Fusion Reactor Technology 2

(459.761, 3 Credits)

Prof. Dr. Yong-Su Na

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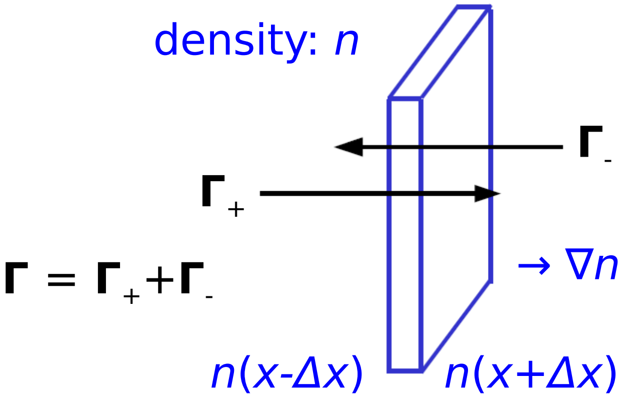
Plasma Transport

- **Classical Transport**

  - Particle transport



density: *n*

Γ_-

Γ_+

$\Gamma = \Gamma_+ + \Gamma_-$

$\rightarrow \nabla n$

$n(x\text{-}\Delta x)$ $n(x+\Delta x)$

$$\Gamma_+ = \frac{1}{4} n(x - \Delta x) v$$

$$\vec{\Gamma}_- = \frac{1}{4} n(x + \Delta x) \vec{v}$$

$$\vec{\Gamma} = \vec{\Gamma}_+ - \vec{\Gamma}_- = \frac{1}{4}\left[n(x - \Delta x) - n(x + \Delta x)\right]\vec{v}$$

$$= -\frac{\Delta x \vec{v}}{2}\frac{\partial n}{\partial x} = -\frac{(\Delta x)^2}{2\tau}\frac{\partial n}{\partial x}$$

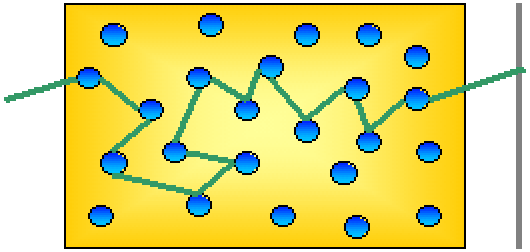
$$= -D\frac{\partial n}{\partial x}$$

: Particle flux- Fick’s law

$$n(x \pm \Delta x) \approx n(x) \pm \Delta x \frac{\partial n}{\partial x}$$

$$D = \frac{(\Delta x)^2}{2\tau}$$

: diffusion coefficient (m^2/s)



The heat and momentum fluxes can be estimated in the similar fashion.

Plasma Transport

- **Classical Diffusion**

  - Momentum transport

    Momentum flux

$$\pi_{\alpha\beta} = -\eta \frac{\partial v_y}{\partial x}$$

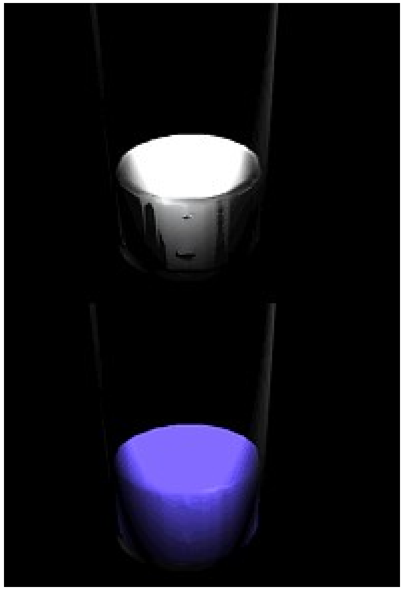
$$\eta \sim \frac{mn(\Delta x)^2}{\tau} \sim mnD$$: viscosity coefficient

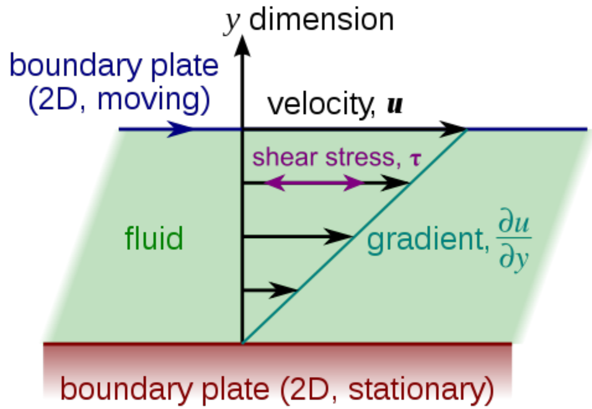
  - Heat transport

    Heat flux

$$q = -\kappa \frac{\partial T}{\partial x}$$: Fourier's law

$$\kappa \sim \frac{n(\Delta x)^2}{\tau} \sim nD$$: thermal conductivity



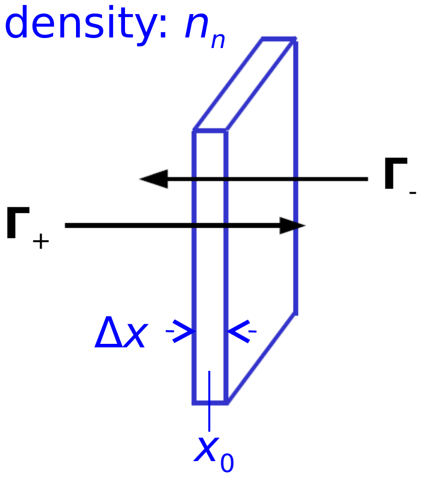


$$F = \mu A \frac{u}{y}, \quad \tau = \mu \frac{du_x}{dy}$$

μ: dynamic viscosity

Plasma Transport

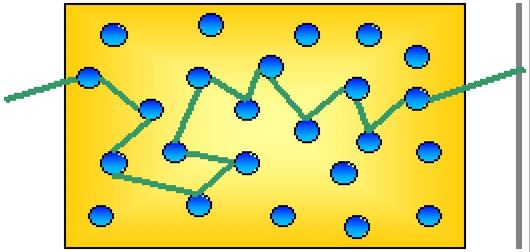
- **Classical Transport**
  - Particle transport in weakly ionised plasmas



$$D = \frac{(\Delta x)^2}{2\tau}$$

$$\Gamma = \Gamma_+ + \Gamma_-$$

Estimate transport coefficients: Δx from mean free path



$$\Delta x = \lambda_m = \frac{1}{n_n \sigma}$$

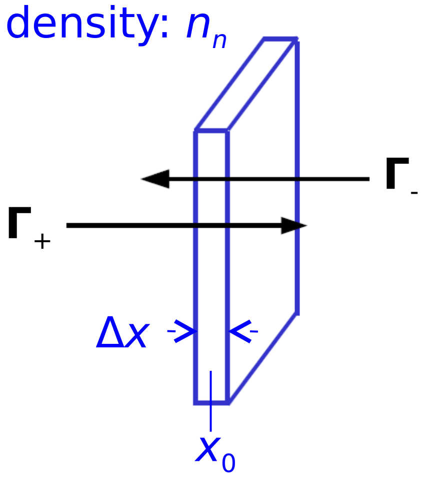
$$\frac{ct}{n_n \pi d^2 ct} = \frac{1}{n_n \pi d^2} = \frac{1}{n_n \sigma}$$: particle approach

$$\Gamma = \Gamma_0 e^{-n_n \sigma x} \equiv \Gamma_0 e^{-x/\lambda_m}$$: fluid approach

Plasma Transport

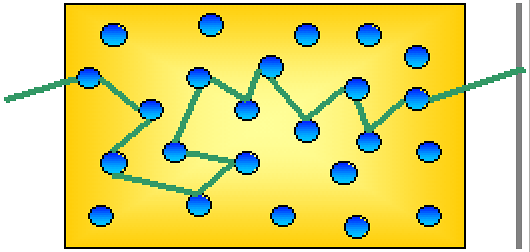
- **Classical Transport**
  - Particle transport in weakly ionised plasmas

$$D = \frac{(\Delta x)^2}{2\tau}$$



$$\Gamma = \Gamma_+ + \Gamma_-$$

Estimate transport coefficients:
τ from collision frequency with neutrals



Plasma Transport

- **Classical Transport**

  - Particle transport in weakly ionised plasmas

$$\Gamma_j = n v_j = \pm \mu_j n E - D_j \nabla n$$

$$\mu \equiv \frac{|q|}{m\nu}$$: Mobility

$$D = \frac{kT}{m\nu} \sim v_{th}^2 \tau \sim \frac{\lambda_m^2}{\tau}$$: Diffusion coefficient

Ambipolar Diffusion

$$\Gamma = - D_a \nabla n$$

$$D_a \equiv \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \sim D_i + \frac{T_e}{T_i} D_i$$

Plasma Transport

- **Classical Transport**

  - Particle transport in weakly ionised plasmas with magnetic field

$$\vec{\Gamma}_{\perp j} = n\vec{v}_{\perp j} = \pm\mu_{\perp j} n\vec{E} - D_{\perp j}\nabla n + \frac{n(v_E + v_D)}{1+(\nu^2/\omega_c^2)}$$

$$\mu_{\perp} \equiv \frac{\mu}{1+\omega_c^2\tau^2}$$: Mobility

$$D_{\perp} = \frac{D}{1+\omega_c^2\tau^2} = \frac{kT\nu}{m\omega_c^2} \sim v_{th}^2\frac{r_L^2}{v_{th}^2} \sim \frac{r_L^2}{\tau}$$: Diffusion coefficient

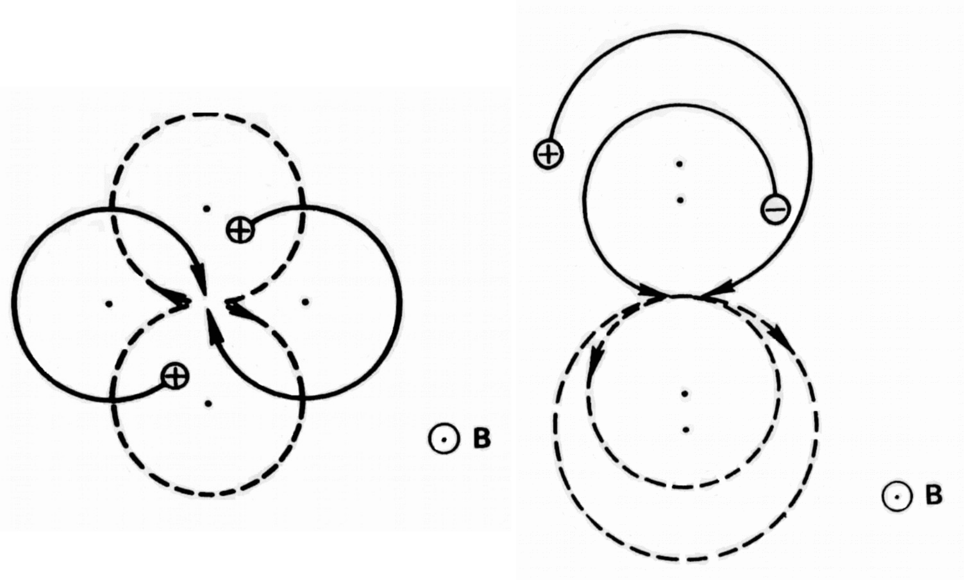
Plasma Transport

- **Classical Transport**

  - Particle transport in fully ionised plasmas with magnetic field

$$\Gamma_\perp = n v_\perp =- D_\perp \nabla n$$

$$D_\perp = \frac{\eta_\perp n \sum kT}{B^2}$$



τ from collision frequency

$$\nu_{ee} \approx \nu_{ei} \propto \frac{ne^4}{\sqrt{m_e} T_e^{3/2}}$$

$$\nu_{ie} = \left(\frac{m_e}{m_i}\right) \nu_{ee}$$

$$\nu_{ii} = \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{T_e}{Ti}\right)^{3/2} \nu_{ee}$$

Plasma Transport

- **Classical Transport**

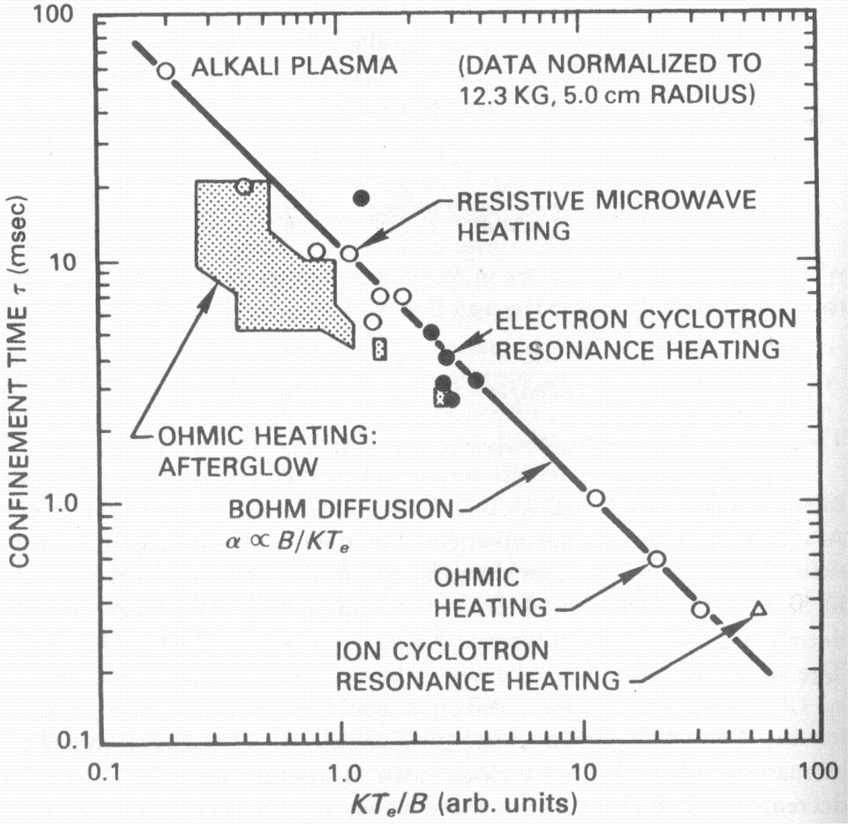
  - Classical thermal conductivity (expectation): $\chi_i \sim 40\chi_e$
  - Typical numbers expected: 10^{-4} m^2/s
  - Experimentally found: 1 m^2/s, $\chi_i \sim \chi_e$

Bohm diffusion (1946): $$D_\perp = \frac{1}{16}\frac{kT_e}{eB}$$

Plasma Transport

- **Classical Transport**

Bohm diffusion: $D_\perp = \frac{1}{16}\frac{kT_e}{eB}$



τ_E in various types of discharges in the Model C Stellarator

F. F. Chen, "Introduction to Plasma Physics and Controlled Fusion" (2006)

Plasma Transport

- **Braginskii Equations**

$$p_e = n_e T_e, \quad p_i = n_i T_i$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla)$$

$$n = n_e = Z n_i$$

assume $Z = 1$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0$$

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0$$

$$m_e n_e \frac{dv_{e\alpha}}{dt} = - \frac{\partial p_e}{\partial x_\alpha} - \frac{\partial \pi_{e\alpha\beta}}{\partial x_\beta} - e n_e (E + \left[\mathbf{v}_e \times \mathbf{B}\right]_\alpha) + R_\alpha$$

$$m_i n_i \frac{dv_{i\alpha}}{dt} = - \frac{\partial p_i}{\partial x_\alpha} - \frac{\partial \pi_{i\alpha\beta}}{\partial x_\beta} - Z e n_i (E + \left[\mathbf{v}_i \times \mathbf{B}\right]_\alpha) - R_\alpha$$

R, $\pi_{\alpha\beta}$, q, Q ?

$$\frac{3}{2} n_e \frac{dT_e}{dt} + p_e \nabla \cdot \mathbf{v}_e = -\nabla \cdot \mathbf{q}_e - \pi_{e\alpha\beta} \frac{\partial v_{e\alpha}}{\partial x_\beta} + Q_e$$

Meaning of each term?

Heat flux: transport of energy associated with the random motion in the coordinate system in which the particle flow velocity is zero.

$$\frac{3}{2} n_i \frac{dT_i}{dt} + p_i \nabla \cdot \mathbf{v}_i = -\nabla \cdot \mathbf{q}_i - \pi_{i\alpha\beta} \frac{\partial v_{i\alpha}}{\partial x_\beta} + Q_i$$

Rate of thermal energy change

Heat generated by collisions between two different species

Plasma Transport

- **Transport / Closure Theories**

	Braginskii	Neoclassical transport	Unified Closure (Ji)
Collisionality	High	High: PS Low: banana	General
Magnetic field strength	General	Strong	Strong
Magnetic geometry	General	Nested	General
Collision operator	Landau	Landau	Landau

Jeong-Young Ji, Lecture at SNU, 2012

Plasma Transport

- **Braginskii Equations**

  - Transfer of momentum from ions to electrons by collisions

$$\mathbf{R} = \mathbf{R}_u + \mathbf{R}_T$$

R$_u$: force of friction due to the existence of a relative velocity

u=v$_e$-**v**$_i$

R$_T$: thermal force which arises by virtue of a gradient in the electron temperature

$$\mathbf{R}_u = -\frac{m_e n_e}{\tau_e}(0.51\mathbf{u}_\parallel + \mathbf{u}_\perp) = en\left(\frac{\mathbf{j}_\parallel}{\sigma_\parallel} + \frac{\mathbf{j}_\perp}{\sigma_\perp}\right)$$

$$\mathbf{R}_T = -0.71 n_e \nabla_\parallel T_e - \frac{3}{2}\frac{n_e}{\omega_e \tau_e}\left(\frac{\mathbf{B}}{B} \times \nabla T_e\right)$$

$$\sigma_\perp = \frac{e^2 n_e \tau_e}{m_e} = \sigma_1 T_e^{3/2}$$

$$\sigma_\parallel = 1.96\sigma_\perp = 1.96\sigma_1 T_e^{3/2}$$

$$\sigma_1 = \frac{0.9 \times 10^{13}}{(\lambda/10)Z} \left[\mathrm{s}^{-1} \cdot \mathrm{eV}^{-3/2}\right]$$

Plasma Transport

- **Braginskii Equations**
  - Heat flux

$$D_\perp = \frac{kT\nu}{m\omega_c^2}$$

$$\mathbf{q}_e = \mathbf{q}_u^e + \mathbf{q}_T^e$$

$$\kappa \sim nD$$

$$\mathbf{q}_u^e = 0.71 n_e T_e \mathbf{u}_\| + \frac{3}{2}\frac{n_e T_e}{\omega_e \tau_e}\left(\frac{\mathbf{B}}{B}\times\mathbf{u}\right)$$

$$\mathbf{q}_T^e = -\kappa_\|^e \nabla_\| T_e - \kappa_\perp^e \nabla_\perp T_e - \frac{5}{2}\frac{n_e T_e}{eB}\left(\frac{\mathbf{B}}{B}\times\nabla T_e\right)$$

$$\kappa_\|^e = 3.16\frac{n_e T_e \tau_e}{m_e}, \quad \kappa_\perp^e = 4.66\frac{n_e T_e}{m_e \omega_e^2 \tau_e}$$

$$\mathbf{q}_i = -\kappa_\|^i \nabla_\| T_i - \kappa_\perp^i \nabla_\perp T_i + \frac{5}{2}\frac{n_i T_i}{ZeB}\left(\frac{\mathbf{B}}{B}\times\nabla T_i\right)$$

$$\omega_i \tau_i >> 1$$

$$\kappa_\|^i = 3.9\frac{n_i T_i \tau_i}{m_i}, \quad \kappa_\perp^i = 2\frac{n_i T_i}{m_i \omega_i^2 \tau_i}$$

Plasma Transport

- **Braginskii Equations**

  - Heat generated as a consequence of collisions

$$Q_i = Q_\Delta = \frac{3m_e}{m_i}\frac{n_e}{\tau_e}(T_e - T_e)$$

$$Q_e = -\mathbf{Ru} - Q_\Delta = \frac{j_\parallel^2}{\sigma_\parallel} + \frac{j_\perp^2}{\sigma_\perp} + \frac{1}{en_e}\mathbf{jR}_T - \frac{3m_e}{m_i}\frac{n_e}{\tau_e}(T_e - T_e)$$

  - Stress tensor in the absence of a magnetic field

$$\pi_{\alpha\beta} = nm\left\langle v'_\alpha v'_\beta - (v'^2/3)\delta_{\alpha\beta}\right\rangle = -\underset{\downarrow}{\eta_0} W_{\alpha\beta}$$

viscosity coefficient

Rate of strain tensor

$$W_{\alpha\beta} = \frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha} - \frac{2}{3}\delta_{\alpha\beta}\nabla\cdot\mathbf{v}$$

Plasma Transport

$$D_\perp = \frac{kT\nu}{m\omega_c^2} \qquad \eta \sim mnD$$

- **Braginskii Equations**

In a strong magnetic field $\omega\tau >> 1$

$$\pi_{zz} = - \eta_0 W_{zz}$$

$$\pi_{xx} = - \eta_0 \frac{1}{2}\left(W_{xx} + W_{yy}\right) - \eta_1 \frac{1}{2}\left(W_{xx} - W_{yy}\right) - \eta_3 W_{xy}$$

$$\pi_{yy} = - \eta_0 \frac{1}{2}\left(W_{xx} + W_{yy}\right) - \eta_1 \frac{1}{2}\left(W_{yy} - W_{xx}\right) + \eta_3 W_{xy}$$

$$\pi_{xy} = \pi_{yx} = - \eta_1 W_{xy} + \eta_3 \frac{1}{2}\left(W_{xx} - W_{yy}\right)$$

$$\pi_{xz} = \pi_{zx} = - \eta_2 W_{xz} - \eta_4 W_{yz}$$

$$\pi_{yz} = \pi_{zy} = - \eta_2 W_{yz} + \eta_4 W_{xz}$$

$$\eta_0^i = 0.96 n_i T_i \tau_i$$

$$\eta_1^i = \frac{3}{10}\frac{n_i T_i}{\omega_i^2 \tau_i}, \quad \eta_2^i = 4\eta_1^i$$

$$\eta_3^i = \frac{1}{2}\frac{n_i T_i}{\omega_i}, \quad \eta_4^i = 2\eta_3^i$$

viscosity coefficients

$$\eta_0^e = 0.73 n_e T_e \tau_e$$

$$\eta_1^e = 0.51 \frac{n_e T_e}{\omega_e^2 \tau_e}, \quad \eta_2^e = 4\eta_1^e$$

$$\eta_3^e = - \frac{1}{2}\frac{n_e T_e}{\omega_e}, \quad \eta_4^i = 2\eta_3^e$$

Plasma Transport

- **Braginskii Equations**
  - Heat generated as a result of viscosity

$$Q_{vis} = - \pi_{\alpha\beta} \frac{\partial v_\alpha}{\partial x_\beta} = - \frac{1}{2} \pi_{\alpha\beta} W_{\alpha\beta}$$

Individual Charge Trajectories

- **Invariant of Motion**

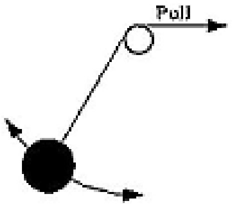
$$\frac{d}{dt}E_0 = \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = 0 \qquad \mu = \frac{mv_\perp^2/2}{B} \qquad m\frac{d\mathbf{v}_\parallel}{dt} = -\frac{\mu}{v_\parallel}\frac{d\mathbf{B}}{dt}$$

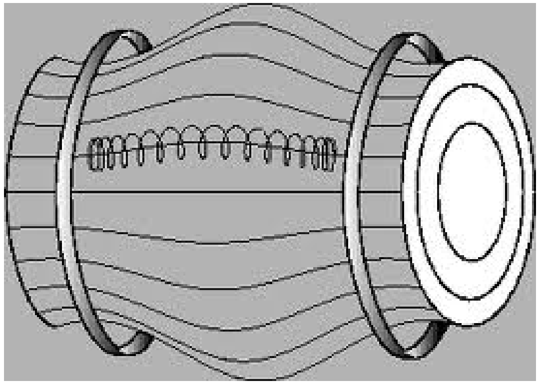
$$\mathbf{F}_\parallel = m\frac{d\mathbf{v}_\parallel}{dt} = -\mu\nabla_\parallel\mathbf{B} = -\mu\frac{\partial\mathbf{B}}{\partial s} = -\mu\frac{\partial\mathbf{B}}{\partial s}\cdot\frac{ds}{dt}\cdot\frac{1}{v_\parallel} = -\frac{\mu}{v_\parallel}\frac{d\mathbf{B}}{dt} \longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{dB}{dt}$$

$$\longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = \frac{d}{dt}(\mu B) + \left(-\mu\frac{dB}{dt}\right) = 0$$

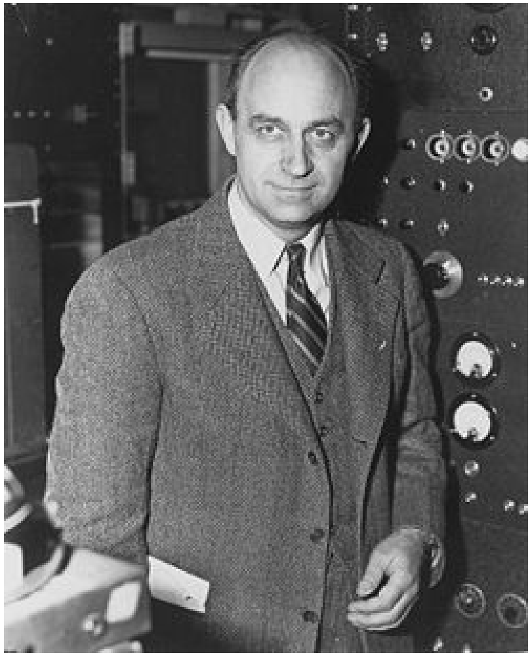
$\longrightarrow \frac{d}{dt}(\mu) = 0$: adiabatic invariant

- If B is constant
- $\frac{r_L}{B}\nabla_\parallel B << 1$
- $\frac{1}{\omega_c B}\frac{dB}{dt} << 1$



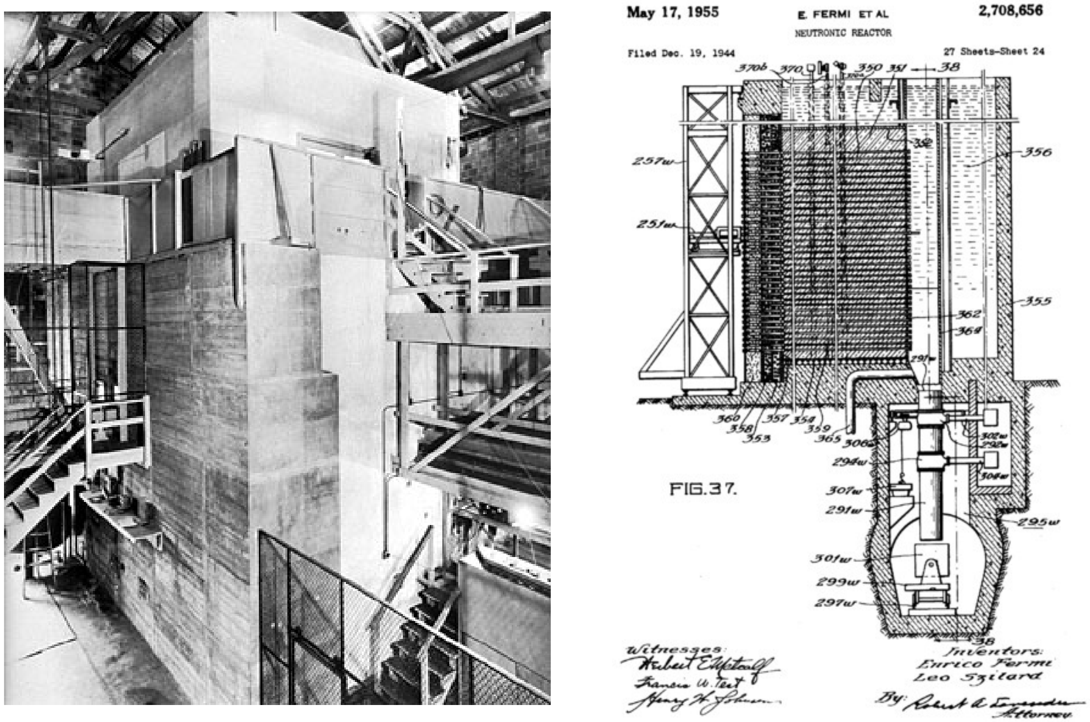


Magnetic Mirror



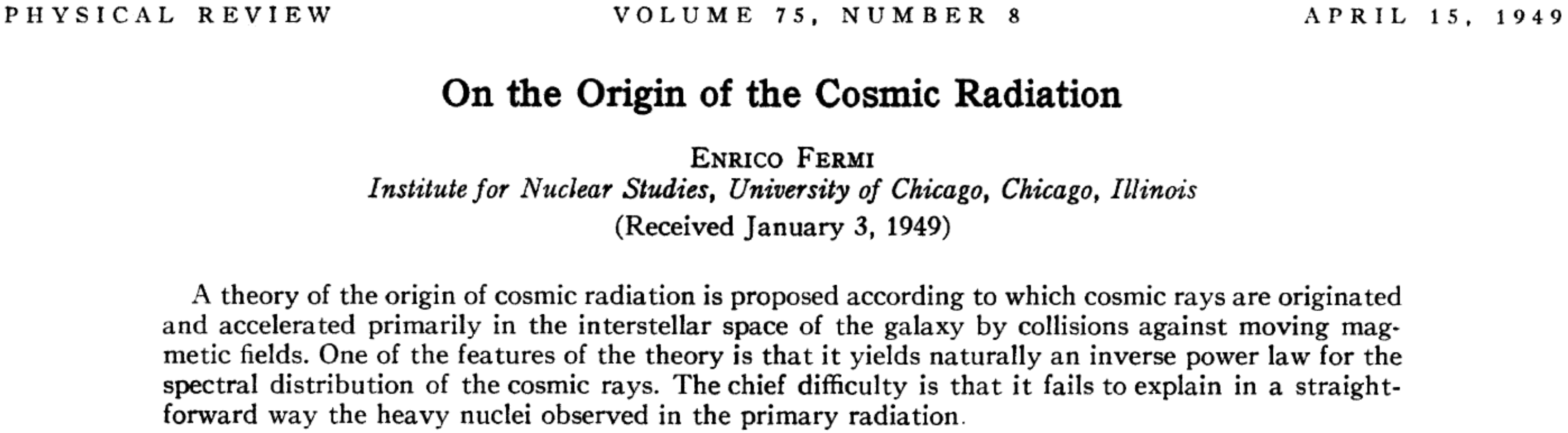
Enrico Fermi (1901-1954)

