Fusion Reactor Technology I
(459.760, 3 Credits)

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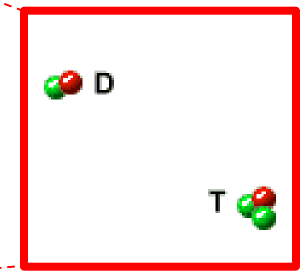
Contents

Fusion Reactor Criterion

What is required to light a fire in a stove?



 Deuterium

 Tritium

- Fuel: D, T
- Amount/density: n
- Heat insulation: τ
- Ignition temperature: T

n … : ?

Cri… …red

(fusi… …duct)

J. D. Lawson

Fusion Reactor Energetics

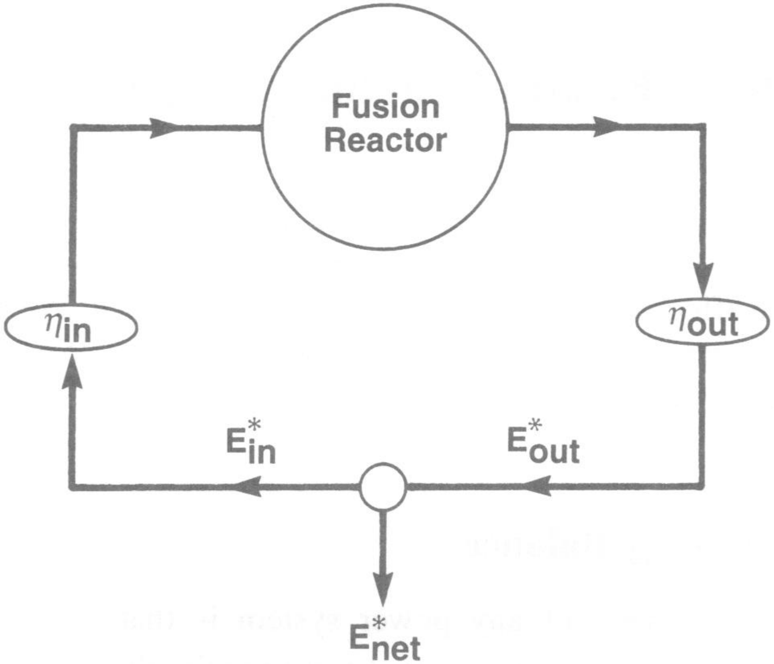
- **Fundamental requirement of a fusion reactor system**

  The overall net energy should be larger than the total energy externally supplied to sustain fusion reactions and associated processes subtracted from the total recovered energy

$$E^*_{net} = E^*_{out} - E^*_{in} > 0$$

*: referring to the entire reaction volume

$$\int_V E_{out}(\vec{r},t)d^3r = E^*_{out}(t)$$



Considering the time variations of power (Particularly for pulsed systems)

$$\int_0^{\tau_b} \left(\frac{dE^*}{dt}\right)_{net} dt = \int_0^{\tau_b} \left(\frac{dE^*}{dt}\right)_{out} dt - \int_0^{\tau_b} \left(\frac{dE^*}{dt}\right)_{in} dt > 0$$

τ_b: burning time

Fusion Reactor Energetics

- **Fusion Plasma Energy Balance**

$$\int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{net} dt = \int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{out} dt - \int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{in} dt > 0$$

Thermal energy content in the total plasma volume

$$\int_o^{\tau_b}\frac{dE_{th}^*}{dt}dt = E_{aux}^* + E_{fu}^* - E_n^* - E_{rad}^* - \int_o^{\tau_b}\frac{E_{th}^*}{\tau_{E^*}}dt$$

$$E_{out}^* = E_{aux}^* + E_{alpha}^* = E_{aux}^* + E_{fu}^* - E_n^*$$

$$E_{in}^* = E_{rad}^* + \int_o^{\tau_b}\frac{E_{th}^*}{\tau_{E^*}}dt$$

$$E_{aux}^* = \eta_{in}E_{in}^*$$

$$\frac{E_{alpha}^*}{E_{fu}^*} = f_c \qquad E_n^* = (1 - f_c)E_{fu}^*$$

f_c: alpha particle fraction of fusion product energy

Fusion Reactor Energetics

$$Q_p = \frac{E^*_{fu}}{E^*_{aux}} = \frac{E^*_{fu}}{\eta_{in} E^*_{in}}$$

Plasma Q-value (fusion multiplication factor): measure for how efficiently an energy input to the plasma is converted into fusion energy

$$\int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = E^*_{aux} + E^*_{fu} - E^*_n - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt \rightarrow \int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = \left(\frac{1}{Q_p} + f_c \right) E^*_{fu} - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt$$

$$E^*_n = (1 - f_c) E^*_{fu}$$

If, steady state

$$E^*_{fu} = \frac{E^*_{rad} + \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt}{f_c + \frac{1}{Q_p}}$$

- if, $Q_p \rightarrow \infty$, the fusion energy delivered to the plasma via the charged reaction products is seen to balance the total energy loss from the plasma.

2.4 핵융합로에서의 에너지 흐름

- **플라즈마 속의 에너지 흐름**

– 핵융합 출력

$$P = dE^* / dt$$

$$P_f = \int n_D n_T \langle \sigma v \rangle_{DT} Q_{DT} dV$$

$$n_e = n_D + n_T + \sum_j Z_j n_j \longrightarrow n_D = n_T \approx \frac{1}{2} n_e$$

$\langle \sigma v \rangle_{DT}$: Assuming Maxwellian distribution for ion

$$\langle n_k \rangle = \int n_k dV / V_P$$

$$\langle T_k \rangle = \int n_k T_k dV / \int n_k dV$$

$$T_k \propto (1 - x^2)$$

$$n_k \propto (1 - x^2)^{0.3}$$

$$V_p = 2\pi R_0 \pi a^2 \kappa$$

- 동일한 평균 온도와 평균 밀도를 갖더라도 공간 분포 형태에 따라 핵융합 출력은 2배 정도 까지 변할 수 있음.

