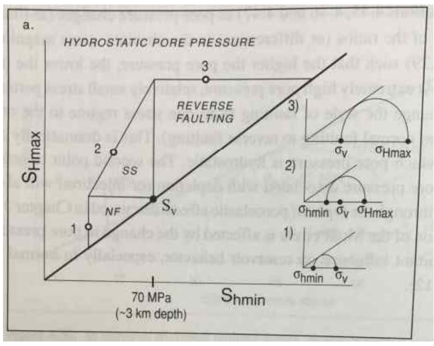
Introduction
Presentation of in situ stress

- Stress polygon



Zoback MD, 2007, Reservoir Geomechanics, Cambridge Univ Press

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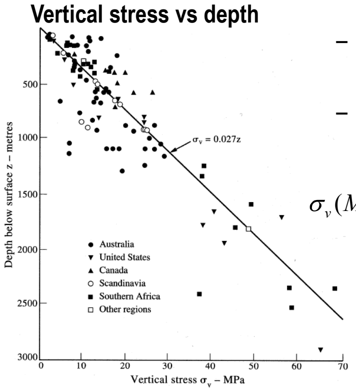
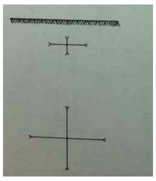
World wide in situ stress data
Magnitude of Vertical stress



- Vertical component of in situ stress
- More or less similar to predicted stress

$$\sigma_v(MPa) = \rho g h = 0.027 \times h(m)$$



Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

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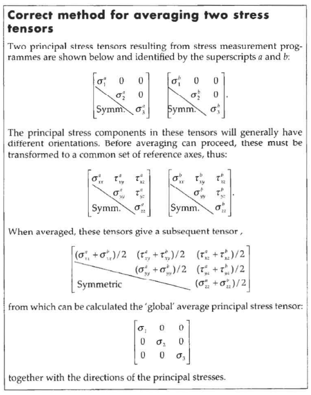
Introduction
Statistical analysis of stress state data

- Averaging must be done in the same reference axis



Correct method for averaging two stress tensors

Two principal stress tensors resulting from stress measurement programmes are shown below and identified by the superscripts *a* and *b*:

$$\begin{bmatrix} \sigma_1^a & 0 & 0 \\ & \sigma_2^a & 0 \\ \text{Symm.} & & \sigma_3^a \end{bmatrix} \quad \begin{bmatrix} \sigma_1^b & 0 & 0 \\ & \sigma_2^b & 0 \\ \text{Symm.} & & \sigma_3^b \end{bmatrix}.$$

The principal stress components in these tensors will generally have different orientations. Before averaging can proceed, these must be transformed to a common set of reference axes, thus:

$$\begin{bmatrix} \sigma_{xx}^a & \tau_{xy}^a & \tau_{xz}^a \\ & \sigma_{yy}^a & \tau_{yz}^a \\ \text{Symm.} & & \sigma_{zz}^a \end{bmatrix} \quad \begin{bmatrix} \sigma_{xx}^b & \tau_{xy}^b & \tau_{xz}^b \\ & \sigma_{yy}^b & \tau_{yz}^b \\ \text{Symm.} & & \sigma_{zz}^b \end{bmatrix}.$$

When averaged, these tensors give a subsequent tensor ,

$$\begin{bmatrix} (\sigma_{xx}^a + \sigma_{xx}^b)/2 & (\tau_{xy}^a + \tau_{xy}^b)/2 & (\tau_{xz}^a + \tau_{xz}^b)/2 \\ & (\sigma_{yy}^a + \sigma_{yy}^b)/2 & (\tau_{yz}^a + \tau_{yz}^b)/2 \\ \text{Symmetric} & & (\sigma_{zz}^a + \sigma_{zz}^b)/2 \end{bmatrix}$$

from which can be calculated the 'global' average principal stress tensor:

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

together with the directions of the principal stresses.

Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

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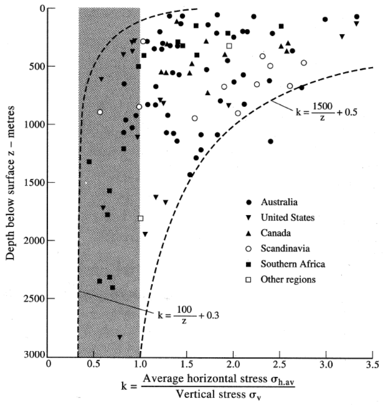
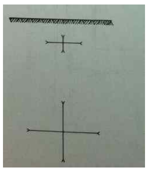
World wide in situ stress data
Magnitude of Horizontal stress



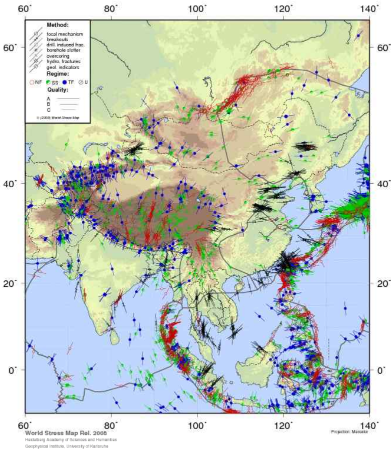
- Horizontal components of insitu stress
- Average horizontal stress is usually 0.3 ~ 4.0 times of vertical stress
- High horizontal stress: tectonic stress, erosion, topography



Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

World wide in situ stress data
World stress map

- http://dc-app3-14.gfz-potsdam.de/



Heidbach, O., Tingay, M., Barth, A., Reinecker, J., Kurfeß, D. and Müller, B., The World Stress Map database release 2008 doi:10.1594/GFZ.WSM.Rel2008, 2008.

Factors affecting in situ stress measurement
Effect of discontinuities

- Discontinuity

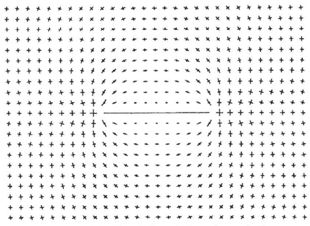
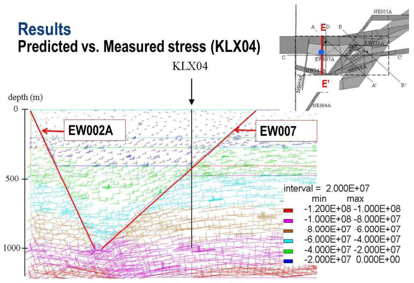


Figure 4.19 Example of the effect of a discontinuity on the near-field stress state, for an applied hydrostatic two-dimensional stress with the discontinuity having a modulus of 10% of the host rock (from Hyett, 1990). The crosses represent the magnitudes and directions of the principal stresses. Note how the stress field close to the discontinuity is quite different from the far-field stress.

Results

Predicted vs. Measured stress (KLX04)



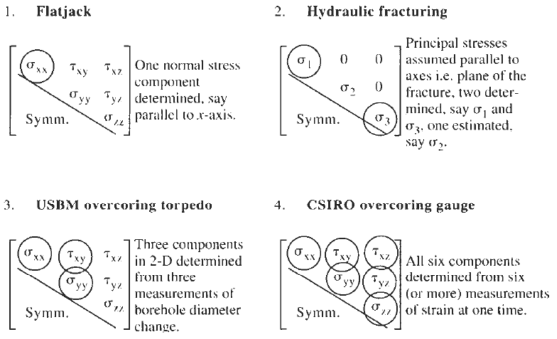
Borehole passes deformation zone EW007 at around 300 ~ 400 m depth
Dramatic change of stress occur due to the deformation zone – smaller stress in the wedge formed by two deformation zones (EW002A and EW007A).

Min KB, Effect of Deformation Zones on the State of *In Situ* Stress at a Candidate Site of Geological Repository of Nuclear Waste in Sweden, *Tunnel & Underground Space: Journal of Korean Society for Rock Mechanics*, 2008;18(2):134-148

Factors affecting in situ stress measurement

- Erosion
- Tectonic activity
- topography
- Rock anisotropy
- Discontinuity

Methods of stress determination



One normal stress component determined, say parallel to x-axis.

Principal stresses assumed parallel to axes i.e. plane of the fracture, two determined, say σ_1 and σ_3, one estimated, say σ_2.

Three components in 2-D determined from three measurements of borehole diameter change.

All six components determined from six (or more) measurements of strain at one time.

Figure 4.3 The four ISRM suggested methods for rock stress determination and their ability to determine the components of the stress tensor *with one application of the particular method*.

Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

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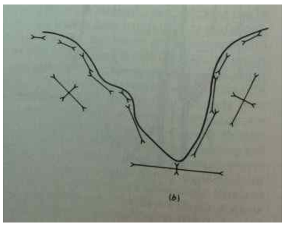
Factors affecting in situ stress measurement
Topography

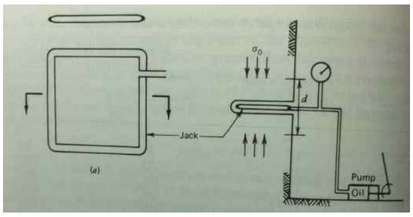
- Topography

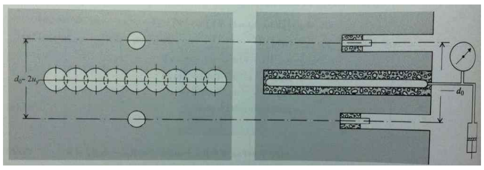


Goodman, 1989, Introduction to Rock Mechanics, Wiley

Methods of stress determination
Flatjack method

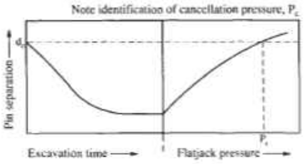
- Directly measure the tangential stress

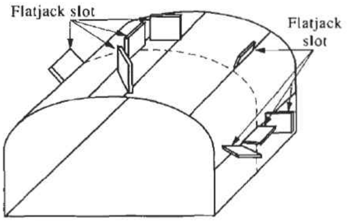




Brady & Brown, 2004, Rock Mechanics for underground mining, Kluwer Academic Publishers

Methods of stress determination
Flatjack method





Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

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Methods of stress determination
USBM overcoring method

- Typical response curve

The traces are the electrical output from the device plotted against time during overcoring and hence illustrate the evolution of diametral change during overcoring.

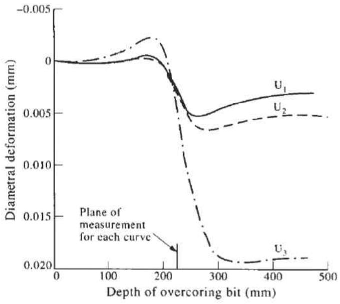

Figure 4.8 Data obtained during a USBM overcoring test.

Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

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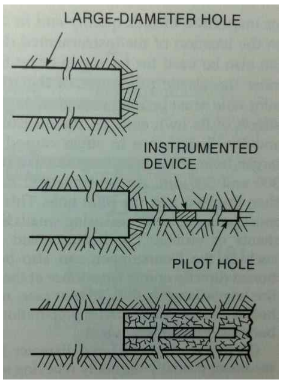
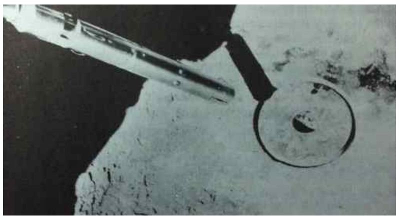
Methods of stress determination
USBM overcoring method

- Typical overcoring procedure





Amadei, B. and O. Stephansson (1997). Rock Stress and its measurement. London, Chapman & Hall

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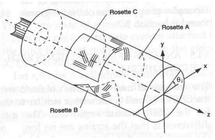
Methods of stress determination
CSIRO type overcoring method

- Complete stress tensor can be determined from minimum of six strain gauges.



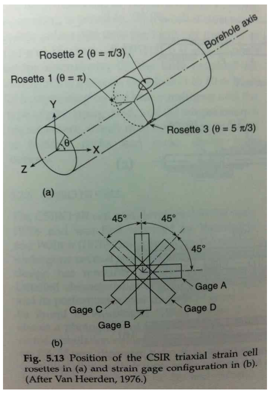


Fig. 5.13 Position of the CSIR triaxial strain cell rosettes in (a) and strain gage configuration in (b). (After Van Heerden, 1976.)

Amadei, B. and O. Stephansson (1997). Rock Stress and its measurement. London, Chapman & Hall

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Methods of stress determination
USBM overcoring method

- USBM deformation gauge – at least three measurements are needed.

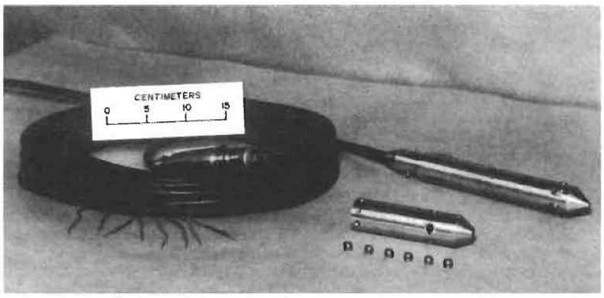

Figure 4.7 The USBM borehole deformation gauge.

Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

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Methods of stress determination
CSIRO type overcoring method

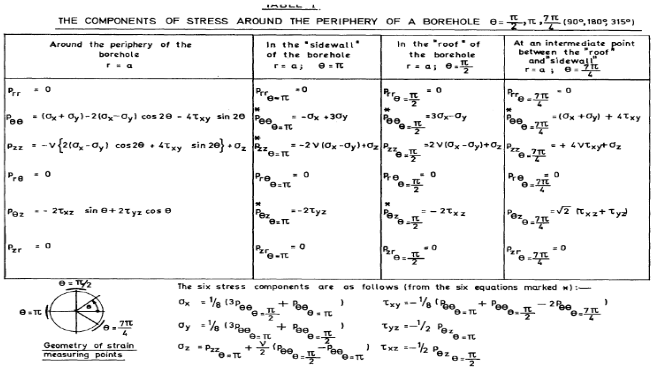


TABLE 1

THE COMPONENTS OF STRESS AROUND THE PERIPHERY OF A BOREHOLE $\theta = \frac{\pi}{2}, \pi, \frac{7\pi}{4}$ (90°, 180°, 315°)

Around the periphery of the borehole $r = a$	In the "sidewall" of the borehole $r = a;\ \theta = \pi$	In the "roof" of the borehole $r = a;\ \theta = \frac{\pi}{2}$	At an intermediate point between the "roof" and "sidewall" $r = a;\ \theta = \frac{7\pi}{4}$
$p_{rr} = 0$	$p_{rr_{\theta=\pi}} = 0$	$p_{rr_{\theta=\frac{\pi}{2}}} = 0$	$p_{rr_{\theta=\frac{7\pi}{4}}} = 0$
$p_{\theta\theta} = (\sigma_x + \sigma_y) - 2(\sigma_x - \sigma_y)\cos 2\theta - 4\tau_{xy}\sin 2\theta$	$\overset{*}{p}_{\theta\theta_{\theta=\pi}} = -\sigma_x + 3\sigma_y$	$\overset{*}{p}_{\theta\theta_{\theta=\frac{\pi}{2}}} = 3\sigma_x - \sigma_y$	$\overset{*}{p}_{\theta\theta_{\theta=\frac{7\pi}{4}}} = (\sigma_x + \sigma_y) + 4\tau_{xy}$
$p_{zz} = -\nu\{2(\sigma_x - \sigma_y)\cos 2\theta + 4\tau_{xy}\sin 2\theta\} + \sigma_z$	$\overset{*}{p}_{zz_{\theta=\pi}} = -2\nu(\sigma_x - \sigma_y) + \sigma_z$	$p_{zz_{\theta=\frac{\pi}{2}}} = 2\nu(\sigma_x - \sigma_y) + \sigma_z$	$p_{zz_{\theta=\frac{7\pi}{4}}} = +4\nu\tau_{xy} + \sigma_z$
$p_{r\theta} = 0$	$p_{r\theta_{\theta=\pi}} = 0$	$p_{r\theta_{\theta=\frac{\pi}{2}}} = 0$	$p_{r\theta_{\theta=\frac{7\pi}{4}}} = 0$
$p_{\theta z} = -2\tau_{xz}\sin\theta + 2\tau_{yz}\cos\theta$	$\overset{*}{p}_{\theta z_{\theta=\pi}} = -2\tau_{yz}$	$\overset{*}{p}_{\theta z_{\theta=\frac{\pi}{2}}} = -2\tau_{xz}$	$p_{\theta z_{\theta=\frac{7\pi}{4}}} = \sqrt{2}(\tau_{xz} + \tau_{yz})$
$p_{zr} = 0$	$p_{zr_{\theta=\pi}} = 0$	$p_{zr_{\theta=\frac{\pi}{2}}} = 0$	$p_{zr_{\theta=\frac{7\pi}{4}}} = 0$

Geometry of strain measuring points

The six stress components are as follows (from the six equations marked *):—

$\sigma_x = \frac{1}{8}(3p_{\theta\theta_{\theta=\frac{\pi}{2}}} + p_{\theta\theta_{\theta=\pi}})$

$\sigma_y = \frac{1}{8}(3p_{\theta\theta_{\theta=\pi}} + p_{\theta\theta_{\theta=\frac{\pi}{2}}})$

$\sigma_z = p_{zz_{\theta=\pi}} + \frac{\nu}{2}(p_{\theta\theta_{\theta=\frac{\pi}{2}}} - p_{\theta\theta_{\theta=\pi}})$

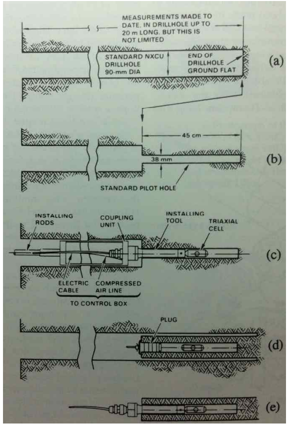
$\tau_{xy} = -\frac{1}{8}(p_{\theta\theta_{\theta=\pi}} + p_{\theta\theta_{\theta=\frac{\pi}{2}}} - 2p_{\theta\theta_{\theta=\frac{7\pi}{4}}})$

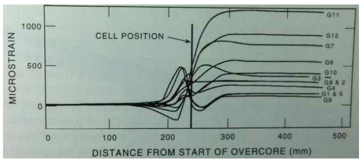
$\tau_{yz} = -\frac{1}{2}\, p_{\theta z_{\theta=\pi}}$

$\tau_{xz} = -\frac{1}{2}\, p_{\theta z_{\theta=\frac{\pi}{2}}}$

Leeman, E. R. (1968). "DETERMINATION OF COMPLETE STATE OF STRESS IN ROCK IN A SINGLE BOREHOLE-LABORATORY AND UNDERGROUND MEASUREMENTS." International Journal of Rock Mechanics and Mining Sciences 5(1): 31-38.