Nobel Laureate in physics in 1938
Cf. Marshall Rosenbluth (Doctoral student)



CP-1 (Chicago Pile-1, the world's first human-made nuclear reactor) and Drawings from the Fermi–Szilárd "neutronic reactor" patent

Magnetic Mirror



PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

On the Origin of the Cosmic Radiation

ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

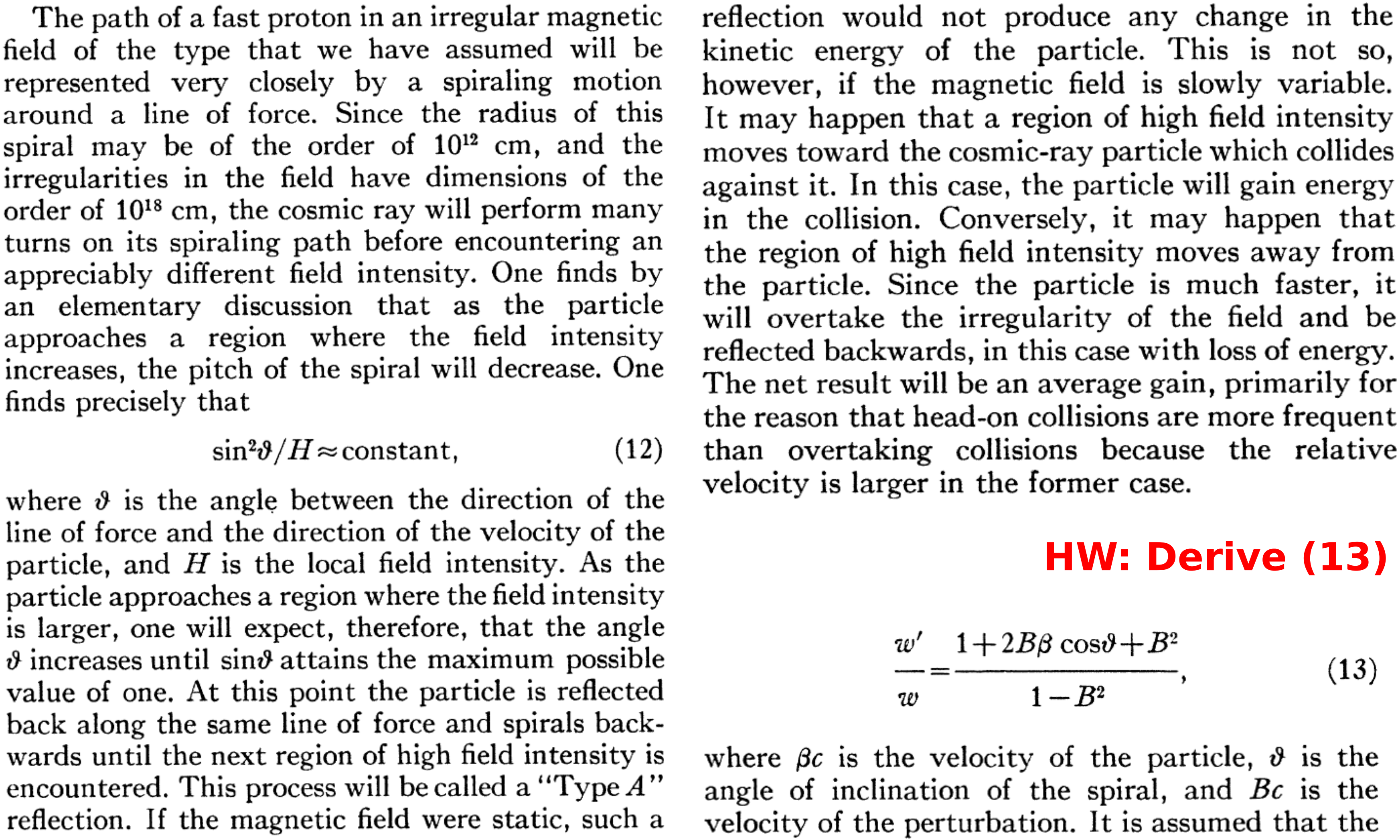
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magmetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

Magnetic Mirror

$$E_0 = \frac{1}{2}mv^2 = \frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2 = const.$$

$$\mu = \frac{\frac{1}{2}mv_\perp^2}{B} = \frac{\frac{1}{2}mv^2\sin^2\theta}{B} = const.$$



The path of a fast proton in an irregular magnetic field of the type that we have assumed will be represented very closely by a spiraling motion around a line of force. Since the radius of this spiral may be of the order of 10^{12} cm, and the irregularities in the field have dimensions of the order of 10^{18} cm, the cosmic ray will perform many turns on its spiraling path before encountering an appreciably different field intensity. One finds by an elementary discussion that as the particle approaches a region where the field intensity increases, the pitch of the spiral will decrease. One finds precisely that

$$\sin^2\vartheta/H \approx \text{constant}, \tag{12}$$

where ϑ is the angle between the direction of the line of force and the direction of the velocity of the particle, and H is the local field intensity. As the particle approaches a region where the field intensity is larger, one will expect, therefore, that the angle ϑ increases until $\sin\vartheta$ attains the maximum possible value of one. At this point the particle is reflected back along the same line of force and spirals backwards until the next region of high field intensity is encountered. This process will be called a "Type A" reflection. If the magnetic field were static, such a reflection would not produce any change in the kinetic energy of the particle. This is not so, however, if the magnetic field is slowly variable. It may happen that a region of high field intensity moves toward the cosmic-ray particle which collides against it. In this case, the particle will gain energy in the collision. Conversely, it may happen that the region of high field intensity moves away from the particle. Since the particle is much faster, it will overtake the irregularity of the field and be reflected backwards, in this case with loss of energy. The net result will be an average gain, primarily for the reason that head-on collisions are more frequent than overtaking collisions because the relative velocity is larger in the former case.

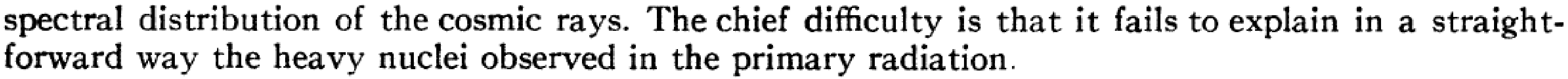
HW: Derive (13)

$$\frac{w'}{w} = \frac{1+2B\beta\cos\vartheta+B^2}{1-B^2}, \tag{13}$$

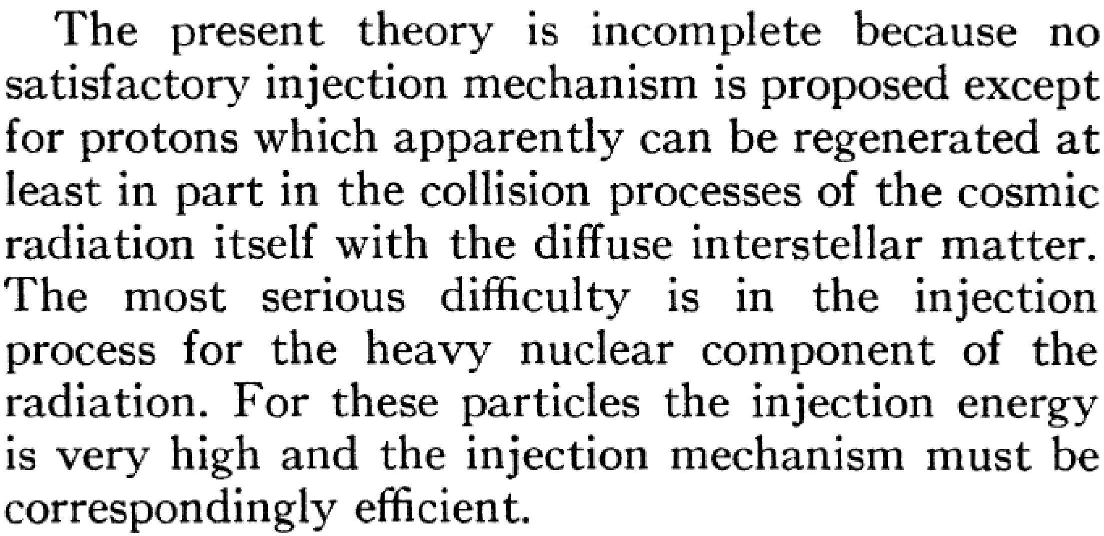
where βc is the velocity of the particle, ϑ is the angle of inclination of the spiral, and Bc is the velocity of the perturbation. It is assumed that the

Magnetic Mirror

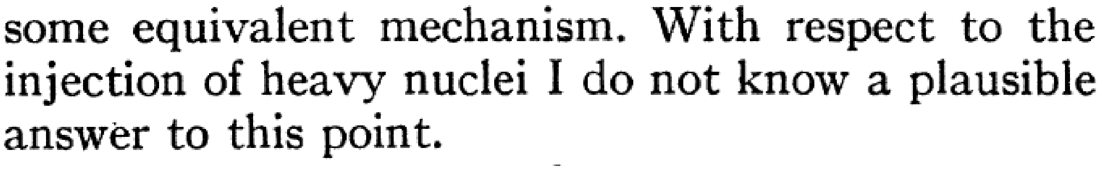
- **Fermi as a genuine scientist**



spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.



The present theory is incomplete because no satisfactory injection mechanism is proposed except for protons which apparently can be regenerated at least in part in the collision processes of the cosmic radiation itself with the diffuse interstellar matter. The most serious difficulty is in the injection process for the heavy nuclear component of the radiation. For these particles the injection energy is very high and the injection mechanism must be correspondingly efficient.



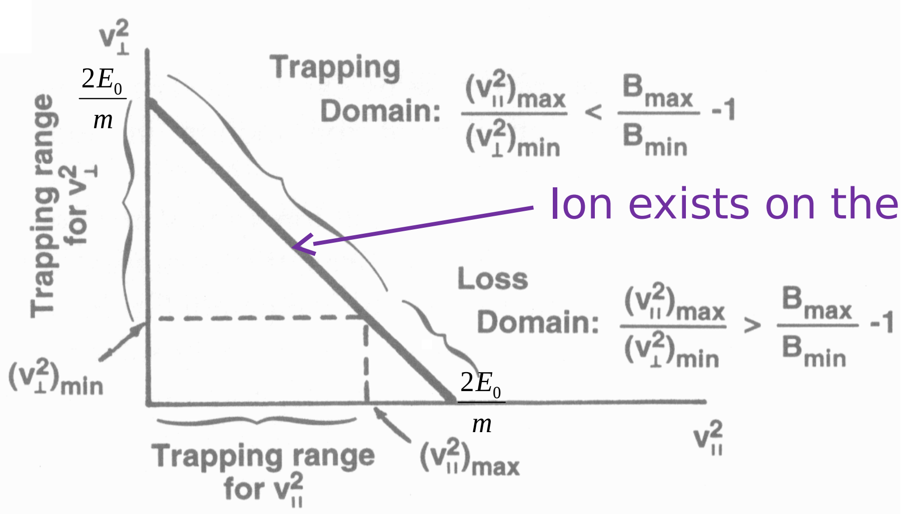
some equivalent mechanism. With respect to the injection of heavy nuclei I do not know a plausible answer to this point.

Magnetic Mirror

- **Condition for Trapping of Particles**

$$E_0 = \frac{1}{2}mv_\parallel^2 + \frac{1}{2}mv_\perp^2 = \frac{1}{2}mv_\parallel^2 + \mu B = \frac{1}{2}m\left(v_\parallel^2\right)_{\max} + \mu B_{\min}$$

$$\mathbf{F}_\parallel = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_\parallel B = -\mu\nabla_\parallel B$$



Ion exists on the E_0=constant line.

$$v_\perp^2 = \frac{2E_0}{m} - v_\parallel^2$$

Condition for trapping of particles

$$v_\parallel\Big|_{B\le B_{\max}} = 0 \longrightarrow E_0 = \frac{1}{2}m\left(v_\parallel^2\right)_{\max} + \mu B_{\min} \le 0 + \mu B_{\max}$$

Magnetic Mirror

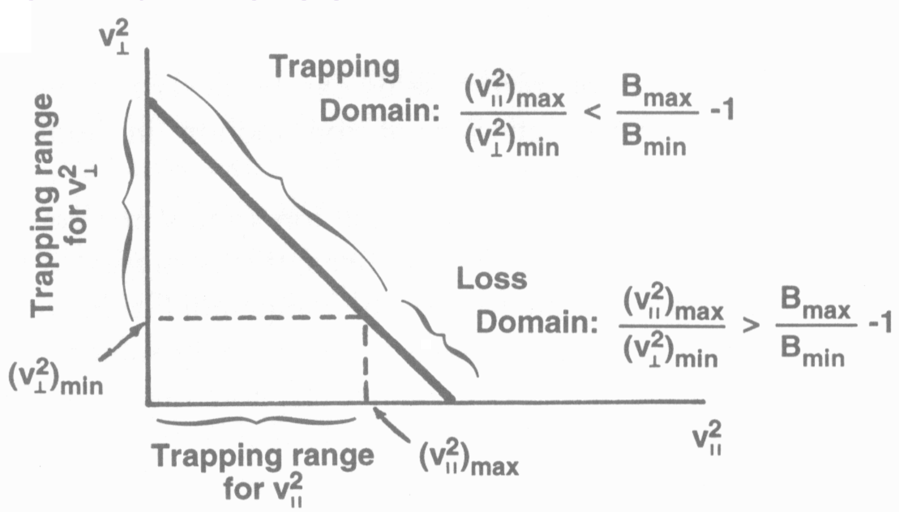
- **Condition for Trapping of Particles**

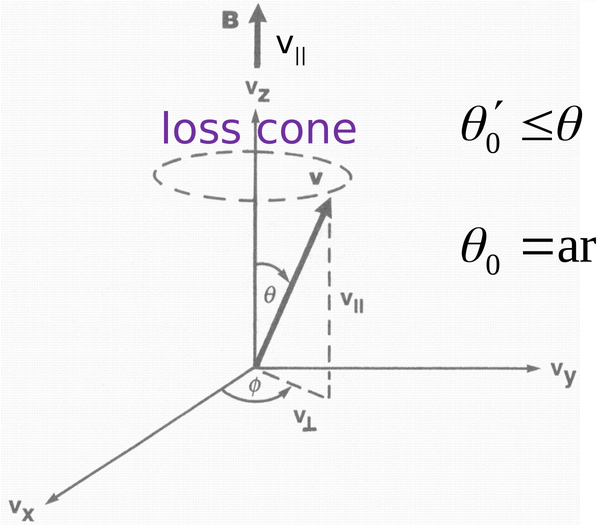
$$\frac{\frac{1}{2}m\left(v_\parallel^2\right)_{\max}}{\mu B_{\min}} \le \frac{B_{\max}}{B_{\min}} - 1$$

$$\frac{\left(v_\parallel^2\right)_{\max}}{\left(v_\perp^2\right)_{\min}} = \left(\frac{v_\parallel^2}{v_\perp^2}\right)_{mid-plane} \le \frac{B_{\max}}{B_{\min}} - 1$$

$$\frac{v_\parallel}{v_\perp} = \frac{\cos\theta}{\sin\theta} \qquad \sin^2\theta = \frac{1}{\frac{v_\parallel^2}{v_\perp^2}+1}$$

$$\sin\theta \ge \sqrt{\frac{B_{\min}}{B_{\max}}}$$





$$\theta_0' \le \theta \le \pi - \theta_0'$$

$$\theta_0 = \arcsin\sqrt{\frac{B_{\min}}{B_{\max}}}$$

Magnetic Mirror

- **Mirror Ratio**

$$f_{loss}=\frac{\int\limits_{\text{doublecone}} f(\vec{v})d^3v}{\int\limits_0^\infty f(\vec{v})d^3v}=\frac{\int\limits_0^{2\pi}d\phi\left[\int\limits_0^{\theta_0'}\sin\theta d\theta+\int\limits_{\pi-\theta_0'}^{\pi}\sin\theta d\theta\right]\int\limits_0^\infty\frac{f(v)}{4\pi v^2}v^2dv}{\int\limits_0^{2\pi}d\phi\int\limits_0^{\pi}\sin\theta d\theta\int\limits_0^\infty\frac{f(v)}{4\pi v^2}v^2dv}$$

$$=1-\cos\theta_0'$$

$$f_{trap}=\cos\theta_0'=\sqrt{1-\frac{B_{\min}}{B_{\max}}}$$

$$\frac{B_{\max}}{B_{\min}}\equiv R_m$$

mirror ratio:
Determining the effectiveness of confinement

Why are particles reflected in the increased field of the mirrors?

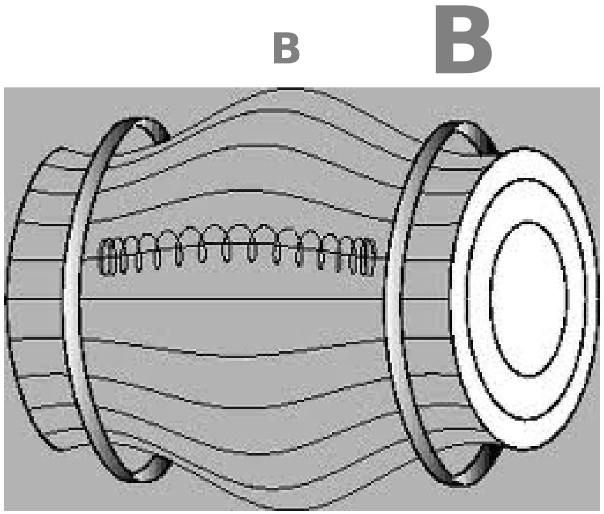
$$\mathbf{F}_{\|}=-\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_{\|}B=-\mu\nabla_{\|}B$$

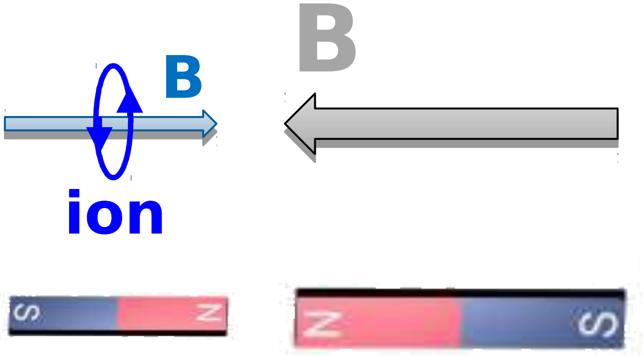
Magnetic Mirror

Why are particles reflected in the increased field of the mirrors?

Adiabatic invariant $\mu = \frac{mv_\perp^2/2}{B}$ $\qquad \frac{d}{dt}(\mu) = 0$

$$\mathbf{F}_\parallel = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_\parallel B = -\mu\nabla_\parallel B$$





Field generated by ion’s gyration

Tokamak Transport

- **Neoclassical theory of transport**

- A. A. Galeev and R. Z. Sagdeev
  “Transport phenomena in a collisionless plasma in a toroidal magnetic system”, Zhurnal Experimentalnoi i Teoreticheskoi Fiziki ***53*** *348 (1967)*
- Major changes arise from toroidal effects characterised by inverse aspect ratio, $\varepsilon = a/R_0$

Tokamak Transport

- **Particle Trapping**

Inverse aspect ratio
$\varepsilon = a/R_0$

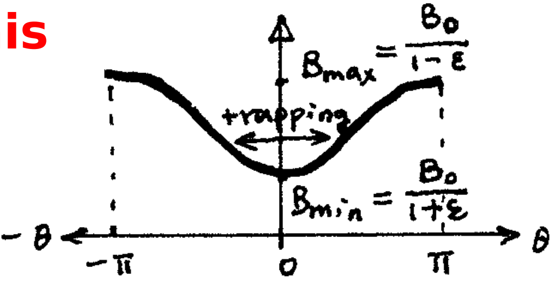
$$\nabla \cdot B = 0$$

$$\Rightarrow \quad \frac{1}{1+\varepsilon\cos\theta}\left\{\frac{1}{r}\frac{\partial}{\partial r}(rB_r)+\frac{1}{r}\frac{\partial}{\partial\theta}\left[(1+\varepsilon\cos\theta)B_\theta\right]+\frac{1}{R_0}\frac{\partial B_\phi}{\partial\phi}\right\}=0$$

$$\Rightarrow \quad B_\theta(r,\theta)=\frac{B_\theta^0(\theta=0)}{1+\varepsilon\cos\theta}$$

HW: Derive this

$$B(r,\theta)=B_\theta(r,\theta)\hat{\theta}+B_\phi(r,\theta)\hat{\phi}=\frac{B_0}{1+\varepsilon\cos\theta}$$



Condition for trapping of particles

$$\frac{\left(v_\parallel^2\right)_{\max}}{\left(v_\perp^2\right)_{\min}}=\left(\frac{v_\parallel^2}{v_\perp^2}\right)_{mid-plane}\le\frac{B_{\max}}{B_{\min}}-1=\frac{\frac{B_0}{1-\varepsilon}}{\frac{B_0}{1+\varepsilon}}-1=\frac{2\varepsilon}{1-\varepsilon}\sim 2\varepsilon$$

$$\Rightarrow \quad v_\parallel^2 \le 2\varepsilon v_\perp^2$$

Tokamak Transport

- **Particle Trapping**

- Particle trapping by magnetic mirrors
  trapped particles with banana orbits
  untrapped (transit or passing) particles with circular orbits

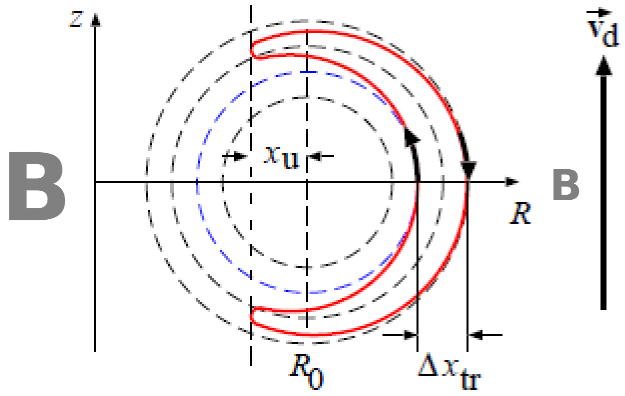
- Trapped fraction: $f_{trap} = \sqrt{1 - \frac{1}{R_m}} = \sqrt{1 - \frac{B_{\min}}{B_{\max}}} = \sqrt{1 - \frac{1-\varepsilon}{1+\varepsilon}} = \sqrt{\frac{2\varepsilon}{1+\varepsilon}} \sim \sqrt{\varepsilon}$

for a typical tokamak, $\varepsilon \sim 1/3 \rightarrow f_{trap} \sim 70\%$

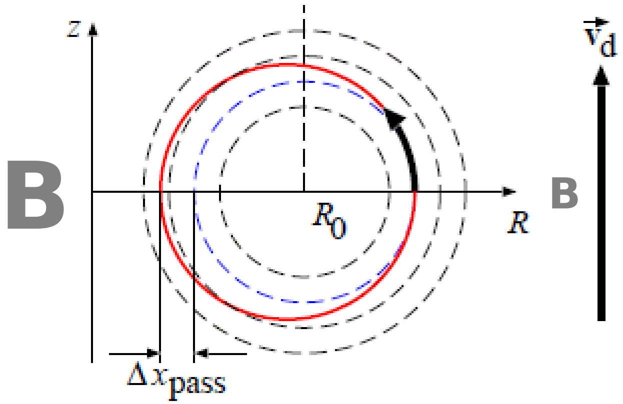
Tokamak Transport

$$\mathbf{v}_d = \frac{m}{q}\left(v_\parallel^2 + \frac{1}{2}v_\perp^2\right)\frac{\mathbf{R}_0 \times \mathbf{B}_0}{R^2 B_0^2}$$

- **Particle Trapping**



trapped particles



Passing (transit) particles

Expelling force of diamagnetic Larmor motion

$$F_D = -\mu\nabla_\perp B = m v_\perp^2 / 2R$$

$$\mathbf{v}_{d,\nabla B} = \pm\frac{v_\perp^2}{2\omega_c}\frac{\mathbf{B}\times\nabla B}{B^2} = \pm\frac{1}{2}v_\perp r_L\frac{\mathbf{B}\times\nabla B}{B^2}$$

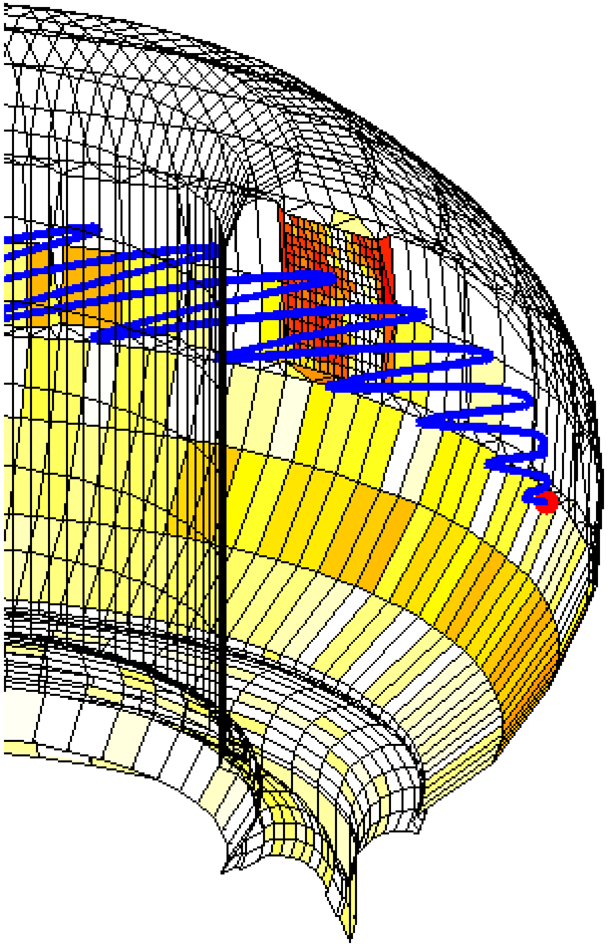
$$F_C = m v_\parallel^2 / R$$

Centrifugal force

$$\mathbf{v}_{d,R} = \frac{m v_\parallel^2}{q B_0^2}\frac{\mathbf{R}_0 \times \mathbf{B}_0}{R^2}$$

Tokamak Transport

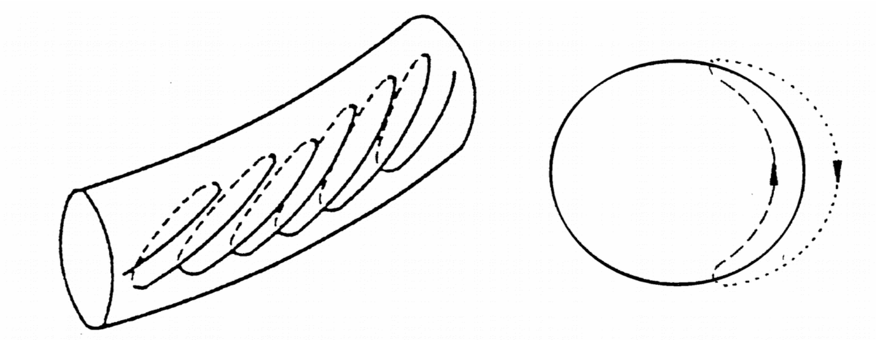
- **Particle Trapping**



HOMEWORK:

- The real particle trajectory is as shown. Why?