2.4 핵융합로에서의 에너지 흐름

- **플라즈마 속의 에너지 흐름**

– 핵융합 출력

$$P_f = 3.57\left\{1 - 1.36\left(1 - \frac{\langle T_i \rangle}{35keV}\right)^{1.7}\right\}\left(\frac{\langle n_i \rangle}{10^{20} m^{-3}}\right)^2 \left(\frac{V_P}{m^3}\right)(MW)$$

Ex. ITER:

$\langle T_i \rangle$ = 8.9 keV

$\langle n_i \rangle$ = 1.0x10^{20} m^{-3}

V_P = 831 m^3

→ 516 MW

2.4 핵융합로에서의 에너지 흐름

- **플라즈마 속의 에너지 흐름**

- Power balance in the plasma

$$n_D = n_T \approx \frac{1}{2} n_e$$

$$T_i = T_e$$

$$\frac{E_{th}^*}{\tau_{E^*}} = \frac{1}{5} P_f + P_{aux} - P_{rad}$$

$$P_f = 3.57 \left\{ 1 - 1.36 \left(1 - \frac{\langle T_i \rangle}{35keV} \right)^{1.7} \right\} \left(\frac{\langle n_i \rangle}{10^{20} m^{-3}} \right)^2 \left(\frac{V_P}{m^3} \right) (MW)$$

$$E_{th}^* = U_p = \frac{3k}{2} \left(\langle n_e \rangle \langle T_e \rangle + \sum_j \langle n_j \rangle \langle T_j \rangle \right) V_P$$

$$\tau_{E^*} \approx 4.81 \times 10^{-2} \frac{\langle n_e \rangle}{10^{20} m^{-3}} \frac{\langle T_e \rangle}{keV} \frac{V_P}{m^3} \Bigg/ \left(\frac{1}{5} P_f + P_{aux} - P_{rad} \right)$$

$$\tau_E \approx 0.0562 I_P^{0.93} B_T^{0.15} P_{loss}^{-0.69} \left(\frac{n_e}{10^{19}} \right)^{0.41} A^{0.19} R^{1.97} \varepsilon^{0.58} \kappa^{0.78}$$

- 밀도가 일정하게 유지된다는 가정
- 핵융합 플라즈마 제어는 power balance와 particle balance를 동시에 제어해야 하는 어려움이 있음.

2.4 핵융합로에서의 에너지 흐름

- **플라즈마 속의 에너지 흐름**

– Energy confinement time in ITER

$$\tau_{E^*} \approx 4.81\times10^{-2}\frac{\langle n_e\rangle}{10^{20}m^{-3}}\frac{\langle T_e\rangle}{keV}\frac{V_P}{m^3}\Bigg/\left(\frac{1}{5}P_f + P_{aux} - P_{rad}\right)$$

n_e = 1.1x10^{20} m^{-3}
T_e = 8.9 keV
V_P = 831 m^3
P_f = 500 MW
P_b = 60 MW
P_{rad} = 0 MW
→ 2.4 s

$$\tau_E \approx 0.0562 I_P^{0.93} B_T^{0.15} P_{loss}^{-0.69}\left(\frac{n_e}{10^{19}}\right)^{0.41} A^{0.19} R^{1.97} \varepsilon^{0.58} \kappa^{0.78}$$

I_P = 15 MA
B_t = 5.3 T
P_{loss} = P_f/5+P_{aux}- P_{rad}
= 160 MW
A = 2.5
R = 6.2 m
a = 2 m
κ = 1.7
→ 2.45 s

2.4 핵융합로에서의 에너지 흐름

- **플라즈마 속의 에너지 흐름**

- 에너지 증배율

$$Q = P_f / P_b$$

$$= \frac{n\tau_E \langle \sigma v \rangle_{DT} / kT}{\frac{12}{Q_{DT}} - \frac{1}{5} n\tau_E \langle \sigma v \rangle_{DT} / kT}$$

$$n_D = n_T \approx \frac{1}{2} n_e$$

$$T_i = T_e$$

$$P_{rad} = 0$$

→ What are requirements of a fusion reactor?

Fusion Reactor Energetics

- **Ignition**

Energy viability of the fusion plasma:

actual self-sustaining engineering reactor condition with no heating power

$$\frac{E^*_{fu}}{\eta_{in} E^*_{in}} = Q_p \rightarrow \infty$$

Considering a D-T plasma with $Q_p \rightarrow \infty$,

$$\int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = \left(\frac{1}{Q_p} + f_c \right) E^*_{fu} - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt > 0$$

$$f_c E^*_{fu} > E^*_{rad} + \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt$$

Fusion Reactor Energetics

$$f_c E^*_{fu} > E^*_{rad} + \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt$$

$$f_{c,dt} \int_V d^3r \int_o^{\tau_b} R_{dt}(\vec{r},t) Q_{dt} dt > \int_V d^3r \left[\int_o^{\tau_b} (P_{br} + P^{net}_{cyc}) dt + \int_o^{\tau_b} \frac{E_{th}(\vec{r},t)}{\tau_E(\vec{r},t)} dt \right]$$

$$P_{br} = A_{br} n_i n_e Z^2 \sqrt{kT_e} \qquad A_{br} \approx 1.6 \times 10^{-38} \left[\frac{m^3 J}{\sqrt{eV} s} \right]$$

$$P^{net}_{cyc} = A_{cyc} n_e B^2 kT_e \psi \qquad A_{cyc} \approx 6.3 \times 10^{-20} \left[J eV^{-1} T^{-2} s^{-1} \right]$$

$$\int_V d^3r \frac{E_{th}(\vec{r},t)}{\tau_E(\vec{r},t)} = \frac{E^*_{th}(t)}{\tau_{E^*}(t)}$$

volume integrated

Fusion Reactor Energetics

- In a homogeneous plasma, local D-T fusion ignition condition:
  Charged particle self-heating power > loss powers
  (radiation + plasma transport)

$$f_{c,dt}P_{dt}(n_i,T_i) > P_{br}(n_i,n_e,T_e) + P_{cyc}^{net}(n_e,T_e) + \frac{3}{2}\frac{(n_iT_i + n_eT_e)}{\tau_{E^*}}$$

$$E_{th.j} = \frac{3}{2}n_jT_j\,, \quad j = i,e$$

$$f_{c,dt}P_{dt}\tau_{E*} \geq (P_{br} + P_{cyc}^{net})\tau_{E*} + 3nT$$ ⟵ $n_i = n_e = n$, $T_i = T_e = T$

