Direct Energy Conversion (cont'd)

— Field-Reversed Theta Pinch

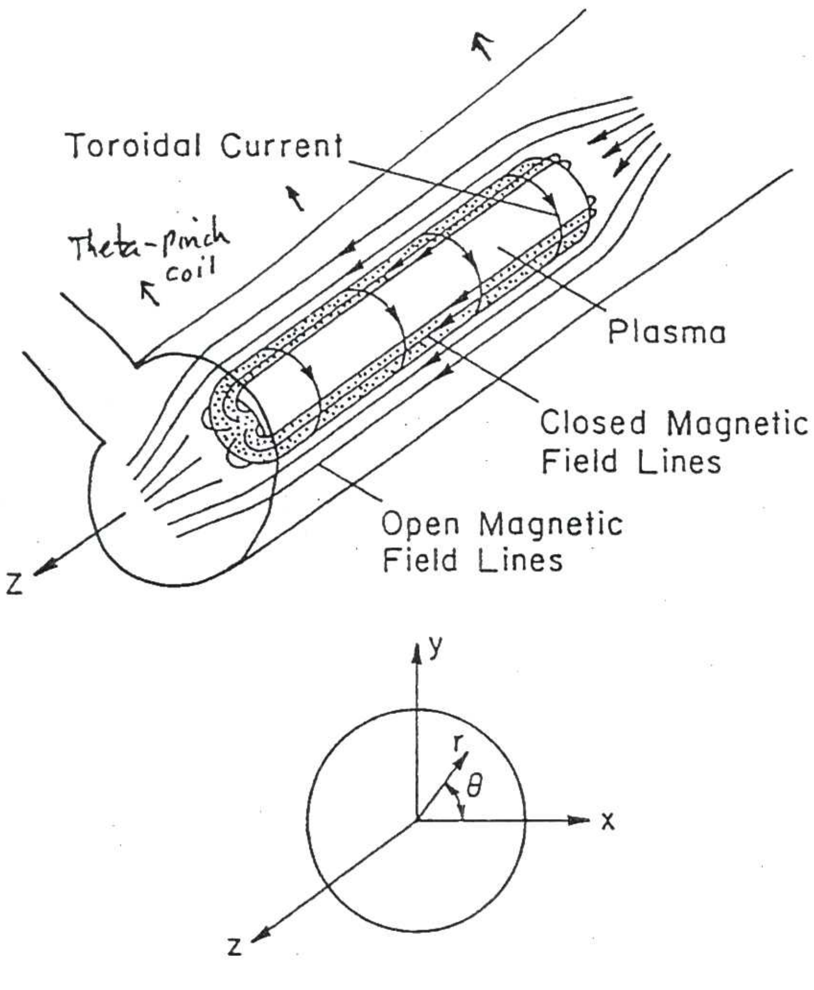


Figure 1.2 The geometry of FRTP configurations.

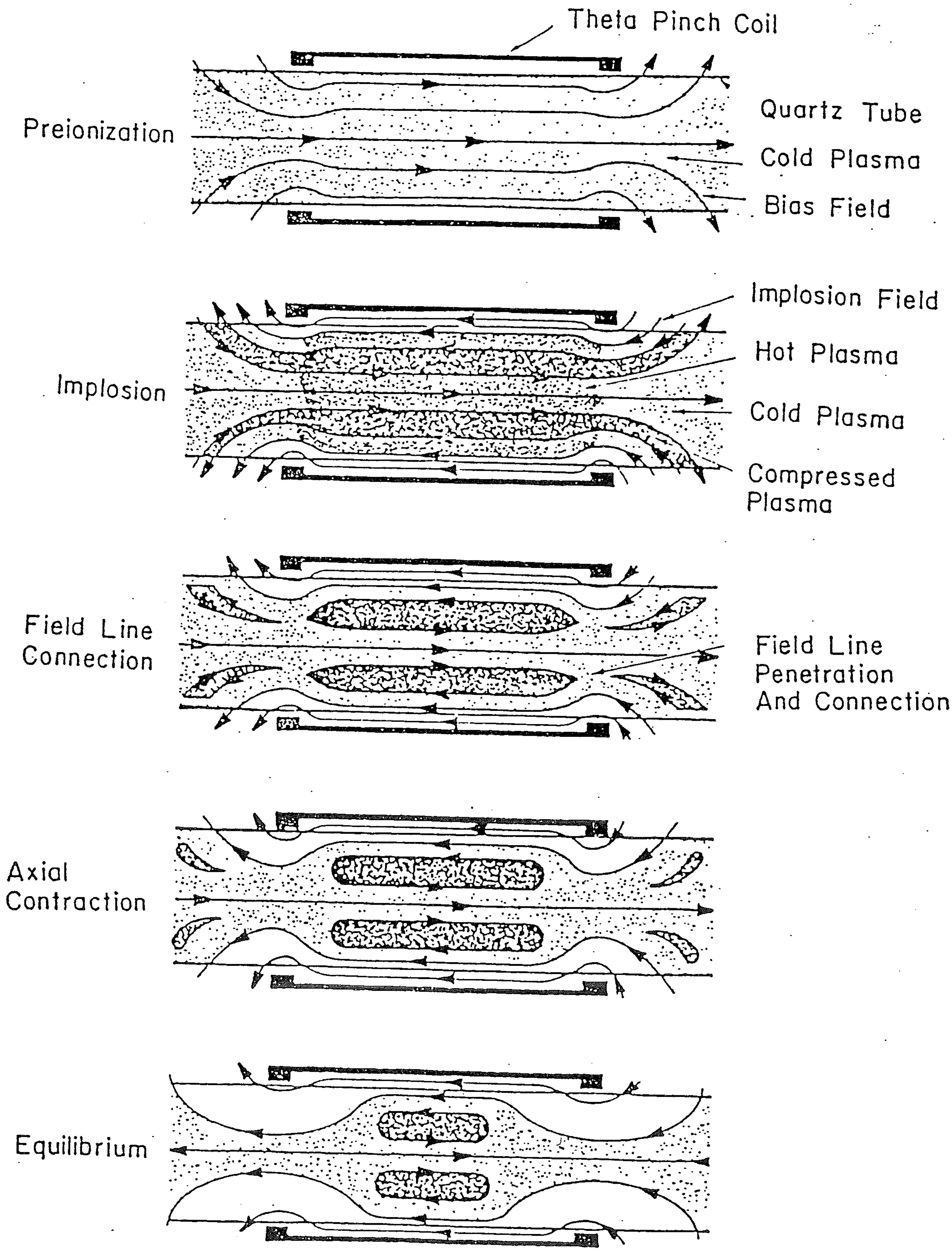


Figure 1.1. Formation of the FRTP.