- In ST, B is small, what is the particle trajectory like?



Tokamak Transport

- **Particle Trapping**

*J. P. Graves et al, Nature Communications **3** 624 (2012)*

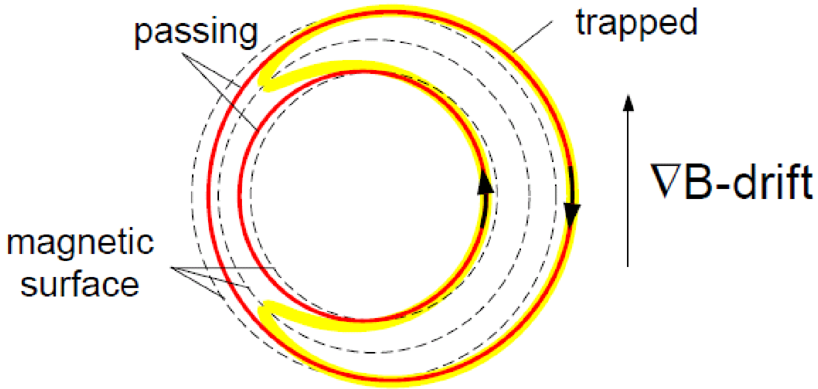
Tokamak Transport

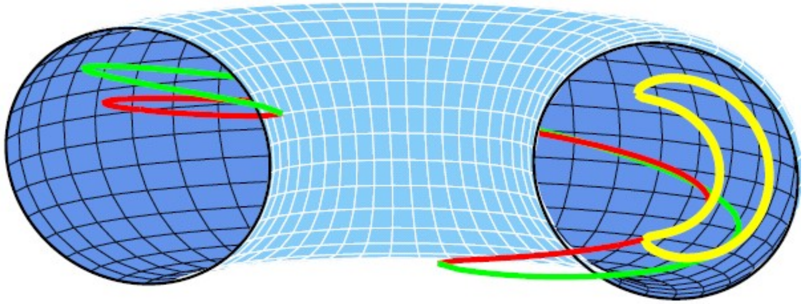
- **Particle Trapping**

*J. P. Graves et al, Nature Communications **3** 624 (2012)*

Tokamak Transport

- **Particle Trapping**





- With known particle trajectories it is possible to find corresponding kinetic coefficients by solving the kinetic equations with Coulomb collisions.
- Rough estimation of transport coefficients: $\delta^2\nu_{eff}$

  δ: particle displacement between collisions

  ν_{eff} : appropriate frequency of collisions

Tokamak Transport

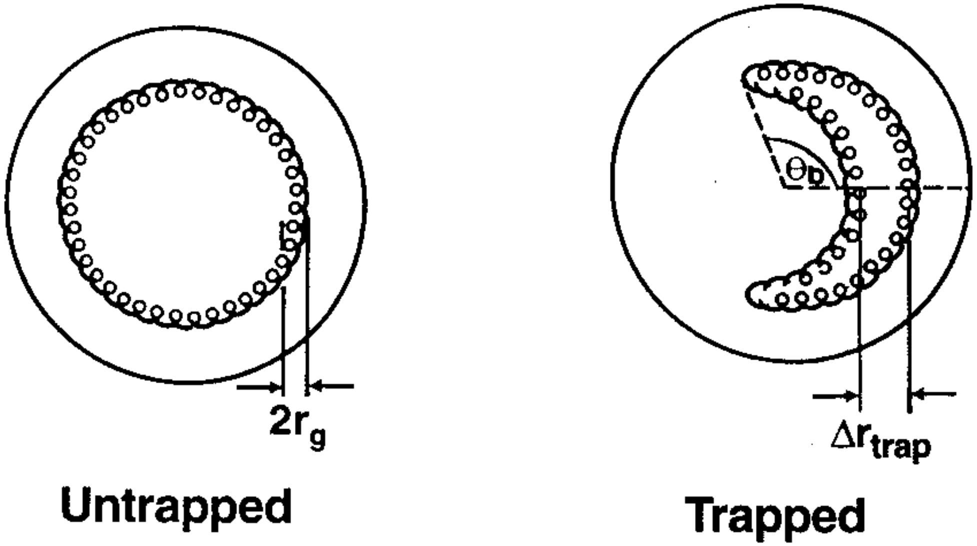
- **Particle Trapping**

- Collisional excursion across flux surfaces

  untrapped particles: $2r_g = 2r_{Li}$

  trapped particles: $\Delta r_{trap} >> 2r_g$

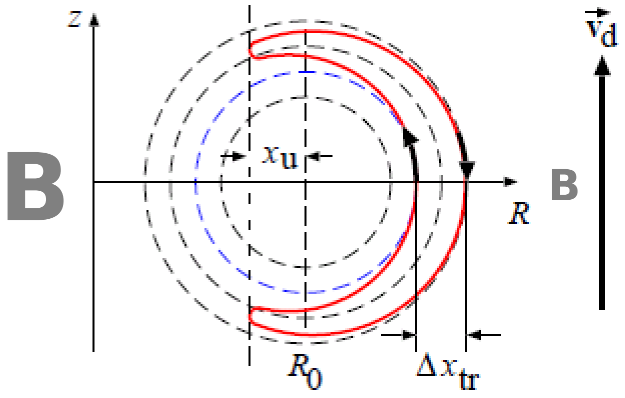
  – enhanced radial diffusion across the confining magnetic field



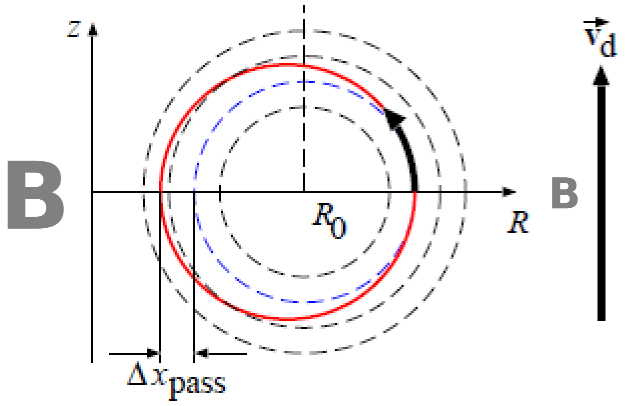
- If the fraction of trapped particle is large, this leakage enhancement constitutes a substantial problem in tokamak confinement.

Tokamak Transport

- **Particle Trapping**



trapped particles



Passing (transit) particles

Banana width: $\Delta x_{tr} \approx v_d t \approx q r_L / \sqrt{\varepsilon}$

$$v_d = \frac{v_{\parallel}^2 + v_{\perp}^2 / 2}{\omega_c R}, \quad v_{\parallel} \sim v_{\perp} \sqrt{\varepsilon}$$

t: transit time of one half of the banana

$$q = \frac{r B_T}{R B_\theta}, \quad r_L = \frac{v_T}{\omega_c}, \quad v_T = \sqrt{2T / m}$$

Displacement of transit particles:

$\Delta x_{pass} \approx q r_L / \sqrt{\varepsilon}$ for particles which have just become transit ones $v_{\parallel} \sim v_{\perp} \sqrt{\varepsilon}$

$\Delta x_{pass} \approx q r_L$ for a typical particle $v_{\parallel} \sim v_{\perp}$

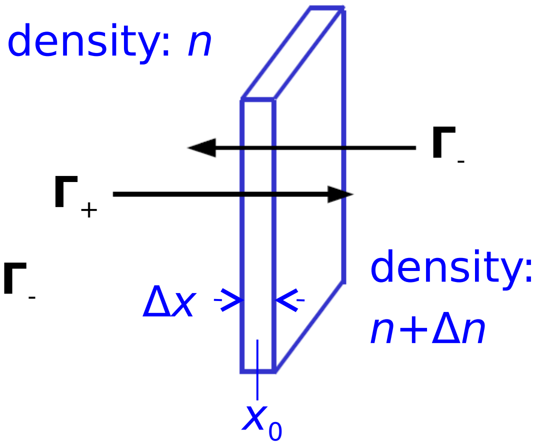
References

- *Francis F. Chen, “Introduction to Plasma Physics and Controlled Fusion”, 2nd Edition, Plenum Press, New York (1984)*
- *Acad. M. A. Leontovich et al, “Reviews of Plasma Physics, Volume 1”, Consultants Bureau, New York (1965)*
- *Jeffrey P. Freidberg, “Plasma Physics and Fusion Energy”, Cambridge University Press (2007)*
- *Hartmut Zohm, “Tokamaks: Equilibrium, Stability and Transport”, IPP Summer University on Plasma Physics, Garching, 18 September, 2001*

Plasma Transport

- **Classical Transport**

  - Particle transport

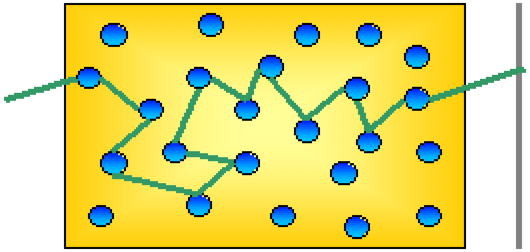


$$\Gamma = \Gamma_+ + \Gamma_-$$

$$n(x) = n(x_0) + \left.\frac{\partial n}{\partial x}\right|_{x=x_0} (x - x_0)$$

$$\Gamma_+ = \frac{1}{2}\int_{x_0-\Delta x/2}^{x_0} \frac{1}{\tau} n(x)dx = \frac{1}{4}\left[n(x_0) - \frac{\partial n}{\partial x}\Delta x\right]\frac{\Delta x}{\tau}$$

$$\Gamma_- = \frac{1}{2}\int_{x_0+\Delta x/2}^{x_0} \frac{1}{\tau} n(x)d(-x) = \frac{1}{4}\left[n(x_0) + \frac{\partial n}{\partial x}\Delta x\right]\frac{\Delta x}{\tau}$$



$$\Gamma = \Gamma_+ - \Gamma_- = -\frac{(\Delta x)^2}{2\tau}\frac{\partial n}{\partial x} = -D\frac{\partial n}{\partial x}$$: Particle flux- Fick’s law

$$D = \frac{(\Delta x)^2}{2\tau}$$: diffusion coefficient (m²/s)

The heat and momentum fluxes can be estimated in similar fashion.

Fusion Reactor Technology 2

(459.761, 3 Credits)

Prof. Dr. Yong-Su Na

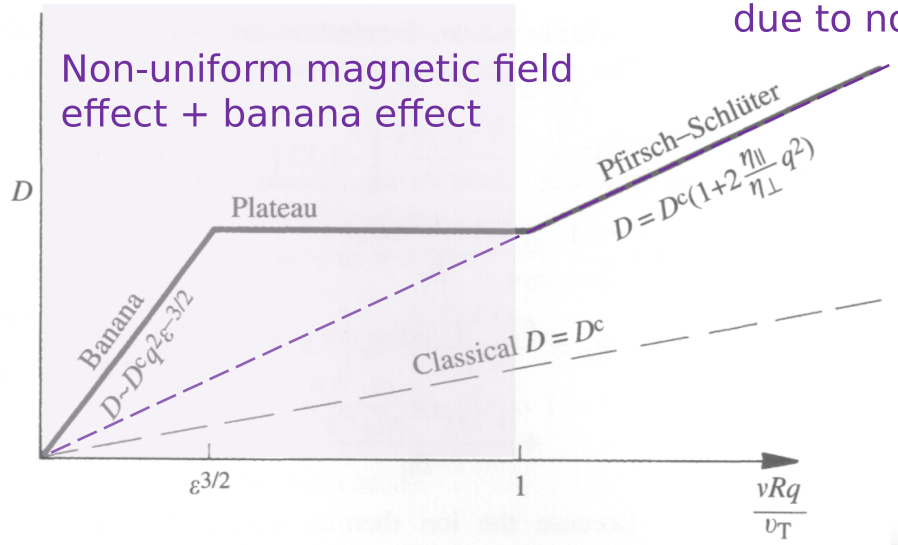
(32-206, Tel. 880-7204)

Tokamak Transport

- **Neoclassical Transports**

$$\Gamma = - D\nabla n \approx - \frac{(\Delta r)^2}{\tau}\nabla n$$: Fick's law

Modified classical diffusion
due to non-uniform magnetic field



Non-uniform magnetic field
effect + banana effect

$$D^c = \frac{\eta_\perp n \sum kT}{B^2}$$

J. Wesson, Tokamaks (2004)

Tokamak Transport

- **Neoclassical Transports**

D. Pfirsch and A. Schlueter, Der Einfluss der elecktrischen Leitfaehigkeit auf das Gleichgewichtsverhalten von Plasmen niedrigen Drucks in Stellaratoren, Max-Planck-Institut, Report MPI/PA/7/62 (1962)

- In the Pfirsch-Schlueter region,

$$j \times B = \nabla p \quad \rightarrow \quad j_\perp = - B^{-1} dp / dr$$

$$E + v \times B = \eta j \quad \rightarrow \quad e\eta_\perp j_\perp = evB$$

Friction force = Lorentz force

(**E**x**B** drift not contributing to diffusion)

- Diffusion flux in a uniform field

$$nv = - n \frac{1}{B^2} \eta_\perp \frac{dp}{dr}$$

$$D_\perp = \frac{\eta_\perp n \sum kT}{B^2}$$

- Modified diffusion flux by the additional flux due to longitudinal current, so-called, the Pfirsch-Schlueter current owing to the toroidal effect

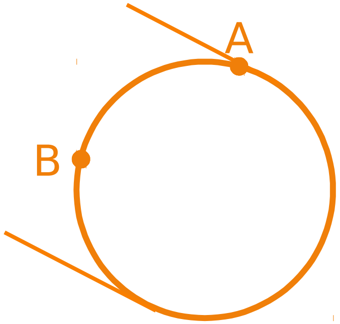
$$nv = - n \frac{1}{B^2} \eta_\perp \frac{dp}{dr} \left(1 + \frac{2\eta_\parallel}{\eta_\perp} q^2 \right)$$

$\eta_\perp \approx 2\eta_\parallel$ in H and D plasmas

- Compared with a uniform magnetic field, the flux in toroidal plasma is enhanced by a factor (1+q^2).

Tokamak Transport

- **Pfirsch-Schlüter Current**



Diamagnetic current or current needed to establish an equilibrium, $J \times B = \nabla p$

$$\vec{J}_{\perp} = \frac{B \times \nabla p}{B^2} = \left(\frac{IB}{B^2} - R\hat{\varphi} \right) \frac{dp}{d\psi} \neq 0$$

$$\left| \vec{J}_{\perp} \right| = \left| \frac{\nabla p}{B} \right|$$

$\left| J_{\perp}^{A} \right| > \left| J_{\perp}^{B} \right|$ → charge separation

No charge accumulation on a flux surface (quasineutrality)

$\longrightarrow \nabla \cdot J = 0 \longrightarrow$ $J_{||}$ needed: Pfirsch-Schlüter current

Tokamak Transport

- **Pfirsch-Schlüter Current (J. Wesson, Tokamaks)**

$$j_\perp = \frac{1}{B}\left|\nabla p\right| = -\frac{1}{B}\frac{dp}{d\psi}\left|\nabla\psi\right| = -\frac{RB_p}{B}\frac{dp}{d\psi} = -\frac{RB_p}{B}p'$$

$$j_p = -\frac{B_p}{B}j_\parallel - \frac{B_\phi}{B}j_\perp$$

$$j_p = \frac{dF}{d\psi}B_p = F'B_p$$

$$\nabla\times E = -\frac{\partial B}{\partial t} \rightarrow \oint E_{ps}ds = 0$$

$$\eta_\parallel j_\parallel = \frac{B_p}{B}E_{ps} + \frac{B_\phi}{B}E_\phi$$

$$F' = \frac{j_\parallel}{B} + \frac{\mu_0 Fp'}{B^2} = \frac{1}{B^2\eta_\parallel / B_p}E_{ps} + \frac{E_\phi B_\phi / B_p}{\eta_\parallel B^2 / B_p} + \frac{\mu_0 Fp' / B_p}{B^2 / B_p}$$

$$= \mu_0 Fp'\frac{\left\langle 1/B_p\right\rangle}{\left\langle B^2/B_p\right\rangle} + \frac{\left\langle E_\phi B_\phi / B_p\right\rangle}{\eta_\parallel\left\langle B^2/B_p\right\rangle}$$

$$\left\langle x\right\rangle = \oint x\,ds / \oint ds$$

Tokamak Transport

- **Pfirsch-Schlüter Current (J. Wesson, Tokamaks)**

$$j_{\|} = \mu_0 F p' \left(\frac{1}{B} - \frac{\langle 1/B_p \rangle}{\langle B^2/B_p \rangle} B \right) + \frac{\langle E_\phi B_\phi / B_p \rangle}{\eta_{\|} \langle B^2 / B_p \rangle} B$$

$$j_{PS} = \mu_0 F p' \left(\frac{1}{B} - \frac{\langle 1/B_p \rangle}{\langle B^2/B_p \rangle} B \right)$$

For the circular CX large aspect ratio toroidal configuration

$$j_{PS} = -2 \frac{1}{B_\theta} \frac{r}{R} \frac{dp}{dr} \cos\theta \qquad B_\phi = \frac{B_0}{1 + \varepsilon \cos\theta}$$

- This flow is dominant in the SOL region (high collisionality regime).
- Pfirsch-Schlüter current removes the main part of the charge separation caused by the curvature and gradient drifts (but residual charge separation still causes transport)

Tokamak Transport

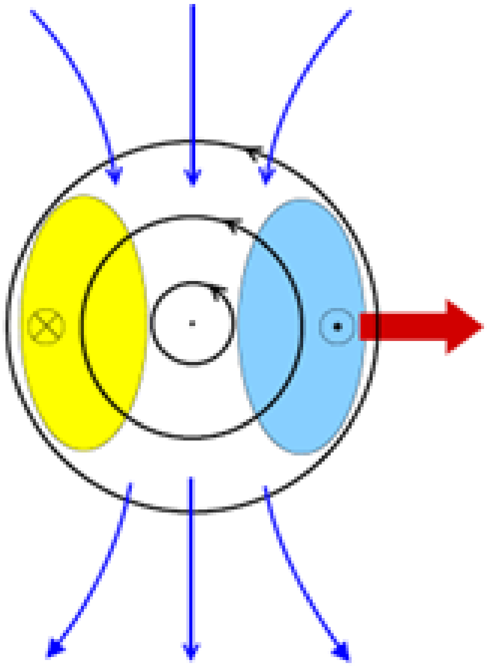
- **Shafranov Shift**

- The Pfirsch-Schlüter current produces vertical field $B_{Z,0}$

→ Plasma shifted outwards

→ Shafranov shift

$$\Delta \approx \frac{R}{B_0 \iota} B_{Z,0}$$



Tokamak Transport

- **Pfirsch-Schlüter Diffusion (J. Wesson, Tokamaks)**

$$v_\perp = \frac{E_{PS}B_\phi - E_\phi B_p}{B^2} - \frac{\eta_\perp \nabla_\perp p}{B^2} \quad \longleftarrow \quad \vec{v}_\perp = \frac{E \times B}{B^2} - \frac{\eta_\perp \nabla p}{B^2}$$

$$= \frac{B_\phi}{B_p B}\eta_\parallel j_\parallel - \frac{\eta_\perp \nabla_\perp p}{B^2} - \frac{E_\phi}{B_p} \quad \longleftarrow \quad \eta_\parallel j_\parallel = \frac{B_p}{B}E_{ps} + \frac{B_\phi}{B}E_\phi$$

$$= \frac{B_\phi}{B_p B}\eta_\parallel j_{PS} - \frac{\eta_\perp \nabla_\perp p}{B^2} + \frac{1}{B_p}\left(\frac{\langle E_\phi B_\phi / B_p\rangle}{\langle B^2 / B_p\rangle}B_\phi - E_\phi\right) \quad \longleftarrow \quad j_\parallel = j_{PS} + \frac{\langle E_\phi B_\phi / B_p\rangle}{\eta_\parallel \langle B^2 / B_p\rangle}B$$

$$\Gamma = 2\pi n \oint v_\perp R ds = 2\pi n \langle v_\perp R\rangle \oint ds$$

$$\langle v_\perp R\rangle_{PS} = -\eta_\parallel \mu_0 F p' \left\langle \frac{RB_\phi}{B_p}\left(\frac{1}{B^2} - \frac{\langle 1/B_p\rangle}{\langle B^2 / B_p\rangle}\right)\right\rangle \quad \longleftarrow \quad j_{PS} = \mu_0 F p' \left(\frac{1}{B} - \frac{\langle 1/B_p\rangle}{\langle B^2 / B_p\rangle}B\right)$$

Tokamak Transport

- **Pfirsch-Schlüter Diffusion (J. Wesson, Tokamaks)**

For the circular CX large aspect ratio toroidal configuration

$$\frac{\langle v_\perp R\rangle_{PS}}{R_0} = -2\left(\frac{r}{R}\right)^2 \eta_\parallel \frac{dp/dr}{B_\theta^2} \qquad j_{PS} = -2\frac{1}{B_\theta}\frac{r}{R}\frac{dp}{dr}\cos\theta$$

$$\frac{\langle v_\perp R\rangle}{R_0} = -\frac{dp/dr}{B^2}\left(\eta_\perp + 2q^2\eta_\parallel\right) - \frac{E_\phi B_\theta}{B^2} \longleftarrow q = \frac{rB_\phi}{RB_\theta}$$

$$D = D^C\left(1 + 2\frac{\eta_\parallel}{\eta_\perp}q^2\right) \qquad D^c = \frac{\eta_\perp n \sum kT}{B^2}$$

Tokamak Transport

- **Neoclassical Transports**

- Rarefied plasma at high temperature:
  trapped particles are the main contributors to transport.
  Diffusion and thermal conductivity are dominated by the collisions
  which correspond to transferring the particles from being trapped
  to
  transit ones and vice versa.

$$\lambda_{eff} = v_T / \nu_{eff} >> qR, \quad \Delta \approx \Delta x_{tr} \approx q r_L / \sqrt{\varepsilon}$$

- Effective collision frequency:

$$\nu_{eff} \approx \left(v_\perp / v_\parallel \right)^2 \nu \approx \nu / \varepsilon \quad \longleftarrow \quad v_\parallel \sim v_\perp \sqrt{\varepsilon}$$

- Transport coefficients:

$$\sqrt{\varepsilon} \left(\Delta x_{tr} \right)^2 \nu_{eff} = \varepsilon^{-3/2} \nu q^2 r_L{}^2 \quad \longleftarrow \quad \Delta x_{tr} \approx v_d t \approx q r_L / \sqrt{\varepsilon}$$

Trapped particle fraction = $\varepsilon^{1/2}$

- The banana diffusion region is limited by the condition:

$$\sqrt{\varepsilon} \lambda_{eff} = \sqrt{\varepsilon} v_T / \nu_{eff} = \varepsilon^{3/2} v_T / \nu >> qR \quad \longleftarrow$$

Trapped particle fraction = $\varepsilon^{1/2}$

$$\nu^* = \nu \varepsilon^{-3/2} qR / v_T << 1$$

Tokamak Transport

- **Neoclassical Transports**

- In the plateau region, $1 < \nu^* < \varepsilon^{-3/2}$
  $(\varepsilon^{3/2} < \nu Rq/v_T < 1$ or $\varepsilon^{3/2} v_T/Rq < \nu < v_T/Rq)$
  - The average collision frequency is less than the mean bounce frequency → only slow-transit particles contribute to the transport
  - The relative number of slow-transit particles: ν/v_T
- Displacement: $\Delta \approx q r_L v_T / v$
- Effective collision frequency: $\nu_{eff} \approx \nu v_T^2 / v^2$
- Transport coefficients:

$$\Delta^2 \nu_{eff} v / v_T \sim q^2 r_L^2 v_T / qR \quad \longleftarrow$$ Slow-transit particle fraction = v/v_T

Tokamak Transport

- **Neoclassical Transports**

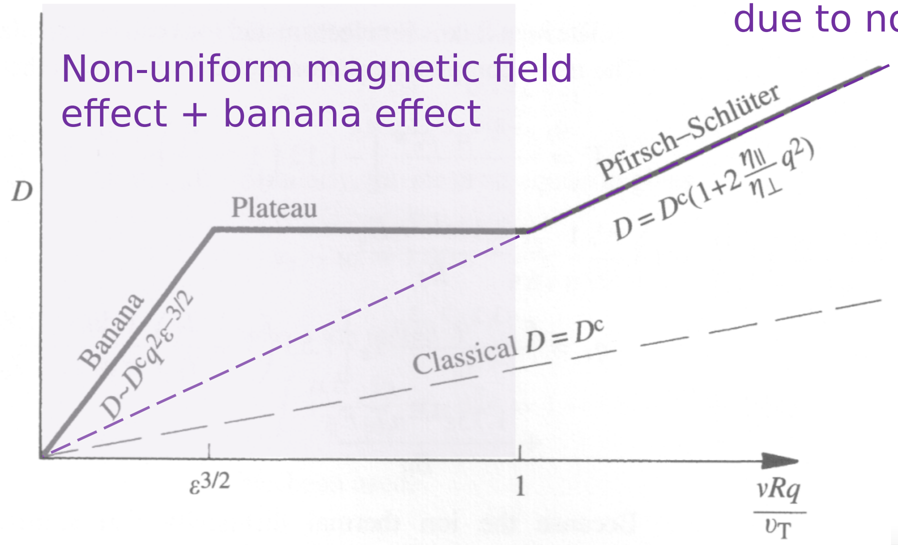
$$\Gamma = - D\nabla n \approx - \frac{(\Delta r)^2}{\tau}\nabla n$$: Fick's law

$$q = - \kappa\nabla T \approx - \frac{(\Delta r)^2 n}{\tau_E}\nabla T$$: Fourier's law

Thermal diffusivity

$$\chi \equiv \frac{\kappa}{n} \approx \frac{(\Delta r)^2}{\tau_E} \approx D \quad \rightarrow \quad \tau_E \approx \frac{a^2}{\chi}$$

Modified classical diffusion
due to non-uniform magnetic field



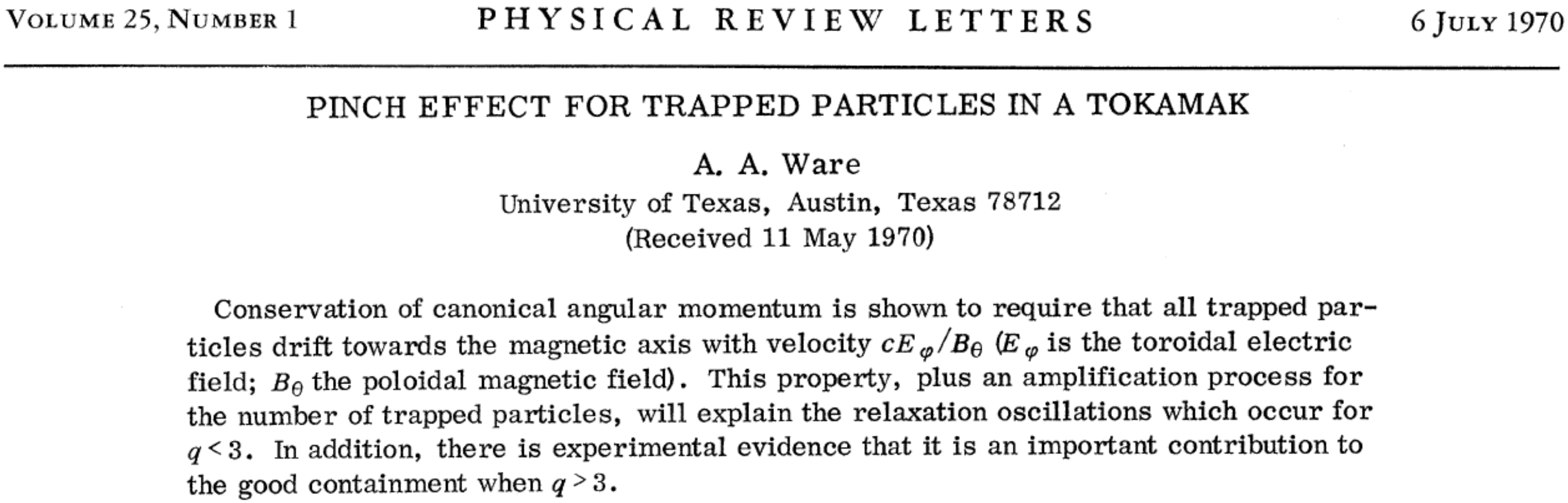
Non-uniform magnetic field
effect + banana effect

$$D^c = \frac{\eta_\perp n \sum kT}{B^2}$$

J. Wesson, Tokamaks (2004)

Tokamak Transport

- **Ware Pinch**



VOLUME 25, NUMBER 1 PHYSICAL REVIEW LETTERS 6 JULY 1970

PINCH EFFECT FOR TRAPPED PARTICLES IN A TOKAMAK

A. A. Ware
University of Texas, Austin, Texas 78712
(Received 11 May 1970)

Conservation of canonical angular momentum is shown to require that all trapped particles drift towards the magnetic axis with velocity cE_φ/B_θ (E_φ is the toroidal electric field; B_θ the poloidal magnetic field). This property, plus an amplification process for the number of trapped particles, will explain the relaxation oscillations which occur for $q<3$. In addition, there is experimental evidence that it is an important contribution to the good containment when $q>3$.

