

Fusion Reactor Technology 2

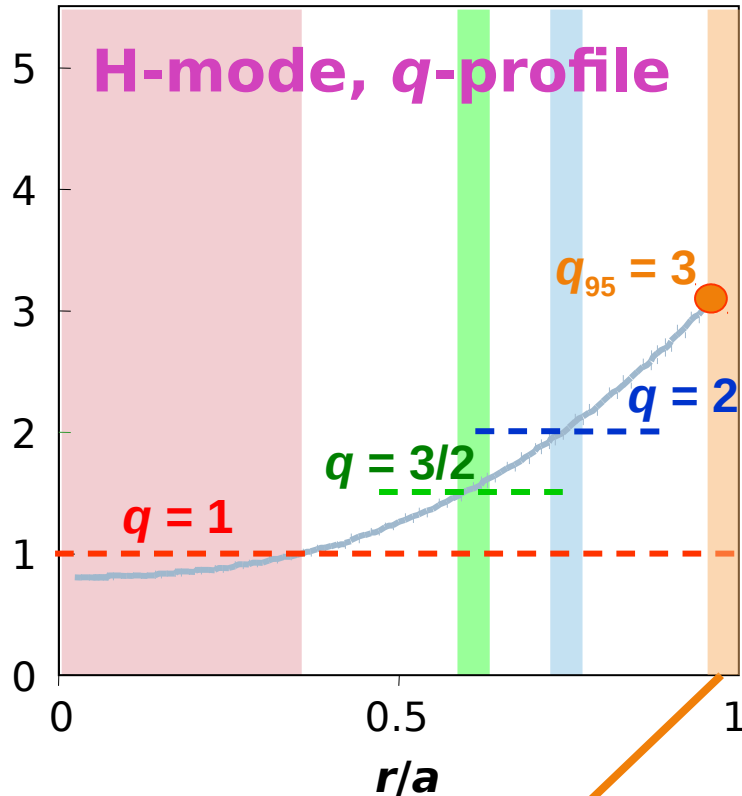
(459.761, 3 Credits)

Prof. Dr. Yong-Su Na

(32-206, Tel. 880-7204)

H-mode: Limitations

- Stability of H-mode plasmas related safety factor profile: $q(r)$



Periodic collapses of the ETB (ELMs)

$q_0 < 1$: Sawtooth instability, periodic flattening of the pressure in the core

$q = 3/2$ and $q = 2$:

Neoclassical Tearing Modes (NTMs):

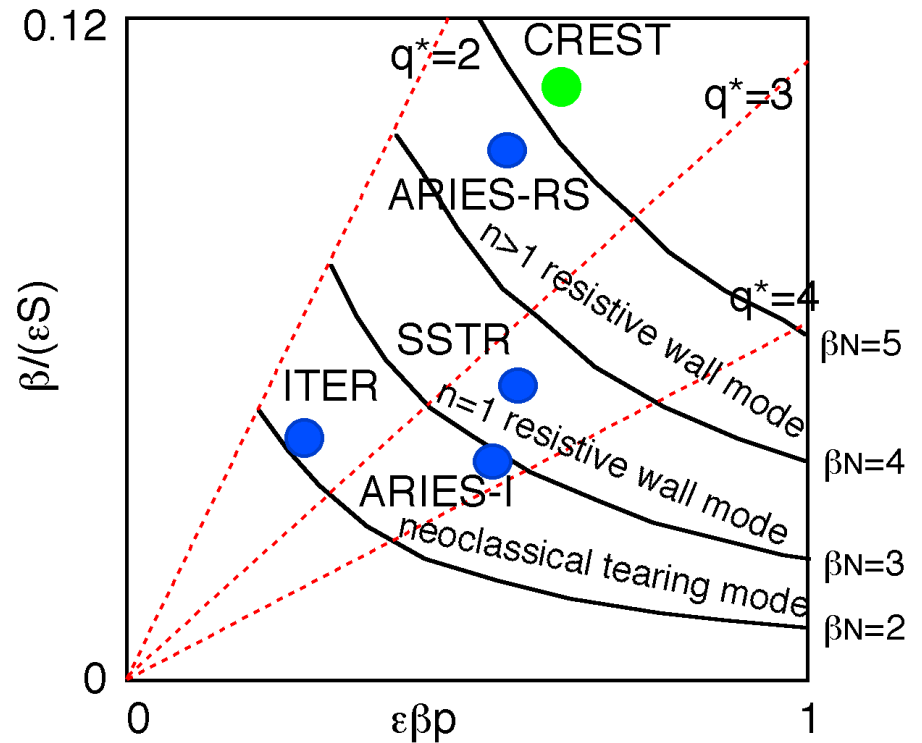
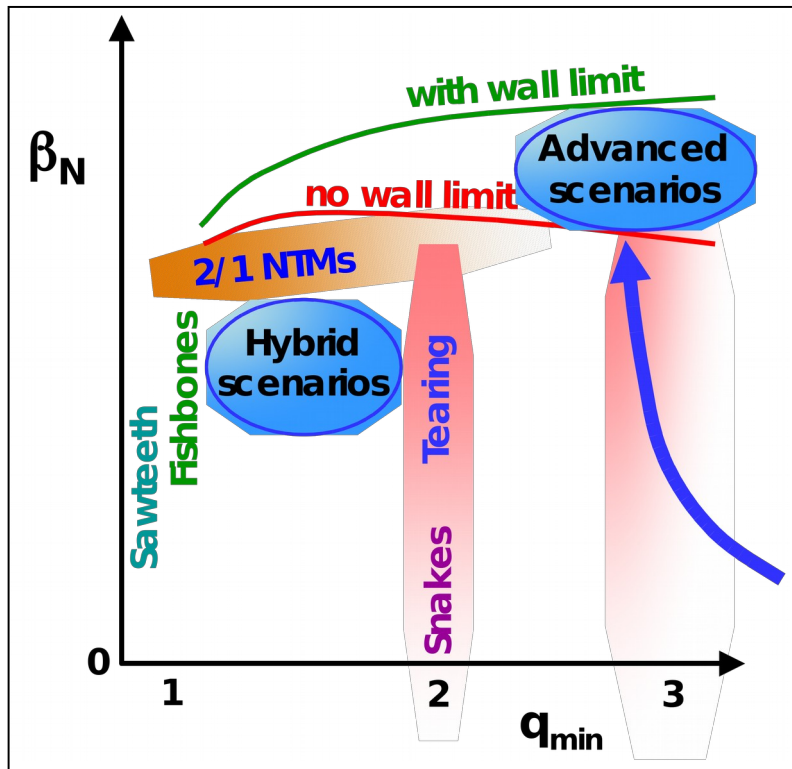
- limit the achievable $\beta \equiv 2\mu_0 p/B^2$
- degrade confinement (+ disruptions)
- often triggered by sawteeth

ITER work point is chosen conservatively: $\beta_N \leq 1.8$

$q_{95} (\propto 1/I_p) = 3$: Safe operation at max. I_p

MHD Instability

- Instabilities limiting beta



R. Buttery, EFPW 05

Neoclassical Tearing Mode (NTM)

LETTERS

The purpose of this Letters section is to provide rapid dissemination of important new results in the fields regularly covered by The Physics of Fluids. Results of extended research should not be presented as a series of letters in place of comprehensive articles. Letters cannot exceed four printed pages in length, including space allowed for title, figures, tables, references and an abstract limited to about 100 words.

Island bootstrap current modification of the nonlinear dynamics of the tearing mode

R. Carrera, R. D. Hazeltine, and M. Kotschenreuther

Institute for Fusion Studies, The University of Texas at Austin, Austin, Texas 78712-1068

(Received 1 November 1985; accepted 13 January 1986)

A kinetic theory for the nonlinear evolution of a magnetic island in a collisionless plasma confined in a toroidal magnetic system is presented. An asymptotic analysis of a Grad–Shafranov equation including neoclassical effects such as island bootstrap current defines an equation for the time dependence of the island width. Initially, the island bootstrap current strongly influences the island evolution. As the island surpasses a certain critical width the effect of the island bootstrap current diminishes and the island grows at the Rutherford rate. For current profiles such that $\Delta' < 0$ the island bootstrap current saturates the island.

R. Carrera et al, Physics of Fluids **29** 899 (1986)

- One of the earliest theoretical paper

Neoclassical Tearing Mode (NTM)

VOLUME 74, NUMBER 23

PHYSICAL REVIEW LETTERS

5 JUNE 1995

Observation of N

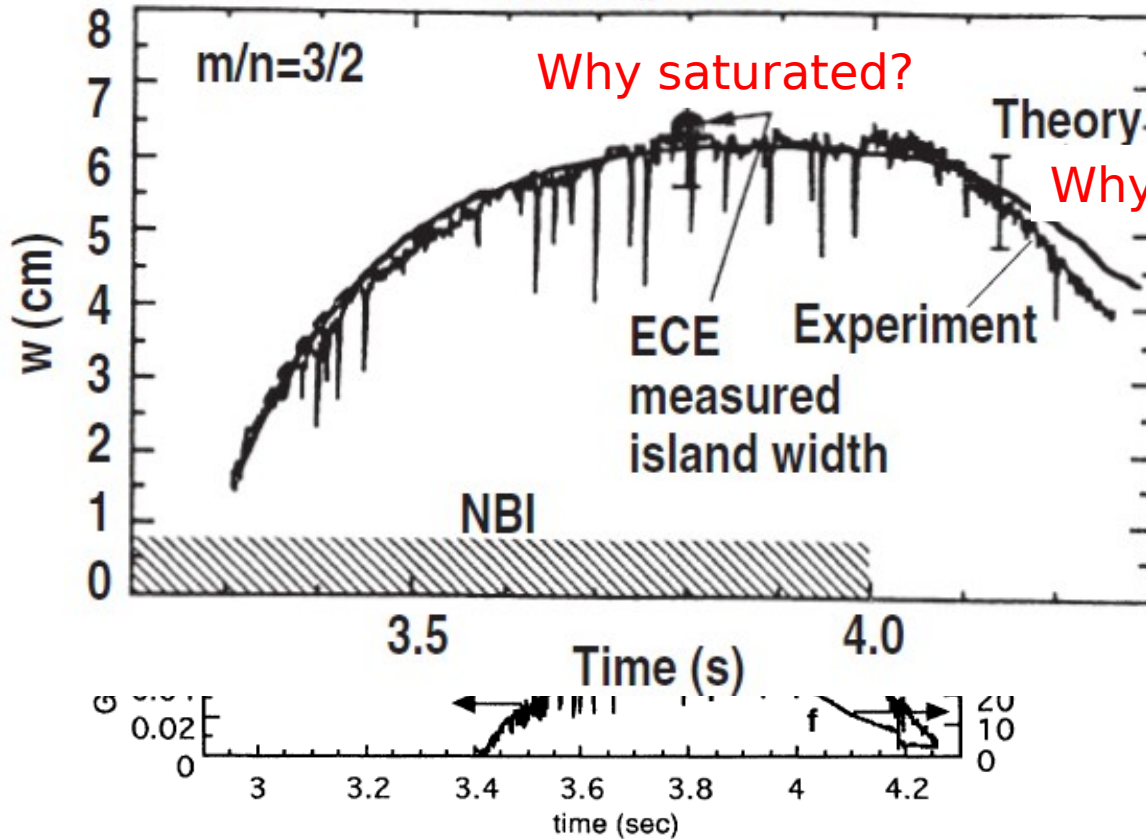
Modes in TFTR

$I_p=1.6$ MA, $B_t=4.8$ T, $R=2.45$ m, $a=0.80$ m, $q_a \sim 5.0$ (a)
 20 μ A A: 66869

TFTR #66869

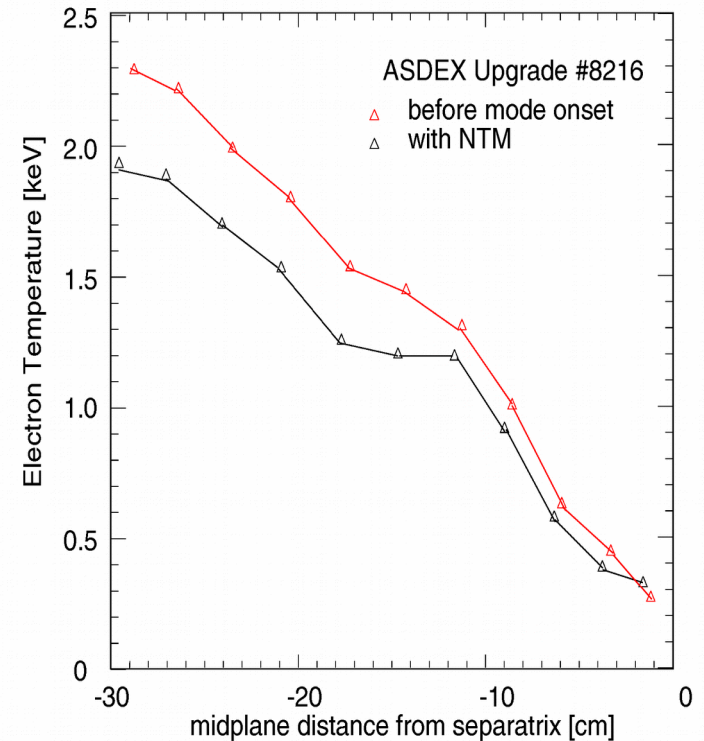
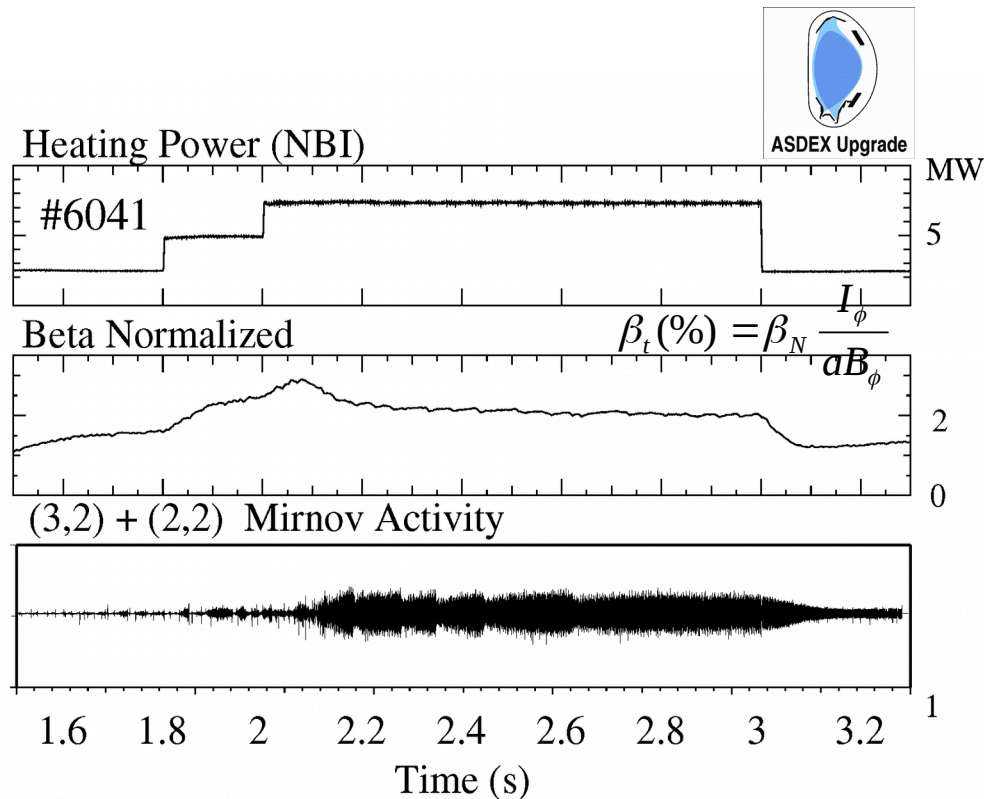
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Neoclassical Tearing Mode (NTM)