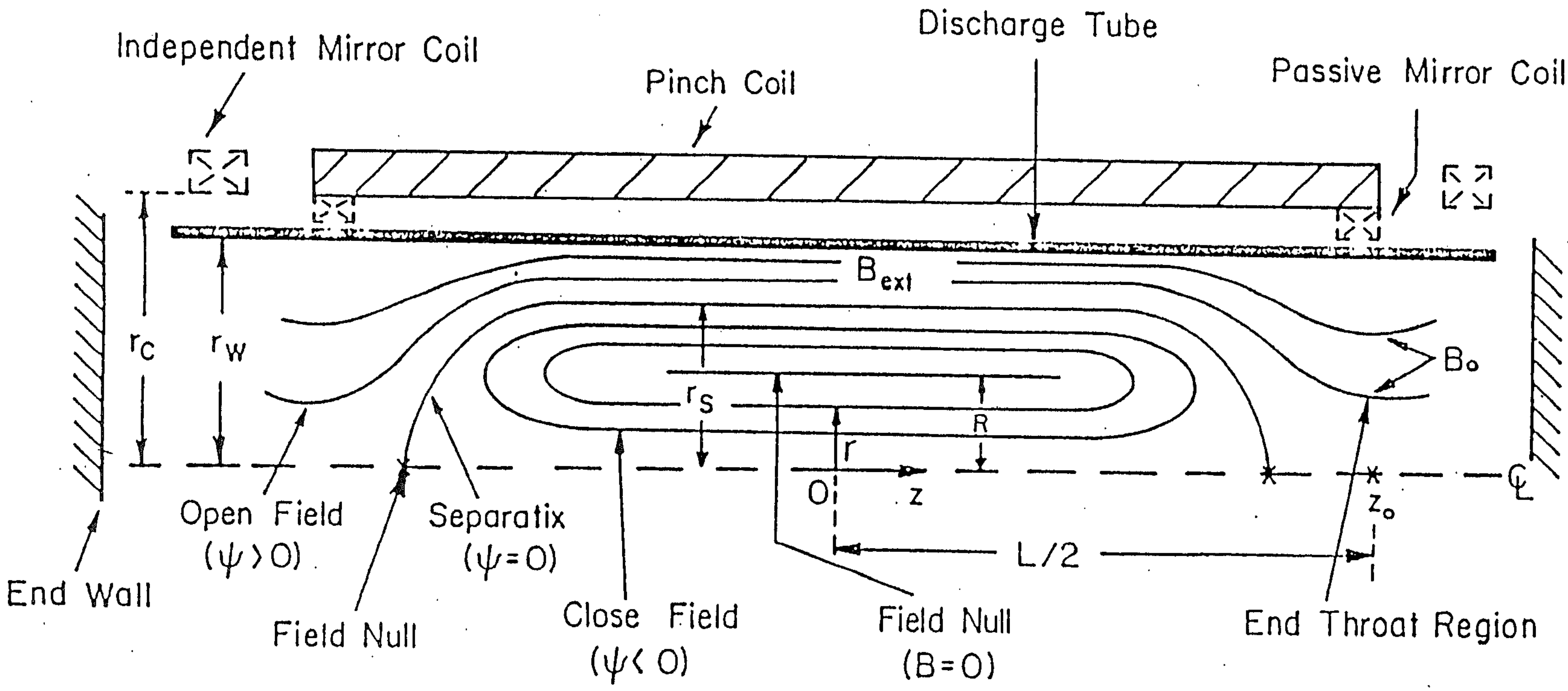
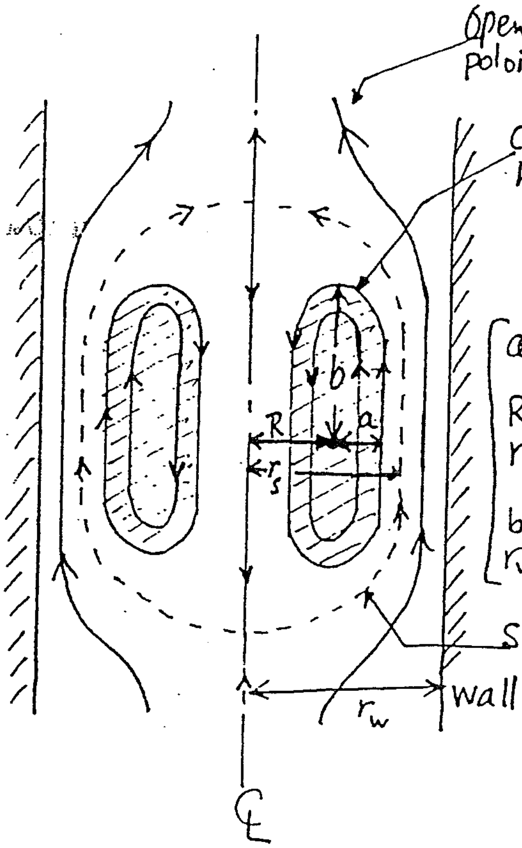


Figure 1.3 The magnetic field configuration and coil structure of a FRTP in the r-z plane

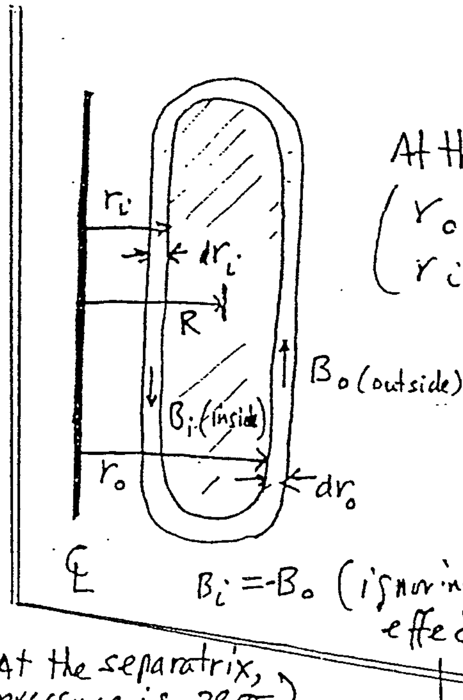
12B. Field-Reversed Theta Pinch (Continued)

equilibrium & stability



Note:
[a = plasma minor radius
R = major radius
r_s = separatrix radius
b = half-length
r_w = wall radius]

Note: At the separatrix, (plasma pressure is zero) and (magnetic field pressure is maximum) → $r_o = r_s$, $r_i = 0$



At the separatrix,
$\left(\begin{array}{l} r_o = r_s \\ r_i = 0 \end{array} \right)$

$B_i = -B_o$ (ignoring curvature effects)

Magnetic flux : $\psi \equiv \pi r^2 \cdot B$

The flux between adjacent surfaces,

$d\psi = 2\pi r_i \, dr_i B_i = 2\pi r_o \, dr_o \, B_o$

(At equil. plasma pressure): $P(r_i) = P(r_o)$

Recall $B_i = -B_o$

$$\boxed{\begin{aligned} \frac{d\psi}{2\pi B_o} &= r_o \, dr_o \\ &= - r_i \, dr_i \end{aligned}}$$

Axial Force Balance Equ.: (§)

$$\boxed{\begin{aligned} &\int_{0}^{r_s} 2\pi r \left(p - \frac{B_{plasma}^2}{2\mu_o} \right) dr \quad (1) \; + \\ &\text{mid plane} \\ &+ \int_{r_s}^{r_w} 2\pi r \left(- \frac{B_{out}^2}{2\mu_o} \right) dr \quad (2) \\ &\text{mid plane} \\ = &\int_{0}^{r_w} 2\pi r \left(- \frac{B_{end}^2}{2\mu_o} \right) dr \quad (3) \\ &\text{end region} \qquad \text{press. along mag. fld.} \end{aligned}}$$

[Note: true pressure ⊥ to fld.]

From $-r_i\,dr_i = r_o\,dr_o$ and after integration wrt r_i and r_o;

$$R^2 - r_i^2 = r_o^2 - R^2 \quad \cdots\cdots \quad (*)$$

At the separatrix; $r_o = r_s$ and $r_i = 0$

& Equ. (*) becomes $R^2 = r_s^2 - R^2$.

$$\therefore \boxed{\frac{r_s}{R} = \sqrt{2}} \quad (\text{independent of pressure } P(\psi)).$$

From radial pressure balance

$$p + \frac{B_{plasma}^2}{2\mu_0} = \frac{B_{out}^2}{2\mu_0} \quad \text{——} \quad (1)$$

Also, from magnetic flux conservation,

$$\underbrace{\pi r_w^2 \cdot B_{end}}_{\text{fld. at the ends}} = \pi(r_w^2 - r_s^2)\underbrace{B_{out}}_{\text{fld. outside the plasma at the mid plane}} \quad \text{——} \quad (2)$$

Then, the axial force balance equ. (§) on the previous page becomes:

[e.g., use eq. (1) and (2) above to write B_{plasma} and B_{end} in terms of B_{out} such as:]

$$① \int_0^{r_s} 2\pi r \underbrace{\left(p - \frac{B_{plasma}^2}{2\mu_0}\right)}_{p - \left(-p + \frac{B_{out}^2}{2\mu_0}\right) \text{ from eq. (1)} \;\Rightarrow\; \left(2p - \frac{B_{out}^2}{2\mu_0}\right)} dr \Rightarrow 2\int_0^{r_s} dr\, 2\pi r\, p - \underbrace{\int_0^{r_s} dr\, 2\pi r \frac{B_{out}^2}{2\mu_0}}_{\left(2\pi \frac{B_{out}^2}{2\mu_0}\right)\cdot\frac{r_s^2}{2} \;=\; \pi r_s^2 \cdot \frac{B_{out}^2}{2\mu_0}}$$

mid plane

$$\Rightarrow 2\int_0^{r_s} dr \cdot 2\pi r p - \pi r_s^2 \cdot \frac{1}{2\mu_0} \cdot B_{out}^2 \quad \text{—} \quad (3)$$

③ $$\int_{0}^{r_w} 2\pi r\left(-\frac{B_{end}^2}{2\mu_0}\right)dr = -\frac{1}{2\mu_0}\left[\frac{(r_w^2-r_s^2)}{r_w^2}\cdot B_{out}\right]^2\cdot\cancel{2}\pi\cdot\underbrace{\frac{r_w^2}{\cancel{2}}}_{(4)}$$

end

$$\frac{B_{end}^2}{2\mu_0}:\quad -\frac{1}{2\mu_0}\left[\frac{\cancel{\pi}(r_w^2-r_s^2)}{\cancel{\pi}r_w^2}\cdot B_{out}\right]^2$$