- Inward particle transport due to the toroidal electric field

Tokamak Transport

- **Ware Pinch (J. Wesson, Tokamaks)**

- The inward flow occurs for trapped particles and their behaviour follows directly from the toroidal equation of motion.

$$\frac{d}{dt}\left(m_j v_\phi\right) = e_j\left[E_\phi + (v \times B)_\phi\right]$$

Zero steady state time average of the left-hand side term for trapped particles (the integral between bounces is zero)

$$\left\langle (v \times B)_\phi \right\rangle = - E_\phi$$

$$\left\langle v_\perp \right\rangle = - \frac{E_\phi}{B_\theta} \longleftarrow \quad (v \times B)_\phi = v_\perp B_\theta$$

$$\Gamma \sim \varepsilon^{1/2} n \frac{E_\phi}{B_\theta}$$ ⟵ Trapped particle fraction = $\varepsilon^{1/2}$

Tokamak Transport

- **Ware Pinch (J. Wesson, Tokamaks)**

- The modified equation of motion along the magnetic field line

$$\frac{d^2 s}{dt^2} = -\omega_b^2 s + \frac{e_j E_\phi}{m_j}$$

$$\longrightarrow \quad s = s_b \sin \omega_b t + \frac{e_j E_\phi}{m_j \omega_b^2}$$

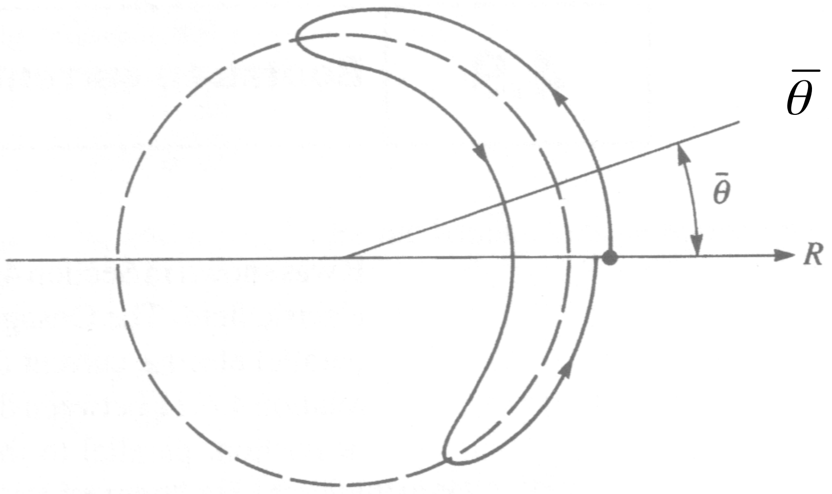
$$\theta = \theta_b \sin \omega_b t + \frac{e_j B_\theta E_\phi}{m_j \omega_b^2 rB} \quad \longleftarrow \quad s = (B / B_\theta) r\theta$$

mean angle: $\bar{\theta} = \dfrac{e_j B_\theta E_\phi}{m_j \omega_b^2 rB}$

⟶ The effect of the ∇*B* and curvature drift is not symmetric about the mid-plane

Tokamak Transport

- **Ware Pinch (J. Wesson, Tokamaks)**



$$\bar{\theta} = \frac{e_j B_\theta E_\phi}{m_j \omega_b^2 r B}$$

$$\langle v_r \rangle = - \frac{E_\phi}{B_\theta}$$

Resulting radial velocity for deeply trapped particles for which *θ* is small

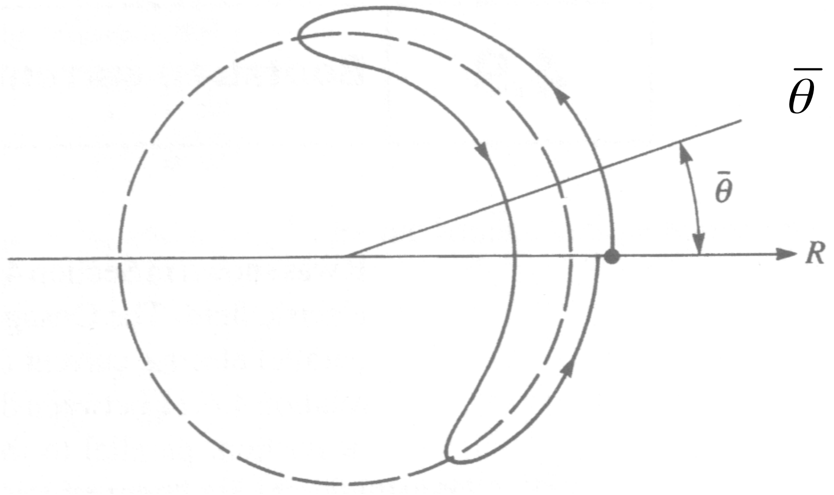
$$v_r = - v_{dj} \sin\theta$$

$$\sim - v_{dj}\theta = - v_{dj}\theta_b \sin\omega_b t - \frac{e_j v_{dj} B_\theta}{m_j \omega_b^2 r B} E_\phi$$

$$\langle v_r \rangle = - \frac{\omega_{cj} v_{dj} B_\theta}{\omega_b^2 r B} E_\phi = - \frac{E_\phi}{B_\theta} \longleftarrow \quad v_{dj} = \frac{1}{2} m_j v_\perp^2 / e_j R B, \quad \omega_{\mathrm{b}} = (v_\perp / qR)(r / 2R)^{1/2}$$

Tokamak Transport

- **Ware Pinch (J. Wesson, Tokamaks)**



$$\bar{\theta} = \frac{e_j B_\theta E_\phi}{m_j \omega_b^2 r B}$$

$$\langle v_r \rangle = -\frac{E_\phi}{B_\theta}$$

- The drift velocity is controlled by the balance of two forces, the electrical field force and the Lorentz force.
- $v_W \sim 0.2$ m/s for $E = 0.1$ V/m, $B_\theta = 0.5$ T
- The effect is much larger ($1/\varepsilon^2$) for trapped particles than that experienced by passing particles. **HW: Why?**

Tokamak Transport

- **Bootstrap current**

bootstrap [búːtstræp]

1. (편상화의) 손잡이 가죽.
2. <재귀용법으로> 노력하여 [자기]를 어떤 상태로 되게 하다.
3. 자동(식)의; 자급(自給)의; 자력의.

- Named after the reported ability of Baron von Munchausen to lift himself by his bootstraps (Raspe, 1785)
- Suggested with ‘Alice in Wonderland’ in mind where the heroine managed to support herself in the air by her shoelaces.

http://en.wikipedia.org/wiki/Bootstrapping

Tokamak Transport

- **Bootstrap**

MEANING:
verb tr.: To help oneself with one's own initiative and no outside help.
noun: Unaided efforts.
adjective: Reliant on one's own efforts.

ETYMOLOGY:
While pulling on bootstraps may help with putting on one's boots, it's impossible to lift oneself up like that. Nonetheless the fanciful idea is a great visual and it gave birth to the idiom "to pull oneself up by one's (own) bootstraps", meaning to better oneself with one's own efforts, with little outside help. It probably originated from the tall tales of Baron Münchausen who claimed to have lifted himself (and his horse) up from the swamp by pulling on his own hair.

Baron Münchausen lifting himself up from the swamp by his own hair
Illustrator: Theodor Hosemann

In computing, booting or bootstrapping is to load a fixed sequence of instructions in a computer to initiate the operating system.
Earliest documented use: 1891.1

http://wordsmith.org/words/bootstrap.html

Tokamak Transport

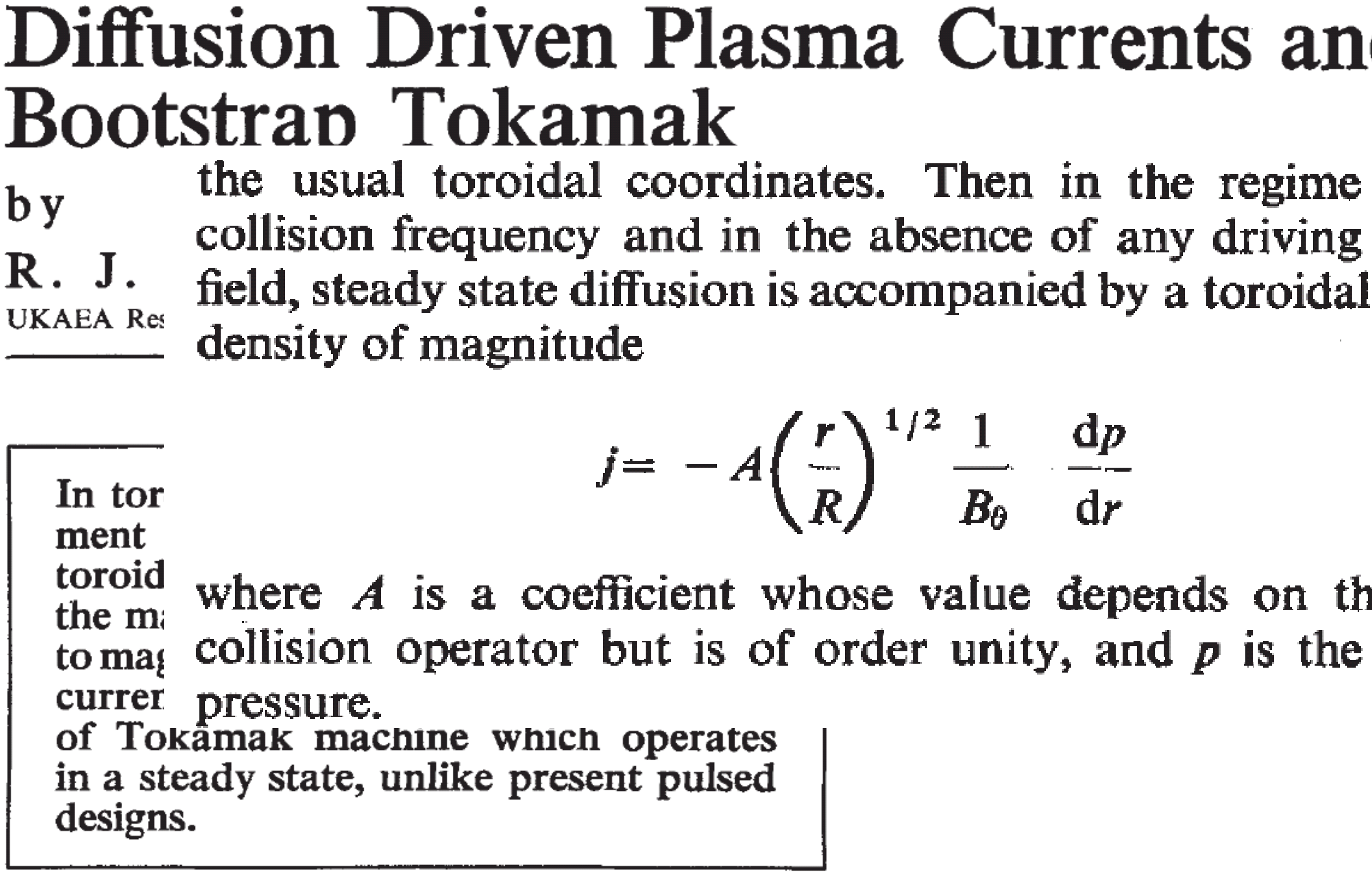
- **Bootstrap**

“I was still a couple of miles above the clouds when it broke, and with such violence I fell to the ground that I found myself stunned, and in a hole nine fathoms under the grass, when I recovered, hardly knowing how to get out again. Looking down, I observed that I had on a pair of boots with exceptionally sturdy straps. Grasping them firmly, I pulled with all my might. Soon I had hoist myself to the top and stepped out on terra firma without further ado.”
- With acknowledgement to R. E. Raspe, *Singular Travels, Campaigns and Adventures of Baron Munchausen*, 1786. Edition edited by J. Carswell. London: The Cresset Press, 1948. Adapted from the story on p. 22(???).

http://wordsmith.org/words/bootstrap.html

Tokamak Transport

- **Bootstrap current**



Diffusion Driven Plasma Currents and Bootstrap Tokamak

by

R. J.

UKAEA Res

R

the usual toroidal coordinates. Then in the regime of low collision frequency and in the absence of any driving electric field, steady state diffusion is accompanied by a toroidal current density of magnitude

$$j = -A\left(\frac{r}{R}\right)^{1/2} \frac{1}{B_\theta} \frac{dp}{dr} \qquad (1)$$

where A is a coefficient whose value depends on the exact collision operator but is of order unity, and p is the plasma pressure.

In tor
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the m
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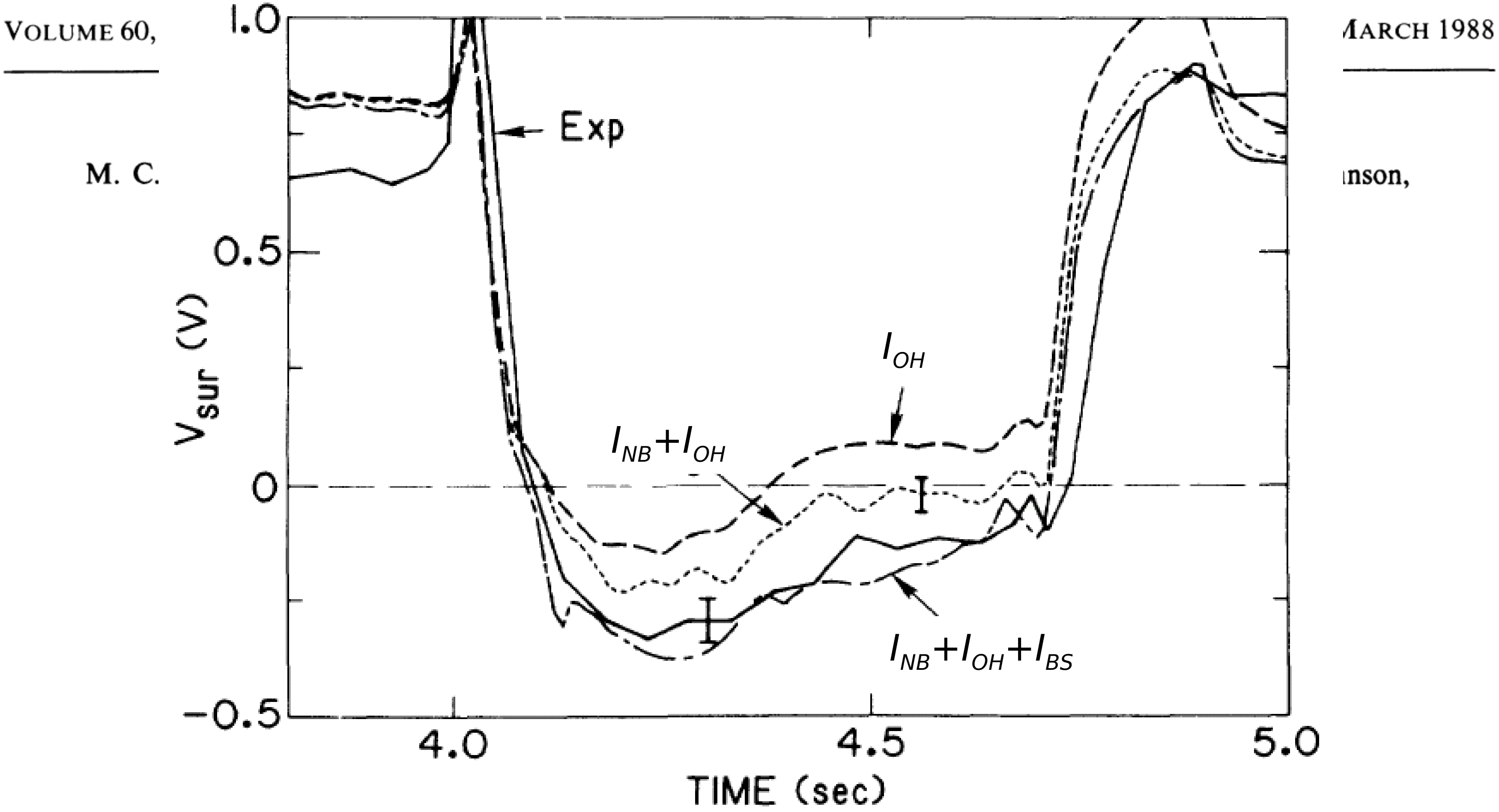
curre
of Tokamak machine which operates in a steady state, unlike present pulsed designs.

Nature Physical Science ***229*** *110 (1971)*

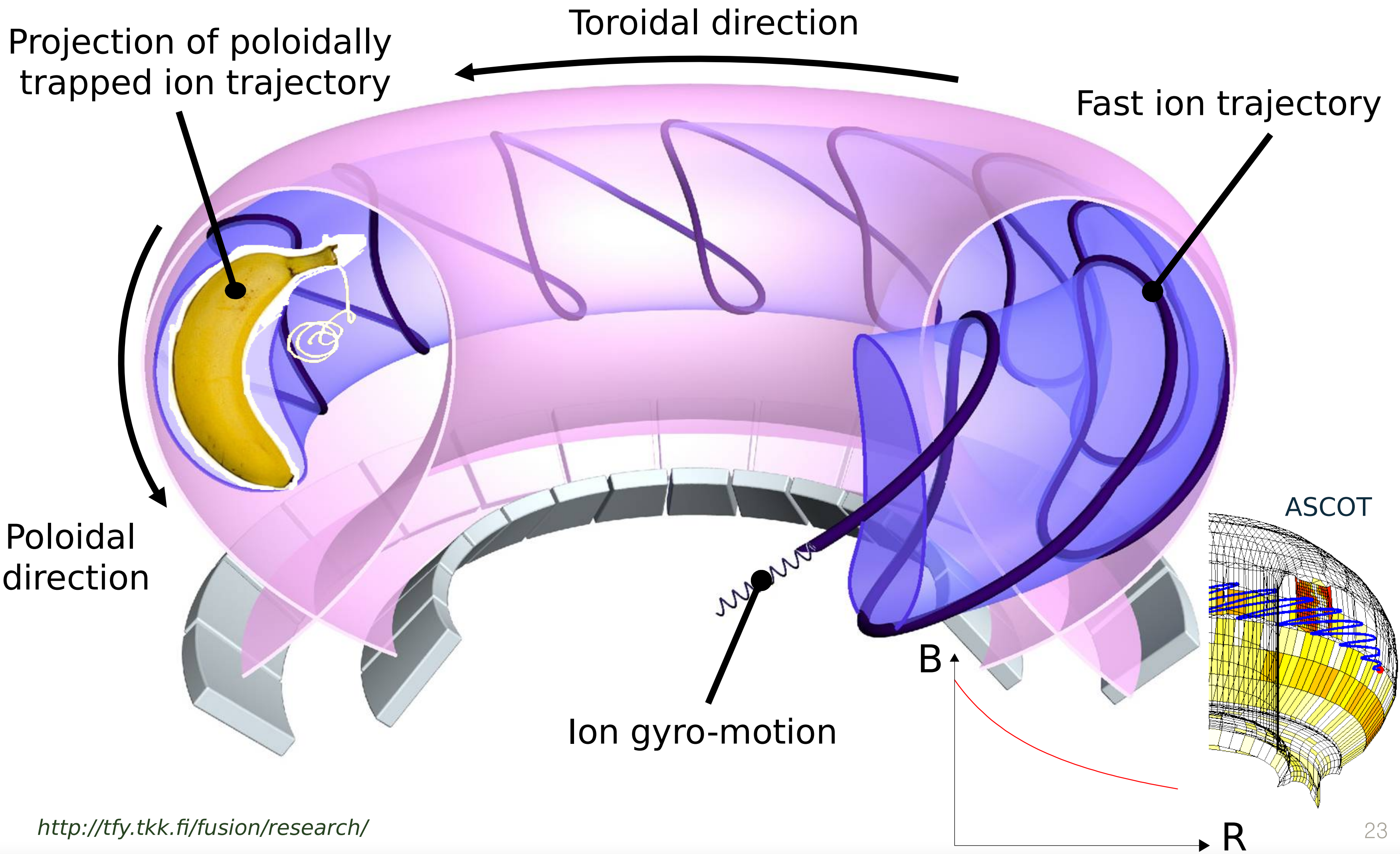
Tokamak Transport

- **Bootstrap current**



Tokamak Transport

- **Neoclassical Bootstrap current**



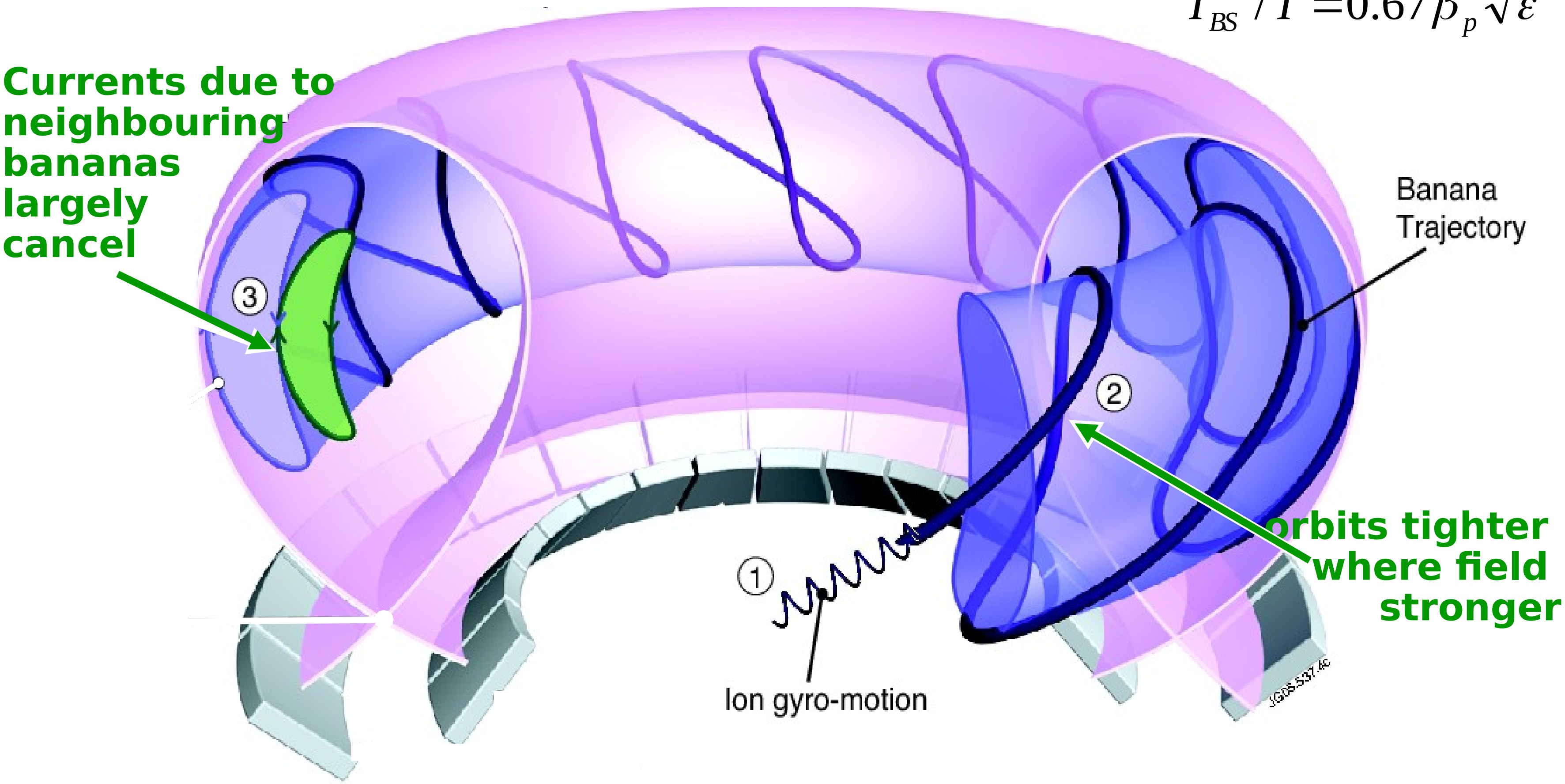
http://tfy.tkk.fi/fusion/research/

Tokamak Transport

$$J_{BS} \approx - \frac{\varepsilon^{1/2}}{B_\theta} \frac{dp}{dr}$$

$$I_{BS} / I = 0.67 \beta_p \sqrt{\varepsilon}$$

- **Neoclassical Bootstrap current**



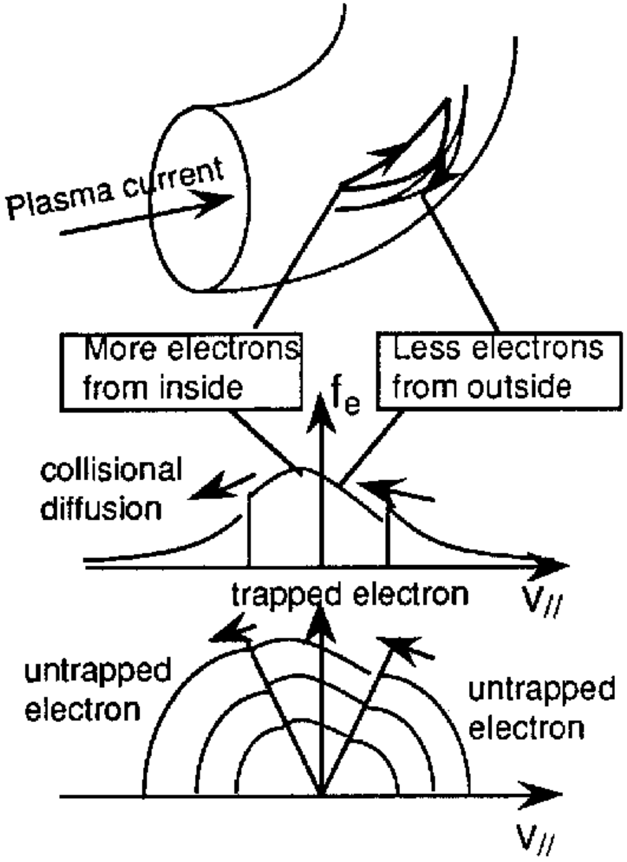
- More & faster particles on orbits nearer the core (green .vs. blue) lead to a net “banana current”.

- This is transferred to a helical bootstrap current via collisions.

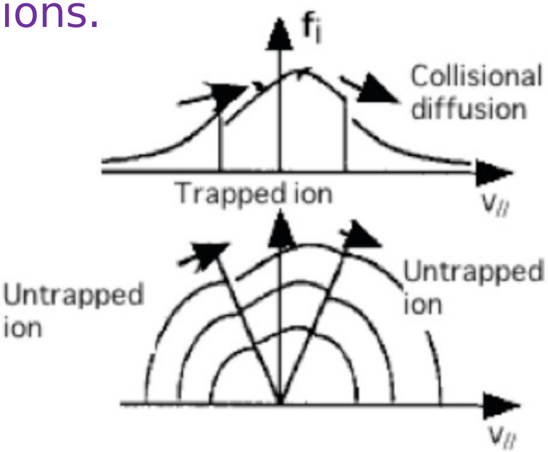
Tokamak Transport

- **Bootstrap Current**

- Trapped-electron orbits and schematics of the velocity distribution function in a collisionless tokamak plasma



Small Coulomb collision smoothes the gap and causes particle diffusion in the velocity space. Collisional pitch angle scattering at the trapped-untrapped boundary produces unidirectional parallel flow/momentum input and is balanced by the collisional friction force between electrons and ions.