Methods of stress determination
CSIRO type overcoring method





stress relief curve

Amadei, B. and O. Stephansson (1997). Rock Stress and its measurement. London, Chapman & Hall

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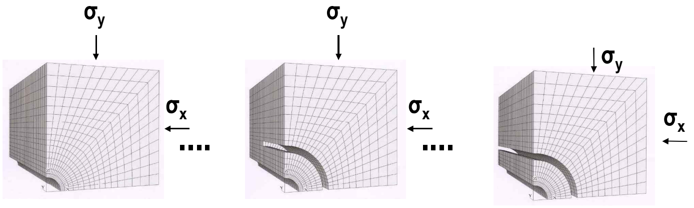
Methods of stress determination
CSIRO type overcoring method

Numerical modeling of stress relief curve



- A 3 dimensional FDM program (FLAC3D, Itasca)
- Elastic transversely isotropic material
- 29 model for each advance of 1 cm overcoring
- Stress on measuring points are monitored and converted to strain

Min KB, Lee CI, Choi HM, An experimental and numerical study of the in-situ stress measurement on transversely isotropic rock by overcoring method, In:Sugawara K et al (eds), 3rd International Symposium on Rock Stress - RS Kumamoto '03, Kumamoto, 2003, pp.189-195.

Methods of stress determination
CSIRO type overcoring method

Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

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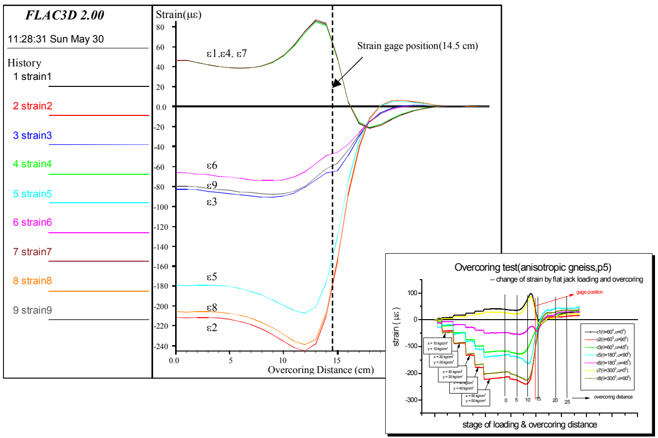
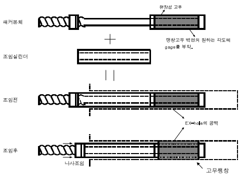
CSIR type overcoring method
Stress relief curve

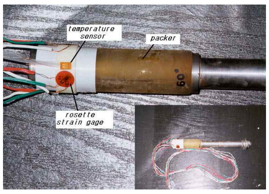


Min KB, Lee CI, Choi HM, An experimental and numerical study of the in-situ stress measurement on transversely isotropic rock by overcoring method, In:Sugawara K et al (eds), 3rd International Symposium on Rock Stress - RS Kumamoto '03, Kumamoto, 2003, pp.189-195.

Methods of stress determination
CSIRO type overcoring method

- Reproduction in the laboratory





Min KB, Lee CI, Choi HM, An experimental and numerical study of the in-situ stress measurement on transversely isotropic rock by overcoring method, In:Sugawara K et al (eds), 3rd International Symposium on Rock Stress - RS Kumamoto '03, Kumamoto, 2003, pp.189-195.

Methods of stress determination
Hydraulic Fracturing for stress determination

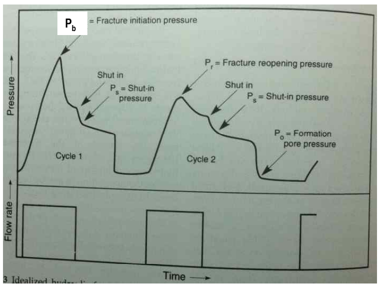
- Principle of stress measurement by hydraulic fracturing (magnitude & orientation)
  - Vertical stress is assumed
  - Knowledge of elastic constants is not needed

In open hole!

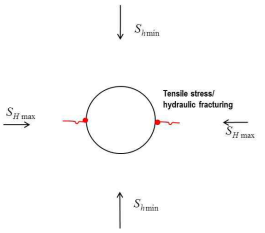
$P_b = 3S_{h\min} - S_{H\max} + T_o$

$P_s = S_{h\min}$

$T_o = P_b - P_r$ **or from Lab Test**



Flowrate-pressure responses



Amadei, B. and O. Stephansson (1997). Rock Stress and its measurement. London, Chapman & Hall

Methods of stress determination
Hydraulic Fracturing for stress determination

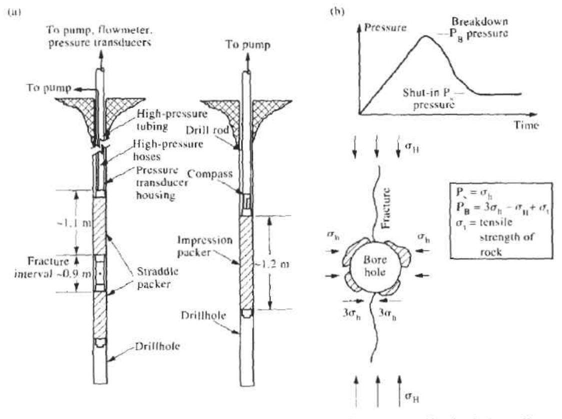


Figure 4.5 (a) The hydraulic fracturing system and (b) associated calculations (from *Suggested Methods for Rock Stress Determination*, Kim and Franklin, 1987).

Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

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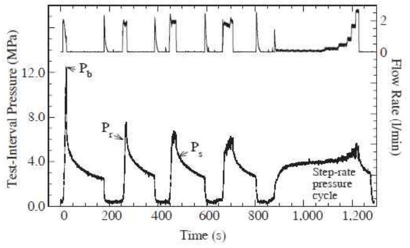
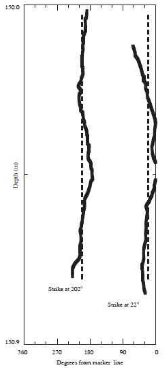
Methods of stress determination
Hydraulic Fracturing for stress determination

- Actual records

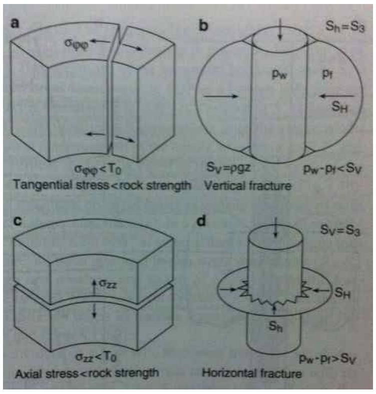




Actual impression packer record (Haimson & Cornet, 2003)

Methods of stress determination
Hydraulic Fracturing for stress determination

- Vertical fracture vs. horizontal fracture (in vertical hole)



Zang A, Stephansson O (2010) Stress field of the Earth's crust. Springer Science and Business Media BV, Dordrecht

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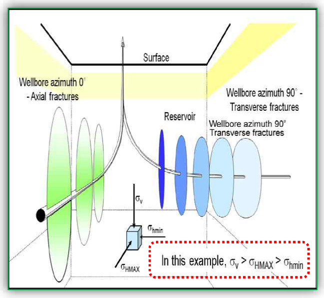
Methods of stress determination
Indirect method

- Borehole breakout
- Anelastic Strain Recovery (ASR)
- Kaiser effect:
- Core disking
- Focal mechanism of earthquake

Methods of stress determination
Hydraulic Fracturing for stress determination

- 'Hydraulic fracturing' is used slightly differently in the industry
  - Hydraulic fracturing for stress measurement: axial fractures <1 m, vertical hole
  - Hydraulic fracturing for shale gas or other petroleum/geothermal engineereing: perforation used, transverse fractures > 100 m, usually horizontal hole



Hydraulic fracturing for shale gas production

- 초기응력의 상태가 수압파쇄 균열의 방향을 "결정"

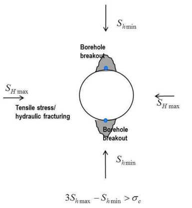
- 최소수평주응력 방향으로 수평정 시추 필요.