(from eq.(2))

∴ Axial force balance equ. becomes from equ (5), (3) + (4) :

$$\left(2\int_0^{r_s} dr\,2\pi r\,p - \pi r_s^2\cancel{\frac{B_{out}^2}{2\mu_0}}\right) + \underbrace{\int_{r_s}^{r_w} 2\pi r\left(-\frac{B_{out}^2}{2\mu_0}\right)dr}_{\cancel{2}\pi\cdot\left(-\frac{B_{out}^2}{2\mu_0}\right)\cdot\frac{(r_w^2-\cancel{r_s^2})}{\cancel{2}}} =$$

$$=\left(-\frac{1}{2\mu_0}\right)\underbrace{\left[\frac{(r_w^2-r_s^2)}{r_w^2}\right]^2\left[B_{out}^2\right]}\cdot\pi r_w^2$$

$$\Rightarrow \boxed{2\int_0^{r_s} dr\cdot 2\pi r\,p - \frac{\pi r_w^2 B_{out}^2}{2\mu_0} = -\underbrace{\pi r_w^2\left[1-\left(\frac{r_s}{r_w}\right)^2\right]^2}\frac{B_{out}^2}{2\mu_0}}$$

Recall $$\langle\beta\rangle \equiv \frac{\int_0^{r_s} 2\pi r\,dr\,p}{\left(\frac{B_{out}^2}{2\mu_0}\cdot\pi r_s^2\right)}$$

Then,

$$2\langle\beta\rangle\cdot\left(\frac{\cancel{B_{out}^2}}{\cancel{2\mu_0}}\cdot\cancel{\pi}r_s^2\right) - \frac{\cancel{\pi}r_w^2\cdot\cancel{B_{out}^2}}{\cancel{2\mu_0}} = -\cancel{\pi}r_w^2\left[1-\left(\frac{r_s}{r_w}\right)^2\right]^2\frac{\cancel{B_{out}^2}}{\cancel{2\mu_0}}$$

$$\Rightarrow \langle\beta\rangle = \frac{1}{2r_s^2}\left\{-r_w^2\left[1-\left(\frac{r_s}{r_w}\right)^2\right]^2 + r_w^2\right\} = \frac{1}{2}\left(\frac{r_w}{r_s}\right)^2\left\{1-[\ ]^2\right\}$$

$$\langle\beta\rangle = \frac{1}{2}\left(\frac{r_w}{r_s}\right)^2 \left\{ \underbrace{1 - \left[1 - \left(\frac{r_s}{r_w}\right)^2\right]^2}_{\underbrace{\cancel{1} - \left[\cancel{1} - 2\left(\frac{r_s}{r_w}\right)^2 + \left(\frac{r_s}{r_w}\right)^4\right]}_{2\left(\frac{r_s}{r_w}\right)^2 - \left(\frac{r_s}{r_w}\right)^4}} \right\}$$

$$\therefore \boxed{\langle\beta\rangle = 1 - \frac{1}{2}\left(\frac{r_s}{r_w}\right)^2}$$ (indep. of pressure $p(\psi)$)

→ smaller r_s, (steeper pressure gradient & more rapid pcle & heat transport)

→ for non-rotating plasma,

$\frac{r_s}{r_w} \gtrsim 0.4$ helps stabilize radial displacement MHD instab modes (w/ $n \lesssim 3$.)

→ plasma rot^n at ω_{rot} produces $n=2$ rotational flute modes which terminate plasma confinement

when $\frac{-\omega_{rot}}{\omega_*} \geq 1.5$ where $\omega_* = k_\theta \underset{\sim}{v_{de}}$

diamag drift freq. — e^- diamag drift vel.

[Note: $\frac{-\omega_{rot}}{\omega_*} \sim \beta$]

Gradients and electron drift waves

Gradients of plasma density and temperature in a cylindrical or toroidal plasma produce "waves" propagating "around" the plasma in azimuthal (θ) direction.

Consider "radial" density gradient for simplicity for:

- uniform T_e
- quasineutrality, $n_e = n_i$
- obey the Boltzmann thermal relation, $n_e = n_0 e^{e\phi/kT_e}$ $(\phi_0 = 0)$
- $\vec{B} = (0, 0, B_z)$ and uniform, $B_z = B$
- uniform plasma in z-direction, $\frac{\partial}{\partial z} = 0$
- $\frac{\vec{E}\times\vec{B}}{B^2}$ drift is dominant ~~for~~ ion $\perp$ motion,
  e.g., $u_{ir} = \frac{E_\theta}{B} = -\frac{1}{r}\left(\frac{\partial\phi}{\partial\theta}\right)/B$, $u_{i\theta} = -\frac{E_r}{B} = \left(\frac{\partial\phi}{\partial r}\right)/B$
- ignore ionization and recombination, $\langle\sigma_i v_i\rangle = 0$, $\langle\sigma_e v_e\rangle = 0$, $\langle\sigma_r v\rangle = 0$
- $\frac{\partial n_0}{\partial\theta} = 0$

Recall particle conservation for ions, (assumed)

$$\frac{\partial n_i}{\partial t} + \vec{\nabla}\cdot(n_i\vec{u}_i) = n_n n_i \cancel{\langle\sigma_i v_i\rangle}^{0} + n_n n_e \cancel{\langle\sigma_e v_e\rangle}^{0} - n_e n_i \cancel{\langle\sigma_r v\rangle}^{0}$$

Note: $\vec{\nabla}\cdot\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial\theta} + \frac{\partial A_z}{\partial z}$ (in cylinder)

$$\frac{\partial n_i}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r n_i u_{ir}) + \frac{1}{r}\frac{\partial}{\partial\theta}(n_i u_{i\theta}) = 0 \quad \cdots\cdots(1)$$

For n_e, replace $n_i \to n_e$ to get

$$\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r n_e u_{ir}) + \frac{1}{r}\frac{\partial}{\partial\theta}(n_e u_{i\theta}) = 0 \quad \cdots\cdots(2)$$

Eq.(2): $\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(r n_e u_{ir}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(n_e u_{i\theta}\right) = 0$

Note: $n_e = n_0 \exp[e\phi/kT_e]$

$u_{ir} = -\frac{1}{r}\left(\frac{\partial \phi}{\partial \theta}\right)/B$

$u_{i\theta} = \left(\frac{\partial \phi}{\partial r}\right)/B$

Substitute

$$\Rightarrow \frac{\partial}{\partial t}\left(n_0 e^{e\phi/kT_e}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(\cancel{r}\cdot n_0 e^{e\phi/kT_e} \cdot \frac{\left(-\frac{\partial \phi}{\partial \theta}\right)}{\cancel{r}B}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(n_0 e^{e\phi/kT_e} \cdot \frac{\left(\frac{\partial \phi}{\partial r}\right)}{B}\right) = 0$$

$$\Rightarrow n_0 \frac{e}{kT_e}\left(\frac{\partial \phi}{\partial t}\right) e^{e\phi/kT_e} + \frac{1}{r}\left(\frac{-1}{B}\right)\left[\left(\frac{\partial n_0}{\partial r}\right)\cdot e^{e\phi/kT_e}\left(\frac{\partial \phi}{\partial \theta}\right) + \cancel{n_0 \frac{e}{kT_e}\left(\frac{\partial \phi}{\partial r}\right)\left(\frac{\partial \phi}{\partial \theta}\right) e^{e\phi/kT_e}} + \cancel{n_0 e^{e\phi/kT_e}\left(\frac{\partial^2 \phi}{\partial \theta \partial r}\right)}\right]$$

$$+ \frac{1}{r}\left(\frac{n_0}{B}\right)\left[\cancel{\frac{e}{kT_e}\left(\frac{\partial \phi}{\partial \theta}\right)\left(\frac{\partial \phi}{\partial r}\right) e^{e\phi/kT_e}} + \cancel{e^{e\phi/kT_e}\frac{\partial^2 \phi}{\partial r \partial \theta}}\right] = 0$$

$$\Rightarrow \left[\underbrace{n_0 \frac{e}{kT_e}\frac{\partial \phi}{\partial t} - \frac{1}{rB}\left(\frac{\partial n_0}{\partial r}\right)\left(\frac{\partial \phi}{\partial \theta}\right)}_{=\,0}\right] e^{e\phi/kT_e} = 0$$

with $\phi(\theta, t) = \phi_0 \exp[i k_\theta r\theta - i\omega t]$ (wave form)

$$\Rightarrow \frac{e n_0}{kT_e}(-\cancel{i}\omega)\phi - \frac{1}{rB}\frac{\partial n_0}{\partial r}(\cancel{i}k_\theta r)\phi = 0$$

$$\Rightarrow \frac{\omega}{k_\theta} = -\frac{kT_e}{e n_0}\cdot\frac{1}{\cancel{r}B}\left(\frac{\partial n_0}{\partial r}\right)\cdot\cancel{r}$$

phase velocity, V_{de} (electron drift velocity)

$$V_{de} \equiv \frac{\omega}{k_\theta} = -\frac{kT_e}{n_0 e B}\left(\frac{\partial n_0}{\partial r}\right) = -\frac{1}{n_0 e B}\frac{\partial (n_0 kT_e)}{\partial r} = -\frac{1}{n_0 e B}\frac{\partial P_e}{\partial r}\left(\frac{m}{sec}\right)$$

where $P_e = n_0 kT_e$ (pressure)