M. Kikuch et al, PPCF ***37*** *1215 (1995)*

Tokamak Transport

Assumption:
- Uniform temperature
- Infinitely massive ions

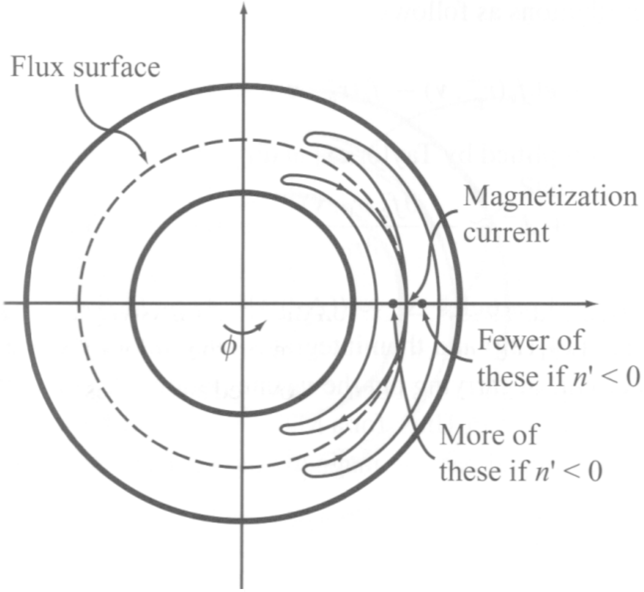
- **Bootstrap Current**

- The trapped electron magnetization current

$$J^{+} + J^{-} = -e\left[f_e\left(r_g^{+}, \mathbf{v}\right) - f_e\left(r_g^{-}, \mathbf{v}\right)\right] v_{\|} d\mathbf{v}, \quad v_{\|} > 0$$

$$\approx -e\frac{\partial f_e\left(r_0, \mathbf{v}\right)}{\partial r_0}\Delta r v_{\|} d\mathbf{v}, \quad v_{\|} > 0$$

$$\Delta r \approx \left\langle (\Delta r)^2 \right\rangle^{1/2} \approx q r_L / \varepsilon^{1/2}$$



$$J_t = -\frac{m_e q}{B_0}\varepsilon^{-1/2}\int \frac{\partial F_M}{\partial r_0} v_{\perp} v_{\|} d\mathbf{v}, \quad v_{\|} > 0$$

$$= -\frac{3}{2} q \varepsilon^{-1/2}\frac{T}{B_0}\frac{\partial n}{\partial r}\int_{\theta_c}^{\pi/2}\sin^2\theta\cos\theta d\theta \qquad \cos\theta_c = \sqrt{2\varepsilon}$$

$$\approx -q\varepsilon^{1/2}\frac{T}{B_0}\frac{\partial n}{\partial r}$$

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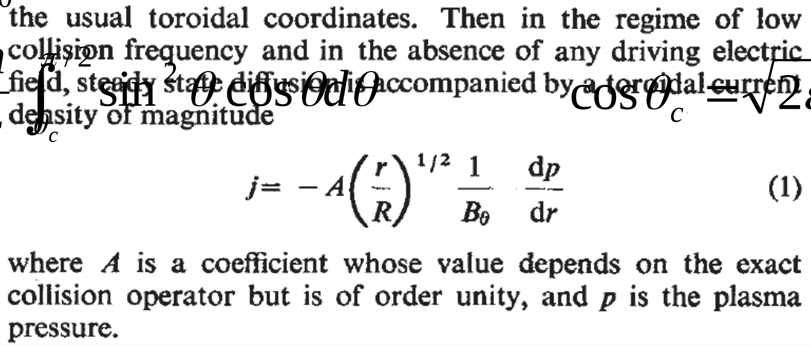
ure)



the usual toroidal coordinates. Then in the regime of low collision frequency and in the absence of any driving electric field, steady state diffusion is accompanied by a toroidal current density of magnitude

$$j = -A\left(\frac{r}{R}\right)^{1/2}\frac{1}{B_\theta}\frac{\mathrm{d}p}{\mathrm{d}r} \qquad (1)$$

where A is a coefficient whose value depends on the exact collision operator but is of order unity, and p is the plasma pressure.

Tokamak Transport

- **Bootstrap Current**

- The passing electron magnetization current

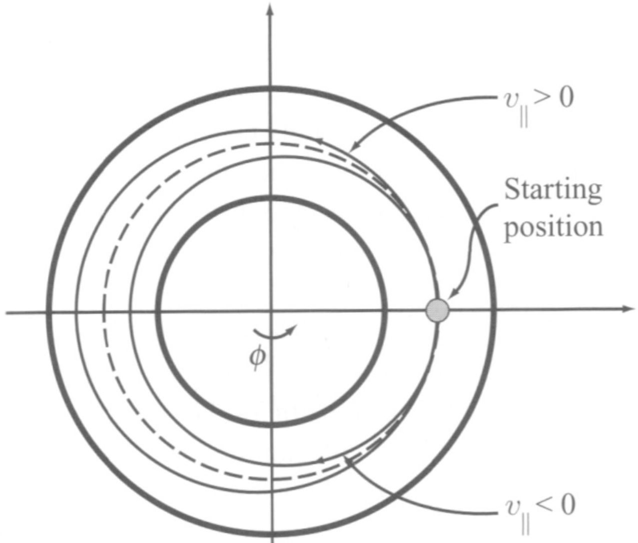
$$J_p \approx - e \frac{\partial f_e(r_0, \mathbf{v})}{\partial r_0} \Delta r v_\| d\mathbf{v}, \quad v_\| > 0$$

$$\Delta r \approx q r_L$$

$$J_p = - \frac{m_e q}{B_0} \int \frac{\partial F_M}{\partial r_0} v_\perp v_\| d\mathbf{v}, \quad v_\| > 0$$

$$= - \frac{3}{2} q \frac{T}{B_0} \frac{\partial n}{\partial r} \int_{\theta_c}^{\pi/2} \sin^2\theta \cos\theta d\theta$$

$$\approx - q \frac{T}{B_0} \frac{\partial n}{\partial r}$$



$$\cos\theta_c = \sqrt{2\varepsilon}$$

Tokamak Transport

- **Bootstrap Current**

- The collision-driven bootstrap current

$$(\Delta P_\parallel)_t = m_e u_t n_t \bar{\nu}_{tp} = m_e\left(-\frac{J_t}{en_t}\right) n_t \bar{\nu}_{tp} \approx -\frac{m_e}{e} J_t \frac{\bar{\nu}_{ee}}{\varepsilon} \approx qT\varepsilon^{-1/2}\frac{\partial n}{\partial r}\frac{\bar{\nu}_{ee}}{|\omega_{ce}|}$$

$$(\Delta P_\parallel)_p = m_e u_p n_p \bar{\nu}_{pt} = m_e\left(-\frac{J_p}{en_p}\right) n_p \bar{\nu}_{pt} \approx -\frac{m_e}{e} J_p \bar{\nu}_{ee} \approx qT\frac{\partial n}{\partial r}\frac{\bar{\nu}_{ee}}{|\omega_{ce}|}$$

$$(\Delta P_\parallel)_p \neq (\Delta P_\parallel)_t$$

Collisional momentum balance violated

$$(\Delta P_\parallel)_p = \varepsilon^{1/2}(\Delta P_\parallel)_t$$

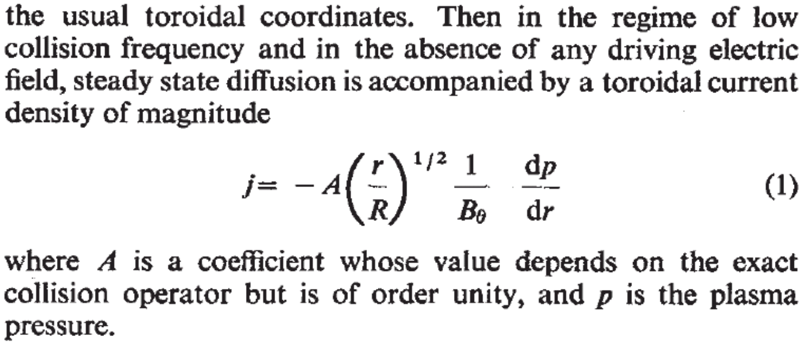
$$f_p(r_g, \mathbf{v}) = \frac{n(r_g)}{\pi^{3/2}v_T^3}\exp\left(-\frac{v_\perp^2+v_\parallel^2}{v_\perp^2}\right) \rightarrow \frac{n(r_g)}{\pi^{3/2}v_T^3}\exp\left[-\frac{v_\perp^2+(v_\parallel-u_B)^2}{v_\perp^2}\right]$$

Shifted Maxwellian

$$\approx \frac{n(r_g)}{\pi^{3/2}v_T^3}\left[1+\frac{v_\parallel}{|v_\parallel|}\left(\frac{1}{n}\frac{\partial n}{\partial r}\right)(\Delta r)_p + 2\frac{v_\parallel u_B}{v_T^2}\right]\exp\left(-\frac{v_\perp^2+v_\parallel^2}{v_\perp^2}\right)$$

- The shift must be in the passing particles since the trapped particles are “trapped” and thus are not allowed to drift toroidally.

Tokamak Transport



the usual toroidal coordinates. Then in the regime of low collision frequency and in the absence of any driving electric field, steady state diffusion is accompanied by a toroidal current density of magnitude

$$j = -A\left(\frac{r}{R}\right)^{1/2}\frac{1}{B_\theta}\frac{\mathrm{d}p}{\mathrm{d}r} \qquad (1)$$

where A is a coefficient whose value depends on the exact collision operator but is of order unity, and p is the plasma pressure.

- **Bootstrap Current**

$$\left(\Delta P_{\|}\right)_p \approx m_e\left(-\frac{J_p}{e}+n_p u_B\right)\bar{\nu}_{ee}$$

$$\left(\Delta P_{\|}\right)_p = \left(\Delta P_{\|}\right)_t \qquad qT\frac{\partial n}{\partial r}\frac{\bar{\nu}_{ee}}{|\omega_{ce}|}+m_e u_B n_p \bar{\nu}_{ee} = qT\varepsilon^{-1/2}\frac{\partial n}{\partial r}\frac{\bar{\nu}_{ee}}{|\omega_{ce}|}$$

$$J_B = -en_p u_B \approx -q\varepsilon^{-1/2}\frac{T}{B_0}\frac{\partial n}{\partial r}$$

$1/\varepsilon$ and $1/\varepsilon^{1/2}$ larger than the trapped and passing particle magnetization current, respectively

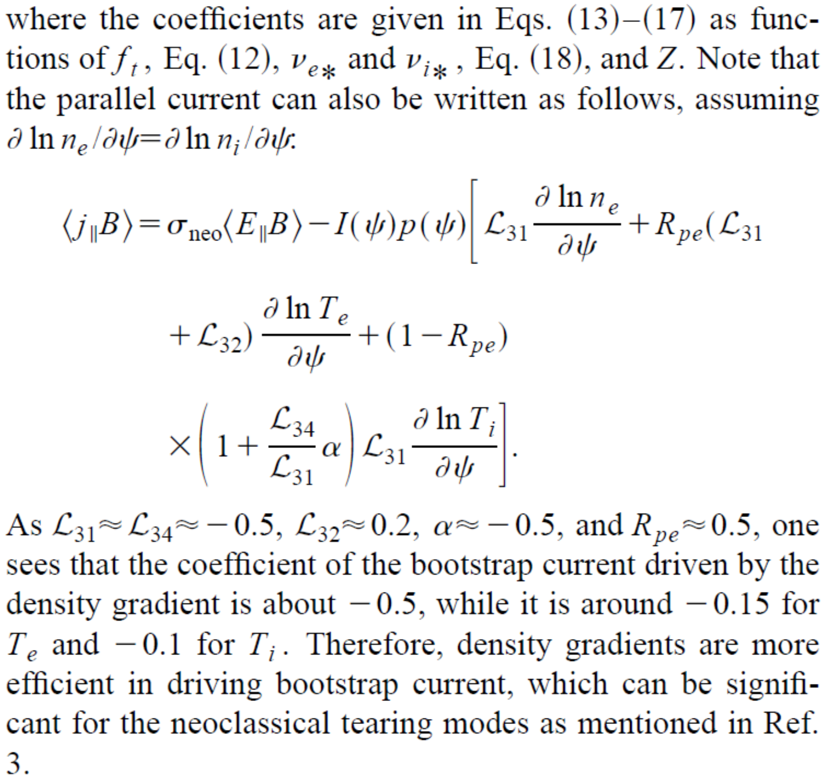
$$J_B = -4.71q\varepsilon^{-1/2}\frac{T}{B_0}\left[\frac{\partial n}{\partial r}+0.04\frac{n}{T}\frac{\partial T}{\partial r}\right]$$

Large aspect ratio
Circular CX
Non-massive ions
Non-uniform temperature

- A transport driven toroidal plasma current carried by the passing electrons generated by collisional friction with the trapped electron magnetization current

Tokamak Transport

- **Bootstrap Current**



where the coefficients are given in Eqs. (13)–(17) as functions of f_t, Eq. (12), ν_{e*} and ν_{i*}, Eq. (18), and Z. Note that the parallel current can also be written as follows, assuming $\partial \ln n_e / \partial\psi = \partial \ln n_i / \partial\psi$:

$$\langle j_\| B\rangle = \sigma_{\rm neo}\langle E_\| B\rangle - I(\psi)p(\psi)\left[\mathcal{L}_{31}\frac{\partial \ln n_e}{\partial\psi} + R_{pe}(\mathcal{L}_{31} + \mathcal{L}_{32})\frac{\partial \ln T_e}{\partial\psi} + (1-R_{pe}) \times\left(1+\frac{\mathcal{L}_{34}}{\mathcal{L}_{31}}\alpha\right)\mathcal{L}_{31}\frac{\partial \ln T_i}{\partial\psi}\right].$$

As $\mathcal{L}_{31} \approx \mathcal{L}_{34} \approx -0.5$, $\mathcal{L}_{32} \approx 0.2$, $\alpha \approx -0.5$, and $R_{pe} \approx 0.5$, one sees that the coefficient of the bootstrap current driven by the density gradient is about -0.5, while it is around -0.15 for T_e and -0.1 for T_i. Therefore, density gradients are more efficient in driving bootstrap current, which can be significant for the neoclassical tearing modes as mentioned in Ref. 3.

- Sauter pop 99

Tokamak Transport

- **Bootstrap Current**

- Bootstrap current fraction

$$f_B(r) \equiv \frac{J_B}{J_\phi} \approx - 1.18 G \varepsilon^{1/2} \beta_P \sim \varepsilon^{1/2} \beta_P$$

$$G(r) = (\ln n + 0.04 \ln T)' / (\ln r B_\theta)'$$

- In high-β tokamak, $\beta_p \sim 1/\varepsilon$, implying that $f_B \sim 1/\varepsilon^{1/2} >>1$:

  The bootstrap current can theoretically overdrive the total current

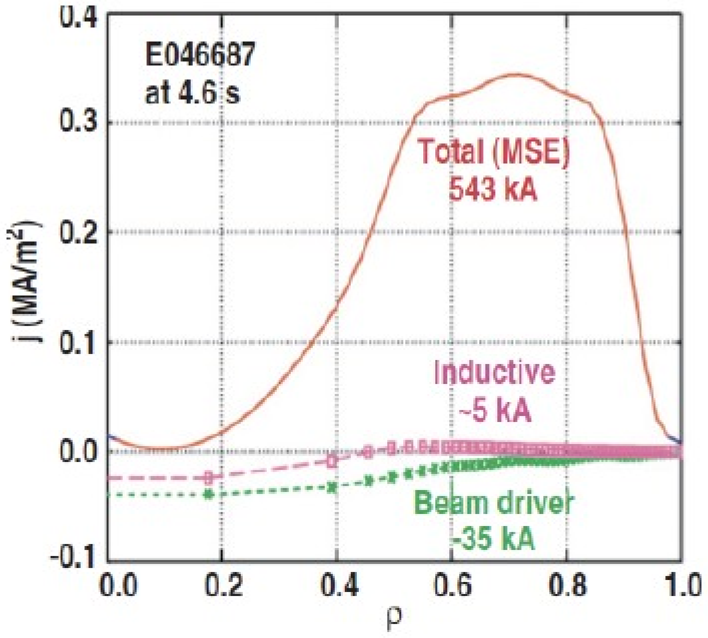
  - No obvious “anomalous” degradation of J_B due to micro-turbulence

- The bootstrap current is capable of being maintained in steady state without the need of an Ohmic transformer or external current drive. This is indeed a favourable result as it opens up the possibility of steady state operation without the need for excessive amounts of external current drive power.
- This is critical since bootstrap current fractions on the order of $f_B > 0.7$ are probably required for economic viability of fusion reactors.

Tokamak Transport

- **100% bootstrap discharges**

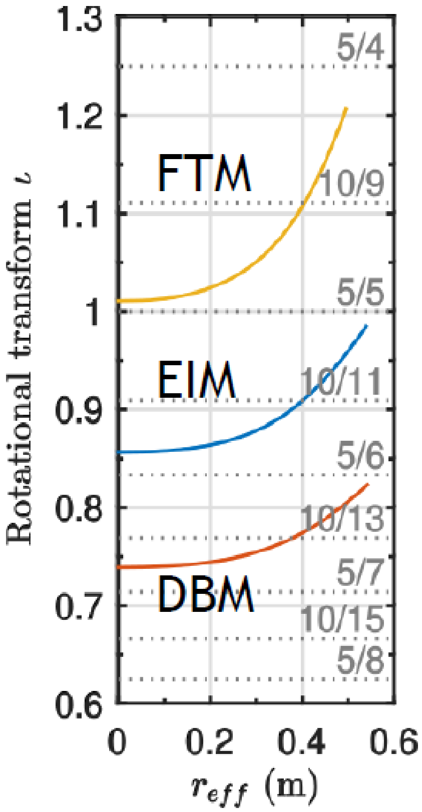
Y. Takase, IAEA FEC 1996, S. Coda, IAEA FEC 2008

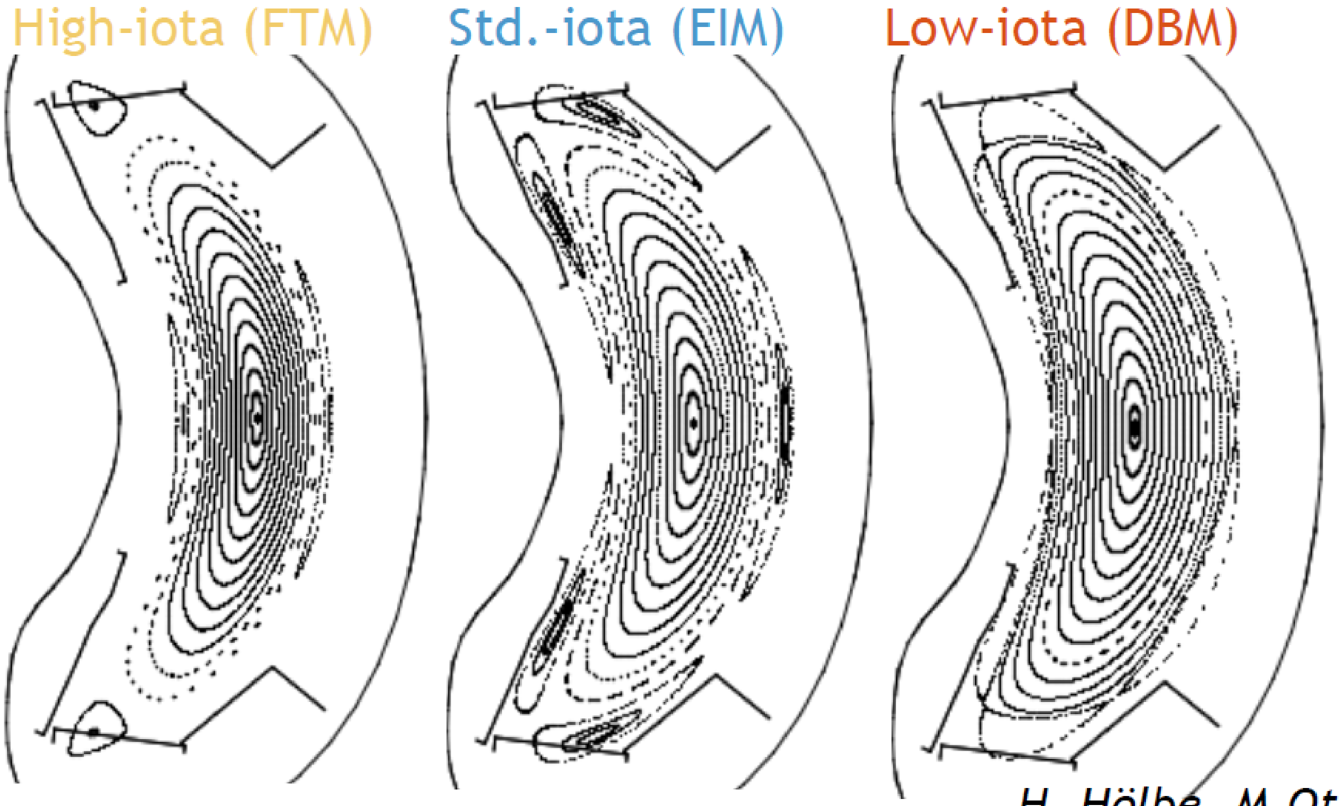


Toroidal current evolution and control

- As for tokamaks, the non-inductive Bootstrap (BS) current is a matter of concern in W7-X.
- The island divertor (ID) concept that we rely on for heat/ particle exhaust is quite sensitive to toroidal currents which need to be kept low (10 kA or less).
- High-density (> 1e20 /m3) plasmas require multi-pass O2 ECR heating, for which the current drive efficiency is low .
- The magnetic configuration needs to be tailored to have low BS in the parameter regime of operational relevance.

Iota profiles and divertor island structure

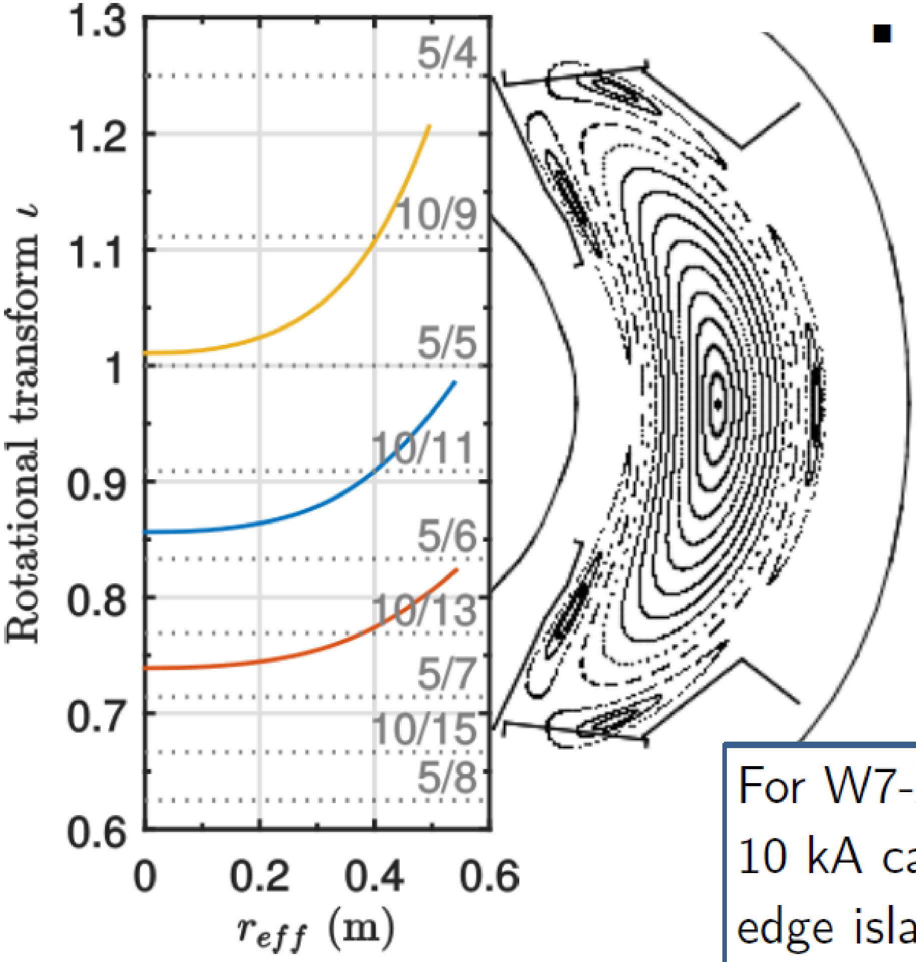




H. Hölbe, M.Otte

- Low-shear iota profiles to avoid low-order rationals in the core region.
- But low shear makes the divertor island sensitive to toroidal currents.

How sensitive?



- What currents are acceptable?

Estimate the change in iota,

$$\Delta\iota = \frac{d\Psi_{pol}/dr}{d\Psi_{tor}/dr} \approx \frac{\mu_0 I_{tor} R/r}{2\pi r B_0}$$

$$\Delta\iota(a) \approx 1.6 \times 10^{-3} I_{tor}[\mathrm{kA}]$$

which causes a radial shift of the edge island

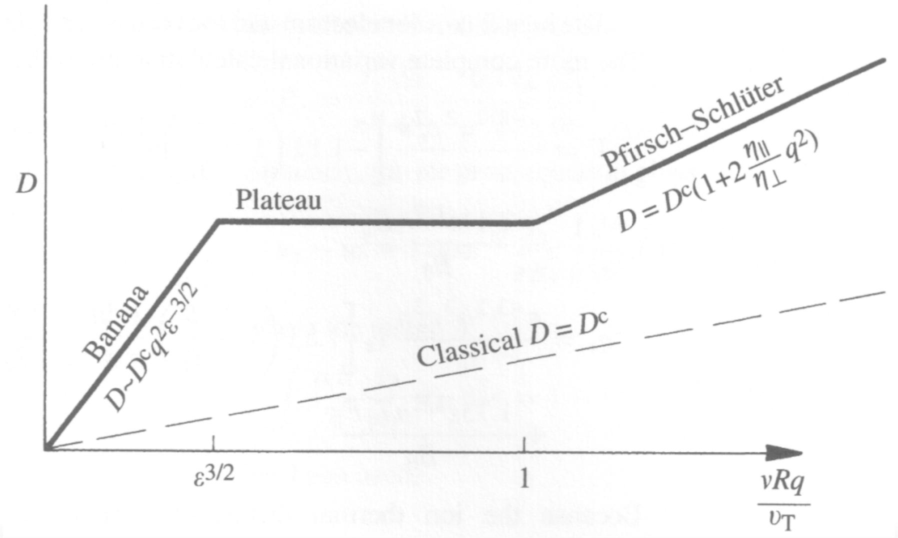
$$\Delta r[\mathrm{cm}] = \Delta\iota/\iota'(a) \approx 0.3 \times I_{tor}[\mathrm{kA}]$$

For W7-X parameters, a toroidal current of 10 kA causes a 3 cm radial movement of the edge island (Note: strike lines in the divertor can move more than that!)