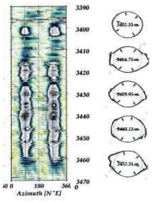
(MA Dusseault, 2011)

Methods of stress determination
Indirect method

- Borehole breakout
  - Enlargements of the borehole wall caused by stress-induced failure of wells occurring 180° apart.
  - In vertical wells, the diametrically faced zones of broken material occur at direction of minimum horizontal stress.

Borehole breakout





Berad and Cornet (2003)

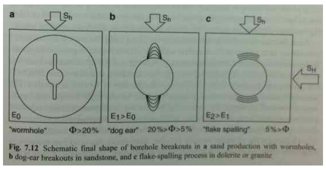


Fig. 7.12 Schematic final shape of borehole breakouts in a sand production with wormholes, **b** dog-ear breakouts in sandstone, and **c** flake-spalling process in dolerite or granite

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Methods of stress determination
Indirect method

- Anelastic Strain Recovery (ASR)
  - Core-based method to estimate in-situ stress magnitudes and orientations from instrumenting a freshly recovered drill core obtained from deep wells.
  - The direction of maximum strain recovery is parallel to the maximum horizontal stress in the borehole.

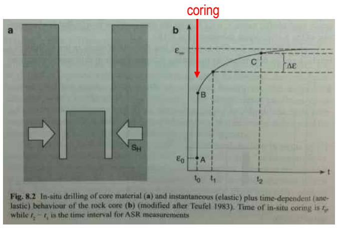

Fig. 8.2 In-situ drilling of core material (**a**) and instantaneous (elastic) plus time-dependent (anelastic) behaviour of the rock core (**b**) (modified after Teufel 1983). Time of in-situ coring is t_0, while $t_2 - t_1$ is the time interval for ASR measurements

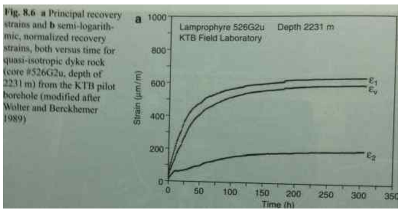

Fig. 8.6 **a** Principal recovery strains and **b** semi-logarithmic, normalized recovery strains, both versus time for quasi-isotropic dyke rock (core #526G2u, depth of 2231 m) from the KTB pilot borehole (modified after Wolter and Berckhemer 1989)

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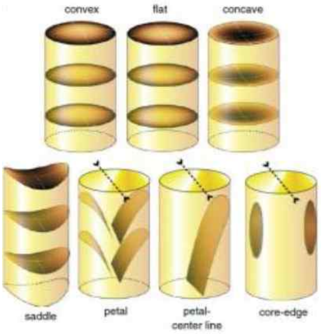
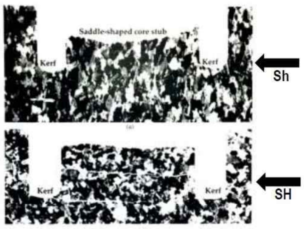
Methods of stress determination
Indirect method





Kim H (2016) presentation material.

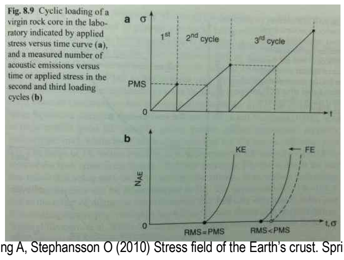
Methods of stress determination
Indirect method

- Kaiser effect
  - phenomenon that a material under stress emits acoustic emissions only after the previous maximum stress is reached.
  - Joseph Kaiser (1950, metal, rock and wood in tension)

PMS: previous maximum stress
RMS: Recalled maximum stress
FE: Felicity effect

$$FR(\text{FelicityRatio}) = \frac{RMS}{PMS}$$

Fig. 8.9 Cyclic loading of a virgin rock core in the laboratory indicated by applied stress versus time curve (**a**), and a measured number of acoustic emissions versus time or applied stress in the second and third loading cycles (**b**)



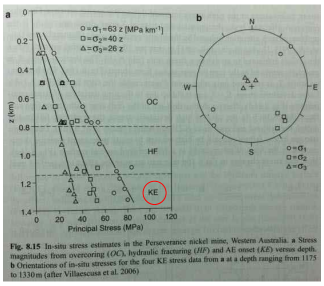


Fig. 8.15 In-situ stress estimates in the Perseverance nickel mine, Western Australia. **a** Stress magnitudes from overcoring (*OC*), hydraulic fracturing (*HF*) and AE onset (*KE*) versus depth. **b** Orientations of in-situ stresses for the four KE stress data from **a** at a depth ranging from 1175 to 1330 m (after Villaescusa et al. 2006)

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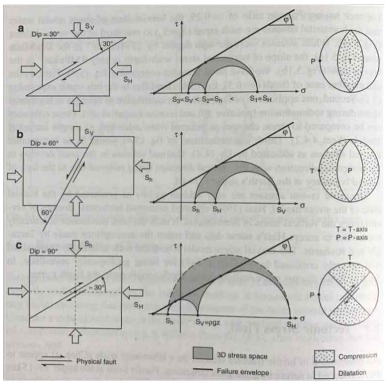
Methods of stress determination
Focal Mechanism

- Focal mechanisms of earthquake
  - Provides the orientation of principal stresses using Coulomb failure criterion
  - Relative magnitude of the three principal stress
  - 77% of WSM data
  - Based on the analysis of observed seismic waveform (first motion of P-wave). Upward: compression, downward: dilational



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Methods of stress determination
Indirect method

- Core disking
  - Assemblage of cored disks in highly stressed rock
  - Often shaped like a horse saddle (axis ~ maximum horizontal stress)
  - The thinner thickness, the greater the horizontal stress

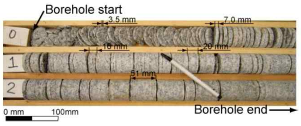
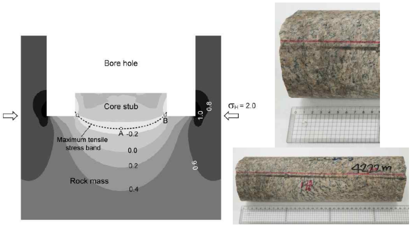


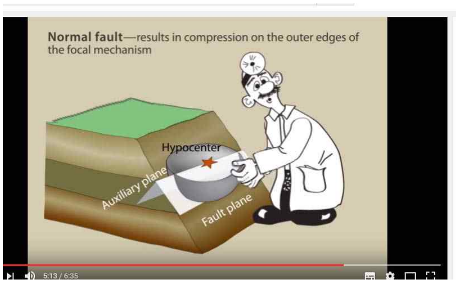
Fig. 1. Typical core disking observed in 75-mm-diameter boreholes drilled from tunnels at the 420-m depth Level of AECL's Underground Research Laboratory.



Lim, S. S., et al. Core Disking Observations and In-Situ Stress Magnitudes, 47 US Rock Mech Symp, Paper No.:13-152

Methods of stress determination
Focal Mechanism

- You tube video
  - https://www.youtube.com/watch?v=MomVOkyDdLo



Methods of stress determination

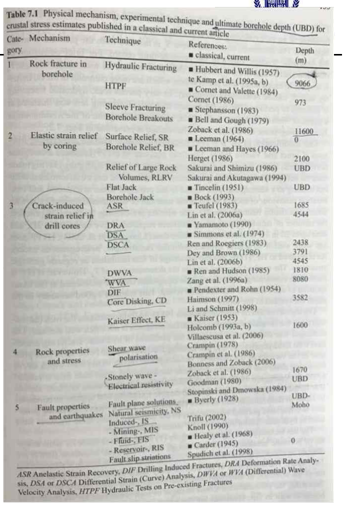


Table 7.1 Physical mechanism, experimental technique and ultimate borehole depth (UBD) for crustal stress estimates published in a classical and current article

Cate-gory	Mechanism	Technique	References: ■ classical, current	Depth (m)
1	Rock fracture in borehole	Hydraulic Fracturing	■ Hubbert and Willis (1957)	
			te Kamp et al. (1995a, b)	9066
		HTPF	■ Cornet and Valette (1984)	
			Cornet (1986)	973
		Sleeve Fracturing	■ Stephansson (1983)	
		Borehole Breakouts	■ Bell and Gough (1979)	
			Zoback et al. (1986)	11600
2	Elastic strain relief by coring	Surface Relief, SR	■ Leeman (1964)	0
		Borehole Relief, BR	■ Leeman and Hayes (1966)	
			Herget (1986)	2100
		Relief of Large Rock Volumes, RLRV	Sakurai and Shimizu (1986)	UBD
			Sakurai and Akutagawa (1994)	
		Flat Jack	■ Tincelin (1951)	UBD
		Borehole Jack	■ Bock (1993)	
3	Crack-induced strain relief in drill cores	ASR	■ Teufel (1983)	1685
			Lin et al. (2006a)	4544
		DRA	■ Yamamoto (1990)	
		DSA	■ Simmons et al. (1974)	
		DSCA	Ren and Roegiers (1983)	2438
			Dey and Brown (1986)	3791
			Lin et al. (2006b)	4545
		DWVA	■ Ren and Hudson (1985)	1810
		WVA	Zang et al. (1996a)	8080
		DIF	■ Pendexter and Rohn (1954)	
		Core Disking, CD	Haimson (1997)	3582
			Li and Schmitt (1998)	
		Kaiser Effect, KE	■ Kaiser (1953)	
			Holcomb (1993a, b)	1600
			Villaescusa et al. (2006)	
4	Rock properties and stress	Shear wave polarisation	Crampin (1978)	
			Crampin et al. (1986)	
			Bonness and Zoback (2006)	
		Stonely wave -	Zoback et al. (1986)	1670
		Electrical resistivity	Goodman (1980)	UBD
			Stopinski and Dmowska (1984)	
5	Fault properties and earthquakes	Fault plane solutions	■ Byerly (1928)	UBD-Moho
		Natural seismicity, NS		
		Induced-, IS	Trifu (2002)	
		- Mining-, MIS	Knoll (1990)	
		- Fluid-, FIS	■ Healy et al. (1968)	
		- Reservoir-, RIS	■ Carder (1945)	0
		Fault slip striations	Spudich et al. (1998)	