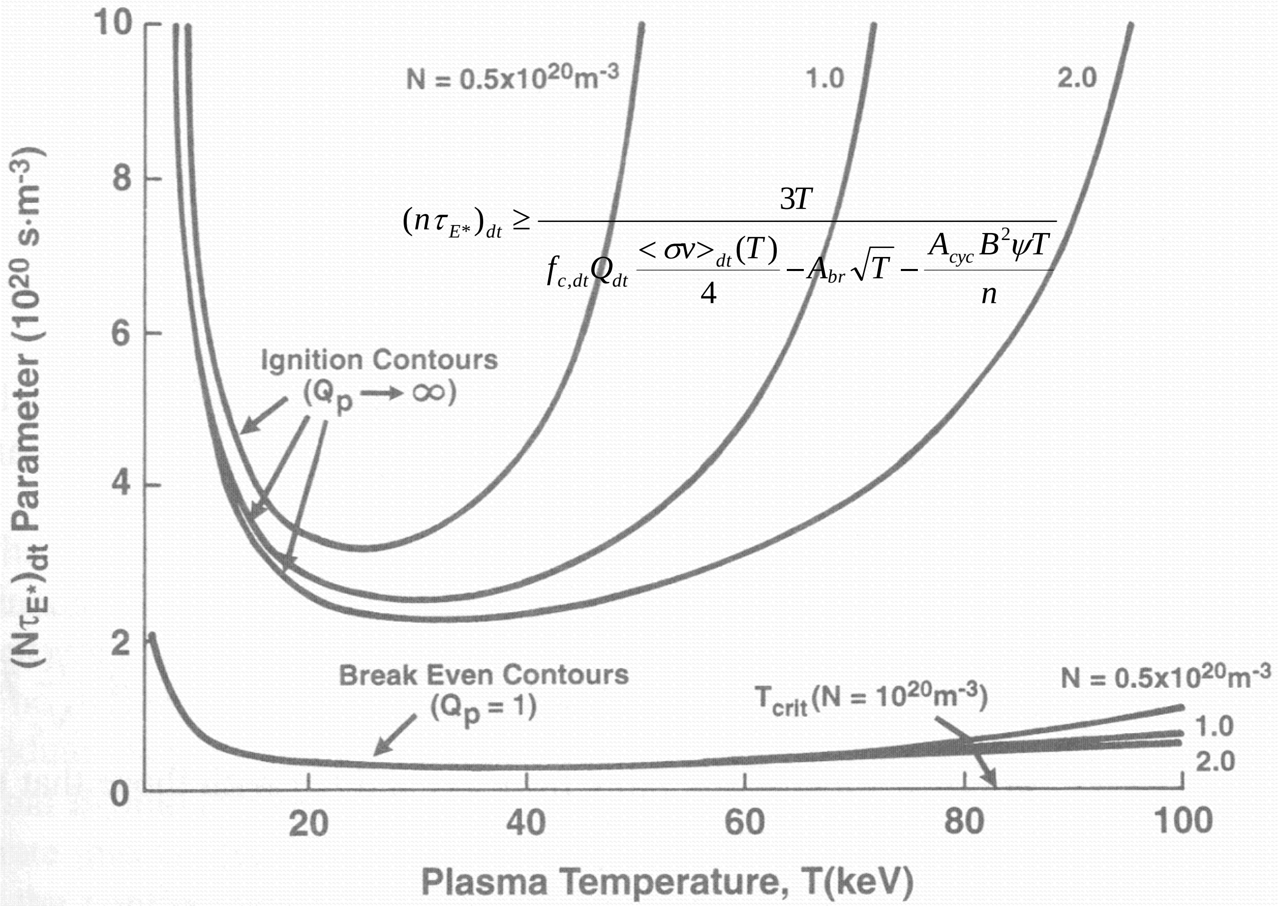
$$(n\tau_{E*})_{dt} \geq \frac{3T}{f_{c,dt}Q_{dt}\frac{<\sigma v>_{dt}(T)}{4} - A_{br}\sqrt{T} - \frac{A_{cyc}B^2\psi T}{n}}$$

no energy conversion efficiency contained

- complex interrelation between the plasma density and its temperature as required for ignition

Plot?