Tokamak Transport

- **Neoclassical Transports**

  - May increase *D*, χ up to two orders of magnitude:
    - χ_i 'only' wrong by factor 3-5
    - *D*, χ_e still wrong by up to two orders of magnitude!



$$D_\perp = \frac{\eta_\perp n \sum kT}{B^2}$$

J. Wesson, Tokamaks (2004)

Tokamak Transport

- **Transport in fusion plasmas is 'anomalous‘.**

- Normal (water) flow: Hydrodynamic equations can develop nonlinear turbulent solutions (Reynolds, 1883)

- Transport mainly governed by MHD turbulence:
  radial extent of turbulent eddy: 1 - 2 cm
  typical lifetime of turbulent eddy: 0.5 - 1 ms

- Anomalous transport coefficients are of the order 1 m^2/s

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- Francis F. Chen, “Introduction to Plasma Physics and Controlled Fusion”, 2nd Edition, Plenum Press, New York (1984)
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- Hartmut Zohm, “Tokamaks: Equilibrium, Stability and Transport”, IPP Summer University on Plasma Physics, Garching, 18 September, 2001
- Tim Hender, “Neoclassical Tearing Modes in Tokamaks”, 2009 Korean Physical Society/ Division of Plasma Physics (KPS/DPP) in Daejun, Korea, 24 April 2009
- Mitsuru Kikuchi, “Frontiers in Fusion Research”, Springer (2011)

Fusion Reactor Technology 2

(459.761, 3 Credits)

Prof. Dr. Yong-Su Na

(32-206, Tel. 880-7204)

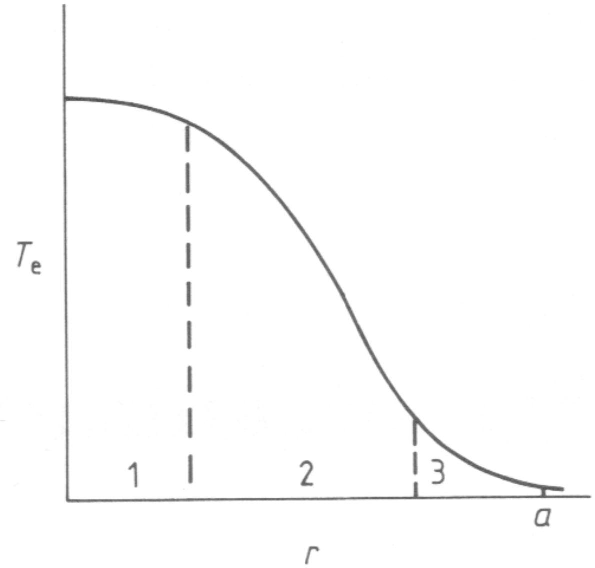
Tokamak Transport

- **Profile consistency (or profile resilience or stiffness)**

- The observation that profiles (of temperature, density, and pressure) often tend to adopt roughly the same shape (in tokamaks), regardless of the applied heating and fueling profiles. B. Coppi, "*Nonclassical Transport and the "Principle of Profile Consistency"*", Comments Plasma Phys. Cont. Fusion **5** 6 261-270 (1980)
  → tendency of profiles to stay close to marginal stability
- Due to plasma self-organisation, i.e., the feedback mechanism regulating the profiles (by turbulence) is often dominant over the various source terms.

Tokamak Transport

- **Profile consistency (or profile resilience or stiffness)**



- Three zones in which transport processes play the dominant part

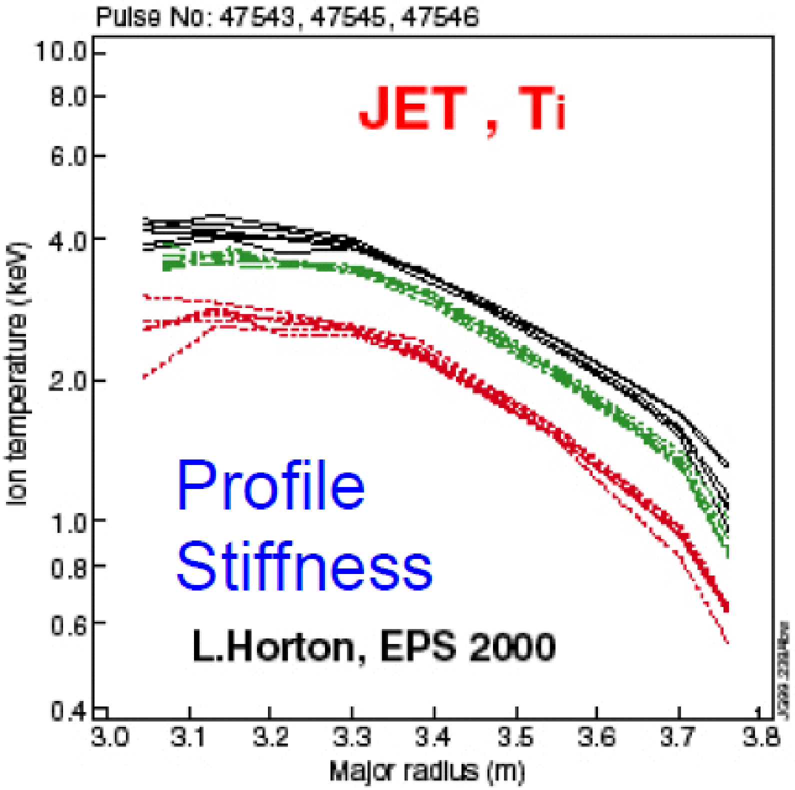
1: sawtooth oscillations - volume depending on the inversion radius which depending on q_a

2: heat transfer - responsible for magnetic confinement

3: atomic processes

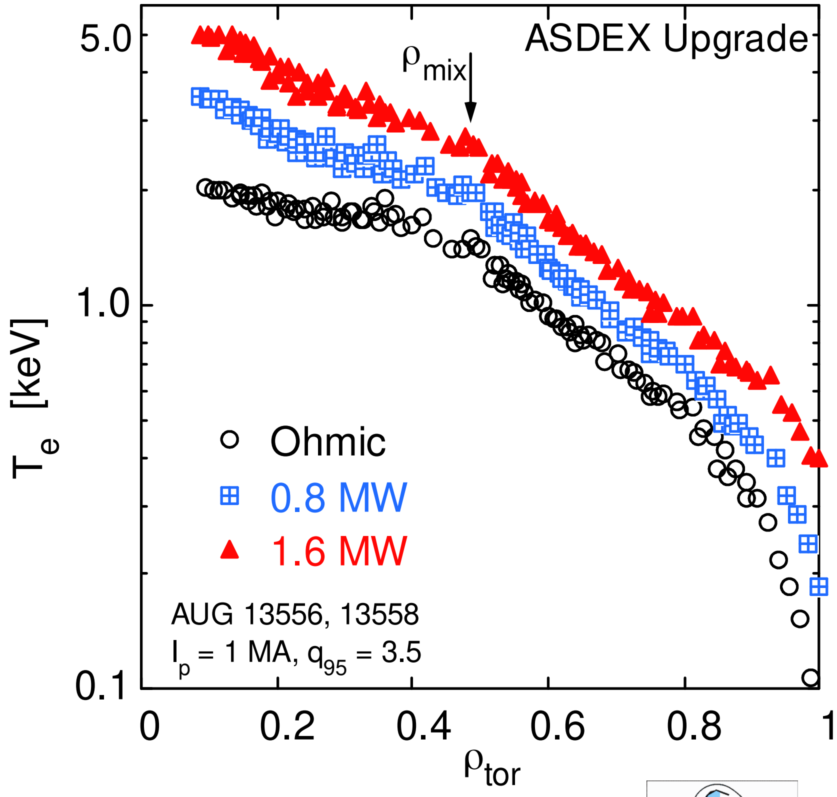
Tokamak Transport

- **Profile consistency (or profile resilience or stiffness)**



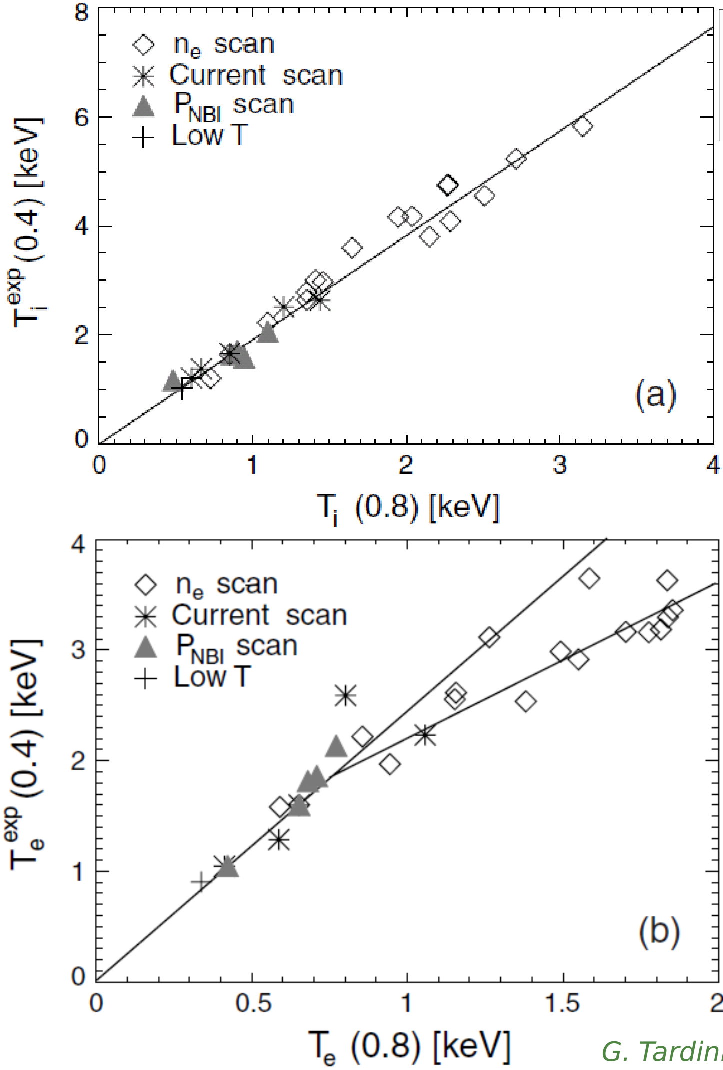
Profile Stiffness

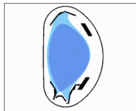
L.Horton, EPS 2000

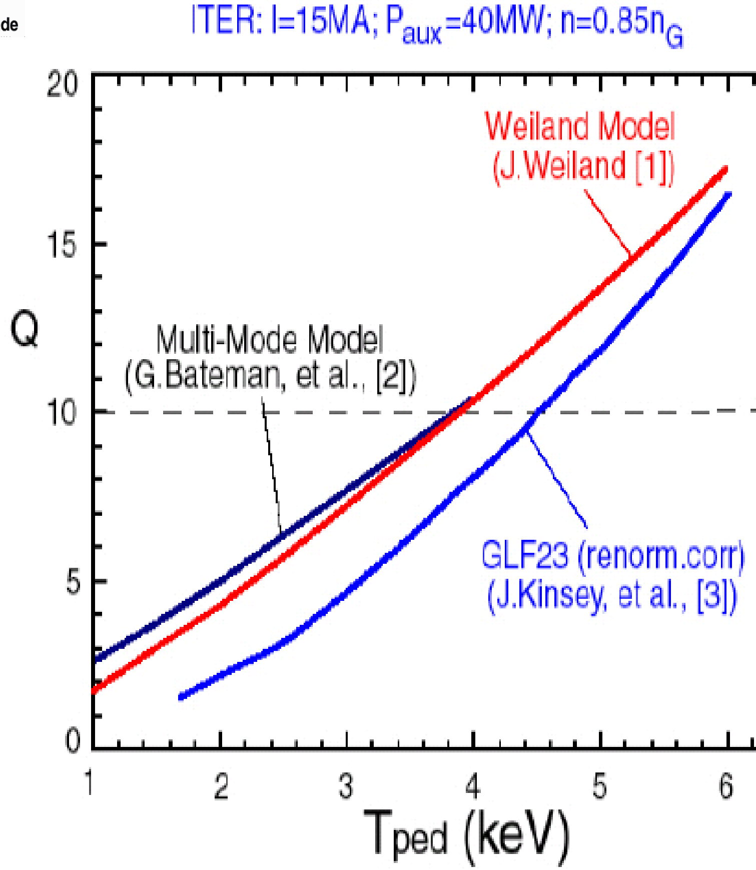


Tokamak Transport

- **Profile consistency (or profile resilience or stiffness)**



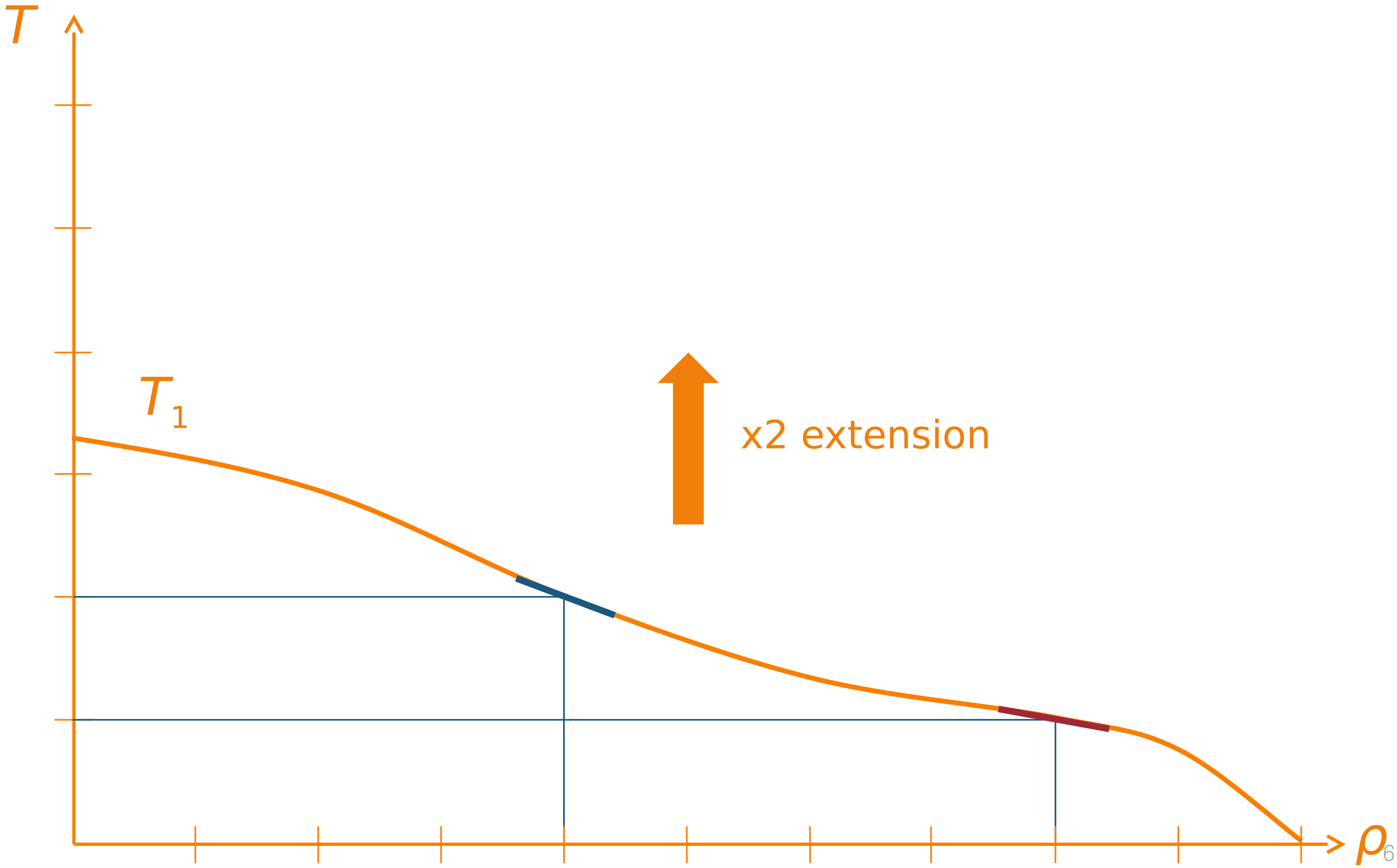




*G. Tardini et al, NF **42** 258 (2002)*

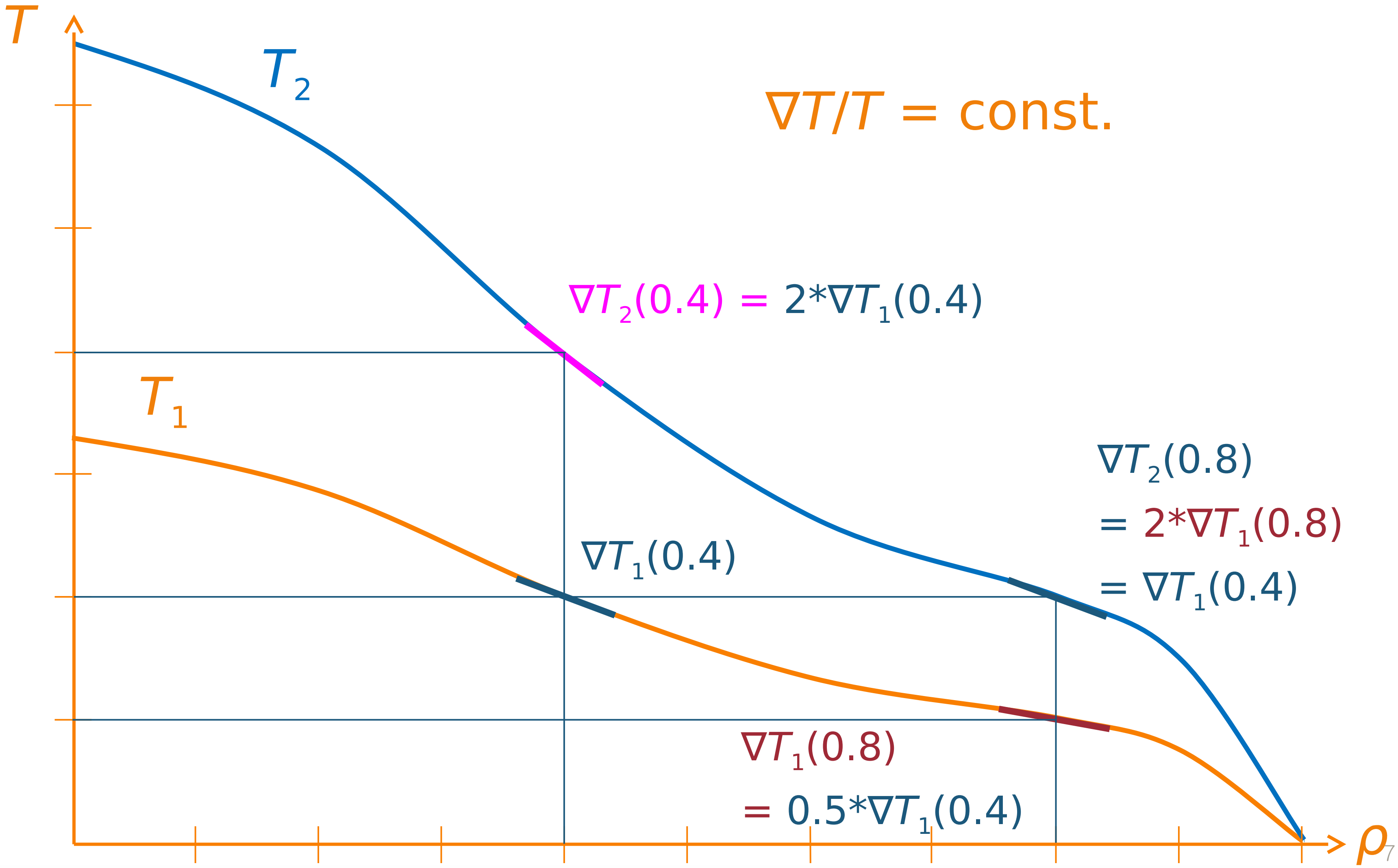
Tokamak Transport

- **Profile consistency (or profile resilience or stiffness)**



Tokamak Transport

- **Profile consistency (or profile resilience or stiffness)**



$\nabla T/T$ = const.

$\nabla T_2(0.4) = 2*\nabla T_1(0.4)$

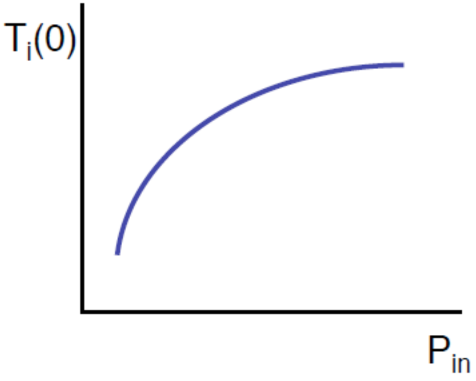
$\nabla T_1(0.4)$

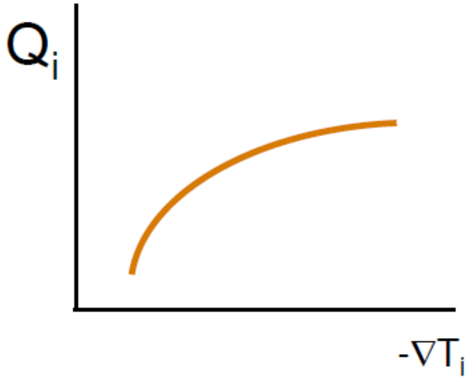
$\nabla T_2(0.8)$
$= 2*\nabla T_1(0.8)$
$= \nabla T_1(0.4)$

$\nabla T_1(0.8)$
$= 0.5*\nabla T_1(0.4)$

Tokamak Transport

- **Flux-Gradient Relation**





Generalization of Fick's Law:

$$\begin{pmatrix} Q_i \\ Q_e \\ \Gamma \\ \Gamma_\phi \end{pmatrix} = -\begin{bmatrix} \chi_i & \cdots & \cdots & \cdots \\ \cdots & \chi_e & \cdots & \cdots \\ \cdots & \cdots & D & \cdots \\ \cdots & \cdots & \cdots & \chi_\phi \end{bmatrix} \begin{pmatrix} \nabla T_i \\ \nabla T_e \\ \nabla n \\ \nabla U_\phi \end{pmatrix}$$

Tokamak Transport

*Ch. P. Ritz et al, PRL **62** 1844 (1989)*
*X. Garbet, C.R. Physique **7** 573 (2006)*

- **Transport dominated by turbulence**

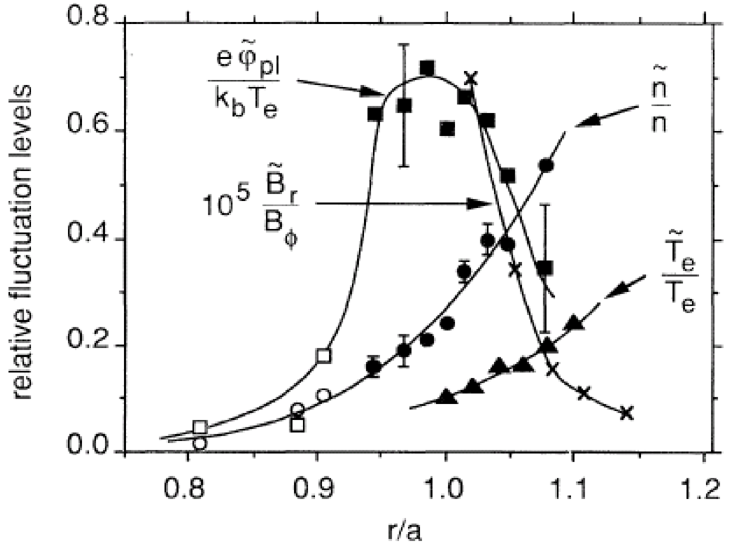


FIG. 1. Relative fluxuation levels of density $\tilde{n}/n$, plasma potential $e\tilde{\varphi}_{\rm pl}/k_B T_e$, electron temperature $\tilde{T}_e/T_e$, and magnetic field $\tilde{B}_r/B_\phi$, as functions of radius. Filled symbols represent data from Langmuir probes, and open symbols from the HIBP.

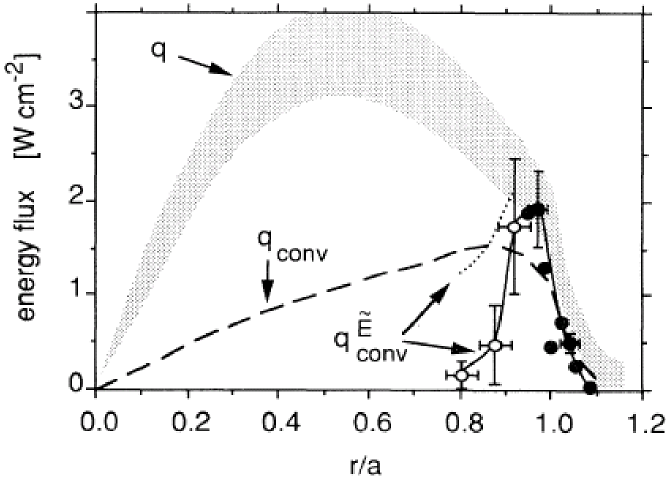
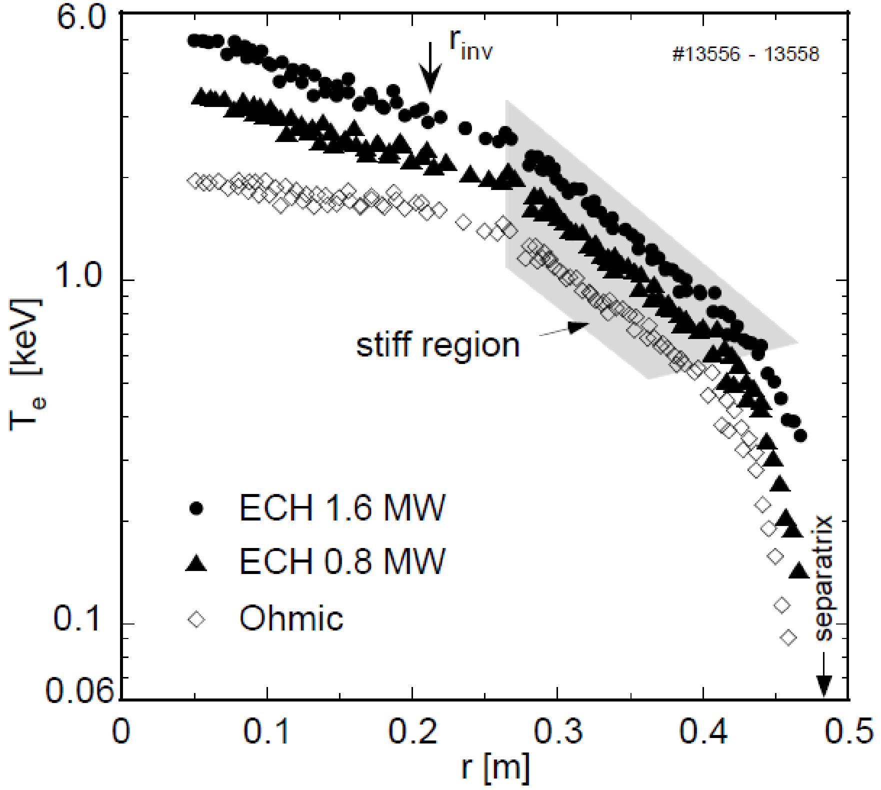


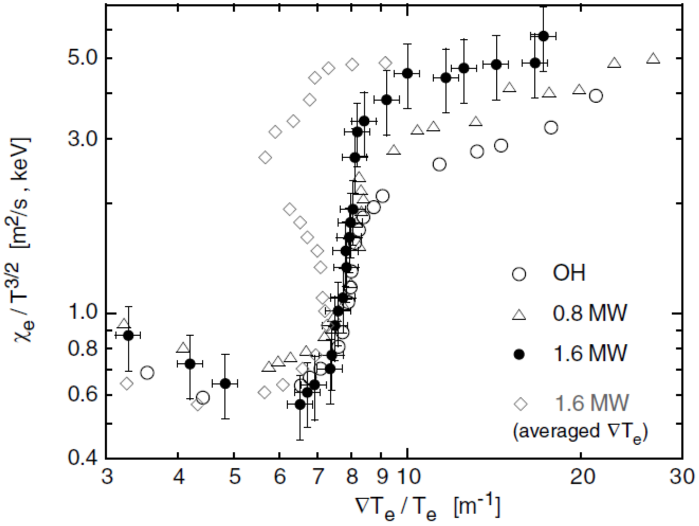
FIG. 2. Radial profiles of the total electron and ion energy flux $q = q_e + q_i$ from power balance (shaded area, defined by the standard deviation), the fluctuation-induced convected flux $q^{\tilde{E}}_{\rm conv}$ (filled circles from Langmuir probes, and open circles from HIBP; dotted line is upper bound in presence of η_i mode), and the total convected energy flux $q_{\rm conv}(r)$ from a neutral-penetration code and H_α measurements.