ASR Anelastic Strain Recovery, *DIF* Drilling Induced Fractures, *DRA* Deformation Rate Analysis, *DSA* or *DSCA* Differential Strain (Curve) Analysis, *DWVA* or *WVA* (Differential) Wave Velocity Analysis, *HTPF* Hydraulic Tests on Pre-existing Fractures

Methods of stress determination

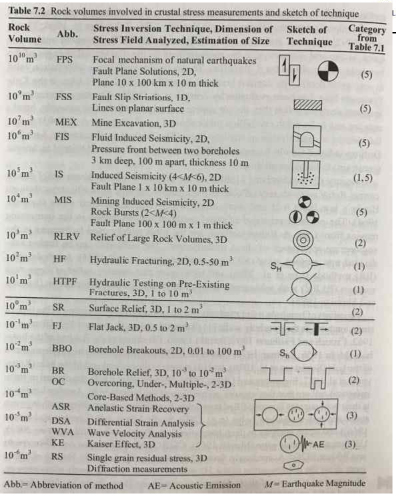


Table 7.2 Rock volumes involved in crustal stress measurements and sketch of technique

Rock Volume	Abb.	Stress Inversion Technique, Dimension of Stress Field Analyzed, Estimation of Size	Sketch of Technique	Category from Table 7.1
$10^{10} m^3$	FPS	Focal mechanism of natural earthquakes Fault Plane Solutions, 2D, Plane 10 x 100 km x 10 m thick		(5)
$10^9 m^3$	FSS	Fault Slip Striations, 1D, Lines on planar surface		(5)
$10^7 m^3$	MEX	Mine Excavation, 3D		
$10^6 m^3$	FIS	Fluid Induced Seismicity, 2D, Pressure front between two boreholes 3 km deep, 100 m apart, thickness 10 m		(5)
$10^5 m^3$	IS	Induced Seismicity ($4<M<6$), 2D Fault Plane 1 x 10 km x 10 m thick		(1,5)
$10^4 m^3$	MIS	Mining Induced Seismicity, 2D Rock Bursts ($2<M<4$) Fault Plane 100 x 100 m x 1 m thick		(5)
$10^3 m^3$	RLRV	Relief of Large Rock Volumes, 3D		(2)
$10^2 m^3$	HF	Hydraulic Fracturing, 2D, 0.5-50 m^3	S_H	(1)
$10^1 m^3$	HTPF	Hydraulic Testing on Pre-Existing Fractures, 3D, 1 to 10 m^3		(1)
$10^0 m^3$	SR	Surface Relief, 3D, 1 to 2 m^3		(2)
$10^{-1} m^3$	FJ	Flat Jack, 3D, 0.5 to 2 m^3		(2)
$10^{-2} m^3$	BBO	Borehole Breakouts, 2D, 0.01 to 100 m^3	S_h	(1)
$10^{-3} m^3$	BR OC	Borehole Relief, 3D, 10^{-3} to $10^{-2} m^3$ Overcoring, Under-, Multiple-, 2-3D		(2)
$10^{-4} m^3$		Core-Based Methods, 2-3D		
	ASR	Anelastic Strain Recovery		
$10^{-5} m^3$	DSA	Differential Strain Analysis		(3)
	WVA	Wave Velocity Analysis		
	KE	Kaiser Effect, 3D	AE	(3)
$10^{-6} m^3$	RS	Single grain residual stress, 3D Diffraction measurements		

Abb.= Abbreviation of method AE= Acoustic Emission M= Earthquake Magnitude

Fundamentals of Geomechanics – Hydromechanics of fractured rock (28 Nov)

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Associate Professor
Department of Energy Resources Engineering
Seoul National University

Introduction
Objectives of the course

- Objective;
  - Understand the importance of Coupled Hydromechanical behavior in fractured rock
  - Flow in a fracture
  - Effect of Stress on a single fracture
  - Effect of stress on fractured rock
  - Application

Rock Mass Properties
Main References

- Hudson JA & Harrison JP, 1997, Engineering Rock Mechanics – An introduction to the principles, Pergamon
  - Chapter 4. In situ stress (p.41-70)
- Jaeger JG, Cook NGW and Zimmerman RW, Fundamentals of Rock Mechanics, 2007, 4th edition, Blackwell Publishing (highly recommended, many typos).
  - Chapter 12. Hydromechanical Behavior of Fractures (p.365-398)
- Council, N. R. (1996). Rock Fractures and Fluid Flow - Contemporary Understanding and Applications. Washington, D.C., National Academy Press.
- Cornet FH, Elements of Crustal Geomechanics, Cambridge Univ Press, 2015 (new and filling the gap in the engineering geoscience)

Rock Mass Properties
Classical References

- Mechanics in a single fracture
  - Bandis SC, Lumsden AC, Barton NR, 1983, Fundamentals of rock joint deformation, Int. J. Rock Mech. Min. Sci. & Geomech. Abstr. 1983;20(6):249-68 (cited>498, in 2016 >36)
  - Brown, S. R. and C. H. Scholz (1986). "CLOSURE OF ROCK JOINTS." Journal of Geophysical Research-Solid Earth and Planets 91(B5): 4939-4948.
  - Fracture dilation?

- Hydraulics in a single fracture/fractured rock
  - Snow, D. T. (1969). "ANISOTROPIC PERMEABILITY OF FRACTURED MEDIA." Water Resources Research 5(6): 1273-1289.(Snow D. T. A parallel plate model of fractured permeable media, Ph.D. thesis. Univ. of California. Berkeley (1965))
  - Witherspoon, P. A., et al. (1980). "VALIDITY OF CUBIC LAW FOR FLUID-FLOW IN A DEFORMABLE ROCK FRACTURE." Water Resources Research 16(6): 1016-1024.
  - Zimmerman RW, Bodvarsson GS, Hydraulic Conductivity of Rock Fractures, Transport in Porous Media 1996;23:1-30
  - Hakami, E. and E. Larsson (1996). "Aperture measurements and flow experiments on a single natural fracture." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 33(4): 395-404.
  - Caine, J. S., et al. (1996). "Fault zone architecture and permeability structure." Geology 24(11): 1025-1028.
  - Kohl, T., et al. (1997). "Observation and simulation of non-Darcian flow transients in fractured rock." Water Resources Research 33(3): 407-418.
  - Elsworth, D. and T. W. Doe (1986). "Application of non-linear flow laws in determining rock fissure geometry from single borehole pumping tests." International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts 23(3): 245-254.
  - Tsang, Y. W. and C. F. Tsang (1987). "CHANNEL MODEL OF FLOW THROUGH FRACTURED MEDIA." Water Resources Research 23(3): 467-479.