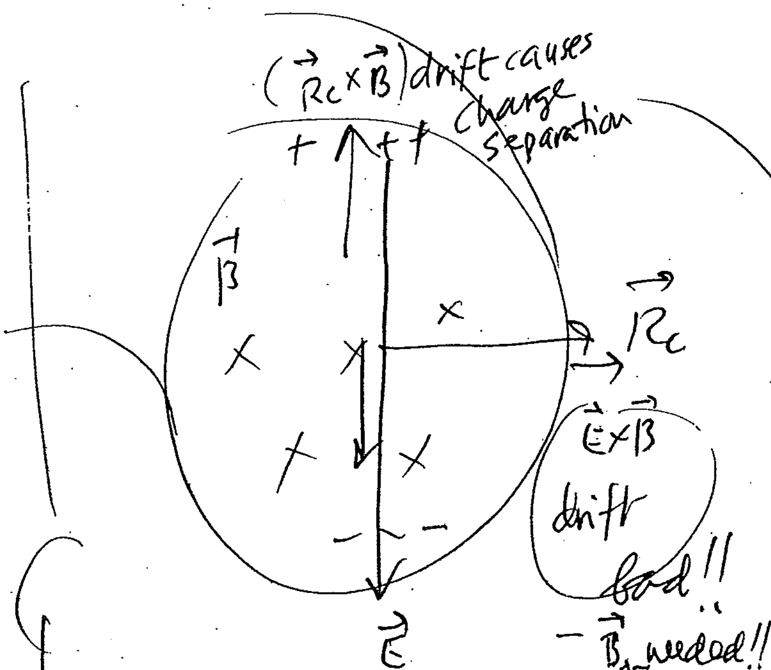
∴ electron drift velocity,

$$\boxed{V_{de} = -\frac{1}{n_0 e B}\left(\frac{\partial P_e}{\partial r}\right)}\left(\frac{m}{sec}\right)$$

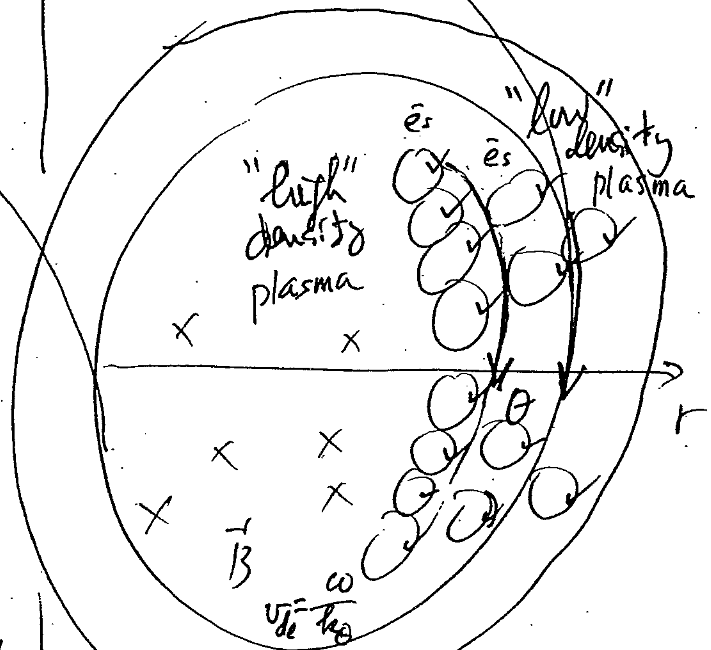
Note: V_{de} is the same velocity from $(-\nabla P) = n_e \cdot e \cdot (\vec{u}_e \times \vec{B})_r$

$$\underbrace{-\frac{\partial P_e}{\partial r} = n_e \cdot e \cdot u_\theta B_z}_{u_{e\theta} = \frac{-1}{n_e e B}\left(\frac{\partial P_e}{\partial r}\right)}$$

Ion Drift due to radius of curvature



Electron Drift due to "radial density gradient"



$$\vec{v}_d = \frac{\vec{F} \times \vec{B}}{qB^2}$$

$[$cf. $(q\vec{v}\times\vec{B} = q\vec{E})\times\vec{B}]$

$$\vec{F} = m\frac{v_{\parallel}^2}{R_c^2}\vec{R}_c$$

$$\vec{v}_d = \frac{m v_{\parallel}^2}{qB^2R_c^2}\vec{R}_c \times \vec{B}$$

$$\vec{v}_d = \frac{q\vec{E}\times\vec{B}}{qB^2}$$

$$\vec{v}_d = \frac{\vec{E}\times\vec{B}}{B^2}$$

drift vel. causes $\leftrightarrow$ oscillation

[Chen p. 220]

while wave moves up & down

$\vec{v}_d = \vec{E}\times\vec{B}$ pushes outward & ∇n pushes inward

stronger θ downward

$$u_{e\theta} = \frac{-\left(\frac{\partial p_e}{\partial r}\right)}{neB}$$

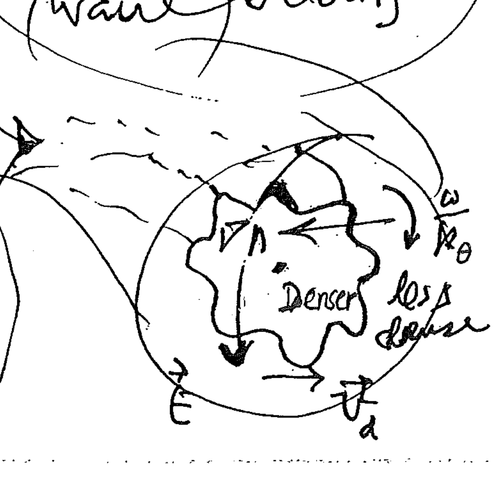
$$\parallel\parallel$$

$$v_{de} = \frac{\omega}{k_\theta}$$

$u_{e\theta}$ drift velocity

electron diamagnetic drift (wave) velocity

[Dolan, p. 202 ch. 8]



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Ref. Dolan, Fusion Research, Vol. I

we have

$$-eu_{e\theta}B - (\partial p_e/\partial r)/n = 0$$

$$u_{e\theta} = -(\partial p_e/\partial r)/neB \equiv v_{de} \quad , \qquad (8E18)$$

which is called the "electron diamagnetic drift velocity". This motion is not a guiding center drift, but an apparent flow, which is a manifestation of the existence of a pressure gradient perpendicular to the magnetic field. This average "flow" in the θ direction, caused by the pressure gradient, is illustrated in Fig. 8E5. Similarly, for the ions $v_{di} = (dp_i/dr)/neB$.

From simple assumptions about ions governed by the $\vec{E}\times\vec{B}/B^2$ drift, electrons obeying the Boltzmann relation, and quasineutrality, we find that a density gradient causes *drift waves* with $\omega/k_\theta = v_{de}$ to flow in the azimuthal direction around the plasma. A temperature gradient has a similar effect. A fluctuation of ϕ affects u_{ir} via the ion equation of motion (8E13); the change of u_{ir} affects n_e via the ion continuity equation (8E14) and quasineutrality; and the change of n_e affects the electrostatic potential via the Boltzmann relation (8E12) to perpetuate the wave. Thus, drift waves involve an oscillating E_θ, which propagates in the θ direction and causes u_{ir} to oscillate. Electrons flow along the magnetic field to compensate for ion space charge. Every confined plasma will have a pressure gradient, and therefore have drift waves, driven by the plasma expansion energy. The drift waves can become unstable if they couple to some means of energy dissipation, such as plasma resistivity or other plasma waves having

$$\omega \approx \omega_* \equiv k_\theta v_{de} \quad \text{(rad/s)}. \qquad (8E19)$$

Other waves available for such coupling include ion acoustic waves, ion cyclotron waves, and Alfven waves. Interactions with trapped particles in Tokamaks may also drive drift waves unstable.