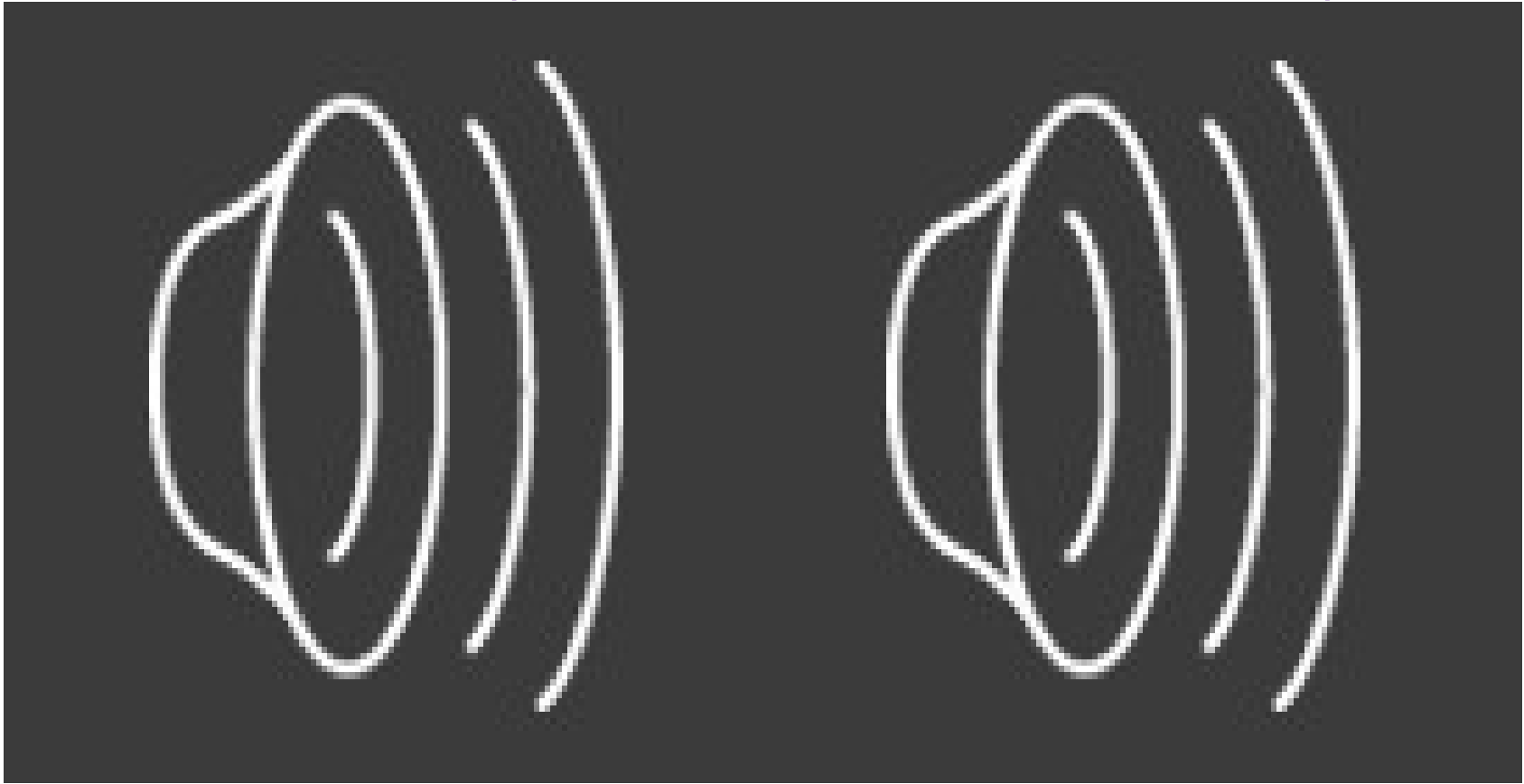
H. Zohm et al., Plasma Phys. Contr. Fusion 37 (1995)

- Neoclassical tearing modes can occur well below ideal limit
- 'practical β -limit' in ITER standard scenario (ELMy H-mode)
- can also lead to disruptive termination (especially at low q)

Neoclassical Tearing Mode (NTM)

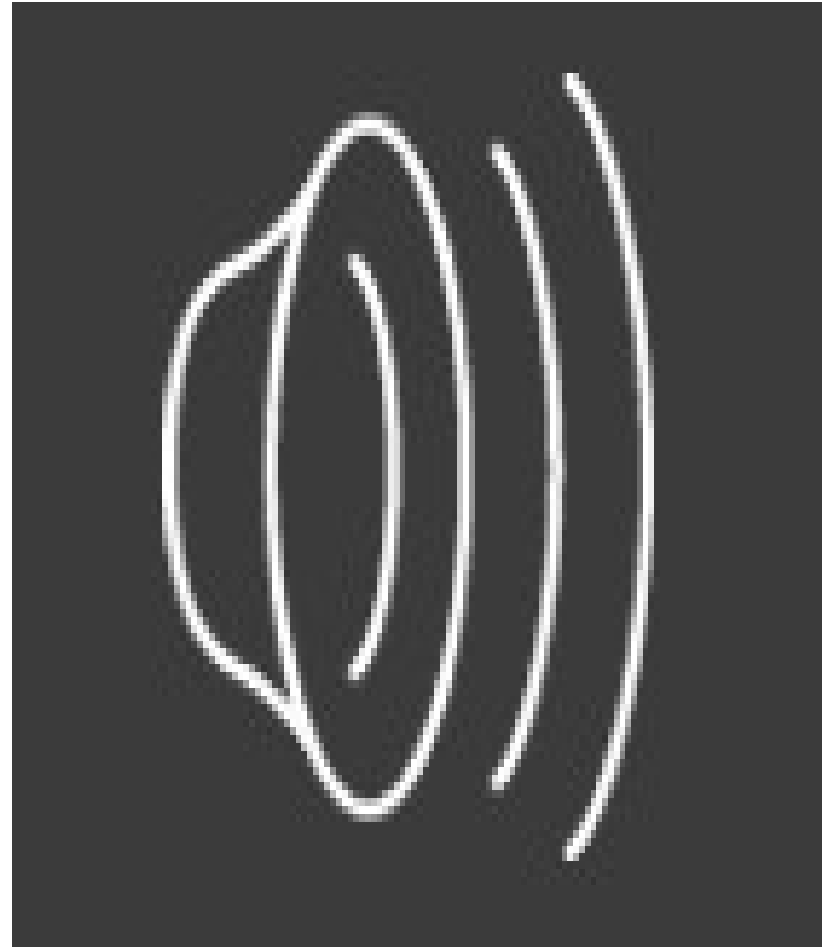
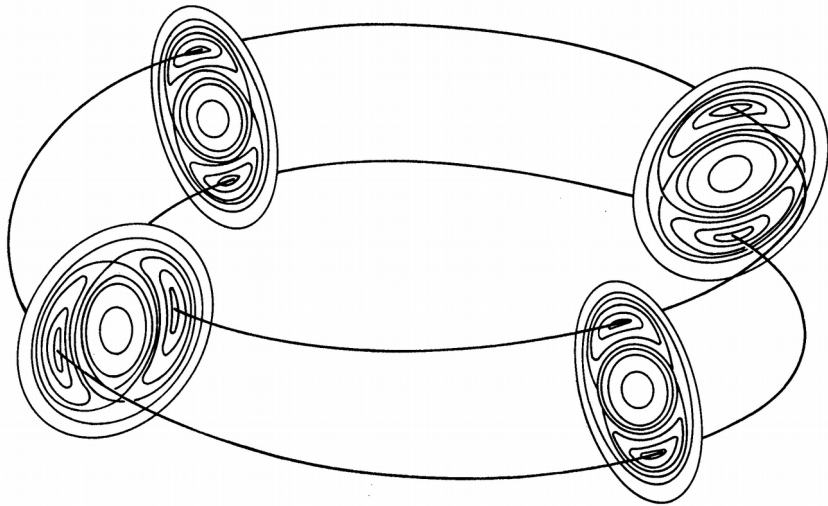
- Ideal MHD: $\eta = 0$

- Resistive MHD: $\eta \neq 0$

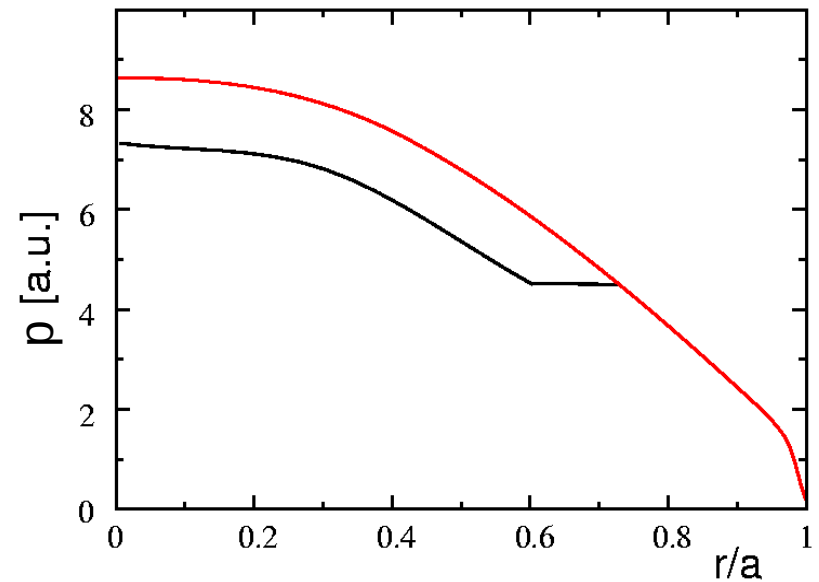


Neoclassical Tearing Mode (NTM)

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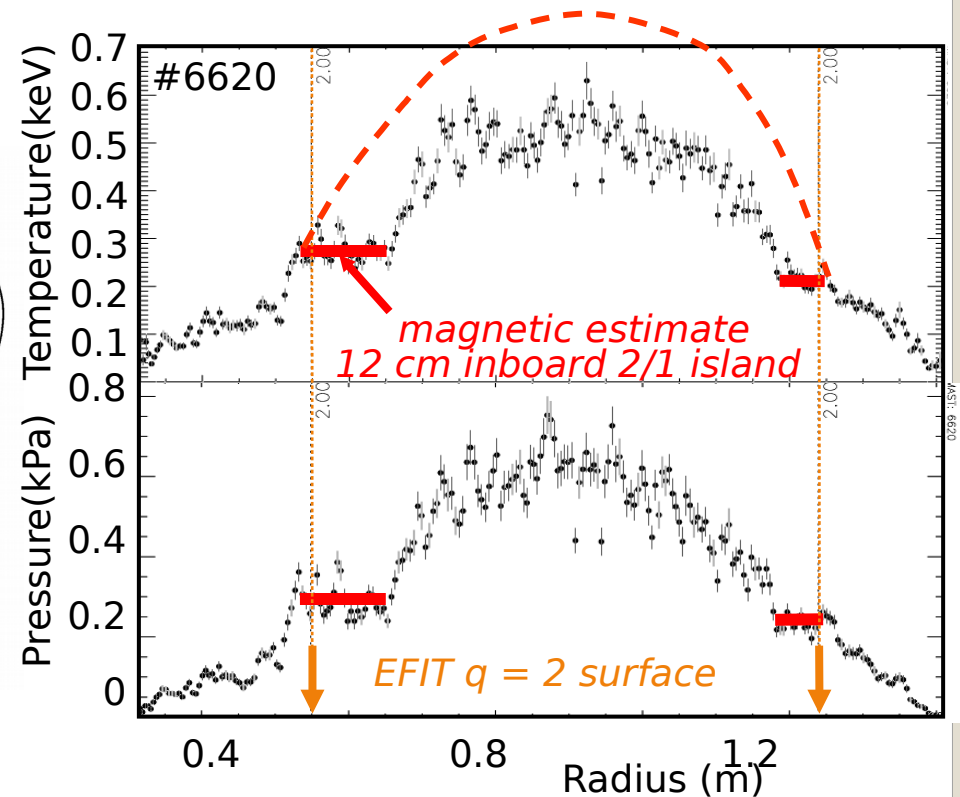


Neoclassical Tearing Mode (NTM)



- Pressure flattening across magnetic islands due to large transport coefficients along magnetic field lines

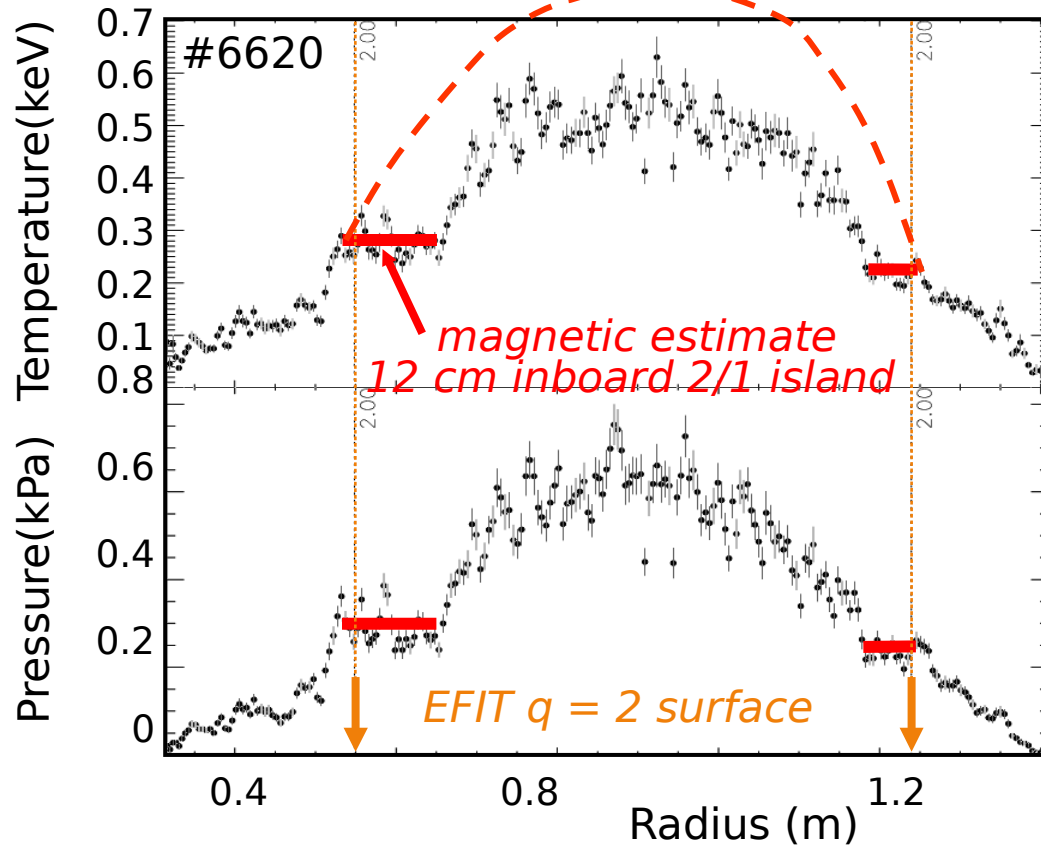
Neoclassical Tearing Mode (NTM)



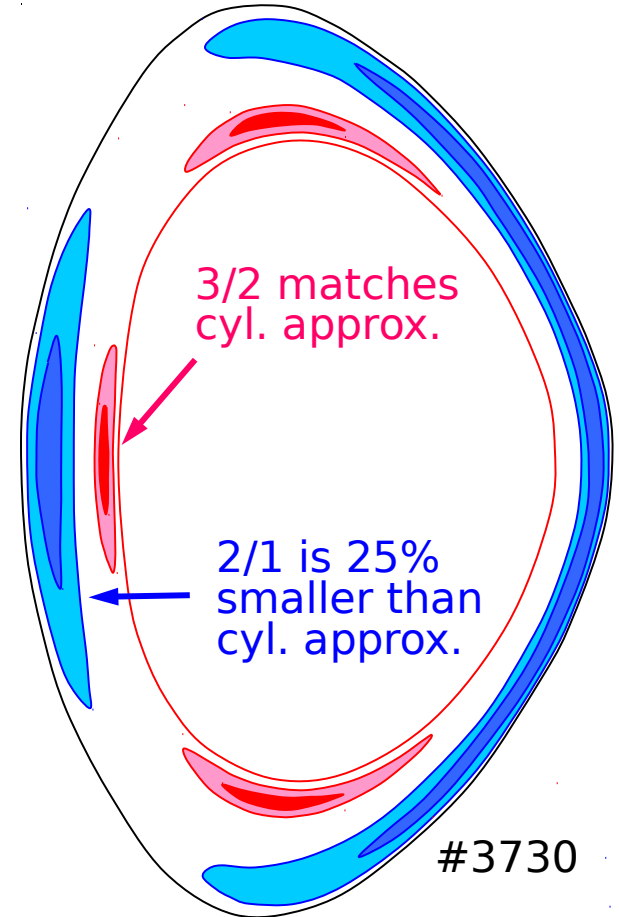
- Pressure flattening across magnetic islands due to large transport coefficients along magnetic field lines