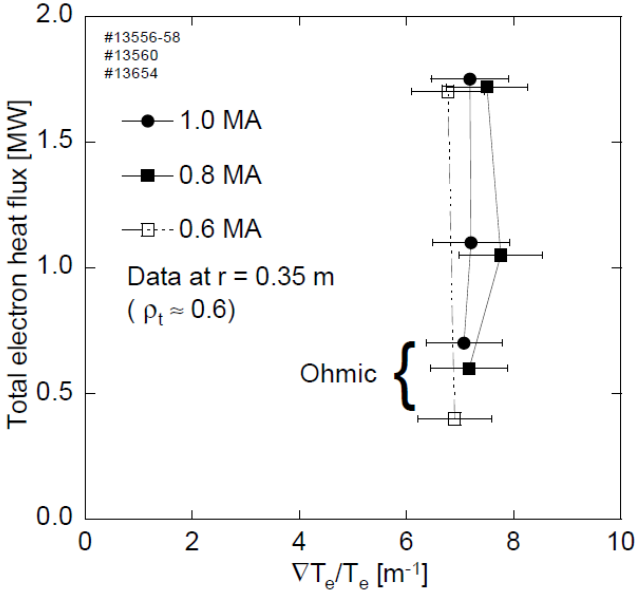
- It was proved that in edge plasmas, turbulence particle and energy fluxes agree with the fluxes deduced from particle and heat balance (i.e., integral of the particle and heating sources). Since then, several studies have confirmed the close connection between turbulence and transport. In particular, a reduction of the fluctuation level is observed when a transport barrier is formed.

Tokamak Transport

- **Anomalous Transport**





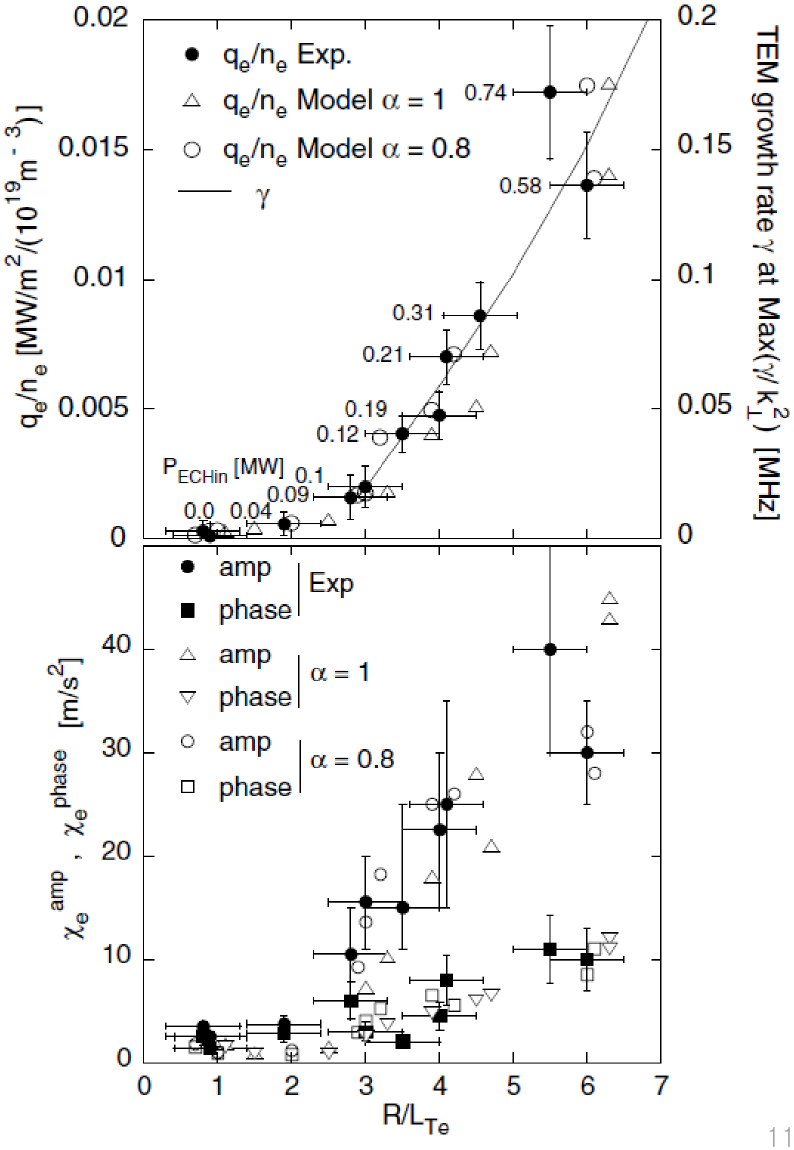


*F. Ryter et al, PRL **86** 2325 (2001),*
*PRL **86** 5498 (2001), NF **41** 537 (2001)*

Tokamak Transport

- **Anomalous Transport**

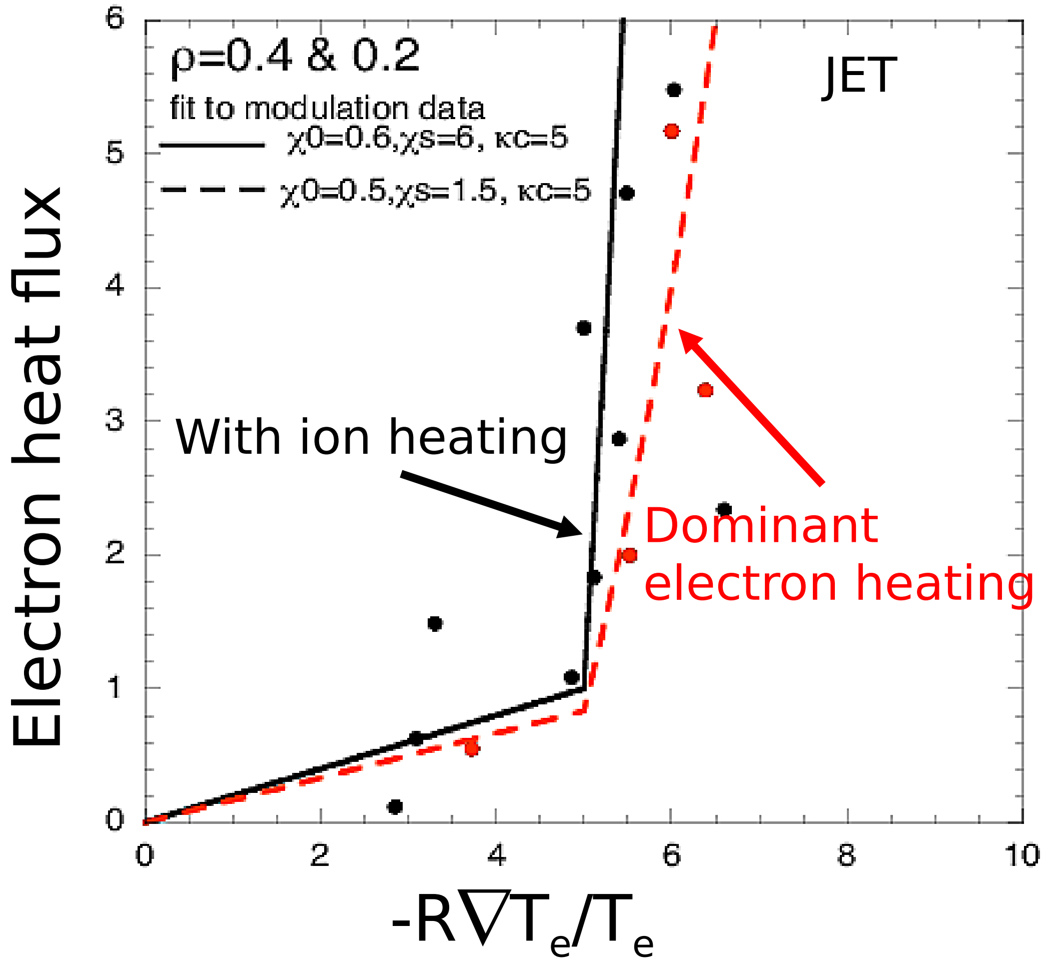
- Trapped electron modes are one of the candidates to explain turbulence driven electron heat transport observed in tokamaks.
- This instability has two characteristics: a threshold in normalized gradient and stabilization by collisions.
- Experiments using modulated ECH in the ASDEX Upgrade tokamak demonstrate explicitly the existence of the threshold.



F. Ryter et al, PRL ***95*** *085001 (2005)*

Tokamak Transport

- **Anomalous Transport**



Tokamak Transport

- **Anomalous Transport - Microinstabilities**

$$D^{\exp} = D^{NC} + D^{anomalous} > D^{NC}$$

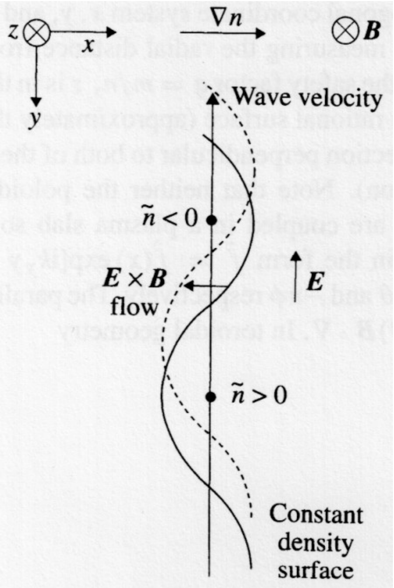
$$\chi^{\exp} = \chi^{NC} + \chi^{anomalous} > \chi^{NC}$$

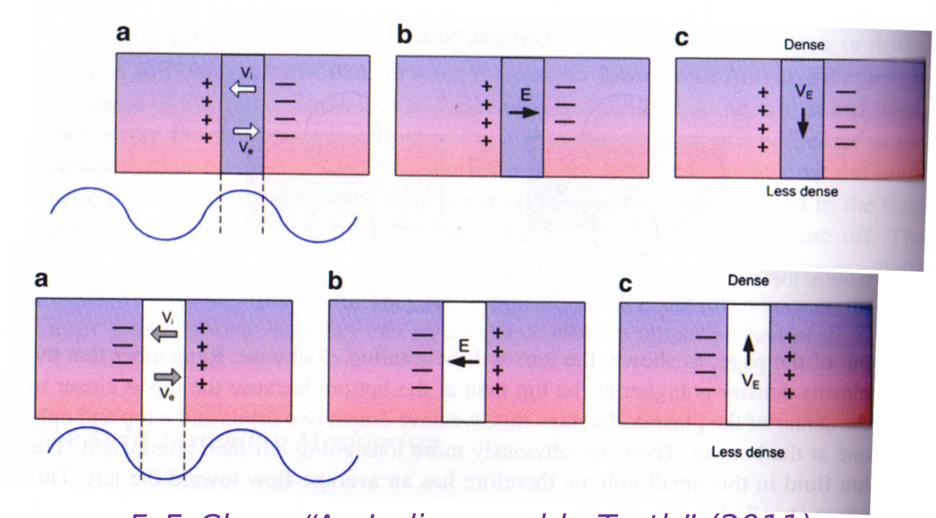
- Plasma waves and their associated instabilities

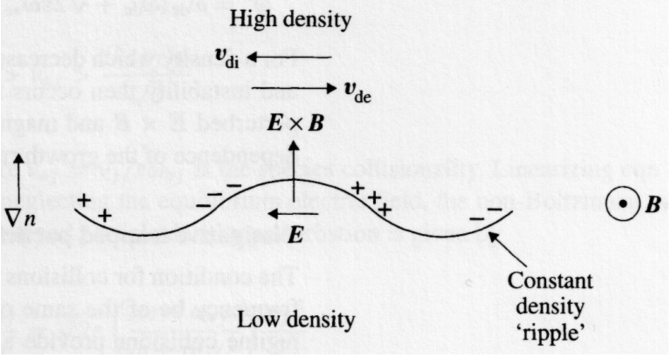
Electron drift wave: ‘Universal’, trapped electron

Sound wave: Ion temperature gradient

Alfven wave: Micro-tearing







F. F. Chen, “An Indispensable Truth” (2011)
J. Wesson, “Tokamaks” (2009)

Tokamak Transport

- **Anomalous Transport - Microinstabilities**

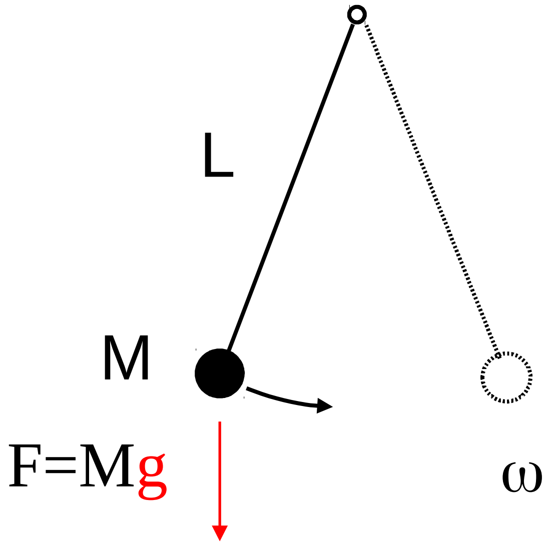
- Electrostatic instabilities: drift wave instabilities
  perturbations of the magnetic field are ignored,
  so that only the perturbed electric field matters.
  Assumption appropriate if the plasma beta is lower than the
  instability threshold for electromagnetic interchange modes
  (called ‘kinetic ballooning modes’)
    Passing particle instabilities
    Trapped particle instabilities
    Ex. Ion Temperature Gradient (ITG) modes,
      Trapped Electron Modes (TEM)
    - Electromagnetic instabilities: micro-tearing modes

Tokamak Transport

- **Anomalous Transport**

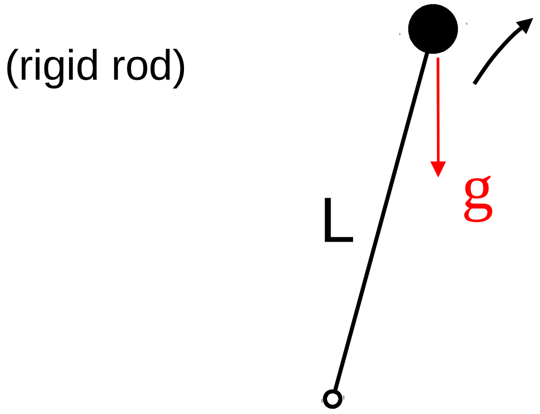
- Main instabilities are interchange modes.

Stable Pendulum



$\omega=(g/L)^{1/2}$

Unstable Inverted Pendulum

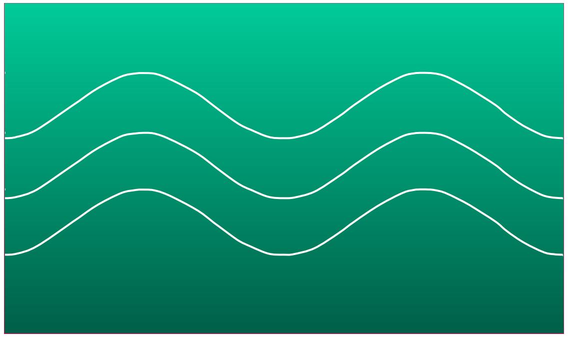


$\omega= (-g/|L|)^{1/2} = i(g/|L|)^{1/2} = i\gamma$

Instability

Density-stratified Fluid

$\rho=\exp(-y/L)$

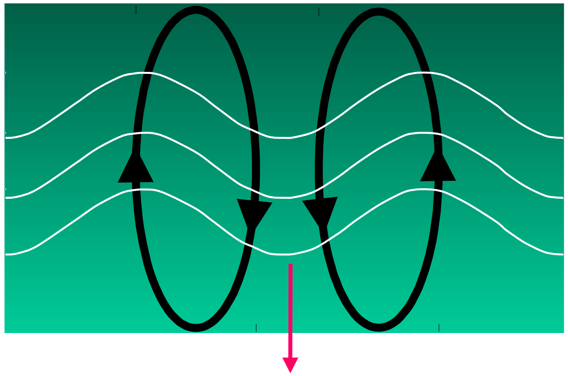


stable $\omega=(g/L)^{1/2}$

Inverted-density fluid

⇒Rayleigh-Taylor Instability

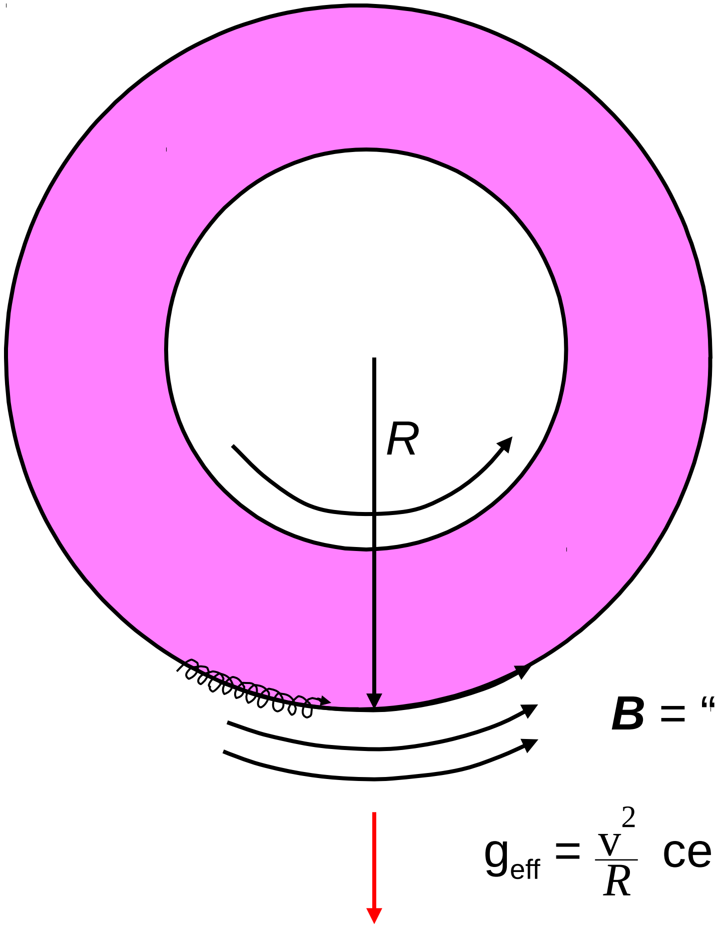
$\rho=\exp(y/L)$



Max growth rate $\gamma=(g/L)^{1/2}$

“Bad Curvature” instability in plasmas
≈ Inverted Pendulum / Rayleigh-Taylor Instability

Top view of toroidal plasma:



plasma = heavy fluid

B = “light fluid”

$g_{eff} = \frac{v^2}{R}$ centrifugal force

Growth rate:

$$\gamma = \sqrt{\frac{g_{eff}}{L}} = \sqrt{\frac{v_t^2}{RL}} = \frac{v_t}{\sqrt{RL}}$$

Similar instability mechanism
in MHD & drift/microinstabilities

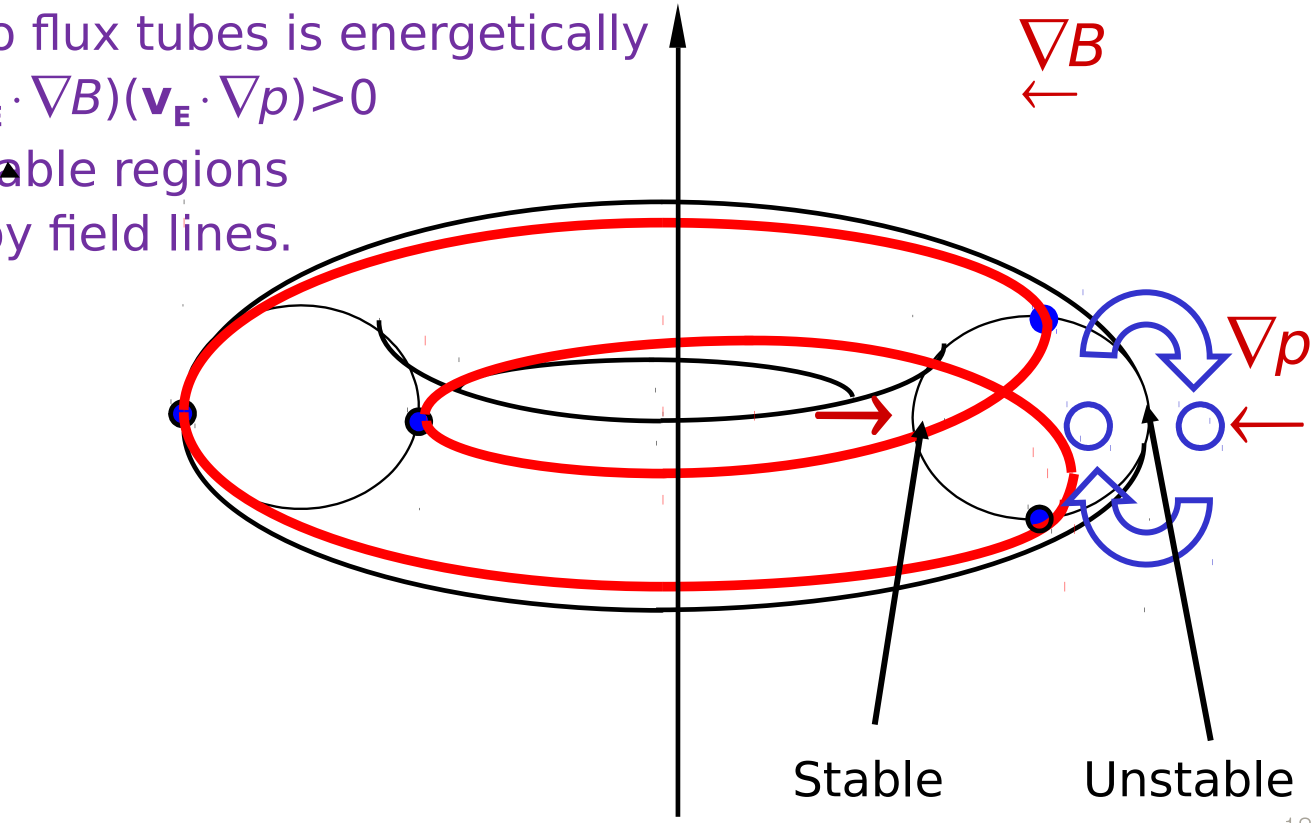
$1/L = \nabla p/p$ in MHD,
$\propto$ combination of ∇n & ∇T
in microinstabilities.

Tokamak Transport

- **Anomalous Transport**

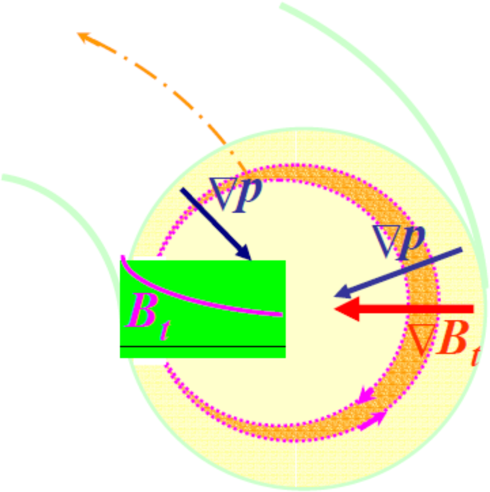
- Main instabilities are interchange modes.
  - The exchange of two flux tubes around a field line releases free energy.

- Exchange of two flux tubes is energetically favourable if $(\mathbf{v}_E \cdot \nabla B)(\mathbf{v}_E \cdot \nabla p) > 0$
- Stable and unstable regions are connected by field lines.



Tokamak Transport

- **Anomalous Transport**



Unstable region: $\nabla B_t \cdot \nabla p > 0$

- Trapped particles are localised on the low field side, as this corresponds to the zone of minimum field along the field lines.
  → Trapped particles are expected to play a prominent role in the interchange process.

Tokamak Transport

- **Anomalous Transport – ITG/TEM**

- Unstable in the limit of large wavelengths: $k_{\perp}\rho_i < 1$
  - Electron and/or ion modes are unstable above a threshold.
  - Underlie particle, electron and ion heat transport: interplay between all channels
- ITG: For a given *q*-profile, the threshold of a pure ion mode (i.e., when the electron response follow a Boltzmann law) appears as a critical ion temperature logarithmic gradient $-R\nabla T_i/T_i$ that depends on the logarithmic density gradient $-R\nabla n_i/n_i$, and on the ratio of electron to ion temperature T_e/T_i.

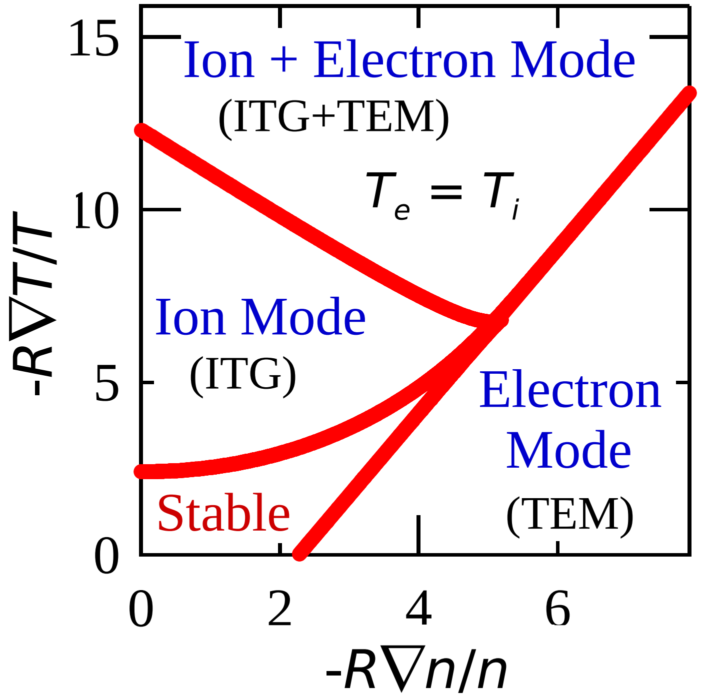
  An ion mode usually rotates in the ion diamagnetic direction.
  - TEM: usually rotate in the electron diamagnetic direction and are mainly driven through a resonant interaction of the modes with trapped electrons at the precession frequency.

    The threshold is a critical value of $-R\nabla T_e/T_e$ that depends on $-R\nabla n_e/n_e$ and the fraction of trapped electrons f_t.

Tokamak Transport

- **Anomalous Transport – ITG/TEM**

- Unstable in the limit of large wavelengths: $k_\perp \rho_i < 1$
  - Electron and/or ion modes are unstable above a threshold.
  - Underlie particle, electron and ion heat transport: interplay between all channels



Stability diagram
(Weiland model)

Tokamak Transport

- **Anomalous Transport**

- Fluctuations of **E**x**B** drift velocity produce turbulent transport.