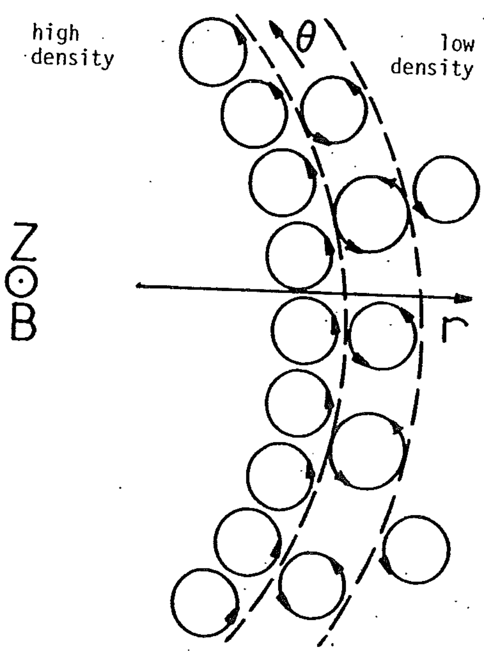

Fig. 8E5. Apparent electron flow due to electron density gradient. The magnetic field is out of the paper, and the circles represent electron Larmor rotation (counterclockwise). Along the dashed lines, it appears that there is more electron motion upward than downward, due to the density gradient.

The damping rate of the turbulent waves caused by microinstabilities is roughly $k_\perp^2 D$, where D is the effective diffusion coefficient. When this damping is comparable to the growth rate γ, the wave amplitude saturates. Hence, an estimate of the diffusion coefficient caused by the waves is

$D \lesssim \gamma/k_\perp^2$

The electi
crease the r
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Table 8E1.

instability

NON-MAXWELL
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ion acousti

ANISOTROPY

ion cyclot
electrosta

loss cone

GRADIENTS

drift wave

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drift

drift cyc

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drift

drift dis

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Ref. Chen, Plasma Phys & Controlled Fusion

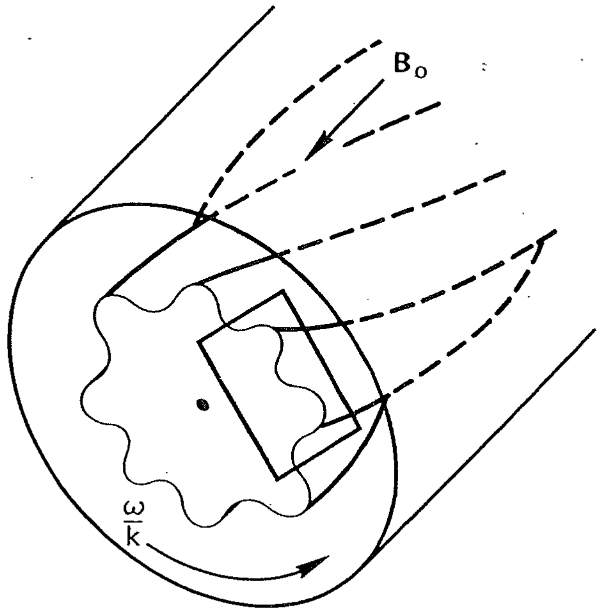
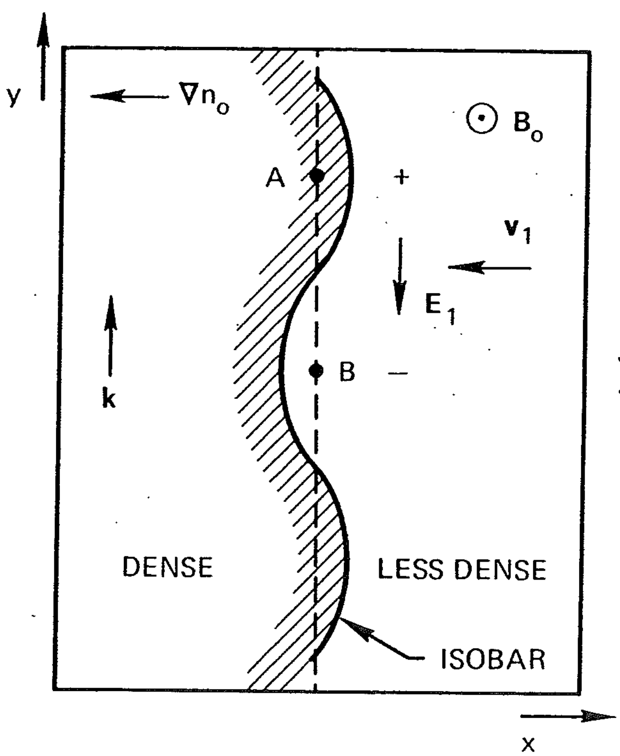


FIGURE 6-13 **Geometry of a drift instability in a cylinder. The region in the rectangle is shown in detail in Fig. 6-14.**



$\vec{E} \times \vec{B}$ drift velocity pushes plasmas outward

Density gradient, ∇n, pushes plasmas inward

Physical mechanism of a drift wave. **FIGURE 6-14**

US007391160B2

(12) **United States Patent**
Monkhorst et al.

(10) **Patent No.: US 7,391,160 B2**
(45) **Date of Patent: Jun. 24, 2008**

(54) **CONTROLLED FUSION IN A FIELD REVERSED CONFIGURATION AND DIRECT ENERGY CONVERSION**

(75) Inventors: **Hendrik J. Monkhorst**, Gainesville, FL (US); **Norman Rostoker**, Irvine, CA (US)

(73) Assignees: **Regents of the University of California**, Oakland, CA (US); **University of Florida Research Foundation**, Gainesville, FL (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 423 days.

(21) Appl. No.: **10/983,012**

(22) Filed: **Nov. 4, 2004**

(65) **Prior Publication Data**

US 2008/0069287 A1 Mar. 20, 2008

Related U.S. Application Data

(60) Division of application No. 10/410,830, filed on Apr. 9, 2003, now Pat. No. 6,850,011, which is a continuation of application No. 10/076,793, filed on Feb. 14, 2002, now Pat. No. 6,611,106.

(60) Provisional application No. 60/297,086, filed on Jun. 8, 2001, provisional application No. 60/277,374, filed on Mar. 19, 2001.

(51) **Int. Cl.**
H01J 7/24 (2006.01)

(52) **U.S. Cl.** **315/111.41**; 315/111.21; 315/111.81; 376/107; 376/127

(58) **Field of Classification Search** 315/111.21, 315/111.41, 111.61, 500–507; 376/107–130; 210/222, 695, 748, 787; 219/121.36, 121.52, 219/123; 250/251, 292
See application file for complete search history.

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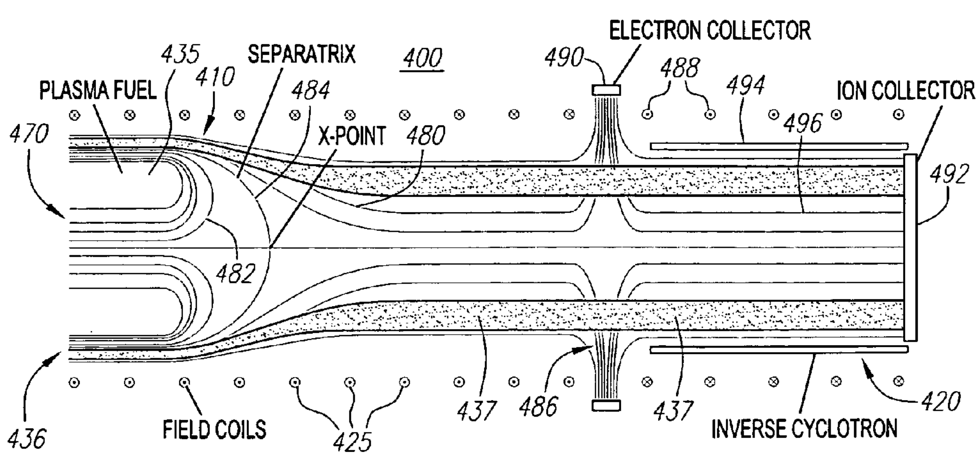
(Continued)

Primary Examiner—Thuy V. Tran
Assistant Examiner—Tung X Le
(74) *Attorney, Agent, or Firm*—Orrick, Herrington & Sutcliffe LLP

(57) **ABSTRACT**

A system and apparatus for controlled fusion in a field reversed configuration (FRC) magnetic topology and conversion of fusion product energies directly to electric power. Preferably, plasma ions are magnetically confined in the FRC while plasma electrons are electrostatically confined in a deep energy well, created by tuning an externally applied magnetic field. In this configuration, ions and electrons may have adequate density and temperature so that upon collisions they are fused together by the nuclear force, thus forming fusion products that emerge in the form of an annular beam. Energy is removed from the fusion product ions as they spiral past electrodes of an inverse cyclotron converter. Advantageously, the fusion fuel plasmas that can be used with the present confinement and energy conversion system include advanced (aneutronic) fuels.