Neoclassical Tearing Mode (NTM)

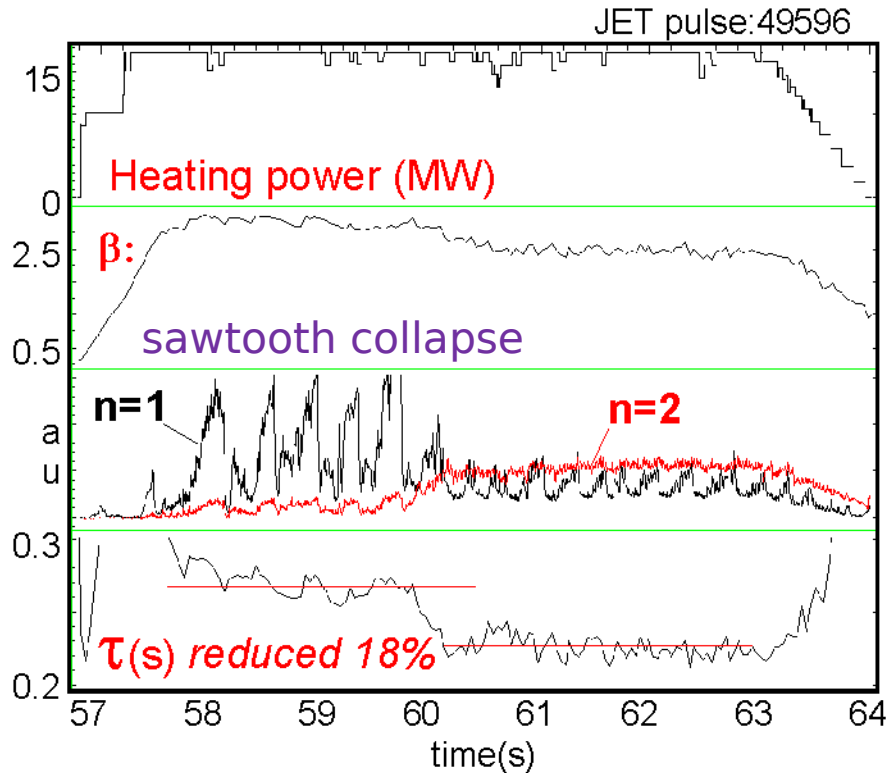
Pressure flattening due to island



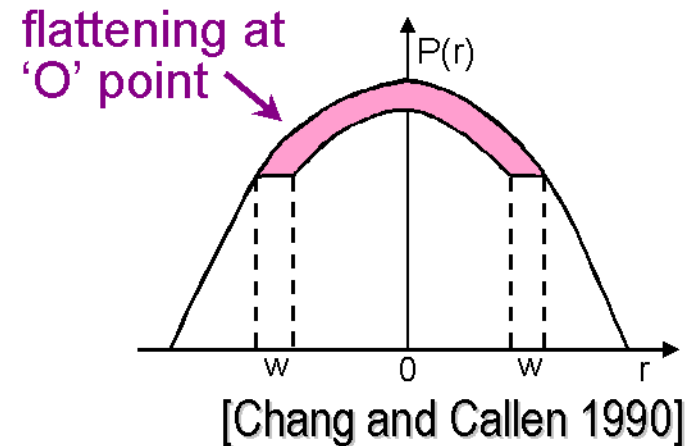
MAST (300 point Thomson scattering)



Neoclassical Tearing Mode (NTM)

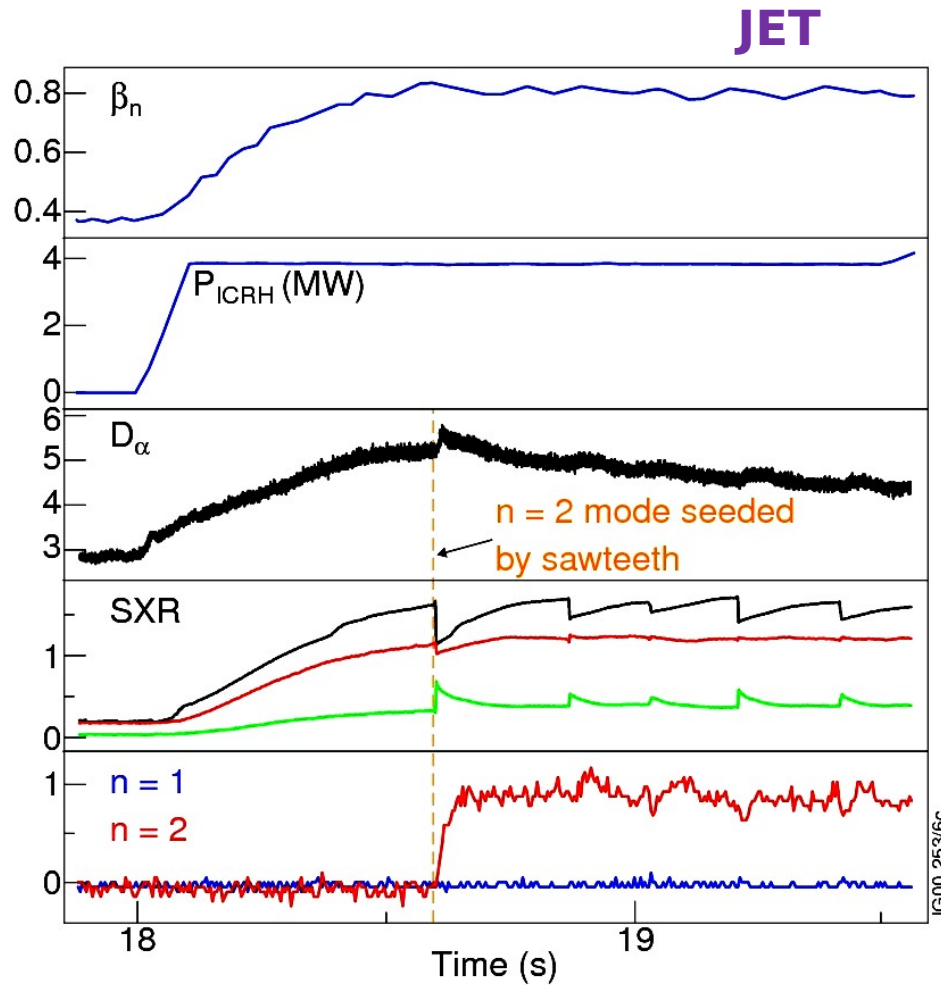


- 3,2 NTM reduces β
- confinement reduced ~15-20%
- explained by 'Belt model':



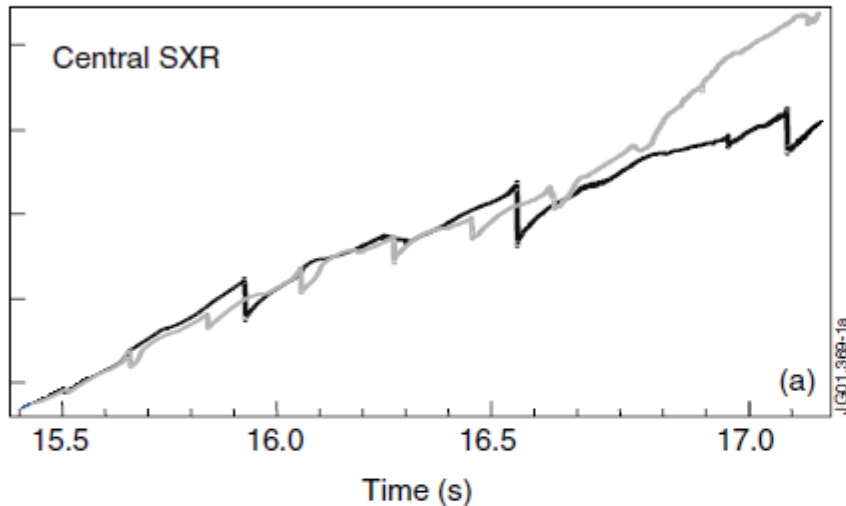
R. Buttery et al, *Plasma Physics and Controlled Fusion* (2000)

Neoclassical Tearing Mode (NTM)



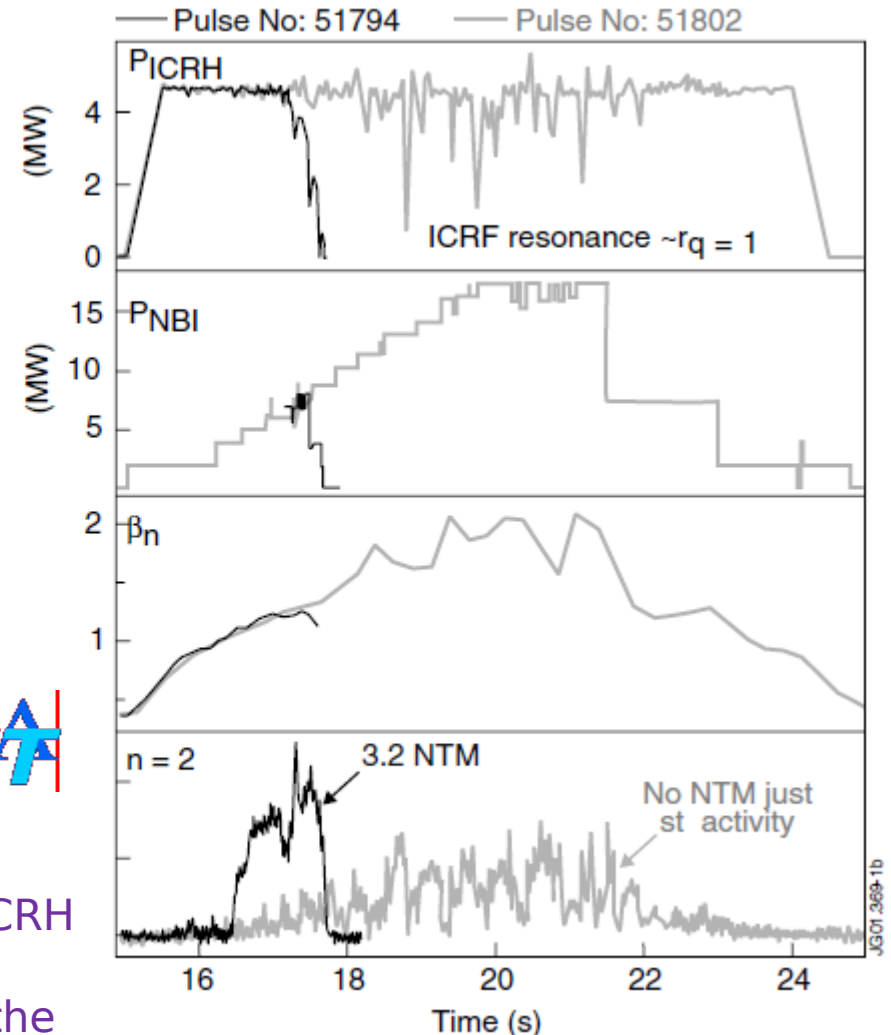
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Neoclassical Tearing Mode (NTM)

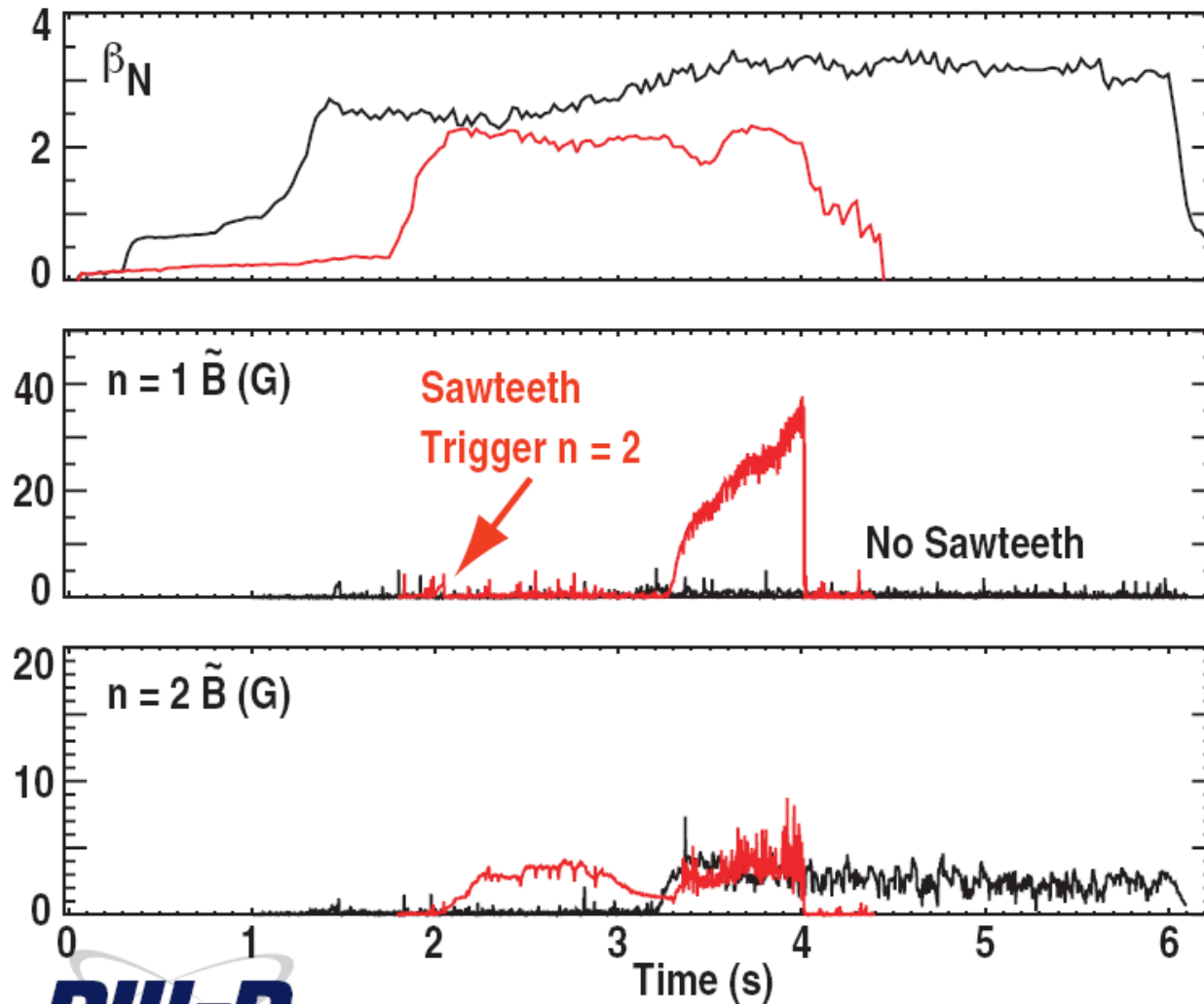


EFDA
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- Increased sawtooth period due to stabilisation by fast ions produced by ICRH leads to the triggering of $n = 2$ NTM activity which causes a termination of the discharge.