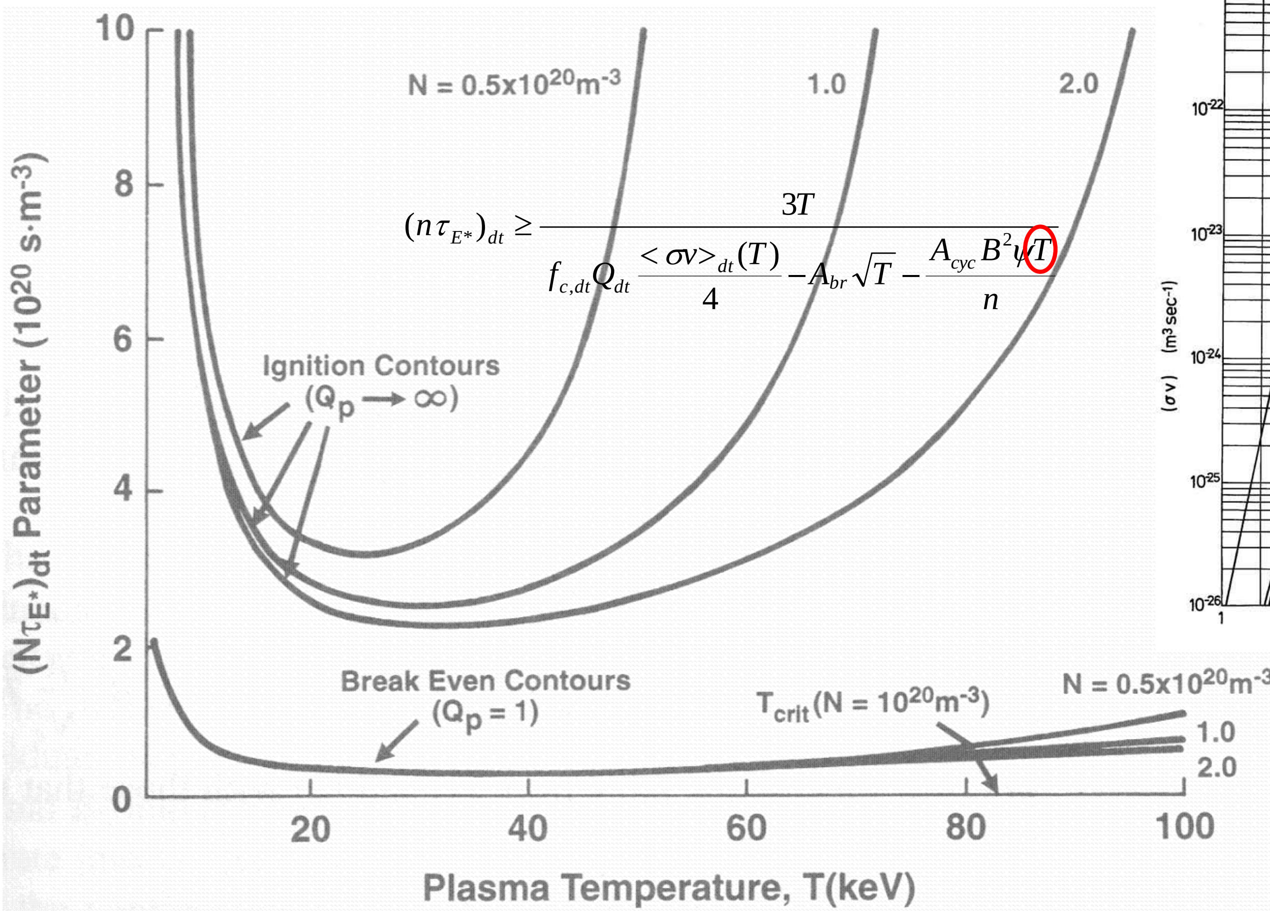
Fusion Reactor Energetics



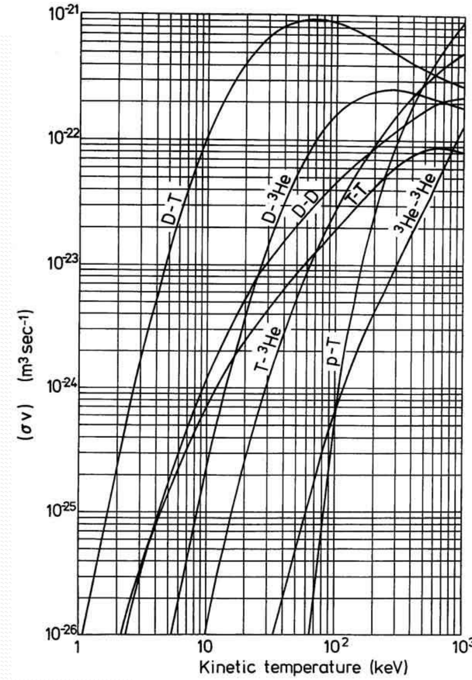
$$(n\tau_{E*})_{dt} \geq \frac{3T}{f_{c,dt}Q_{dt}\frac{<\sigma v>_{dt}(T)}{4} - A_{br}\sqrt{T} - \frac{A_{cyc}B^2\psi T}{n}}$$

- n = 10^{20} m^{-3}: T ~ 30 keV, $n\tau_{E*}$ ~ 2.7x10^{20} m^{-3}s, τ_{E*} ~ 2.7 s
- Ignition contours tend towards infinity as T approaches T_{crit}. **Why?**

Fusion Reactor Energetics



$$(n\tau_{E*})_{dt} \geq \frac{3T}{f_{c,dt}Q_{dt}\frac{<\sigma v>_{dt}(T)}{4} - A_{br}\sqrt{T} - \frac{A_{cyc}B^2\sqrt{T}}{n}}$$



$<\sigma v>_{dt} \propto T^2$ at 10-20 keV but ~constant above 50 keV

- n = 10^{20} m^{-3}: T ~ 30 keV, $n\tau_{E*}$ ~ 2.7x10^{20} m^{-3}s, τ_{E*} ~ 2.7 s
- Ignition contours tend towards infinity as T approaches T_{crit} due to high cyclotron radiation.

Fusion Reactor Energetics

- **Break-even (scientific)**

The total fusion energy production amounts to a magnitude equal to the effective plasma energy input.

$$\frac{E_{fu}^{*}}{\eta_{in} E_{in}^{*}} = Q_p = 1$$

$$\int_{o}^{\tau_b} \frac{dE_{th}^{*}}{dt} dt = \left(\frac{1}{Q_p} + f_c \right) E_{fu}^{*} - E_{rad}^{*} - \int_{o}^{\tau_b} \frac{E_{th}^{*}}{\tau_{E^*}} dt > 0$$

- 핵분열로의 경우 임계조건은 중성자의 수지 밸런스가 취해져 연쇄반응이 안정적으로 계속된다는 물리 현상 상의 임계점이지만, 핵융합의 break-even condition은 물리적으로 크게 변하는 점이 아님.
- Ignition condition의 경우 외부 가열 없이 자기 가열만으로 핵융합을 계속할 수 있다는 점에서 물리적 의미가 있음.
- 실용로에 있어서 Q 값이 무한대가 될 필요는 없으며 steady state operation을 위한 current drive power를 고려할 때 $20 < Q < 50$ 정도로 고려됨.
- Q 정의에 이용한 외부가열 이외에도 핵융합 플랜트에 필요한 보조 기기의 전력이 수십 MW 정도 필요하다. (초전도 냉동기, 펌프 동력 등)