39 Claims, 16 Drawing Sheets



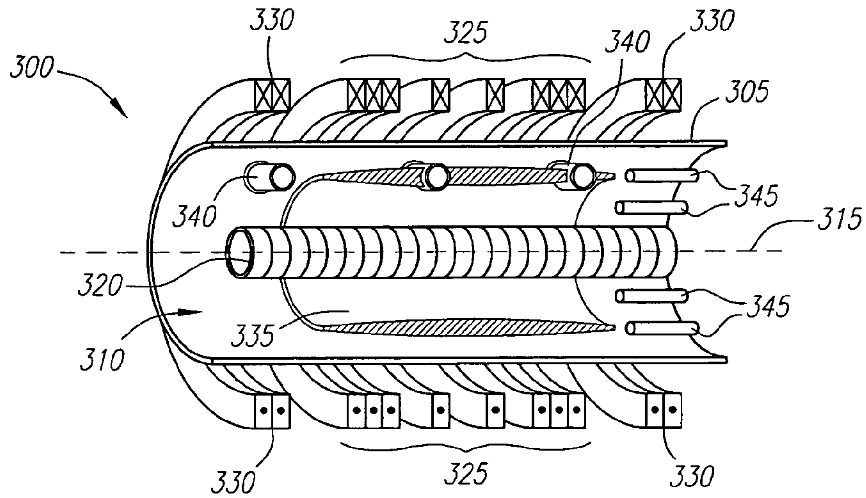

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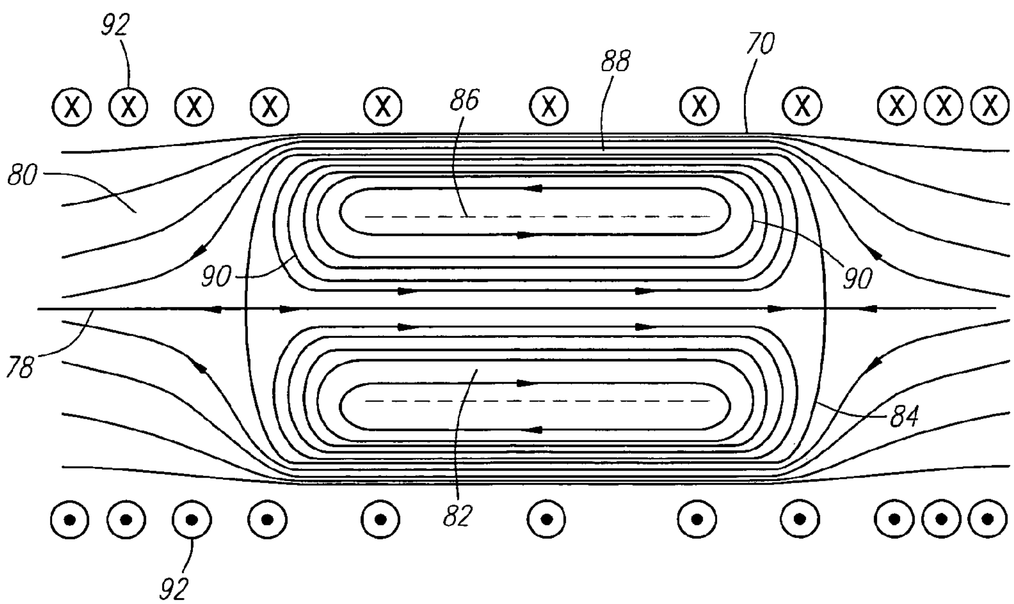
FIG. 1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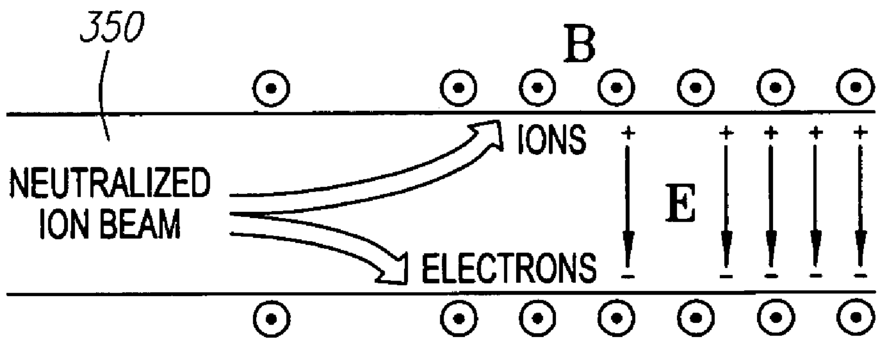
90
78
84
82
92