Neoclassical Tearing Mode (NTM)

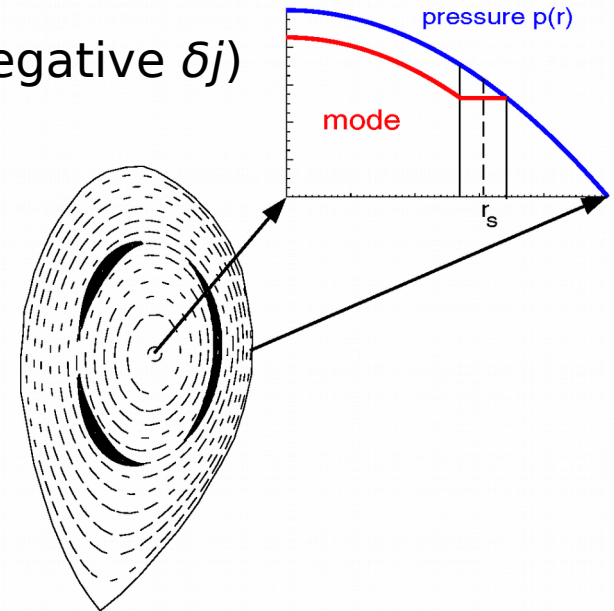
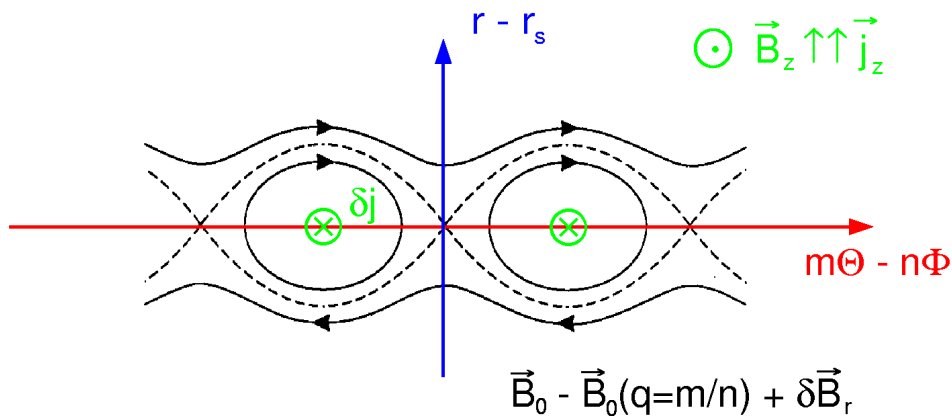


Neoclassical Tearing Mode (NTM)

- At high β_p , pressure gradient drives plasma current by thermo-electric effects (Bootstrap current):

$$j_{BS} \propto \nabla p$$

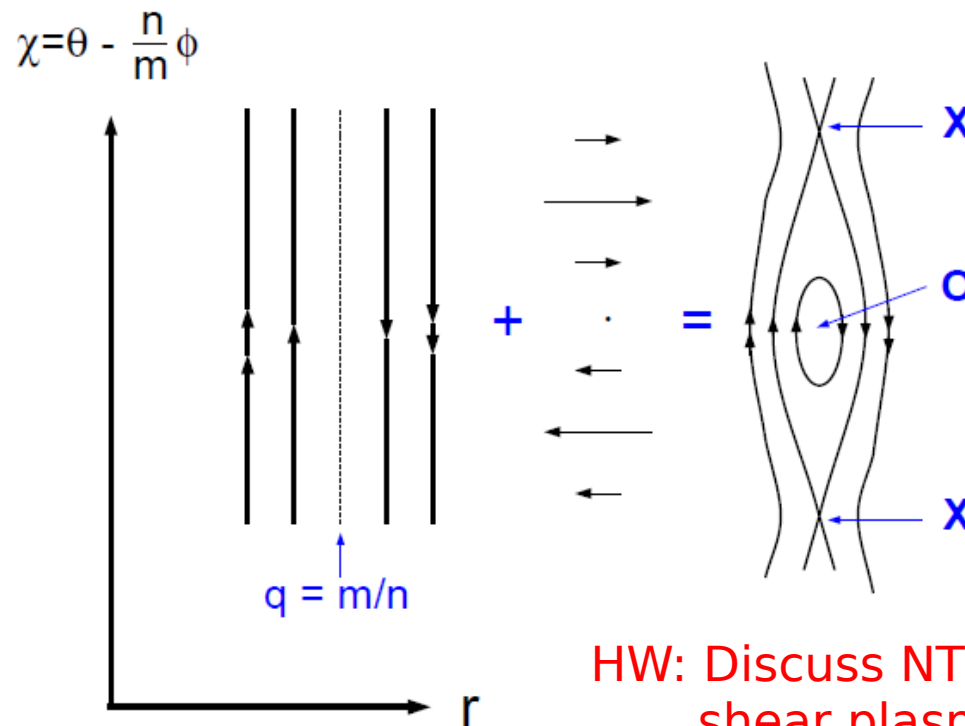
- Inside islands ∇p and thus j_{BS} vanish (negative δj)



- Loss of BS current inside magnetic islands (helical hole) acts as helical perturbation current driving the islands – so once seeded, island is sustained by lack of bootstrap current

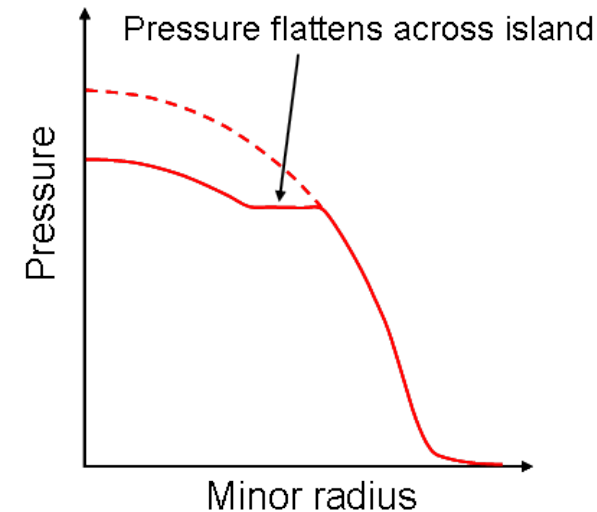
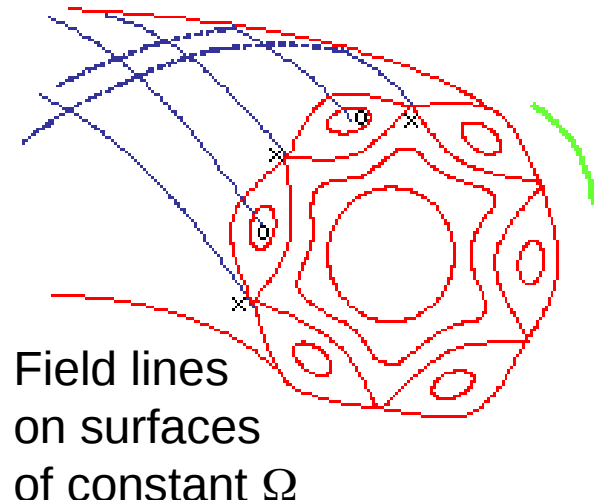
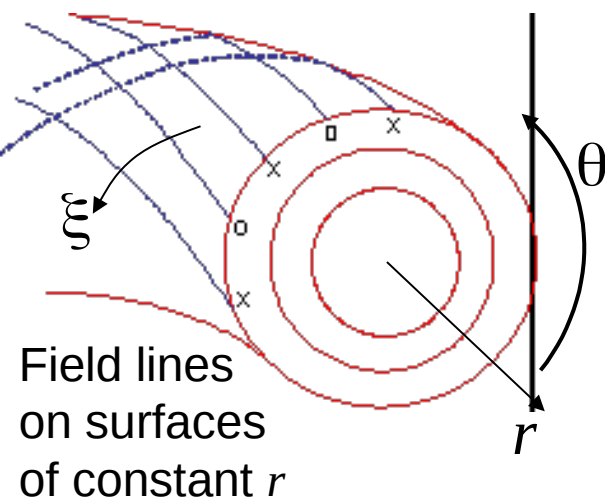
Resistive MHD Instabilities

- growing more slowly compared with the ideal instabilities (10^{-4} - 10^{-2} s)
- resulting from the diffusion or tearing of the magnetic field lines relative to the plasma fluid
- destroying the nested topology of the magnetic flux surfaces



HW: Discuss NTM in reversed shear plasmas

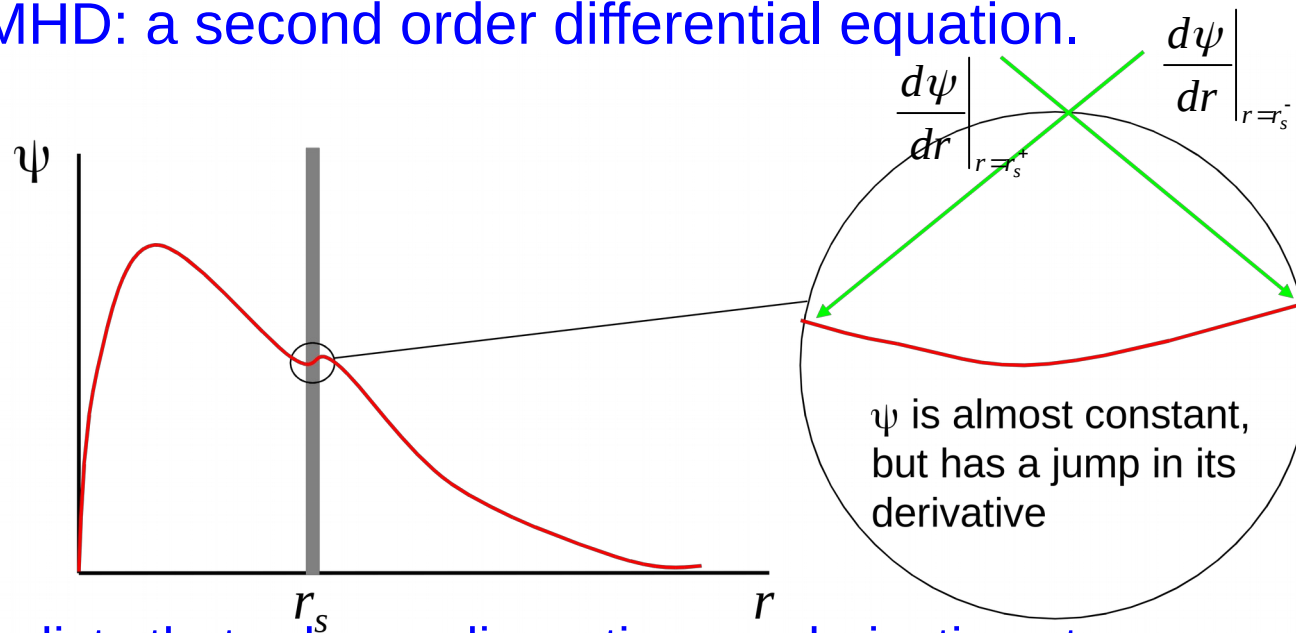
- Tokamaks have good confinement because the flux surfaces lie on nested tori.
- If current flows preferentially along certain field lines, magnetic islands form
- The plasma is then 'short-circuited' across the island region.
- As a result, the plasma pressure is flattened across the island region, and the confinement is degraded:



- We begin by defining the perturbed flux:

$$\delta B = \nabla \phi \times \nabla \psi = B_r \sin m \xi \quad \psi = \tilde{\psi} \cos m \xi \quad B_r = \frac{m \tilde{\psi}}{rR}$$

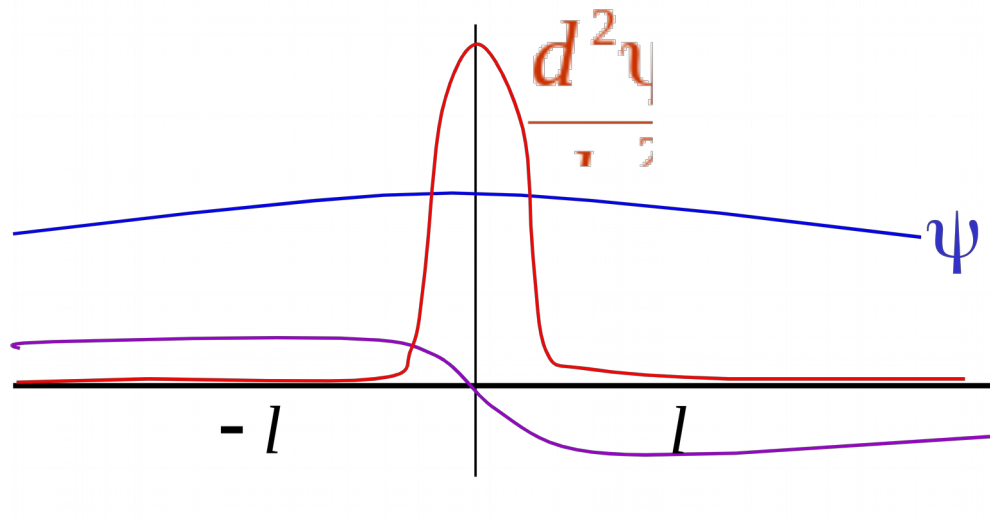
- Away from the rational surface, ψ is determined by the equations of ideal MHD: a second order differential equation.



- it predicts that ψ has a discontinuous derivative at $r=r_s$
- this is conventionally parameterised by Δ' :

$$\Delta' = \frac{1}{\psi} \left[\frac{d\psi}{dr} \Big|_{r=r_s^+} - \frac{d\psi}{dr} \Big|_{r=r_s^-} \right]$$

- The discontinuous derivative arises because of currents, localised around the rational surface, where ideal MHD breaks down.



Ampere's law provides:

$$\frac{1}{R} \frac{d^2\psi}{dr^2} \approx \frac{\mathbf{B} \cdot (\nabla \times \delta\mathbf{B})}{B} = \mu_0 J_{\parallel}$$

$$\Delta' = \frac{1}{\psi} \left[\frac{d\psi}{dr} \Big|_{r=r_s^+} - \frac{d\psi}{dr} \Big|_{r=r_s^-} \right]$$

- Integrate this over a period in ξ and out to a large distance, l , from the rational surface ($w \ll l \ll r_s$): basic tearing mode equation.

$$\Delta' \tilde{\psi} = 2\mu_0 R \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} d\xi J_{\parallel} \cos m\xi$$

$$x = r - r_s$$

- The different models for non-linear tearing mode evolution employ different models for J_{\parallel} .

- Basic 'Rutherford' model: take a simple Ohm's law for J_{\parallel}

$$\eta J_{\parallel} = E_{\parallel} = \frac{\partial \tilde{\psi}}{\partial t} \cos m\xi - \nabla_{\parallel} \varphi$$

- In the absence of perpendicular drifts, perpendicular currents are zero, and so we have $\nabla \cdot \mathbf{J} = \nabla_{\parallel} J_{\parallel} = 0$, or $J_{\parallel} = J_{\parallel}(\Omega)$

- Thus, by averaging around flux surfaces $\langle \dots \rangle$, we eliminate φ to derive.

$$J_{\parallel} = \frac{1}{\eta} \frac{\partial \tilde{\psi}}{\partial t} \langle \cos m\xi \rangle \quad \Delta' \tilde{\psi} = 2\mu_0 R \int_{-\infty}^{\infty} dx \int d\xi J_{\parallel} \cos m\xi$$

- Relating ψ to the island width, w , we then arrive at Rutherford's eqn:

$$\boxed{0.82\tau_r \frac{dw}{dt} = r_s^2 \Delta' \tilde{\psi}} \quad \tau_r = \frac{\mu_0 r_s^2}{\eta} \quad w = 2 \left(\frac{q\tilde{\psi}}{RB_{\theta} dq/dr} \right)^{1/2}$$

→ Classical tearing mode with $\Delta' > 0$

- The bootstrap current in banana collisionality regime is approximately:

$$J_{bs} = -2.44 \frac{\sqrt{\varepsilon}}{B_\theta} \frac{dp}{dr}$$

- Recall that the pressure gradient is removed from inside the island:
 - there is a ‘hole’ in the bootstrap current around the island O-points.
 - provides another contribution to J_{\parallel} perturbation to drive the tearing mode.