Rock Mass Properties
Classical References

- HM coupling in a single fracture
  - Barton N., Bandis S. and Bakhtar K., Strength, Deformation and Conductivity Coupling of Rock Joints, Int J Rock Mech Min Sci, 1985;22(3):121-40
  - Olsson, R. and N. Barton (2001). "An improved model for hydromechanical coupling during shearing of rock joints." International Journal of Rock Mechanics and Mining Sciences 38(3): 317-329.
  - Renshaw CE, On the relationship between mechanical and hydraulic apertures in rough-walled fractures, J Geophy Res 1995;100(B12):24629-24636
  - Yeo, I. W., et al. (1998). "Effect of shear displacement on the aperture and permeability of a rock fracture." International Journal of Rock Mechanics and Mining Sciences 35(8): 1051-1070.
  - Lee, H. S. and T. F. Cho (2002). "Hydraulic characteristics of rough fractures in linear flow under normal and shear load." Rock Mechanics and Rock Engineering 35(4): 299-318.
  - Cappa, F., et al. (2006). "Hydromechanical modelling of pulse tests that measure fluid pressure and fracture normal displacement at the Coaraze Laboratory site, France." International Journal of Rock Mechanics and Mining Sciences 43(7): 1062-1082.
  - Guglielmi, Y., et al. (2014). "ISRM Suggested Method for Step-Rate Injection Method for Fracture In-Situ Properties (SIMFIP): Using a 3-Components Borehole Deformation Sensor." Rock Mechanics and Rock Engineering 47(1): 303-311.
  - Guglielmi, Y., et al. (2015). "Seismicity triggered by fluid injection-induced aseismic slip." Science 348(6240): 1224-1226.
- HM coupling in fractured rock
  - Rutqvist, J. and O. Stephansson (2003). "The role of hydromechanical coupling in fractured rock engineering." Hydrogeology Journal 11(1): 7-40.
  - Rutqvist, J. (2015). "Fractured rock stress-permeability relationships from in situ data and effects of temperature and chemical-mechanical couplings." Geofluids 15(1-2): 48-66.
  - Min, K. B., et al. (2004). "Stress-dependent permeability of fractured rock masses: a numerical study." International Journal of Rock Mechanics and Mining Sciences 41(7): 1191-1210.
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Mechanics of a single fracture
Normal loading

- Full stress defromation curve in cyclic normal loading/unloading

$$\Delta V_j = \frac{\sigma_n a}{1+\sigma_n a} = \frac{\sigma_n V_m}{K_{ni} V_m + \sigma_n} \qquad V_m = A + B(\mathrm{JRC}) + C\left(\frac{\mathrm{JCS}}{a_j}\right)^D.$$

$$\begin{aligned} A_1 &= -0.2960 \pm 0.1258 & A_2 &= -0.1005 \pm 0.0530 & A_3 &= -0.1032 \pm 0.0680 \\ B_1 &= -0.0056 \pm 0.0022 & B_2 &= -0.0073 \pm 0.0031 & B_3 &= -0.0074 \pm 0.0039 \\ C_1 &= -2.2410 \pm 0.3504 & C_2 &= -1.0082 \pm 0.2351 & C_3 &= 1.1350 \pm 0.3261 \\ D_1 &= -0.2450 \pm 0.1086 & D_2 &= -0.2301 \pm 0.1171 & D_3 &= -0.2510 \pm 0.1029 \\ r_1^2 &= 0.675 & r_2^2 &= 0.546 & r_3^2 &= 0.589 \end{aligned}$$

$$K_{ni} = -7.15 + 1.75\,\mathrm{JRC} + 0.02\left(\frac{\mathrm{JCS}}{a_j}\right), \quad r^2 = 0.573$$

$$a_j = \frac{\mathrm{JRC}}{5}\left(0.2\frac{\sigma_c}{\mathrm{JCS}} - 0.1\right)$$

Bandis SC, Lumsden AC, Barton NR, 1983, Fundamentals of rock joint deformation, Int. J. Rock Mech. Min. Sci. & Geomech. Abstr. 1983;20(6):249-68

Mechanics of a single fracture
Normal loading

- Highly nonlinear
  - Normal stiffness increases with stress
- Not reversible
  - Permanent aperture increase

$$\sigma_n = \left(\frac{\Delta V_j}{V_m - \Delta V_j}\right)\sigma_i + \sigma_i$$

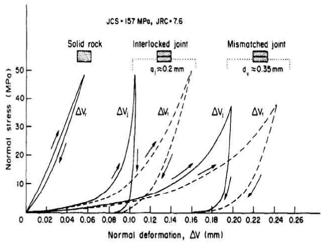
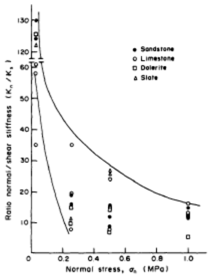


Fig. 6. Comparison of total deformation (ΔV_t) and net closure curves $\Delta V_j = \Delta V_t - \Delta V_r$) from the first loading cycle of the same joint tested in fully interlocked and mismatched positions.

Bandis SC, Lumsden AC, Barton NR, 1983, Fundamentals of rock joint deformation, Int. J. Rock Mech. Min. Sci. & Geomech. Abstr. 1983;20(6):249-68

Mechanics of a single fracture
Normal loading

- Ratio of normal stiffness/shear stiffness



Bandis SC, Lumsden AC, Barton NR, 1983, Fundamentals of rock joint deformation, Int. J. Rock Mech. Min. Sci. & Geomech. Abstr. 1983;20(6):249-68

Mechanics of a single fracture
Normal loading

- Cyclic loading

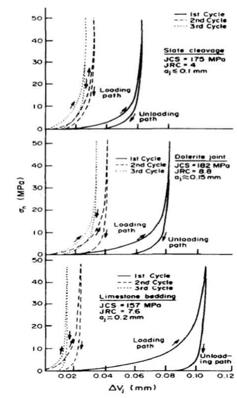


Fig. 3. Normal stress (σ_n) vs closure (ΔV_j) curves for a range of *fresh* joints in different rock types, under repeated loading cycles.

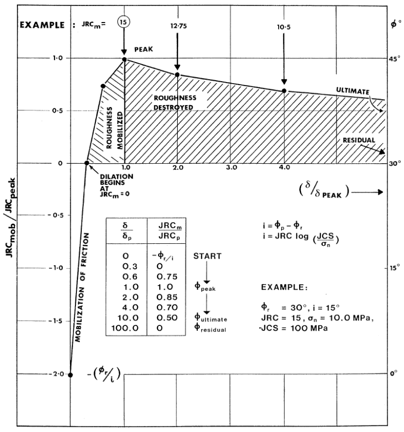
Table 1. Index properties and sampling locations

Properties Rock materials	Tensile strength σ_t (MPa)	Compressive strength σ_c (MPa)	Young's modulus $E_{t_{50}}$ (GPa)
Slate (Ordovician from Skiddaw Slates—Quarry nr Keswick, Lake District)	14.9	159.0	66.0
Dolerite (Permo-Carboniferous —natural exposure at Horwick Crags, Teesdale)	17.3	165.0	78.0
Limestone (Lower Carboniferous —natural exposure, N. Yorks)	10.6	152.0	49.0
Siltstone (Upper Coal Measures —NCB pit, W. Yorks)	6.3	84.0	28.5
Sandstone (Lower Coal Measures —Shibden Dale Quarry, W. Yorks)	5.1	78.0	24.0

Bandis SC, Lumsden AC, Barton NR, 1983, Fundamentals of rock joint deformation, Int. J. Rock Mech. Min. Sci. & Geomech. Abstr. 1983;20(6):249-68

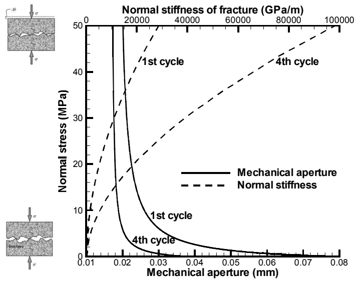
Mechanics of a single fracture
Shear loading

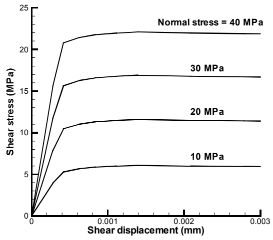
$$u_{sp} = \frac{L_n}{500}\left(\frac{\mathrm{JCS}_n}{L_0}\right)^{0.33}$$



Barton NR, 1982, Modelling Rock Joint Behavior from In Situ Block Tests: Implications for Nuclear Waste Repository Design, Technical Report ONWI-308

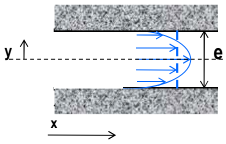
Mechanics of a single fracture
Normal & Shear Stress-Deformation Relationship





JRC: 3.85 (0.3 m scale)
JCS: 112.21
Initial aperture: 77 µm
Residual Friction angle: 27.2°

Hydraulics in a single fracture
Cubic law



Velocity (v)distribution between parallel plates

$$v=-\frac{1}{8\mu}\left(e^2-4y^2\right)\frac{d(\rho_w gh)}{dx}$$
$$=-\frac{1}{8\mu}\left(e^2-4y^2\right)\frac{dp}{dx}$$

- Navier-Stokes' equation for laminar flow.
- Most of geological application involves laminar flow (low Reynolds number, <2000, de Marsily, 1986)

$$\mathrm{Re}=\frac{\rho Vd}{\mu}$$

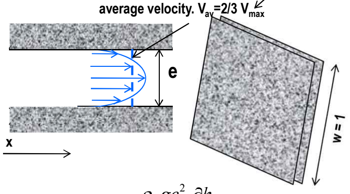
ρ: density of fluid
V: mean velocity of fluid
D: diameter of the pipe
μ: viscosity

Hydraulics in fractured rock
Importance

- In many rock types (especially hard rocks), fractures are the main pathways of fluid flow – note that hard rocks are attractive for many applications.
- Understandings on fluid flow in fractures are essential for;
  - Underground structure (mines, tunnels and oil storages)
  - Geological repository of high level nuclear waste
  - Enhanced Geothermal System
  - Fractured Oil Reservoir

Hydraulics in a single fracture
Cubic law

$$v=-\frac{1}{8\mu}\left(e^2-4y^2\right)\frac{d(\rho_w gh)}{dx}$$



$$Q=\int_{-\frac{e}{2}}^{\frac{e}{2}} v(wdy)=-\frac{we^3}{12\mu}\frac{d}{dx}(\rho_w gh)$$

$$Q=-\frac{e^3}{12\mu}\frac{d}{dx}(\rho_w gh)=-\frac{\rho_w ge^3}{12\mu}\frac{dh}{dx}$$

Hydraulic conductivity (K) of parallel plate model

$$q=Q/A=Q/ew=-\frac{\rho_w ge^2}{12\mu}\frac{\partial h}{\partial x}$$

$$\boxed{Q=-\frac{\rho_w ge^3}{12\mu}\frac{\partial h}{\partial x}}\qquad\boxed{Q=-\frac{e^3}{12\mu}\frac{\partial p}{\partial x}}\longleftarrow$$

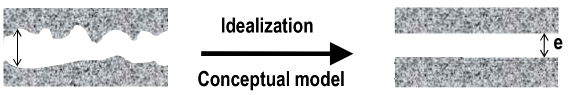
with zero elevation

ρ_w: density of fluid
g: acceleration of gravity
μ: viscosity

- Cubic law: for a given gradient in head and unit width (w), flow rate through a fracture is proportional to the **cube** of the fracture aperture.