FIG. 2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B
NEUTRALIZED
ION BEAM
IONS
E
ELECTRONS

FIG. 14

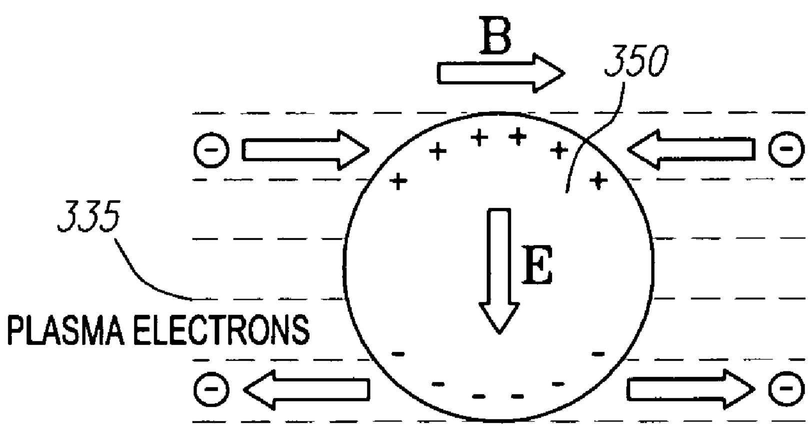

B
350
335
E
PLASMA ELECTRONS

FIG. 15

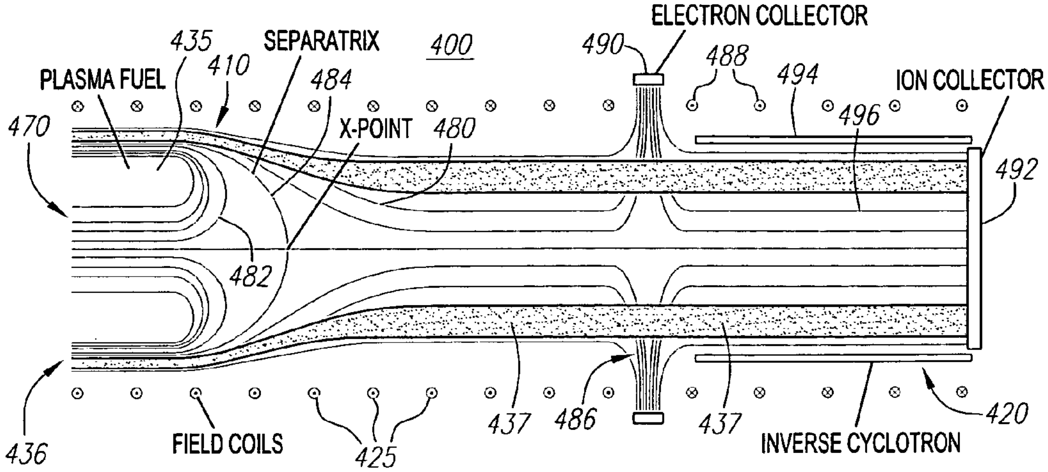


FIG. 19A

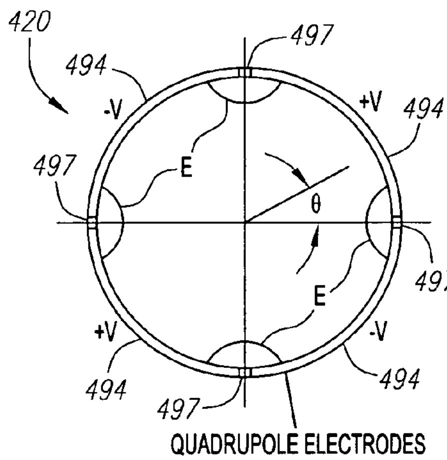


FIG. 19B

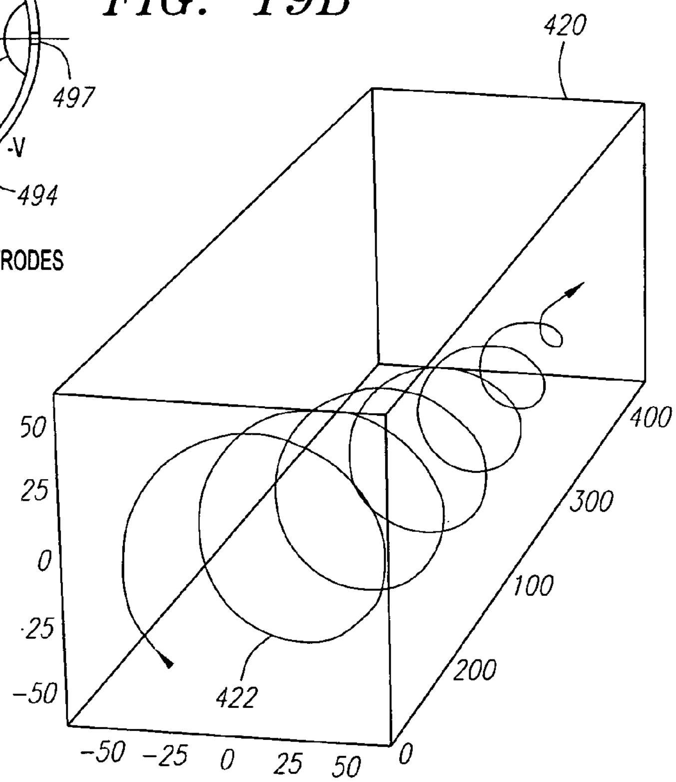


FIG. 19C

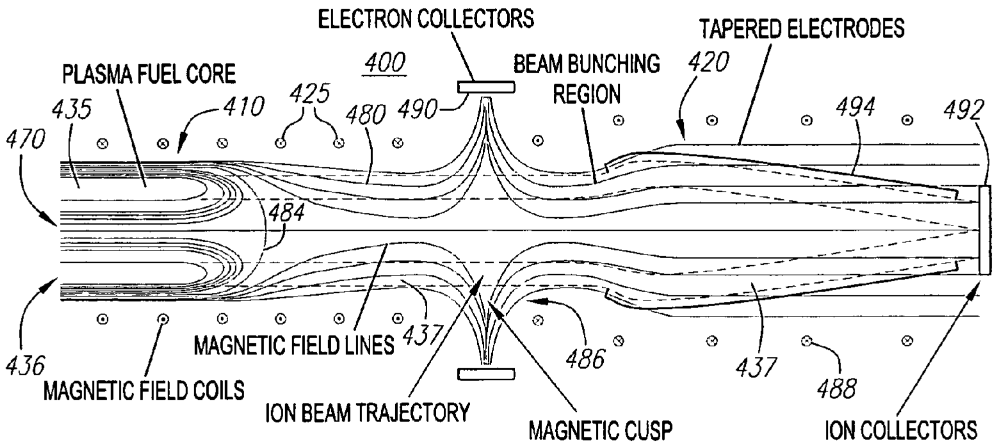


FIG. 20A

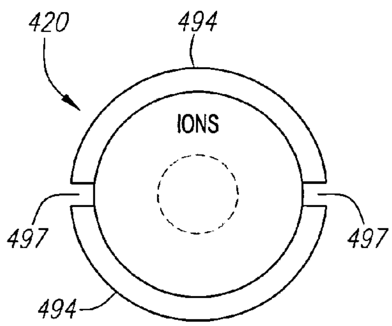


FIG. 20B

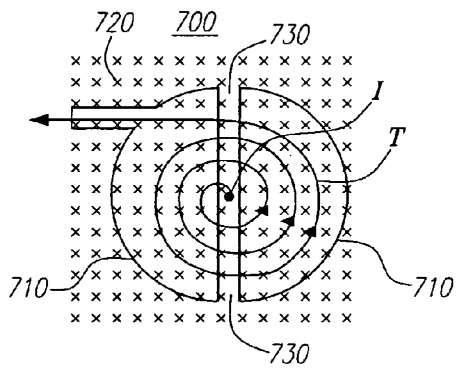


FIG. 21A

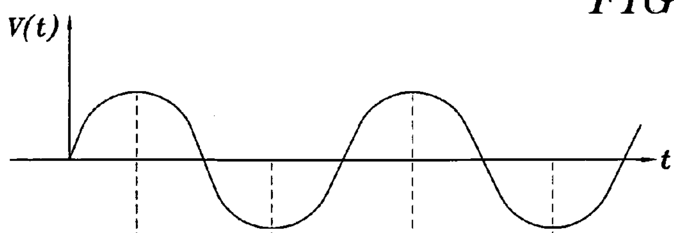


FIG. 21B