- Using the above expression, we derive the neoclassical tearing mode equation:

$$0.82 \frac{\tau_r}{r_s^2} \frac{dw}{dt} = \Delta' + a_2 \sqrt{\varepsilon} \frac{\beta_\theta}{w} \frac{L_q}{L_p}$$

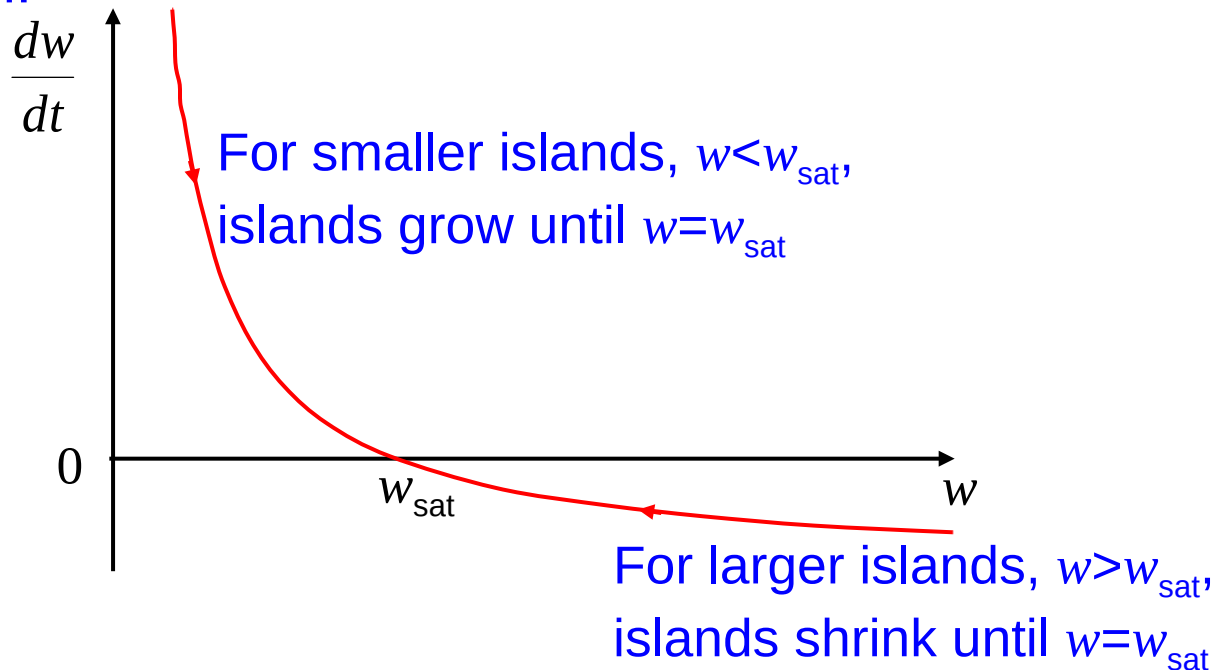
$$\beta_\theta = \frac{2\mu_0 p}{B_\theta^2}$$

$$L_p^{-1} = - \frac{d \ln p}{dr}$$

$$L_q^{-1} = \frac{d \ln q}{dr}$$

$$0.82 \frac{\tau_r}{r_s^2} \frac{dw}{dt} = \Delta' + a_2 \sqrt{\varepsilon} \frac{\beta_\theta}{w} \frac{L_q}{L_p}$$

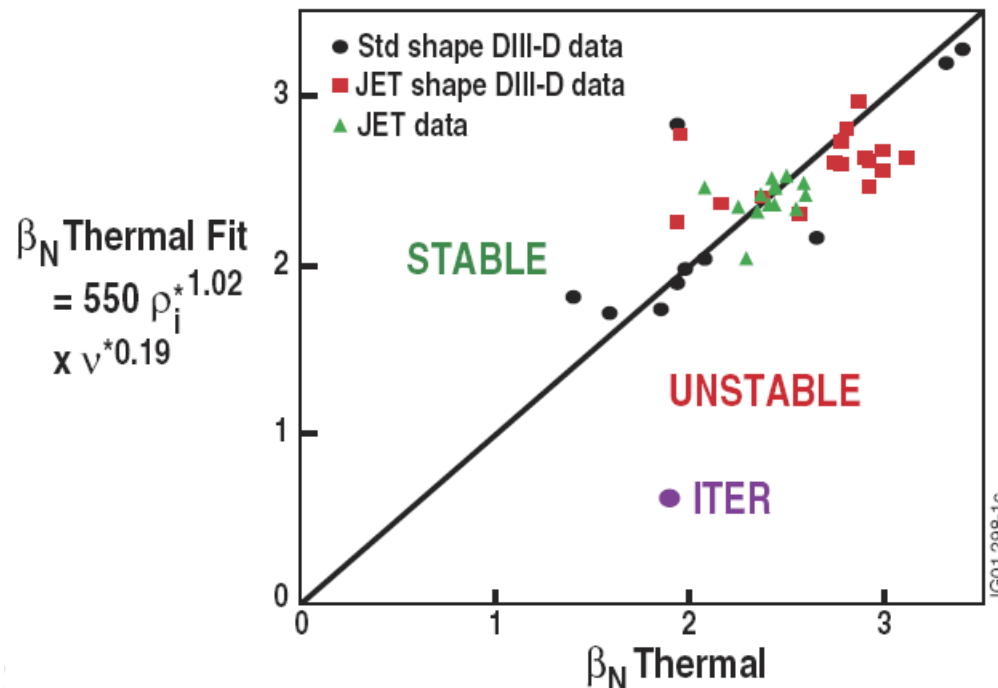
- For typical tokamak profiles bootstrap contribution *drives* island growth.
- When $\Delta' < 0$ (Rutherford stable), there exists a stable, saturated island width solution:



- The saturated island width is:
$$\frac{w_{\text{sat}}}{r_s} = a_2 \sqrt{\varepsilon} \frac{\beta_\theta}{(-r_s \Delta')} \frac{L_q}{L_p}$$
- The saturated island width increases with β_p .
 - as the confinement will deteriorate with increasing island size, this sets an effective β -limit in tokamaks.
 - the saturated island width can become a large fraction of the plasma radius, and this can lead to disruption.
 - the saturated island width can reduce with β_p (see TFTR case) which is different from classical tearing modes.
- As Δ' becomes more negative for increasing poloidal mode number, m , it is the lowest m modes which are most dangerous ($r_s \Delta' \sim -m$).
- Nevertheless, the above model predicts magnetic islands at all rational surfaces:
 - why does the tokamak work? All the small islands will grow!
 - additional “threshold” physics is important at small island width.

Neoclassical Tearing Mode (NTM)

- Critical β_N (beta-limit)



T.C. Hender et al. Nucl. Fusion **44** 788 (2004)

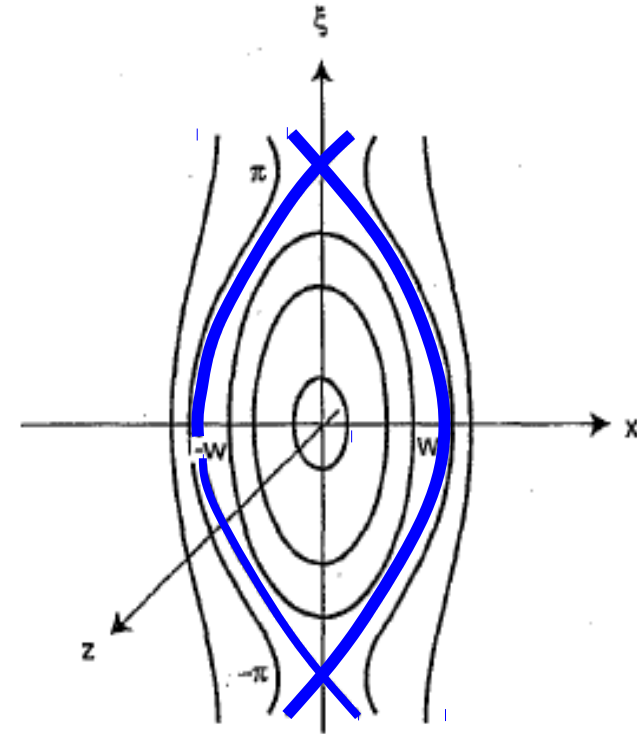
- Critical beta for $m/n=2/1$ NTM: scales with ρ_i^* , ν^*



- For sufficiently small island widths, the pressure is not completely flattened inside the island.
 - ⇒ the bootstrap current drive is not so effective for small islands.
 - ⇒ we refer to this as ‘finite radial diffusion effects’.

- The expression for the bootstrap current is based on an expansion in the ratio of the banana width to the equilibrium length scales
 - ⇒ the theory must therefore be questioned for islands with a width comparable to the ion banana width.
 - ⇒ we refer to this as ‘finite orbit width effects’.

- The connection length L_c is the distance along a field line from one side of the island to the other - i.e. the route for the enhanced transport that flattens the temperature. $L_c \sim 1/w$ so the **enhanced transport is reduced for small islands.**
- When w is close to a critical width w_c , both the flattening and hence the bootstrap drive are reduced, **giving rise to a threshold.**





Finite radial diffusion: (Kieran will discuss more detail)

• For a simple illustration, consider diffusive electron heat fluxes parallel and perpendicular to field lines: $Q_{\parallel} = -\chi_{\parallel} \nabla_{\parallel} T$ $Q_{\perp} = -\chi_{\perp} \nabla_{\perp} T$

• In the absence of heat sources $\nabla \cdot \mathbf{Q} = 0$, so that

$$\chi_{\parallel} \nabla_{\parallel}^2 T + \chi_{\perp} \nabla_{\perp}^2 T = 0$$

• Now $\chi_{\parallel} \gg \chi_{\perp}$

⇒ generally radial diffusion can be ignored

⇒ $\nabla_{\parallel} T = 0$, so that the temperature is flattened across the island

• However, the gradient operators depend on island size:

— $\nabla_{\perp} \sim \partial / \partial r \sim 1/w$

— $\nabla_{\parallel} = (\mathbf{B} \cdot \nabla) / B \sim mw / (RqL_q)$

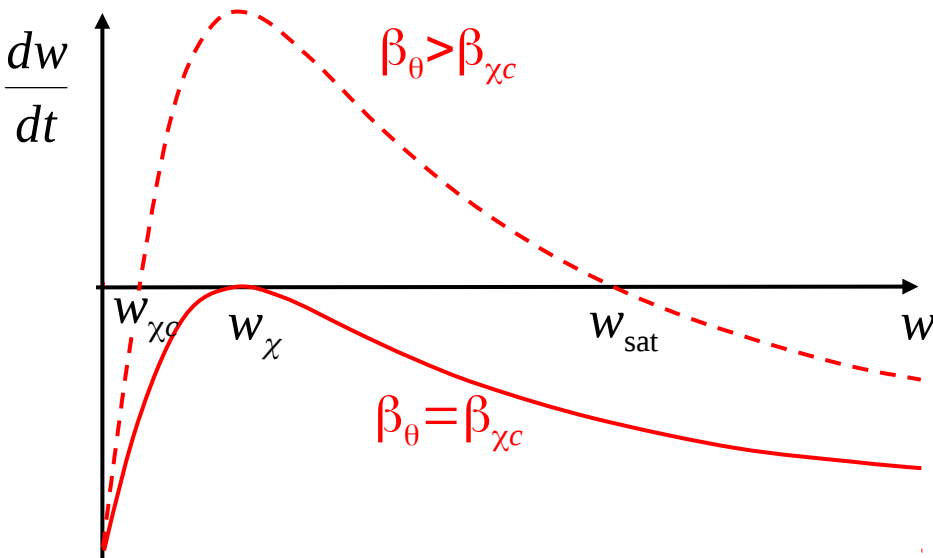
• Balancing terms ⇒ radial diffusion is important for $w < w_{\chi}$, where

$$w_{\chi} = \sqrt{\frac{RqL_q}{m} \left(\frac{\chi_{\perp}}{\chi_{\parallel}} \right)^{1/4}} \quad \text{the island width for which perpendicular and parallel transport are equal.}$$

• Needs much more care for ion thermal transport and particle transport

- Thus, for sufficiently small islands, $w < w_{\chi}$, the temperature is not flattened across the island, and the bootstrap drive is weakened:

$$0.82 \frac{\tau_r}{r_s^2} \frac{dw}{dt} = \Delta' + a_2 \sqrt{\varepsilon} \frac{\beta_{\theta}}{w} \frac{L_q}{L_p} \left(\frac{w^2}{w^2 + w_{\chi}^2} \right)$$



- For $\beta_{\theta} < \beta_{\chi c}$, $dw/dt < 0$ for all $w \Rightarrow$ all islands decay away.
- For $\beta_{\theta} > \beta_{\chi c}$, an additional, unstable, root for $dw/dt = 0$ at $w = w_{\chi c}$.

\Rightarrow an island will only grow to its saturated state provided.

$$w > w_{\chi c} \quad \text{AND} \quad \beta_{\theta} > \beta_{\chi c}$$

\Rightarrow a 'seed' island is required .

$$\beta_{\chi c} = - \frac{2\Delta' w_{\chi} L_p}{a_2 \sqrt{\varepsilon} L_q} \quad w_{\chi c} \approx \frac{w_{\chi} \beta_{\chi c}}{2\beta_{\theta}}$$

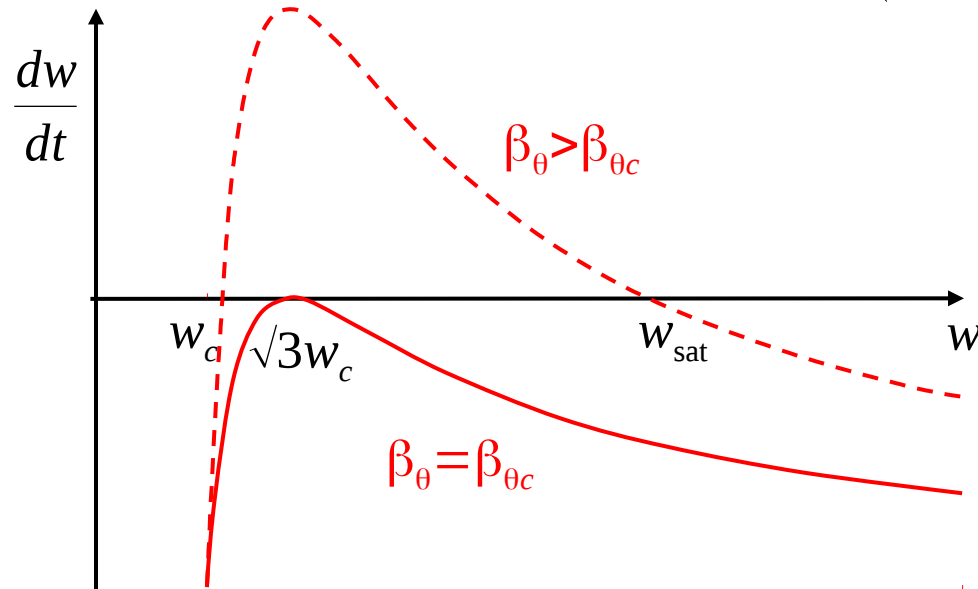


- For small islands of width comparable to the ion banana width, ions and electrons respond differently to the island:
 - an electrostatic potential is required to maintain quasi-neutrality in the vicinity of the island.
 - ions and electrons experience $\mathbf{E} \times \mathbf{B}$ drifts.
 - the ions experience an orbit averaged drift, which differs from the local drift experienced by the electrons for island width \sim ion banana width.
 - a perpendicular current is generated; this is the polarisation current.
 - the polarisation current is not divergence-free.
 - sets up an electric field to drive a current parallel to field lines.
 - this current can influence island evolution.
- The theory is still under development
- Consider island width much greater than the ion banana width
 - led to the inclusion of the so-called 'polarisation term' in the modified Rutherford equation.