Fusion Reactor Energetics

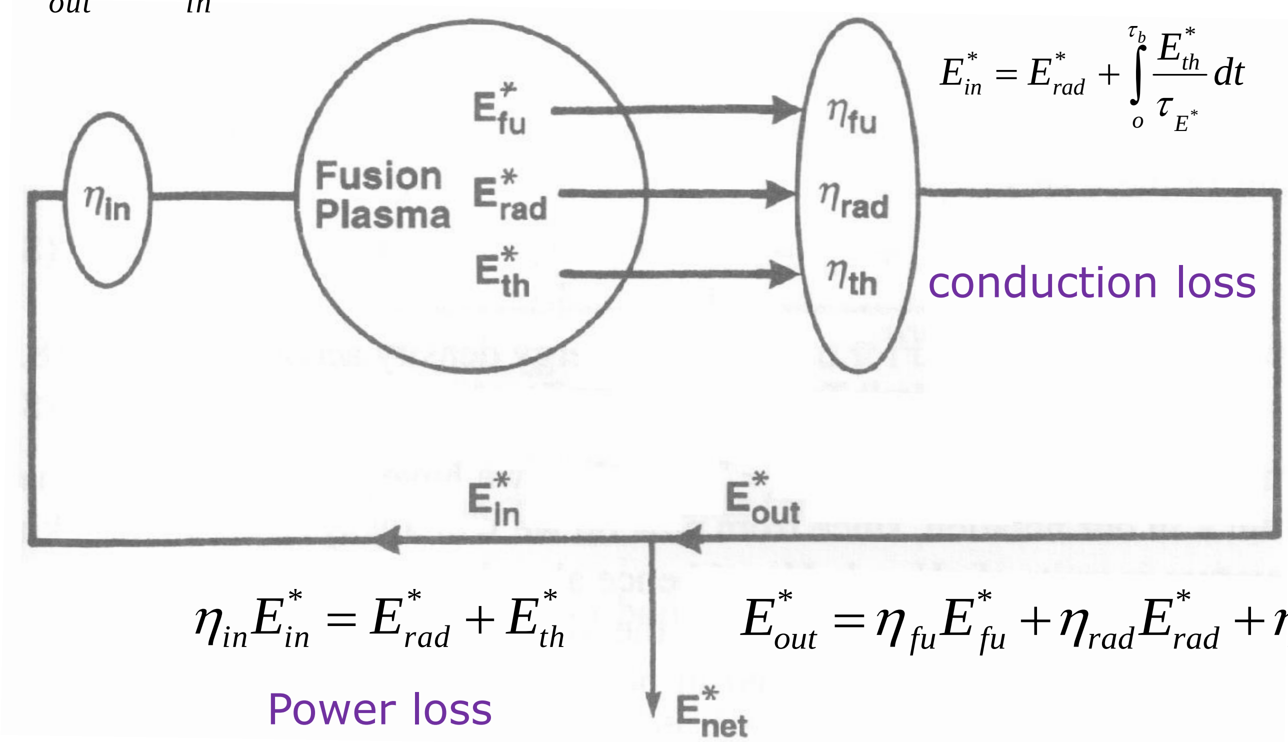
- **Lawson criterion**

  - The recoverable energy from a fusion reactor must exceed the energy which is supplied to sustain the fusion reaction.

$$E^*_{out} > E^*_{in}$$

$$E^*_{out} = E^*_{aux} + E^*_{fu} = \eta_{in} E^*_{in} + E^*_{alpha} + E^*_n$$

$$E^*_{in} = E^*_{rad} + \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt$$



conduction loss

$$\eta_{in} E^*_{in} = E^*_{rad} + E^*_{th}$$

Power loss

$$E^*_{out} = \eta_{fu} E^*_{fu} + \eta_{rad} E^*_{rad} + \eta_{th} E^*_{th}$$

conduction loss

Fusion Reactor Energetics

- **Lawson criterion**

- output electric energy (recoverable energy) > required input energy

$$E^*_{out} > E^*_{in}$$

$$E^*_{out} = \eta_{fu} E^*_{fu} + \eta_{rad} E^*_{rad} + \eta_{th} E^*_{th}$$

$$\eta_{in} E^*_{in} = E^*_{rad} + E^*_{th}$$

$$\eta_{fu} E^*_{fu} + \eta_{rad} E^*_{rad} + \eta_{th} E^*_{th} > \frac{E^*_{rad} + E^*_{th}}{\eta_{in}}$$

$$\eta_{in} \eta_{out} (E^*_{fu} + E^*_{rad} + E^*_{th}) > E^*_{rad} + E^*_{th}$$

$$\eta_{out} = \frac{\sum_l \eta_l E_l}{\sum_l E_l}, \; l = fu, rad, th$$

average conversion efficiency

$$E^*_l = \tau_{E*} \int_V P_l(\vec{r}) d^3 r$$

global energy terms

Fusion Reactor Energetics

Assuming, Bremsstrahlung only

$$\eta_{in}\eta_{out}\int_V d^3r(\tau_{E*}P_{fu} + \tau_{E*}P_{br} + 3nT) > \int_V d^3r(\tau_{E*}P_{br} + 3nT)$$

$$E_{th}(\vec{r}) = \frac{3}{2}(n_i T_i + n_e T_e) = 3nT$$

Assuming, homogeneity throughout the plasma volume *V*

$$\eta_{in}\eta_{out}\left(\frac{n_a n_b}{1+\delta_{ab}} < \sigma v >_{ab} Q_{ab}\tau_{E*} + A_{br} n^2\sqrt{T}\tau_{E*} + 3nT\right) > A_{br} n^2\sqrt{T}\tau_{E*} + 3nT$$

Kronecker-δ introduced to account for the case of indistinguishable reactants

$$n\tau_{E*} > \frac{3(1-\eta_{in}\eta_{out})T}{\eta_{in}\eta_{out}\frac{< \sigma v >_{ab}(T)Q_{ab}}{4(1+\delta_{ab})} - (1-\eta_{in}\eta_{out})A_{br}\sqrt{T}}$$