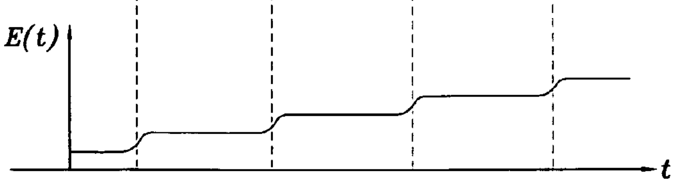


FIG. 21C

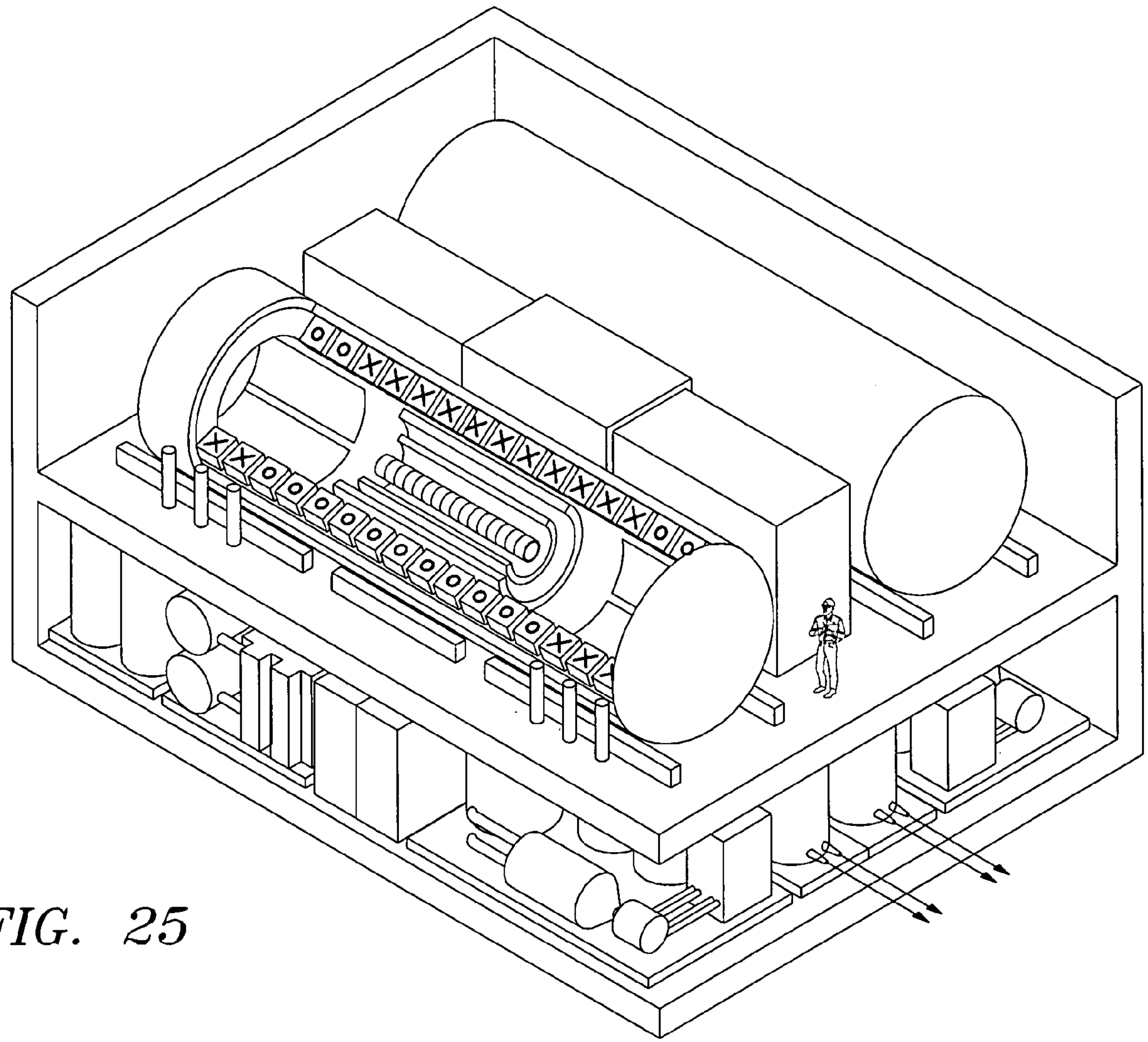
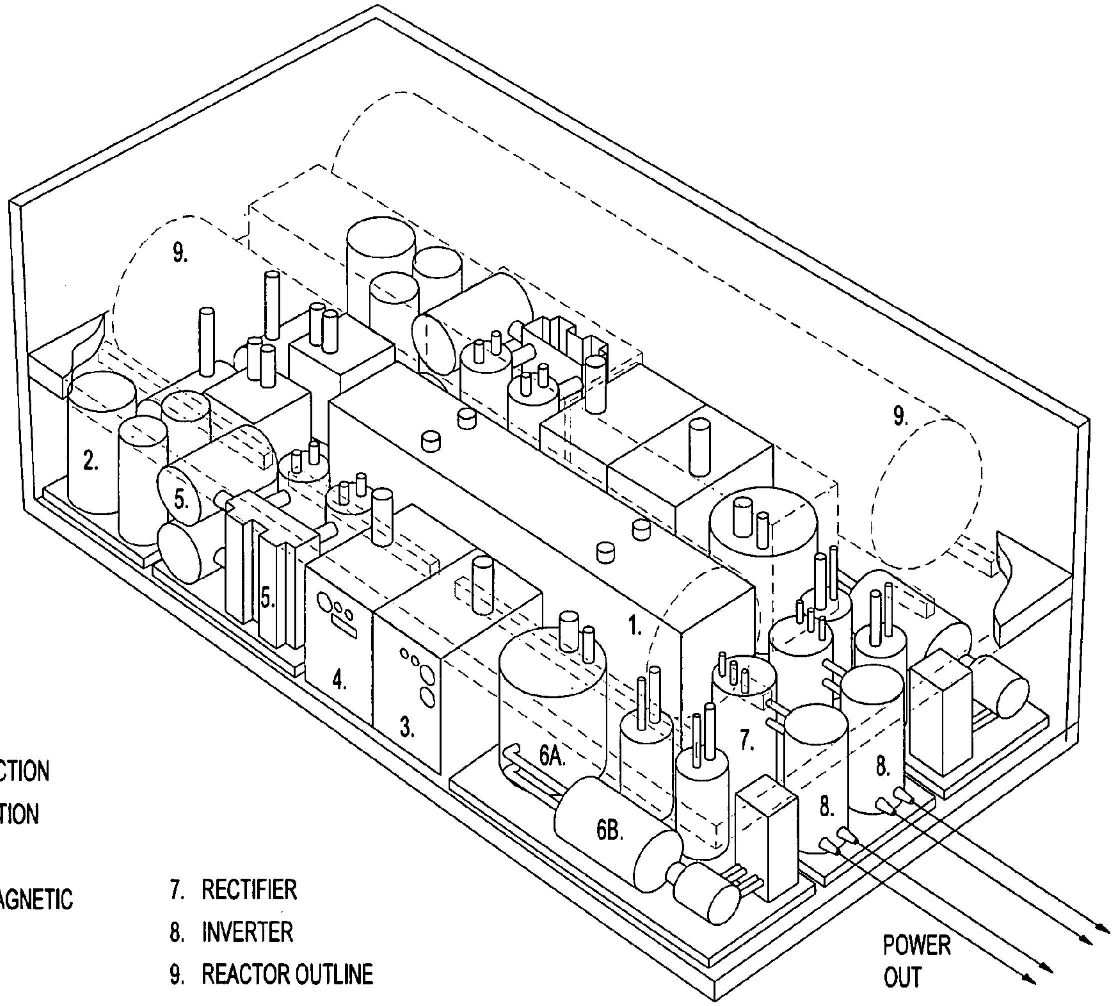


FIG. 25

FIG. 26



REACTOR
SUPPORT
EQUIPMENT

1. CAPACITOR BANK FOR INDUCTIVE START-UP
2. CRYOGENIC EQUIPMENT FOR SUPERCONDUCTING MAGNETS
3. ACCELERATOR FOR PROTON INJECTION
4. ACCELERATOR FOR BORON INJECTION
5. VACUUM PUMPS

6A. LITHIUM OR NAK TANK ELECTROMAGNETIC PUMP AND HEAT EXCHANGER

6B. STEAM TURBIN AND GENERATOR

7. RECTIFIER
8. INVERTER
9. REACTOR OUTLINE

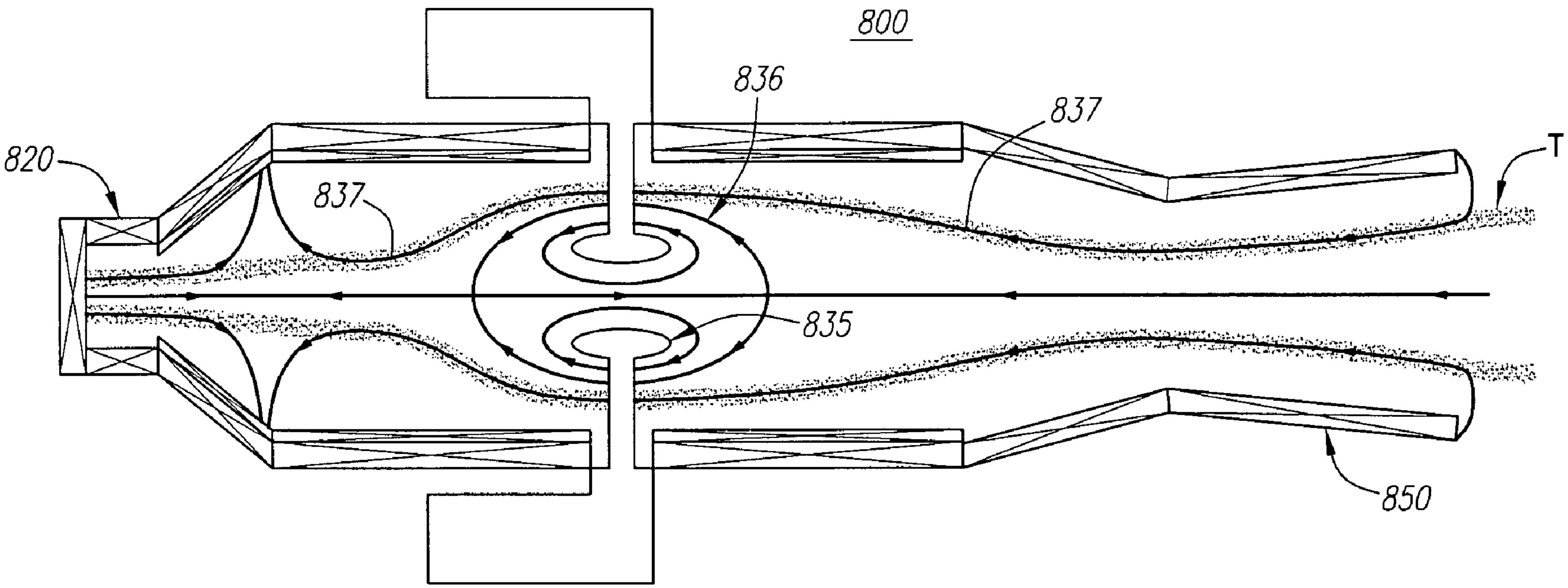


FIG. 27