- Allowing for the polarisation term, the modified Rutherford equation is:

$$0.82 \frac{\tau_r}{r_s^2} \frac{dw}{dt} = \Delta' + a_2 \sqrt{\varepsilon} \frac{\beta_\theta}{w} \frac{L_q}{L_p} \left(\frac{w^2}{w^2 + w_\chi^2} \right) - a_3 g(\varepsilon, \nu_i) \left(\frac{\rho_{bi}}{w} \right)^2 \left(\frac{L_q}{L_p} \right)^2 \frac{\beta_\theta}{w}$$



$$g(\varepsilon, \nu_i) = \begin{cases} 1.64\varepsilon^{1/2} & \nu_i / \varepsilon\omega \ll 1 \\ \varepsilon^{-1} & \nu_i / \varepsilon\omega \gg 1 \end{cases}$$

Note:

$$w_c = \sqrt{\frac{a_3}{3a_2}} \frac{\sqrt{g(\varepsilon, \nu_i)}}{\varepsilon^{1/4}} \sqrt{\frac{L_p}{L_q}} \rho_{bi} \quad \beta_{\theta c} = \frac{3\sqrt{3}}{2a_2\sqrt{\varepsilon}} \frac{L_p}{L_q} (-w_c \Delta') \propto \rho_*$$

- In general, the full story is more complicated.
- The transport and polarisation terms interact (especially ion thermal and particle transport).



- Both the transport model and the polarisation current provide a threshold island width comparable to the ion banana width:
 - Kinetic theory with full ion banana widths is essential
- This provides a rich, essentially unexplored vein of physics.
- Gyrokinetic models are being developed to address this issue.



Tearing Modes – nonlinear growth



Consider various helical currents on resonant surface...

$$B_{\theta}(r_s^+) - B_{\theta}(r_s^-) \propto \delta I = I_{Ohm} + I_{bs} + I_{extern}$$

$$I_{Ohm} \propto j_{Ohm} W \propto \sigma W \frac{d\psi}{dt} \propto \sigma W^2 \frac{dW}{dt}$$

inductive

$$I_{bs} \propto j_{bs} W \propto - \frac{\nabla p}{B_{\theta}} W$$

pressure driven

$$I_{extern}$$

externally driven

...leads to the so-called Rutherford equation

$$\tau_{res} \frac{dW}{dt} = a_1 \Delta' + a_2 \frac{\nabla p}{W} - a_3 \frac{I_{extern}}{W^2}$$

where $\Delta' = (B_{\theta}(r_s^+) - B_{\theta}(r_s^-)) / \psi$



Interpretation of the different terms

$$\tau_{res} \frac{dW}{dt} = a_1 \Delta' + a_2 \frac{\nabla p}{W} - a_3 \frac{I_{extern}}{W^2}$$

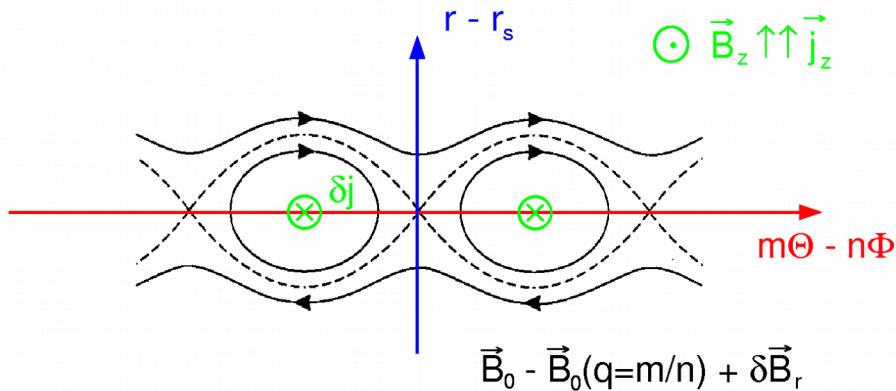
- for small ∇p , current gradient (Δ') dominates
⇒ 'classical Tearing Mode', current driven
- for larger ∇p , pressure gradient dominates:
⇒ 'neoclassical Tearing Mode', pressure driven
- adding an externally driven helical current can stabilise

Neoclassical Tearing Mode (NTM)

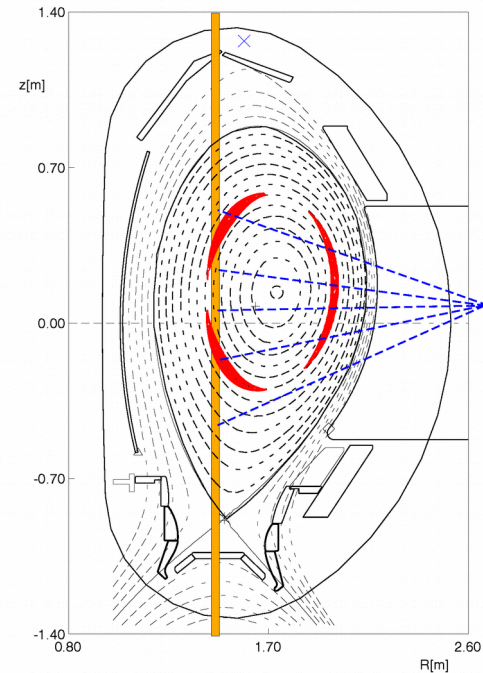
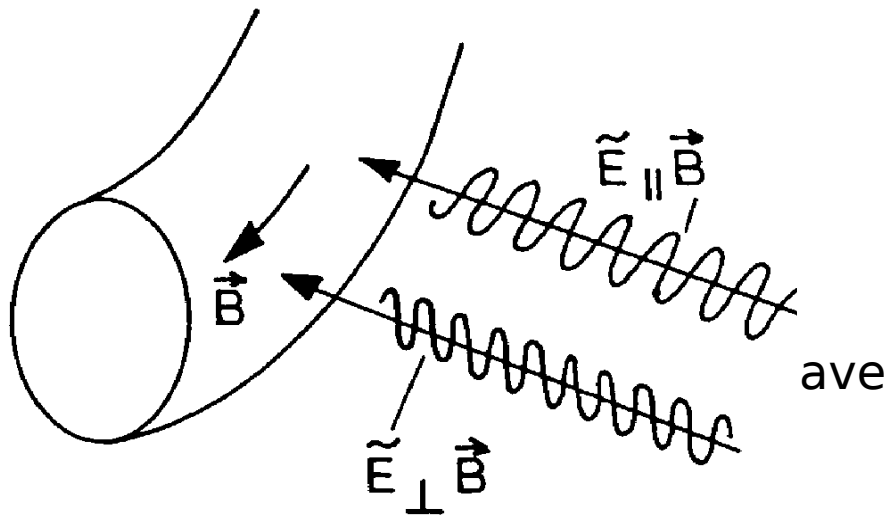
- **How to stabilize NTM?**

If the modes cannot be avoided, a limit of $\beta_N = 2 - 2.2$ ($\beta_p \approx 0.7$) is predicted for ITER since, at this beta value, the saturated island size at the $q = 2$ surface is of the order of the distance to the edge and will cause a disruption. However, due to the very large resistive time in ITER, the island will need 150–200 s to grow to its full size. Therefore one has operational time to detect the mode and end the discharge before the disruption, since one needs less than 30 s to decrease β by a factor 10,³⁹ or control the final saturated size with an external feedback mechanism. Such a system could be ECCD, in order to replace the reduction of the bootstrap current inside the island. We have shown that this is possible with 50 MW of ECRH power, using the present design specifications for the gyrotrons. However, this needs to be first demonstrated in the present experiments. Other possible control mechanisms are keeping $q_0 > 1$, reverse shear configurations, since the neoclassical tearing modes are stable for $q' < 0$, and profile control since they are very sensitive to local parameters.

Neoclassical Tearing Mode (NTM)



- Missing bootstrap current inside island can be replaced by localised external current drive.



Helical current can be driven by electron cyclotron resonance waves
 Deposition controlled by local B-field \Rightarrow very good localisation
 Feedback control of position possible via launch angle of ECCD beam

Neoclassical Tearing Mode (NTM)

- The Modified Rutherford Equation (MRE)**

$$\frac{\tau_R}{r_s} \frac{d\omega}{dt} = \Delta'_0 r_s + \delta \Delta' r_s + a_2 \frac{j_{bs}}{j_{\parallel}} \frac{L_q}{\omega} \left[1 - \frac{\omega_{marg}^2}{3\omega^2} - K_1 \frac{j_{ec}}{j_{bs}} \right]$$

HW: derive Δ'

$$\Delta' \equiv \frac{1}{B_r} \left[\frac{\partial B_r}{\partial r} \right]_{r=r_s}$$

1st: Conventional tearing mode stability: assumed as $\Delta'_0 r_s \approx -m$ for m/n NTM

2nd: Tearing mode stab. enhancement by ECCD: Westerhof's model with no-island assumption

$$\delta \Delta' r_s \approx - \frac{5\pi^{3/2}}{32} a_2 \frac{L_q}{\delta_{ec}} F(e) \frac{j_{ec}}{j_{\parallel}} \tau, \text{ where the misalignment function } F(e) = 1 - 2.43e + 1.40e^2 - 0.23e^3$$

3rd: Destabilization from perturbed bootstrap current:

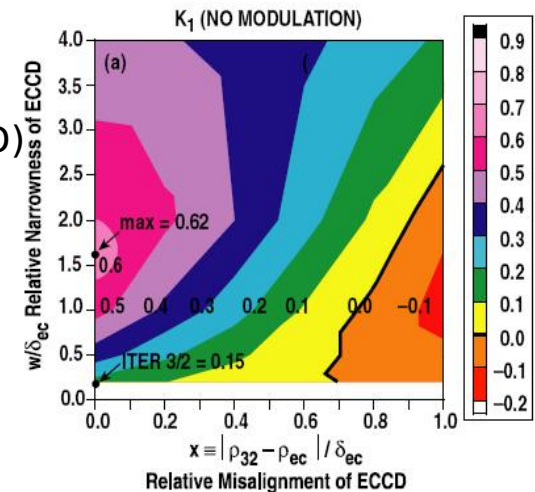
a_2 fitted by inferred size of saturated NTM island (e.g. ISLAND)

4th: Stabilization from small island & polarization threshold:

$$w_{marg} \approx 2\varepsilon^{1/2} \rho_{\theta i} \quad (= \text{twice ion banana width})$$

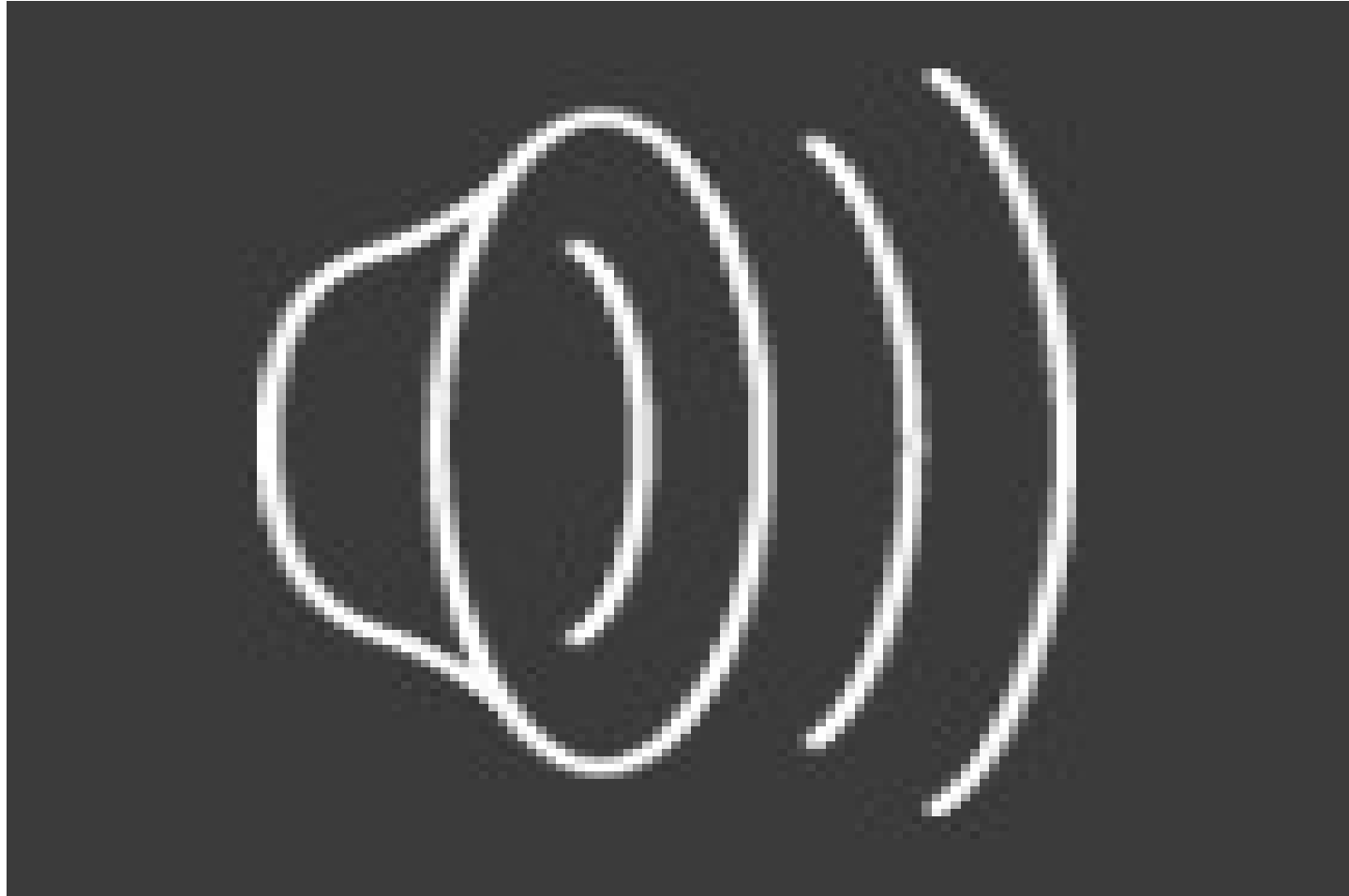
5th: Stabilization from replacing bootstrap current by ECCD:

K_1 calculated from improved Perkins' current drive model

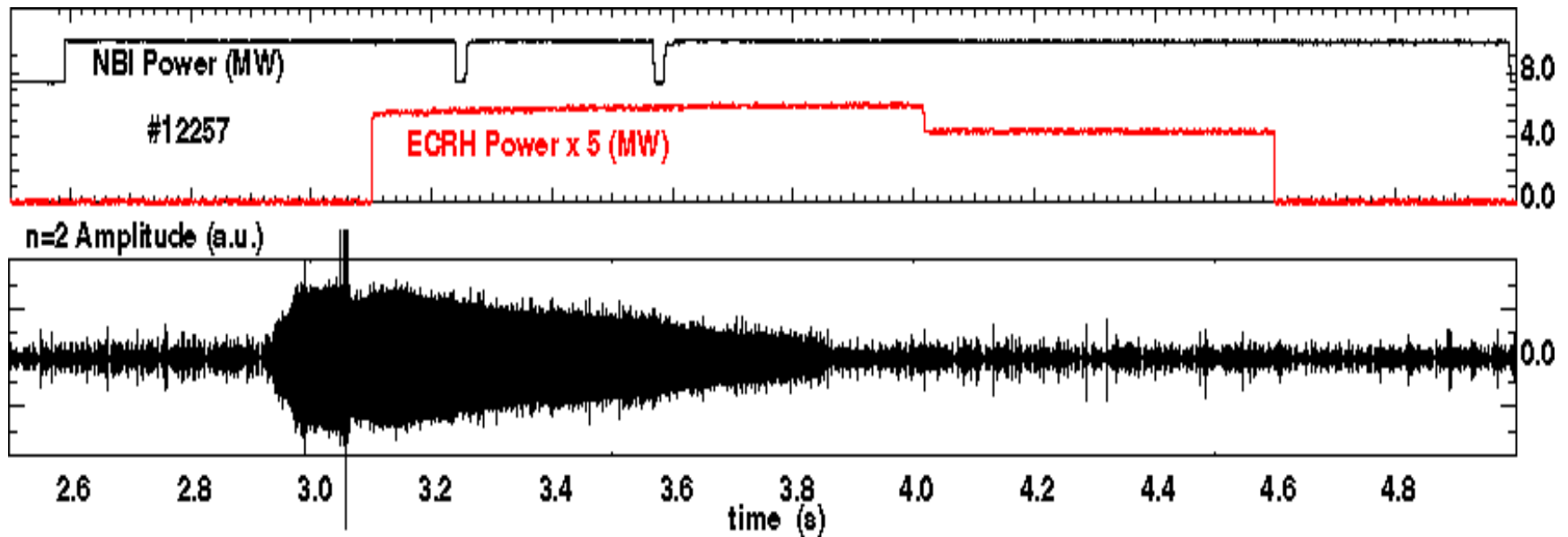


NTM Stabilisation by ECCD

- JT-60U

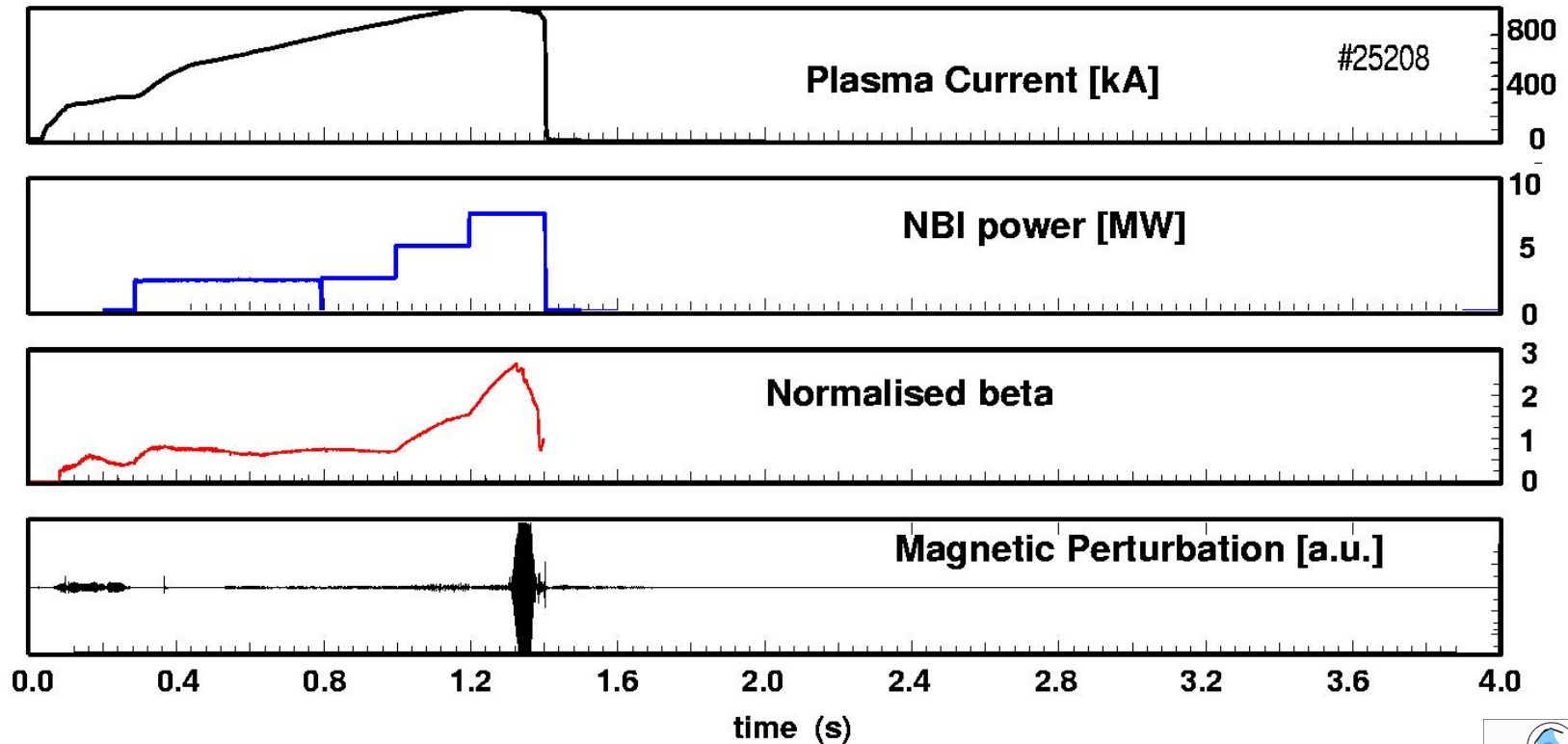


Neoclassical Tearing Mode (NTM)



- Complete stabilisation by searching the position of the magnetic island by scanning magnetic field in quantitative agreement with theory.

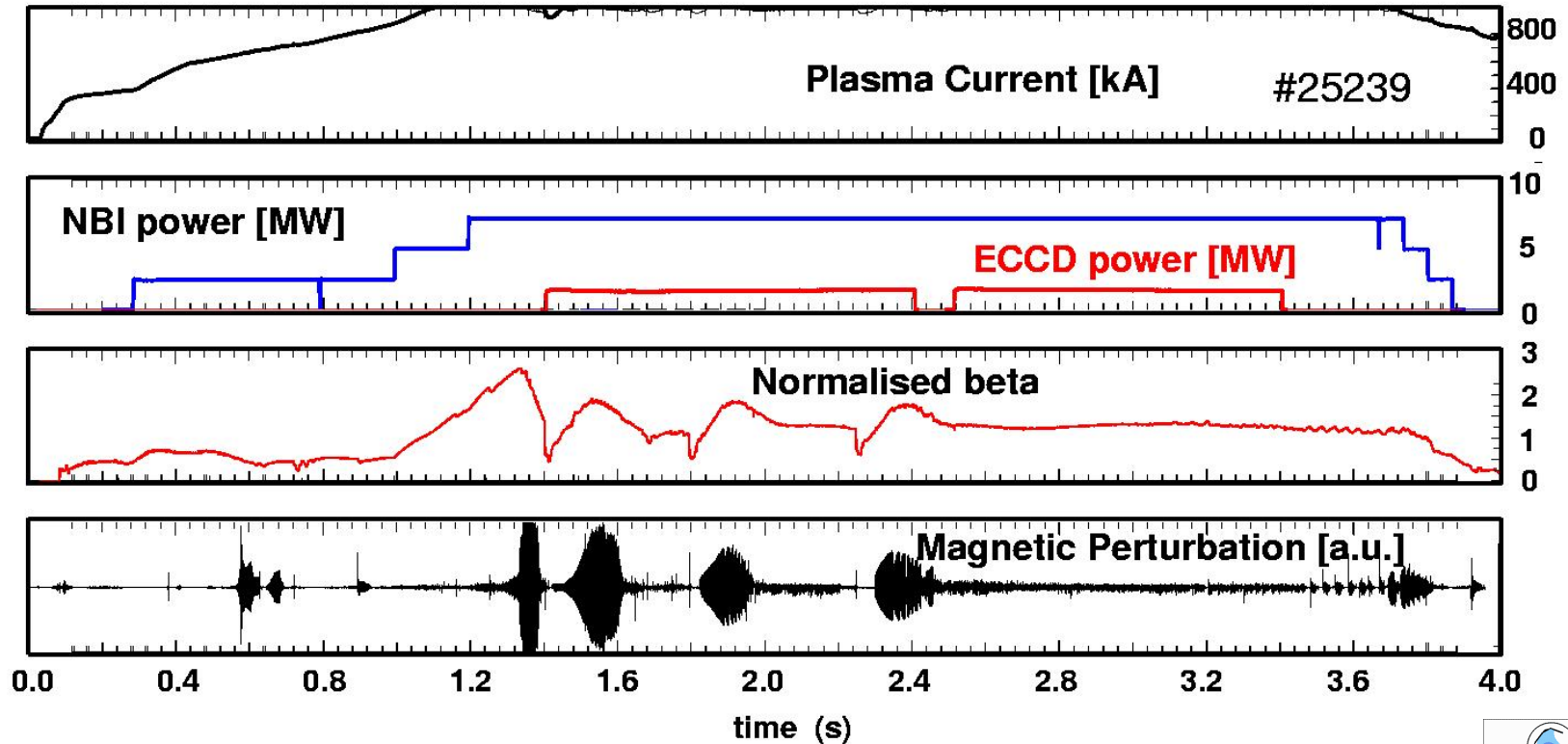
Neoclassical Tearing Mode (NTM)



- Disruption avoidance by ECRH
Target: a discharge that disrupts due to an early (2,1) NTM
($q = 3.9$, $\beta_N = 2.6$)



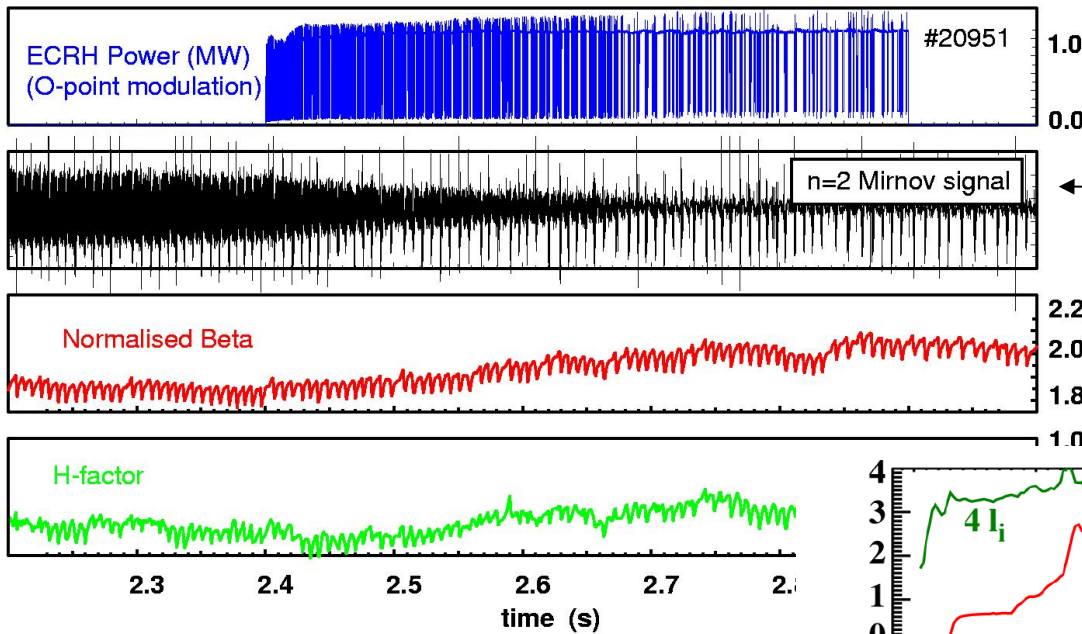
Neoclassical Tearing Mode (NTM)



- Disruption avoidance by ECRH
1.5 MW of ECCD sufficient to avoid disruption, prepare safe landing
note: discharge never recovers performance - need to develop strategy
analysis of 'scalability' ongoing



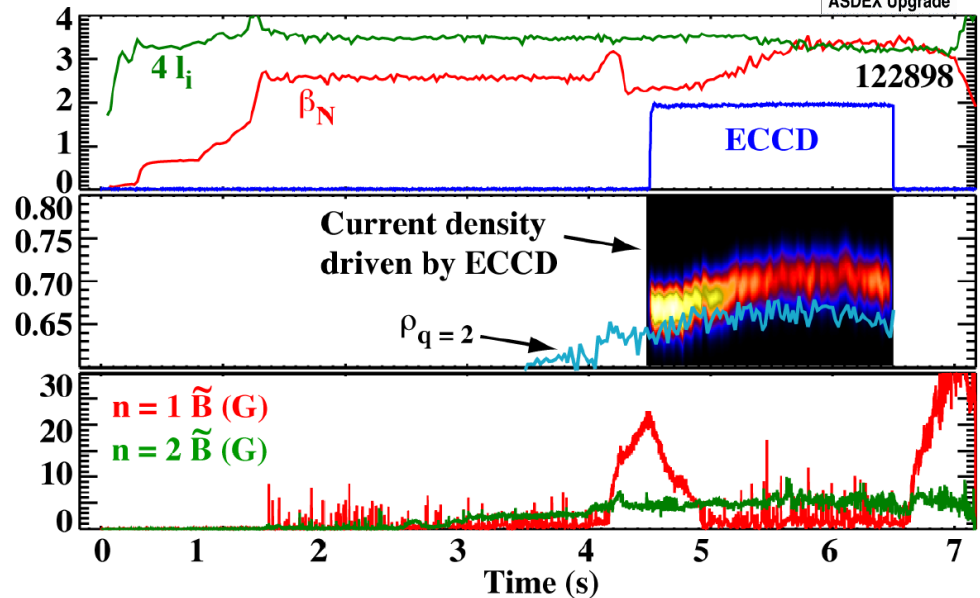
NTM Stabilisation by ECCD



NTM stabilisation with ITER relevant broad deposition in ASDEX Upgrade

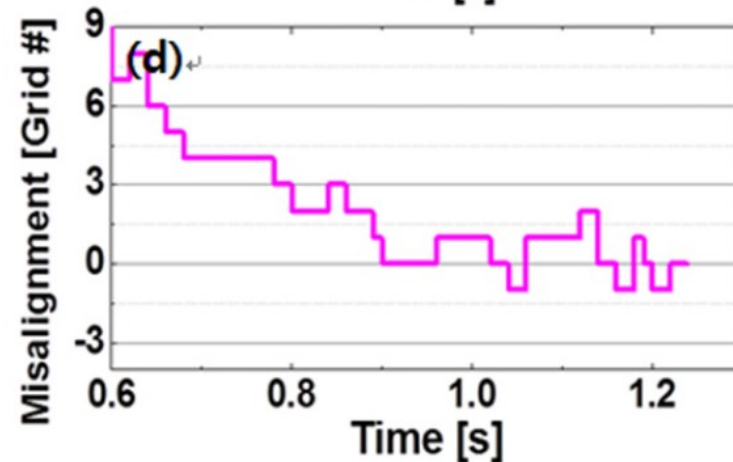
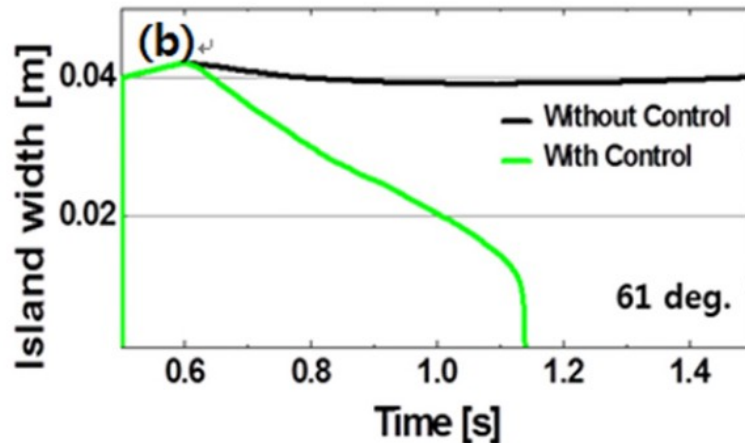
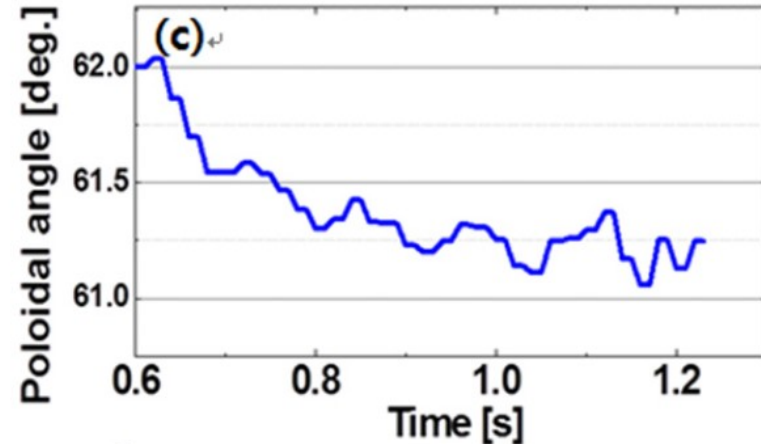
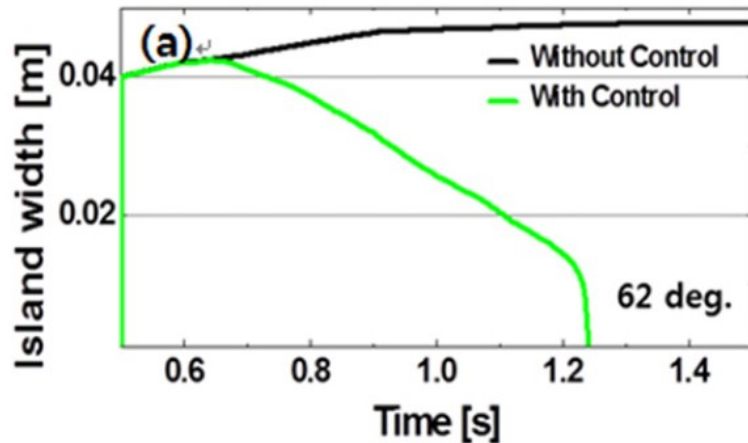