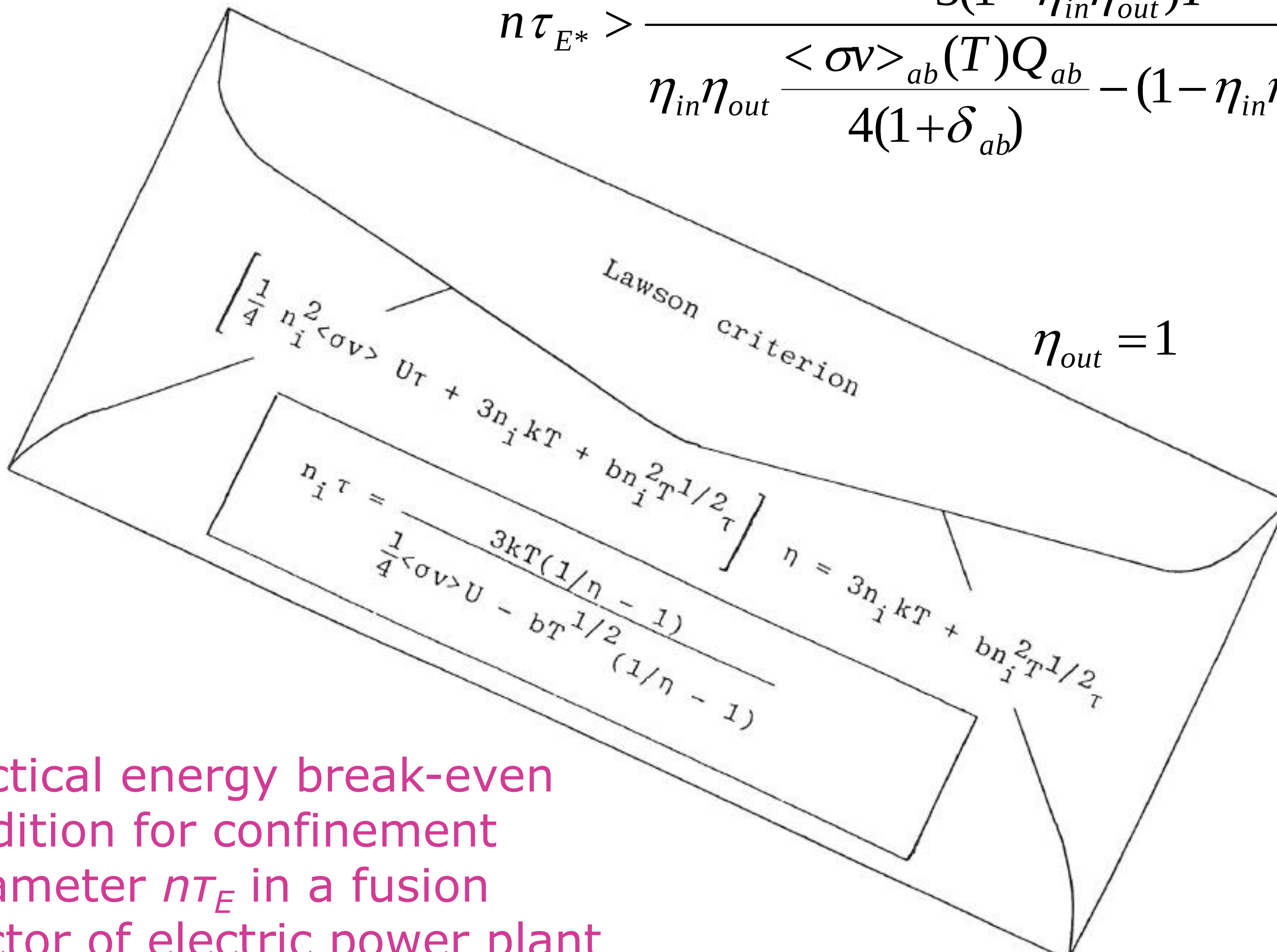
$$\eta_{in}\eta_{out} \approx 1/3$$

Fusion Reactor Energetics

- **Lawson criterion**

$$n\tau_{E*} > \frac{3(1-\eta_{in}\eta_{out})T}{\eta_{in}\eta_{out}\dfrac{<\sigma v>_{ab}(T)Q_{ab}}{4(1+\delta_{ab})} - (1-\eta_{in}\eta_{out})A_{br}\sqrt{T}}$$

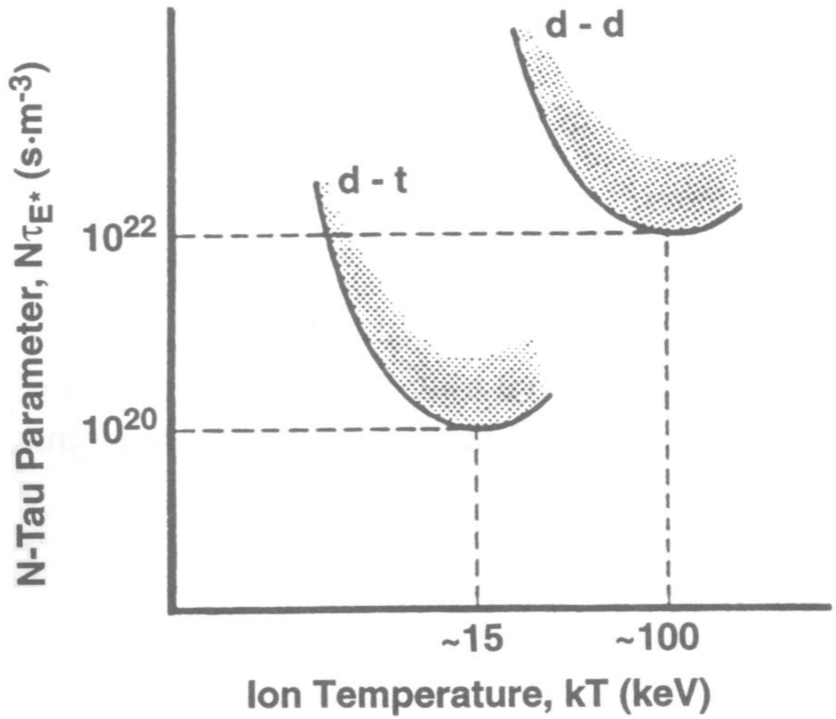
$$\eta_{out} = 1$$



- Practical energy break-even condition for confinement parameter $n\tau_E$ in a fusion reactor of electric power plant