Hydraulics in a single fracture
Cubic law

Aperture (e): size of opening measured normal to the fracture wall



Idealization

Conceptual model

Real rock fracture

Idealized rock fracture

- Hard to estimate Q due mainly to complex geometry

-Analytical solution exist to calculate Q and velocity profile

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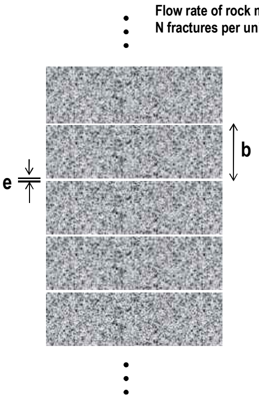
Hydraulics in a single fracture
Cubic law - Equivalent permeability

Equivalent hydraulic conductivity (K) of multiple parallel plate models

Flow rate of rock mass with N fractures per unit length

$$Q=-\frac{\rho_w ge^3}{12\mu}\frac{\partial h}{\partial x}\times N=-\frac{\rho_w ge^3}{12\mu}\frac{1}{b}bN\frac{\partial h}{\partial x}$$

$$K=\frac{\rho_w gNe^3}{12\mu}=\frac{\rho_w ge^3}{12\mu b}\qquad k=\frac{Ne^3}{12}=\frac{e^3}{12b}$$

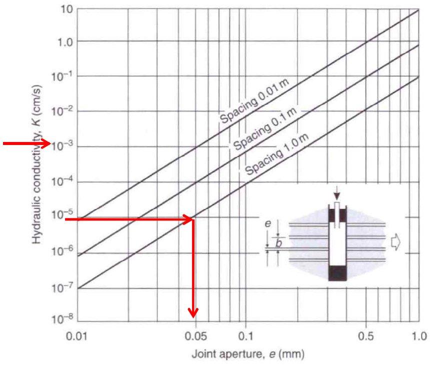


N = 1/b

- K: hydraulic conductivity
- k: permeability
- e: aperture
- N: number of fracture per unit distance = frequency, (L^{-1})
- b: spacing (L)

Hydraulics in a single fracture
Cubic law - Equivalent permeability

10^{-5} m/s
= 10^{-12} m2
= 1 D



$$K = \frac{\rho_w g e^3}{12 \mu b}$$

A sandstone with K of 10^{-5} cm/s (which is 10^{-7} m/s ~10^{-14} m^2~10^{-2}Darcy~10mD) correspond to aperture 50µm in 1 m interval.

Influence of fracture aperture e and spacing b on hydraulic conductivity K in the direction of a set of smooth parallel fractures in a rock mass (Hoek et al., 2004)

Hydromechanics in a single fracture
Flow change due to shear deformation

- Mechanical Aperture vs. Hydraulic Aperture

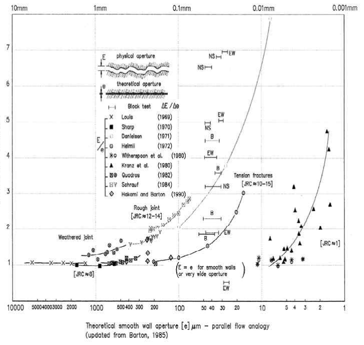


Fig. 3. Comparison of real mechanical apertures (E) with theoretical smooth wall conducting apertures (e). The mismatch is caused by flow losses due to tortuosity and surface roughness. After Barton [14], with updating by Barton and Quadros [20]. The references listed in this figure are given in full by Barton and Quadros [20].

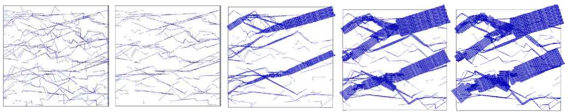
Olsson, R. and N. Barton (2001). "An improved model for hydromechanical coupling during shearing of rock joints." International Journal of Rock Mechanics and Mining Sciences 38(3): 317-329.

Hydromechanics in fractured rock
Importance

CO2 Geosequestration → **Increase of injected CO2 pressure**

Nuclear waste repository → **Generation of thermal stress**

THERMOSHEARING



K=1.0 K=2.0 K=3.0 K=4.0 K=5.0

Geothermal Energy – Enhanced Geothermal System → **Increased hydraulic pressure → hydraulic stimulation**

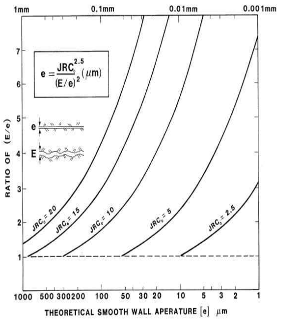
HYDROSHEARING

Hydromechanics in a single fracture
Flow change due to shear deformation

- Mechanical Aperture vs. Hydraulic Aperture

$$e = \frac{E^2}{\mathrm{JRC}_0^{2.5}}, \quad u_s \le 0.75 u_{sp},$$

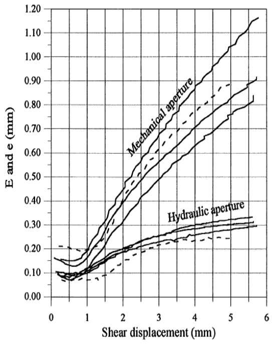
$$e = E^{1/2}\,\mathrm{JRC}_{\mathrm{mob}}, \quad u_s \ge u_{sp}.$$

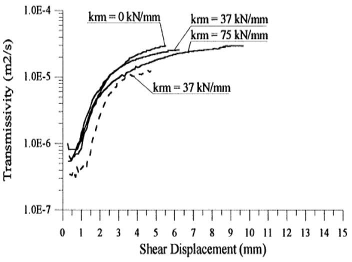


Olsson, R. and N. Barton (2001). "An improved model for hydromechanical coupling during shearing of rock joints." International Journal of Rock Mechanics and Mining Sciences 38(3): 317-329.

Hydromechanics in a single fracture
Flow change due to shear deformation

- Coupled Shear Flow Test (Olsson & Barton, 2001)



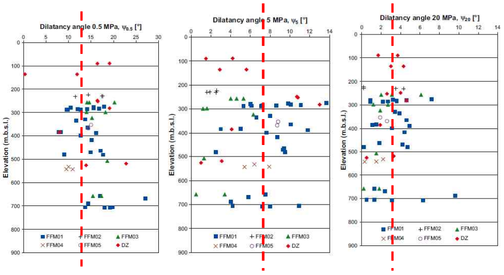


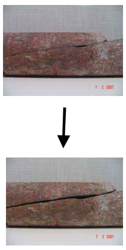
Olsson, R. and N. Barton (2001). "An improved model for hydromechanical coupling during shearing of rock joints." International Journal of Rock Mechanics and Mining Sciences 38(3): 317-329.

Hydromechanics in a single fracture
Flow change due to shear deformation – dilation angle

- Direct shear test on 57 single fractures (Glamheden, 2007)

σ_n =	0.5 MPa	5 MPa	20 MPa
Ψ_{mean} =	14.6°	7.7°	3.2°





Elmar & Stefan, 2010

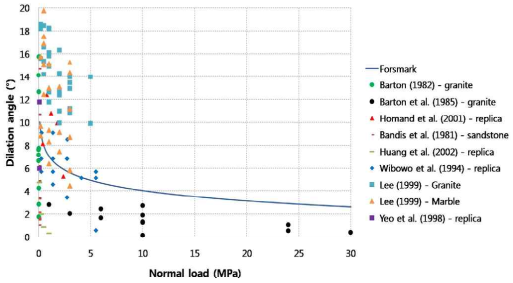
Shear dilation is important even at moderate normal stress (~ 20 MPa, ~ 500 m)

Glamheden R, Fredriksson A, Röshoff K, Karlsson J, Hakami H and Christiansson R (2007), Rock Mechanics Forsmark. Site descriptive modelling Forsmark stage 2.2. SKB.