Feedback controlled Deposition in DIII-D



- Demonstration of individual elements as well as integrated feedback

NTM Stabilisation by ECCCD



- Feedback control simulation with minimum seeking method in KSTAR (island growth rate control)

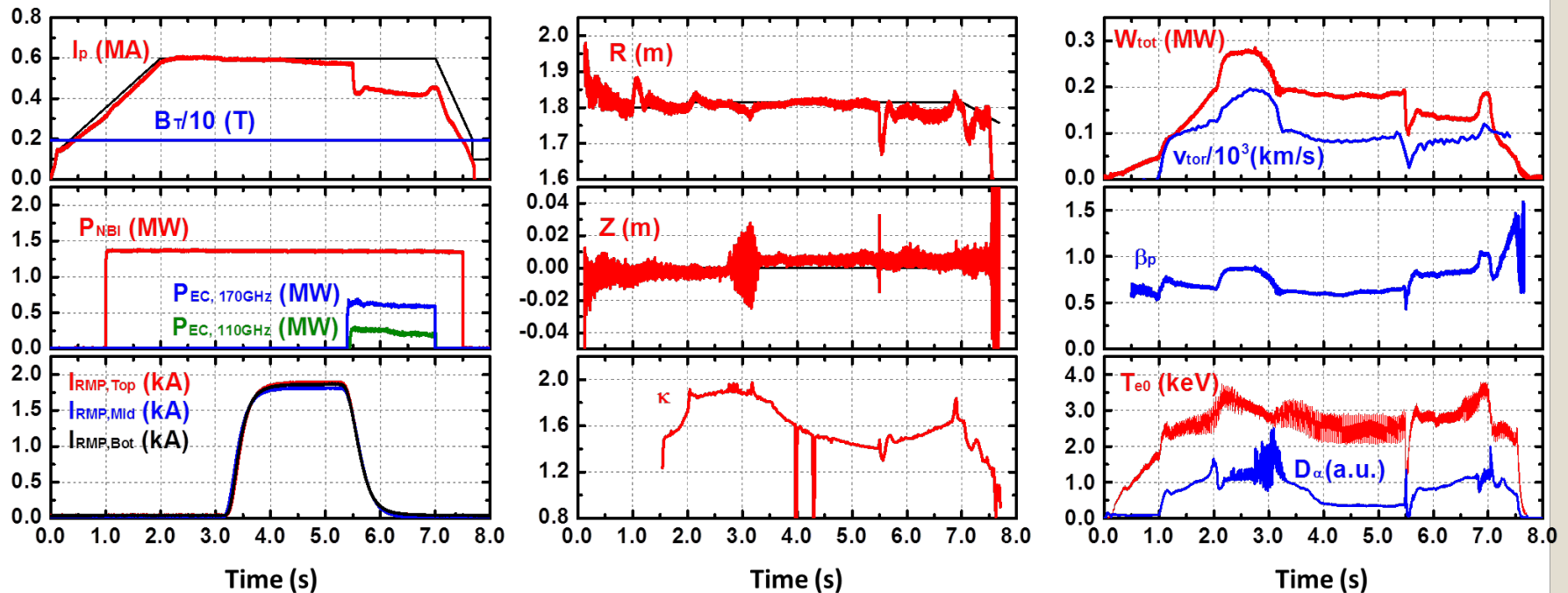
Minhwa Kim et al, Nuclear Fusion **55** 023006 (2015)

NTM Stabilisation by ECCD

- KSTAR

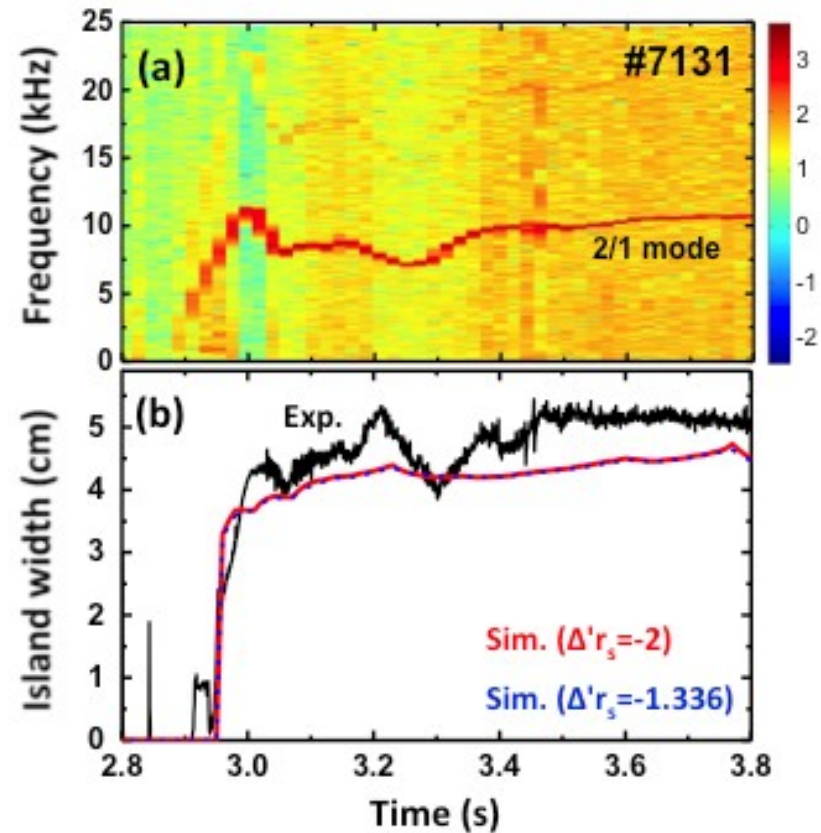
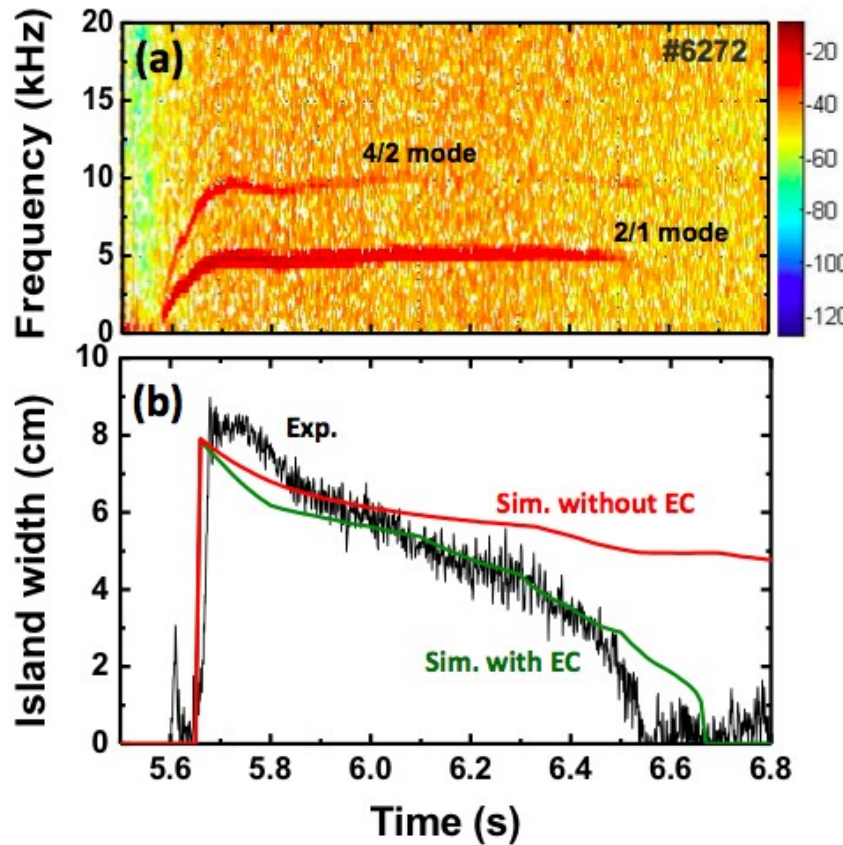


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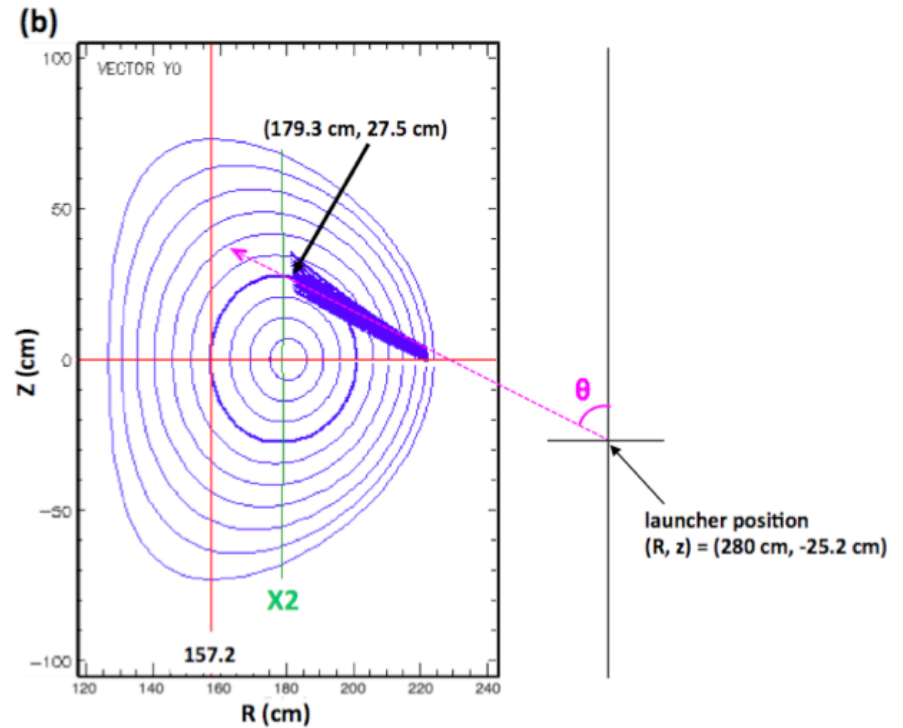
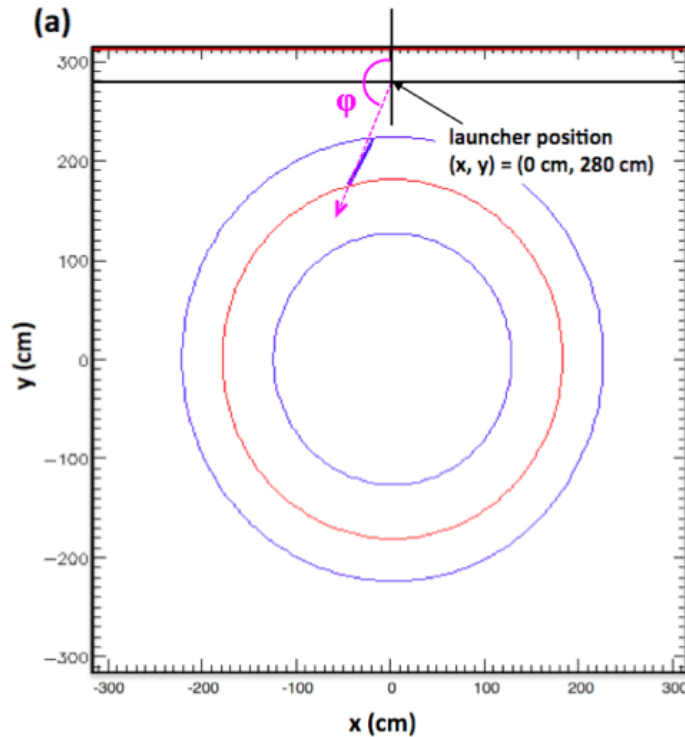
NTM Stabilisation by ECCD

- KSTAR



NTM Stabilisation by ECCD

- KSTAR



NTM Stabilisation by ECCD

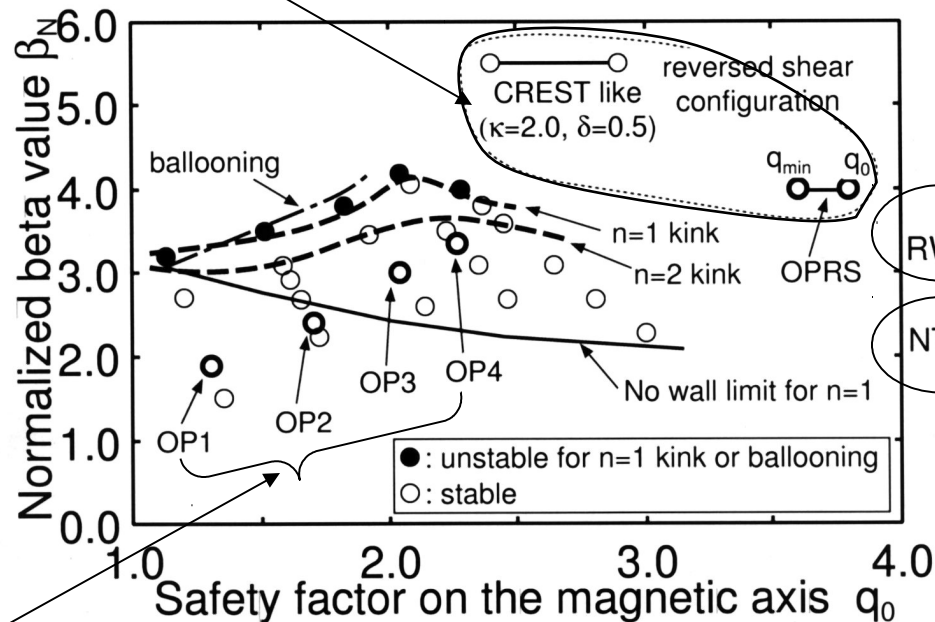


Courtesy from R. J. La Haye, APS (2005)

Development issues on plasma MHD control

The increase of plasma shape parameters from ($\kappa \sim 1.85$, $\delta \sim 0.35$) to ($\kappa \sim 2.0$, $\delta \sim 0.5$) is required to achieve $\beta_N \sim 5.0$ with $r_{\text{wall}} = 1.15a$, however, several improvement for positional instability is supposed to be required in the present design of Demo-CREST

When the plasma performance exceeds the no wall limit (OP3, OP4, OPRS), the suppression of RWM has to be considered.



In the demonstration phase, plasma performance is improved from OP1 to OP4, assisted by the conducting wall at $r_{\text{wall}} = 1.3a$ just behind the blanket modules.

NTM probably appears even in the low β_N region corresponding to OP1 and OP2

References

- *H. R. Wilson, "Modelling the Neoclassical Tearing Mode: An Overview", NTM Miniworkshop, POSTECH, October 31-November 1, 2011, POSTECH, Korea*
- *K. Gibson, "Experimental studies of electron temperature profiles around NTM islands on MAST", NTM Miniworkshop, POSTECH, October 31-November 1, 2011, POSTECH, Korea*
- *H. Zohm, "The Physics of Tearing Modes in Magnetically confined Fusion Plasmas and Their Impact on Plasma Performance", SNU Seminar, March 22, 2013, SNU, Korea*