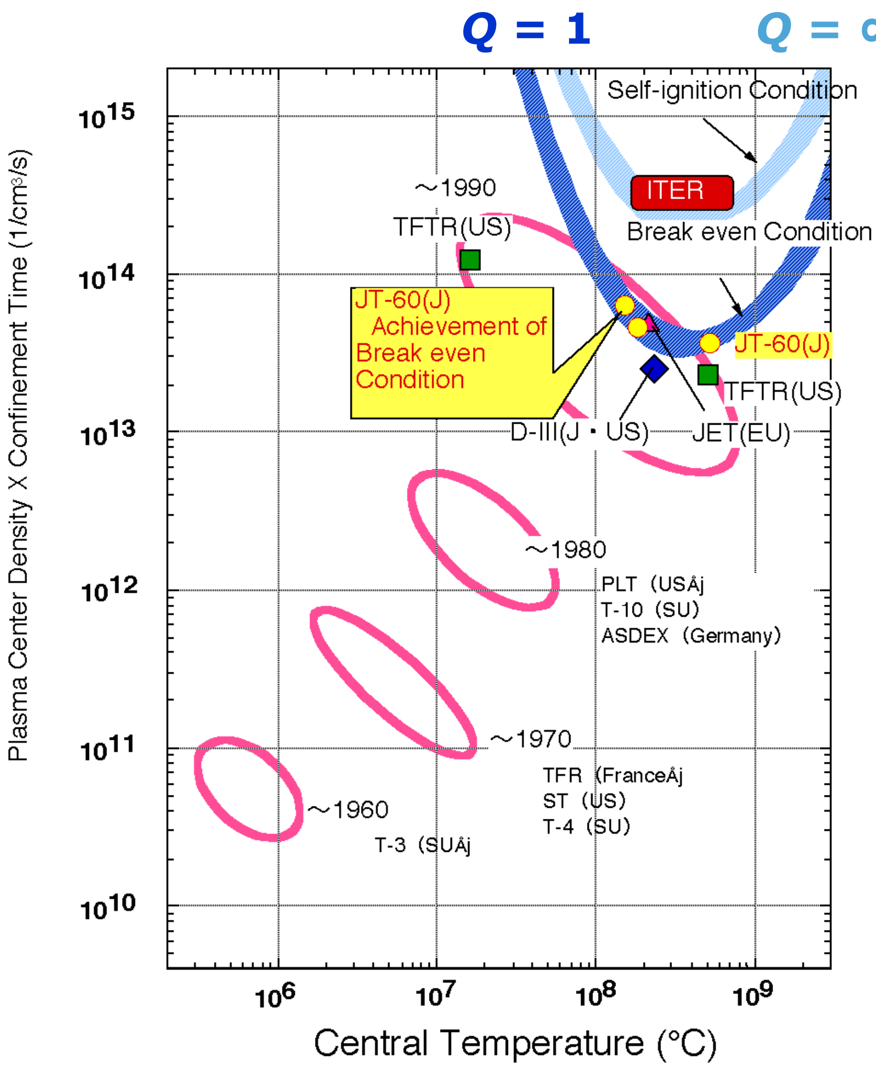
Fusion Reactor Energetics

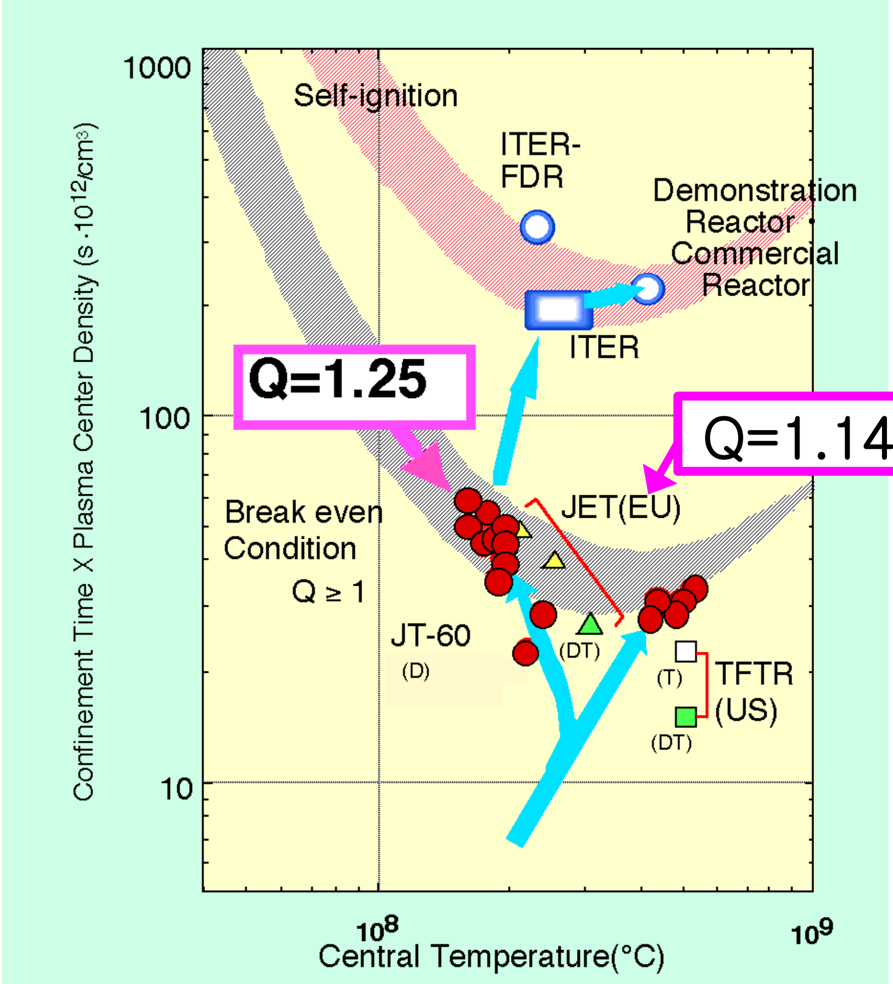
- **Lawson criterion**



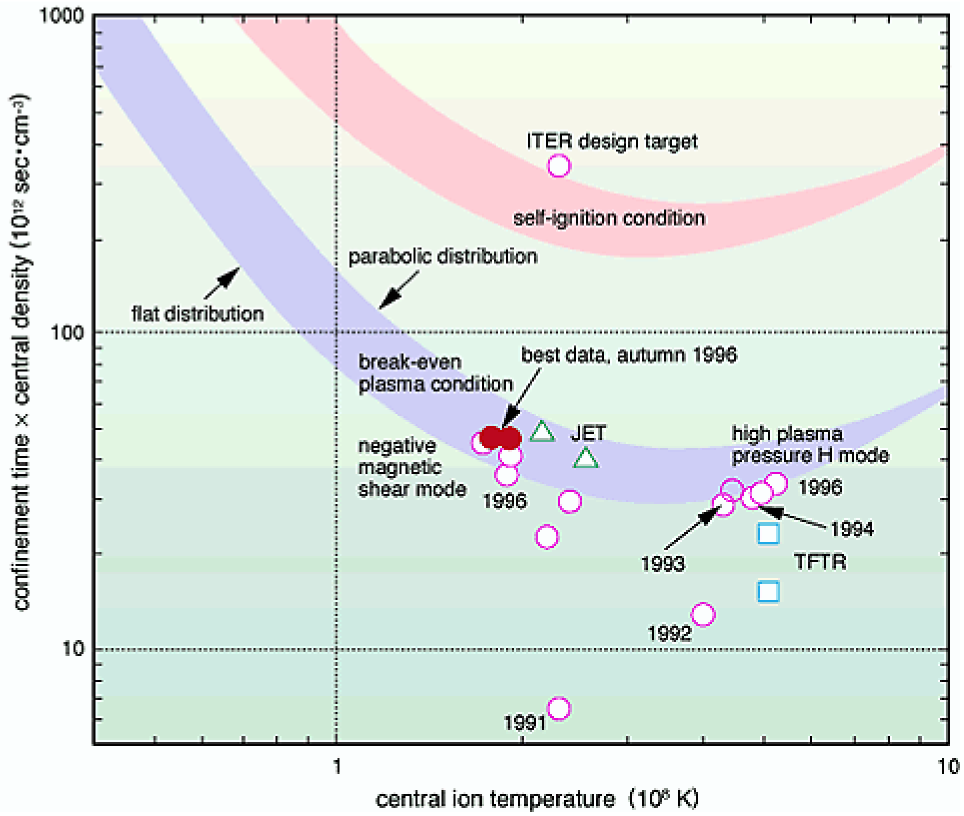
- No particular fusion design was necessary in the derivation of this criterion.
- Although it does not contain all relevant processes such as cyclotron radiation, it is a useful and widely employed criterion.
- For commercial power applications, it would be necessary to exceed the minimum Lawson limit by perhaps a factor of ten or better.