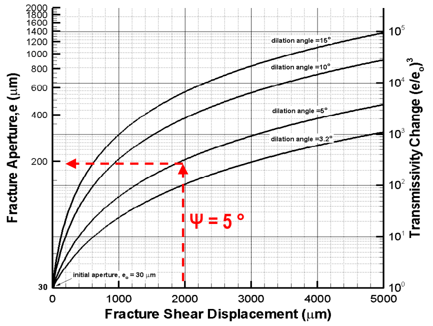
Hydromechanics in a single fracture
Flow change due to shear deformation – dilation angle

- Prediction of dilation angle

$$d_{\text{nmob}} = \frac{1}{M}\text{JRC}_{\text{mob}}\log_{10}(\text{JCS}/\sigma_{\text{n}}),$$



Hydromechanics in a single fracture
Flow change due to shear deformation – dilation angle



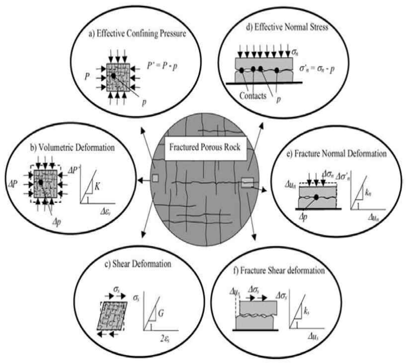
1 μm ~ $8.3\text{x}10^{-12}$ m^2/s
10 μm ~ $8.3\text{x}10^{-9}$ m^2/s

Dilation angle 5° ~ 10 % of fracture opening relative to shear displacement

- Transmissivity

$$T = \frac{\rho g}{12\mu}e^3 \qquad \frac{T}{T_0} = \left(\frac{e}{e_0}\right)^3$$

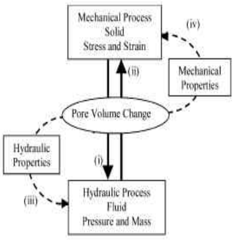
Hydromechanics in fractured rock
HM process in fractured-porous rock



Rutqvist, J. and O. Stephansson (2003). "The role of hydromechanical coupling in fractured rock engineering." Hydrogeology Journal 11(1): 7-40.

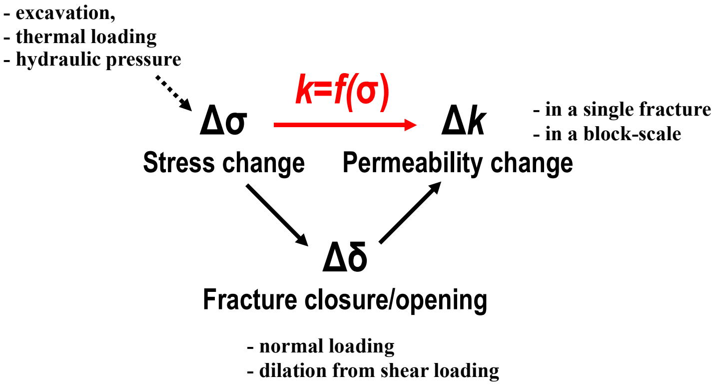
Hydromechanics in fractured rock
HM process in fractured-porous rock

- Direct Coupling vs. Indirect Coupling



Rutqvist, J. and O. Stephansson (2003). "The role of hydromechanical coupling in fractured rock engineering." Hydrogeology Journal 11(1): 7-40.

Hydromechanics in fractured rock
HM process in fractured-porous rock



- excavation,
- thermal loading
- hydraulic pressure

$\Delta\sigma$ $\xrightarrow{k=f(\sigma)}$ **Δk**

Stress change — Permeability change

- in a single fracture
- in a block-scale

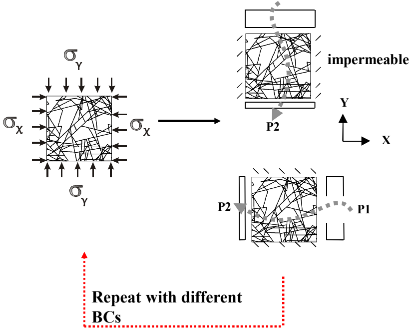
$\Delta\delta$

Fracture closure/opening

- normal loading
- dilation from shear loading

Hydromechanics in fractured rock
HM process in fractured-porous rock

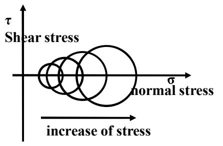
- Stress dependent permeability of fractured rock (Min et al., 2004) - methodology



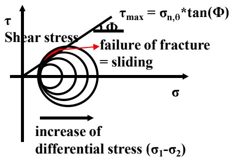
(1) Apply Stress

(2) Measure permeability

Hydromechanics in fractured rock
HM process in fractured-porous rock



- σ_x & σ_y are increased with the same ratio of K (σ_x/σ_y) 1.3
- This can relate the permeability change to the depth
- Dilation is NOT anticipated.

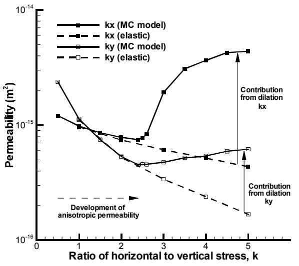


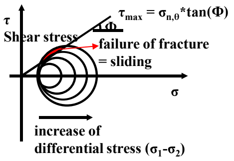
- σ_x is increased with the fixed σ_y (5 MPa)
  - K(σ_x/σ_y) :0.5 ~ 5.0.
- This can relate the permeability change to the differential stress
- Dilation is anticipated.

Min, K. B., et al. (2004). "Stress-dependent permeability of fractured rock masses: a numerical study." International Journal of Rock Mechanics and Mining Sciences 41(7): 1191-1210.

Hydromechanics in fractured rock
HM process in fractured-porous rock

- Stress dependent permeability of fractured rock (Min et al., 2004) – Permeability change due to shearing



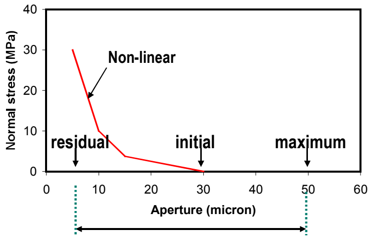


- Deformation of aperture occur not uniformly
- Shear dilation is dominating the k_x,k_y change

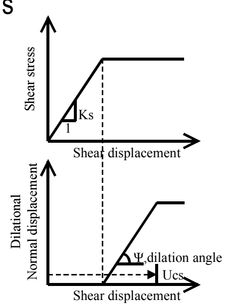
Min, K. B., et al. (2004). "Stress-dependent permeability of fractured rock masses: a numerical study." International Journal of Rock Mechanics and Mining Sciences 41(7): 1191-1210.

Hydromechanics in fractured rock
HM process in fractured-porous rock

- Stress dependent permeability of fractured rock (Min et al., 2004) – Input parameters and models

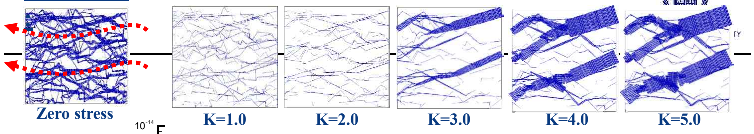


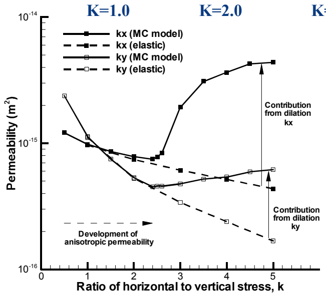
Range of hydraulic aperture (5-50 μm)



- Step-wise non-linear Fracture normal stiffness (Kn)
- Fracture friction angle : 24.9 °
- Fracture dilation angle : 5 °
- Dilation start after failure
- Critical shear displacement:3mm

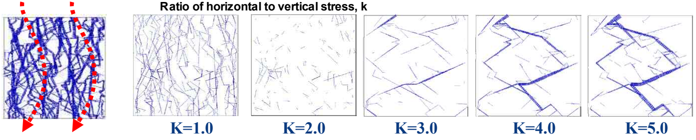
Flowrates





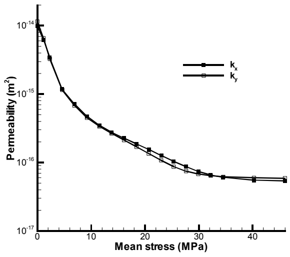
- stress-induced channelling is reproduced.

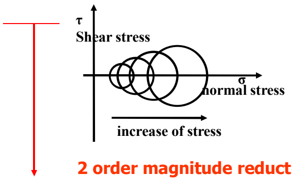
- This partly explains why fluid flow in a few fractures are dominating the fluid behaviour



Hydromechanics in fractured rock
HM process in fractured-porous rock

- Stress dependent permeability of fractured rock (Min et al., 2004) – Permeability decrease by normal closure





2 order magnitude reduction

More sensitive at low stress

Deformation of aperture occur uniformly

Normal closure is dominating the k_x,k_y change

Min, K. B., et al. (2004). "Stress-dependent permeability of fractured rock masses: a numerical study." International Journal of Rock Mechanics and Mining Sciences 41(7): 1191-1210.