Status of the Tokamak Research





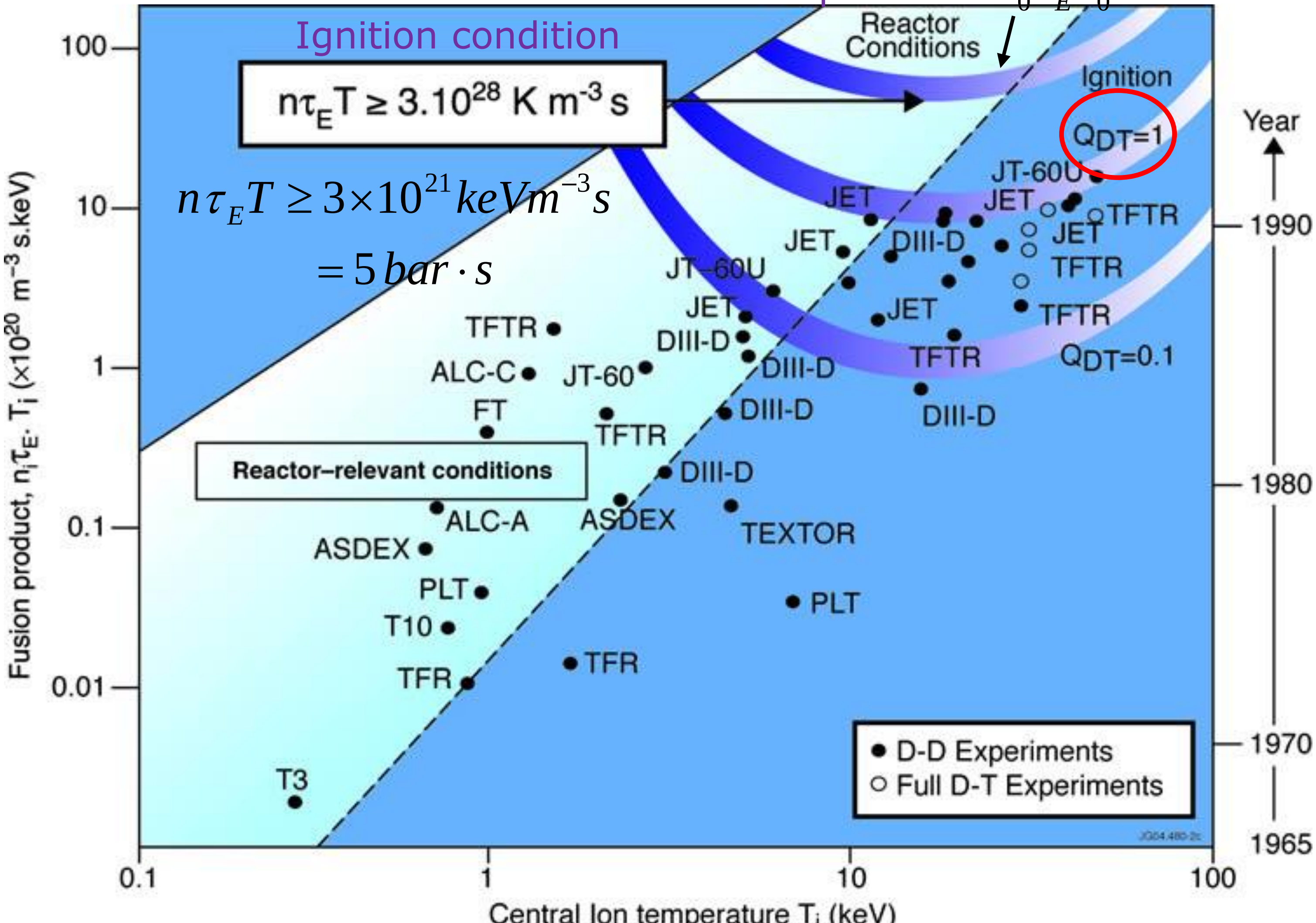
Status of the Tokamak Research



Status of the Tokamak Research

with realistic profiles: $n_0 \tau_E T_0 \geq 5\times10^{21} keV m^{-3} s$

Ignition condition



$$n\tau_E T \geq 3\times10^{21} keV m^{-3} s$$

$$= 5\, bar \cdot s$$

$$1\, atm = 1.01325\, bar$$

$<\sigma v>_{dt} \propto T^2$ at 10-20 keV $\rightarrow n\tau_{E*}T$

Status of the Tokamak Research

- DT-Experiments only in
- JET
- TFTR

- with world records in JET:
- P_{fusion} = 16 MW
- Q = 0.65

Q_{tot} ?