- **E**x**B** drift

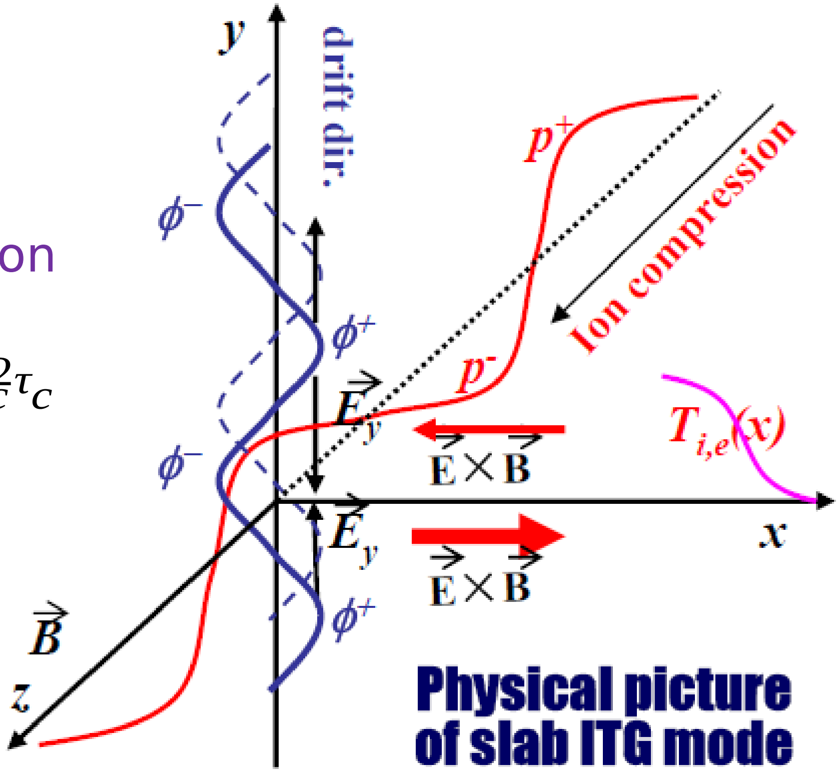
$$v_E = \frac{B\times\nabla\phi}{B^2}$$

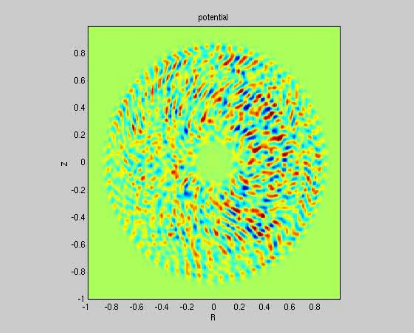
- Turbulent diffusion

$$D_{turb} \propto \left|v_E\right|^2 \tau_c \propto L_c^2 \tau_c$$

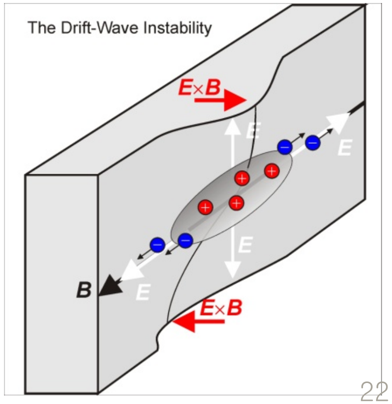
- Turbulent flux

$$\phi_E = \frac{3}{2}\left\langle p v_E \right\rangle$$



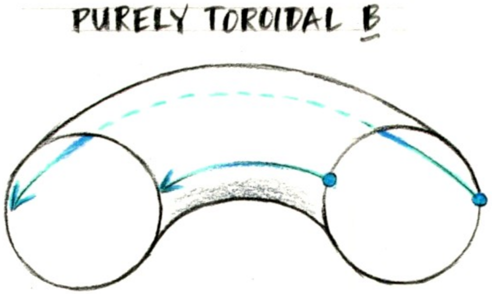


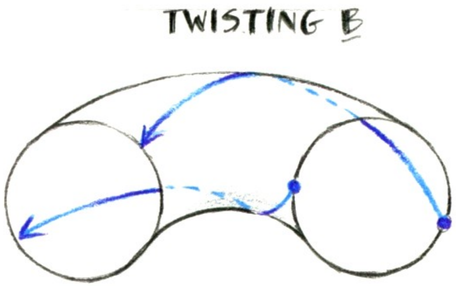
Contour lines of electric potential ϕ (TRB simulation)



The Secret for Stabilizing Bad-Curvature Instabilities

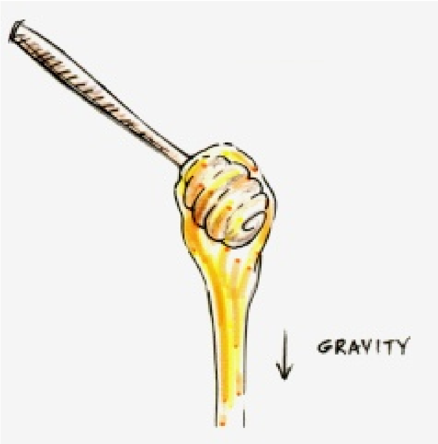
Twist in ***B*** carries plasma from bad curvature region to good curvature region:

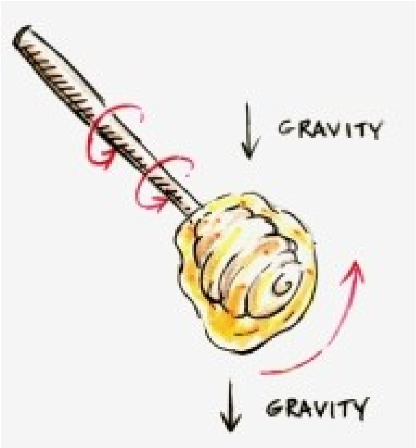




Unstable

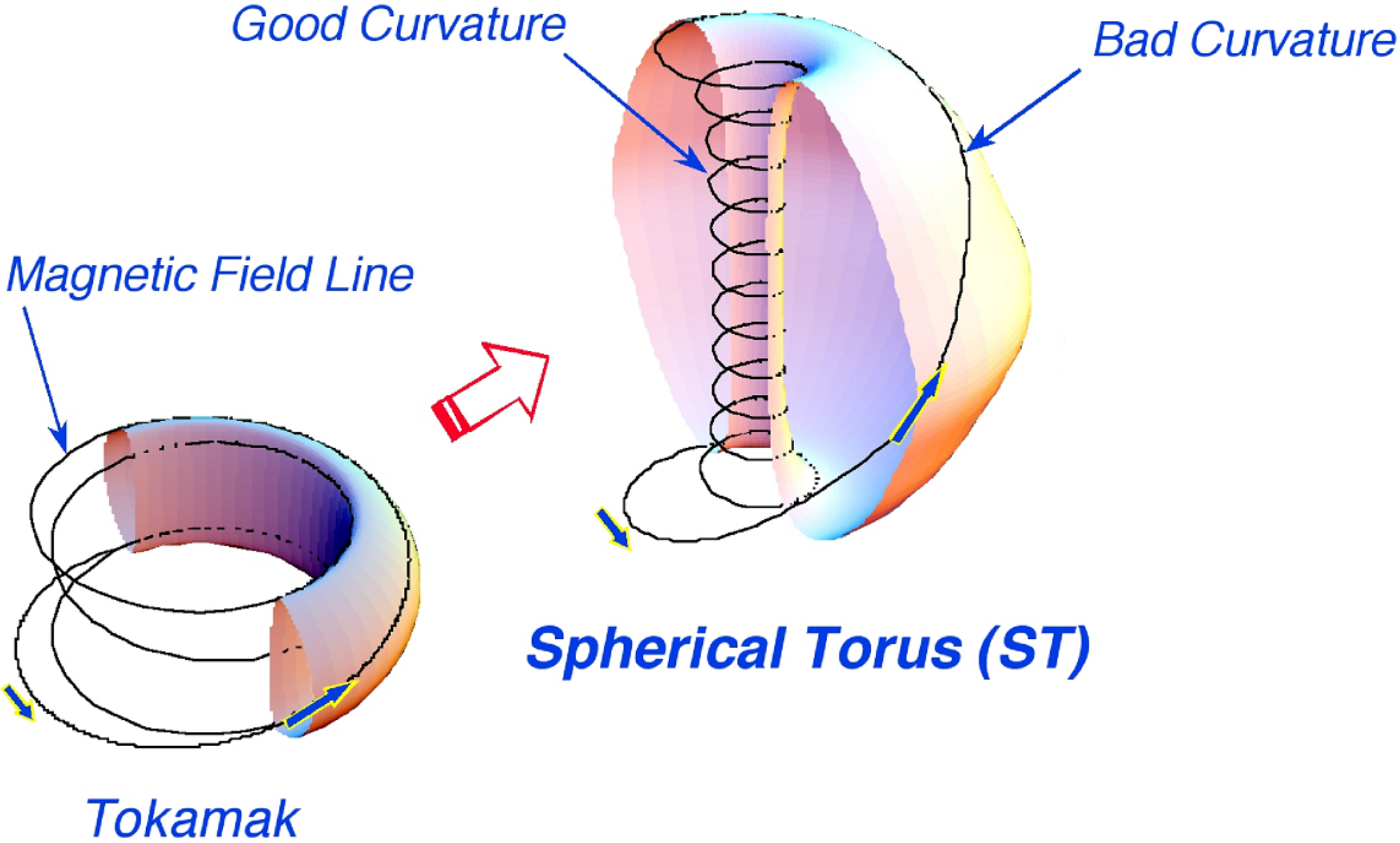
Stable





Similar to how twirling a honey dipper can prevent honey from dripping.

Spherical Torus has improved confinement and pressure limits (but less room in center for coils)



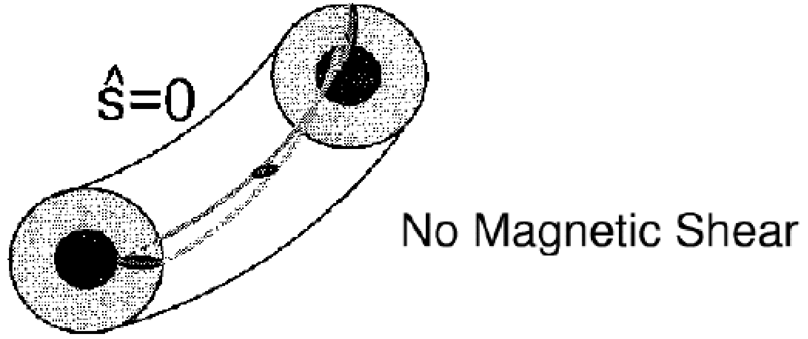
Simple picture of reducing turbulence by negative magnetic shear

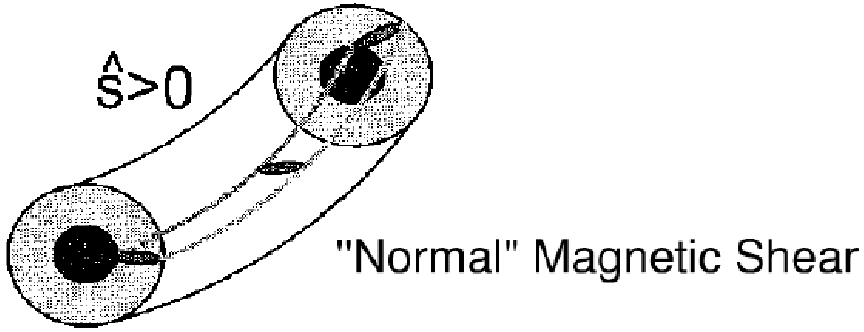
Particles that produce an eddy tend to follow field lines.

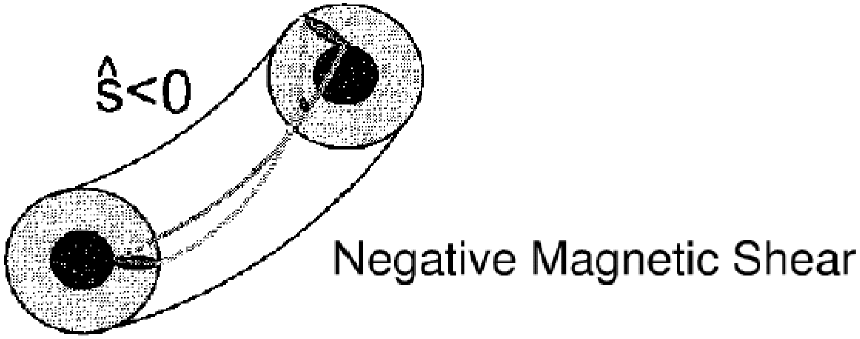
Reversed magnetic shear twists eddy in a short distance to point in the ``good curvature direction".

Locally reversed magnetic shear naturally produced by squeezing magnetic fields at high plasma pressure: ``Second stability" Advanced Tokamak or Spherical Torus.

Shaping the plasma (elongation and triangularity) can also change local shear







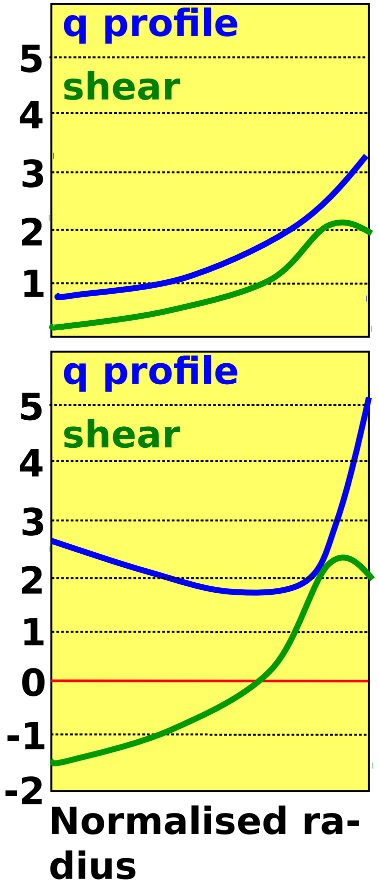
Antonsen, Drake, Guzdar et al. Phys. Plasmas 96
Kessel, Manickam, Rewoldt, Tang Phys. Rev. Lett. 94

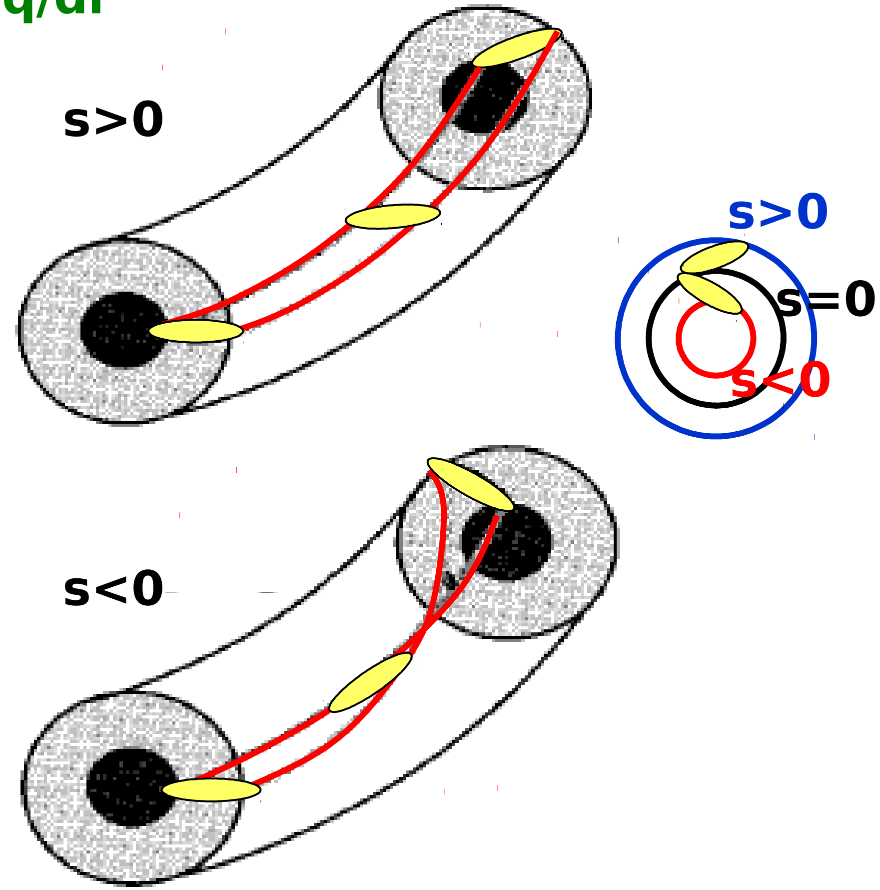
Turbulence Stabilisation

q & magnetic shear: s = r/q dq/dr

Standard scenario

Advanced tokamak scenario

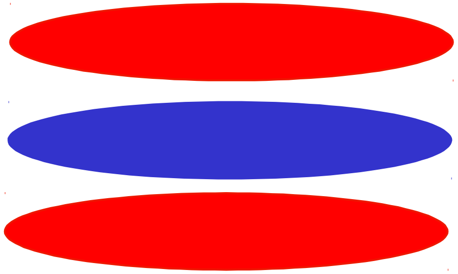




Magnetic shear can twist plasma disturbances

Sheared flows can suppress or reduce turbulence

Most Dangerous Eddies:
Transport long distances
In bad curvature direction



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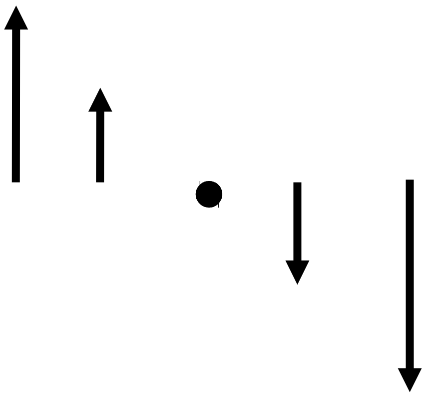
Sheared Flows



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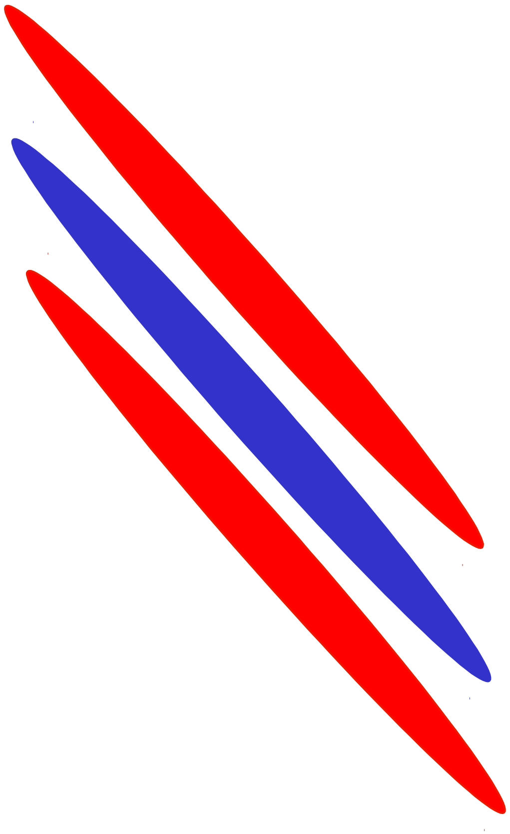
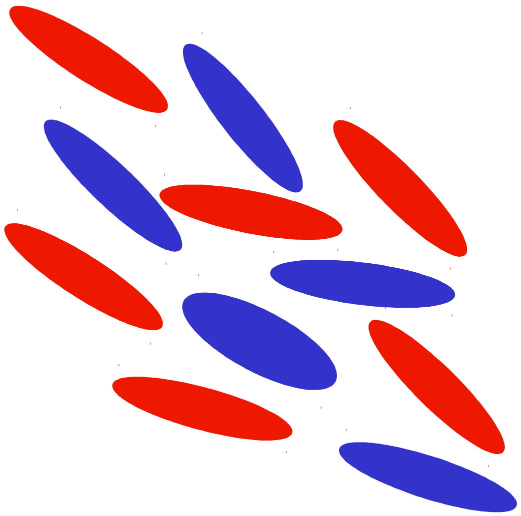
Sheared Eddies
Less effective

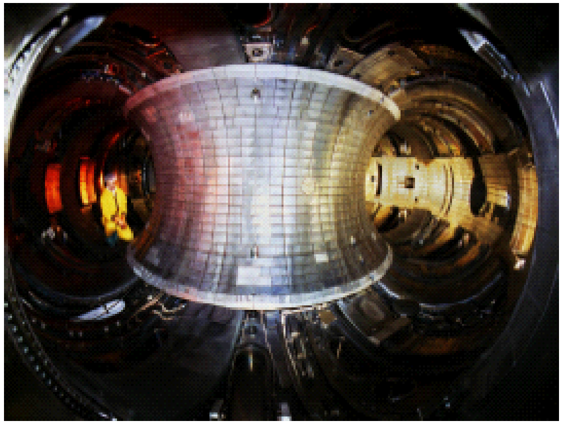


Eventually break up



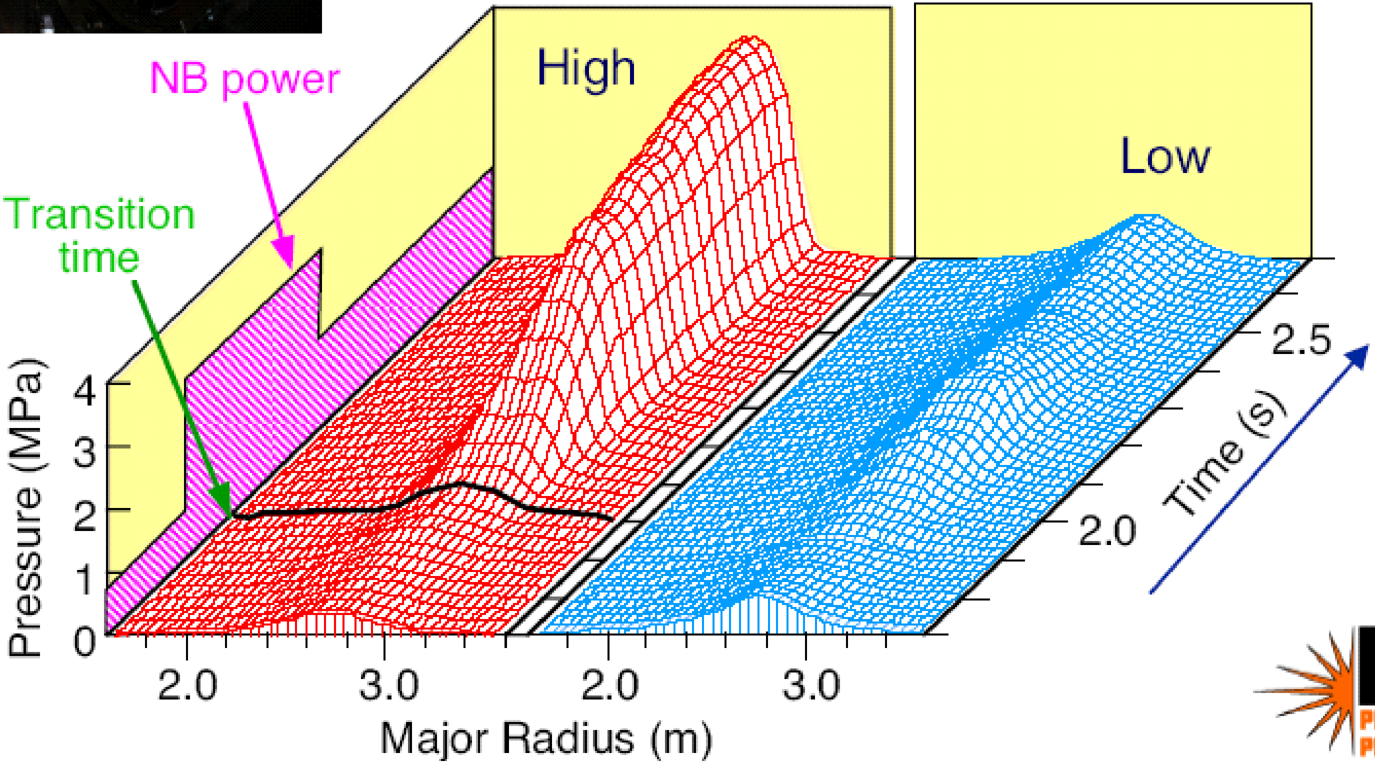
Biglari, Diamond, Terry (Phys. Fluids1990),
Carreras, Waltz, Hahm, Kolmogorov, et al.

Fascinating Diversity of Regimes in Fusion Plasmas. What Triggers Change? What Regulates Confinement?



TFTR

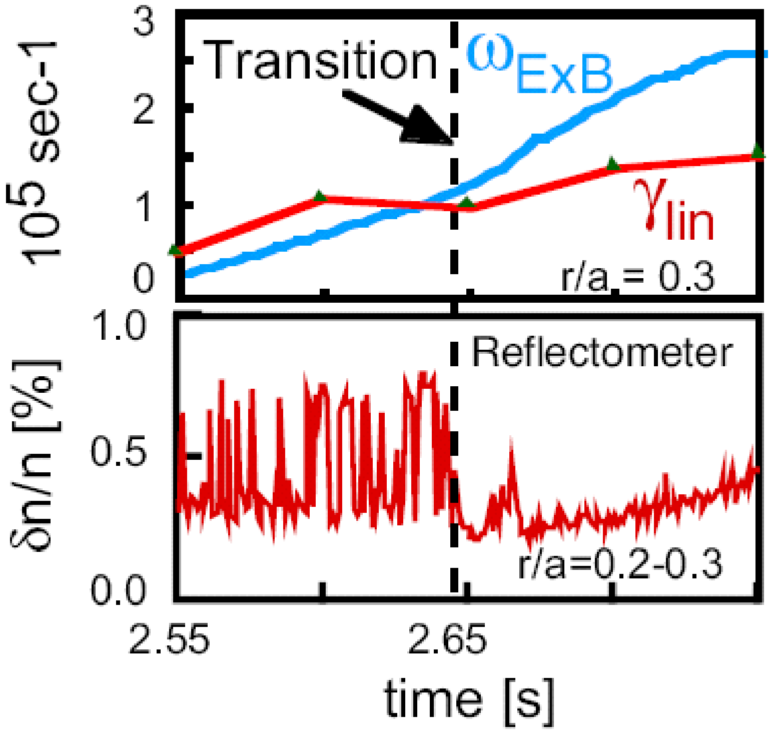
- Two regimes with very different confinement for similar initial conditions and neutral beam heating
- Access depends on plasma heating and reducing current density on axis
- Can we attribute a difference in turbulence to these two different confinement regimes?



PPPL PRINCETON PLASMA PHYSICS LABORATORY

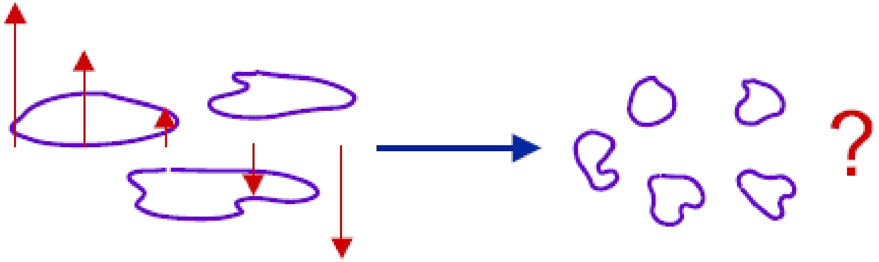
R. Nazikian et al.

Transition to Enhanced Confinement Regime is Correlated with Suppression of Core Fluctuations in TFTR



- Theory predicts fluctuation suppression when rate of shearing (ω_{ExB}) exceeds rate of growth (γ_{lin})

- Outstanding issue: Is suppression accompanied by radial decorrelation?



- Similar suppression observed on JET (X-mode reflectometer) and DIII-D (FIR Scattering)

Hahm, Burrell, Phys. Plas. 1995, E. Mazzucato et al., PRL 1996.

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PRINCETON PLASMA PHYSICS LABORATORY

Tokamak Transport

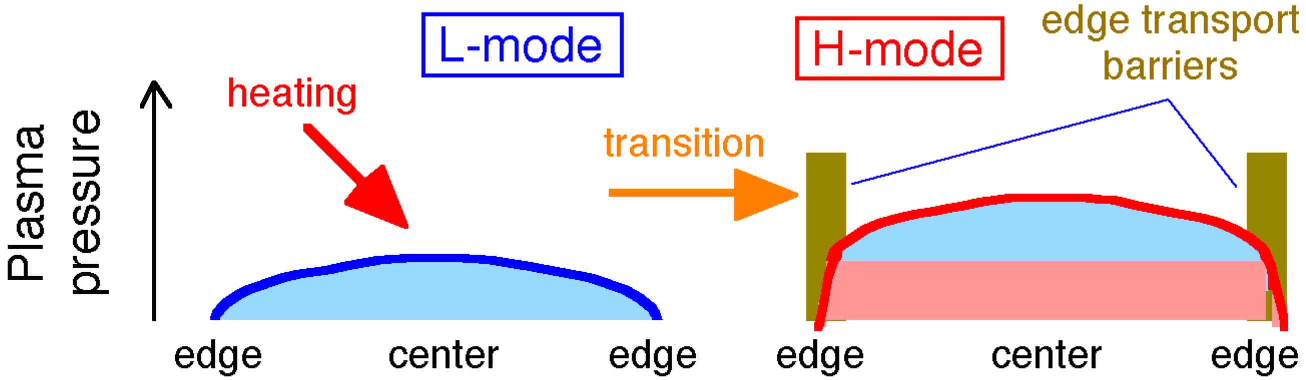
- **Anomalous Transport**

*F. Ryter et al, PRL **86** 2325 (2001),*
*PRL **86** 5498 (2001), NF **41** 537 (2001)*

$$\chi = T^{3/2}\left[\xi_0 + \xi_{TG} G\left(R/L_T - R/L_{T,c}\right)\right]$$

- ξ_0: transport when TG turbulence is not active
- $\xi_0 G$: transport caused by TG turbulence
- $T^{3/2}$: reflecting the gyro-Bohm assumption
- $G = 0$ for $L_T \geq L_{T,c}$ but increases strongly when $L_T < L_{T,c}$ and eventually saturates. When $L_T < L_{T,c}$ transport is high to keep L_T close to $L_{T,c}$, providing the stiffness.

Confinement scaling



$\tau_E^{ITER89P} = 0.048 M^{0.5} I_p^{0.85} B_t^{0.2} R^{1.2} a^{0.3} \kappa^{0.5} n_{20}^{0.1} P^{-0.5}$

$\chi \sim \frac{a^2}{\tau_E} \sim \chi_{Bohm} (\rho *)^{\mu} F(\beta, \nu *)$: nearly “Bohm” scaling ($\mu \sim 0$)

$$\chi_{Bohm} = \frac{cT}{eB}$$

M for DT mixture?

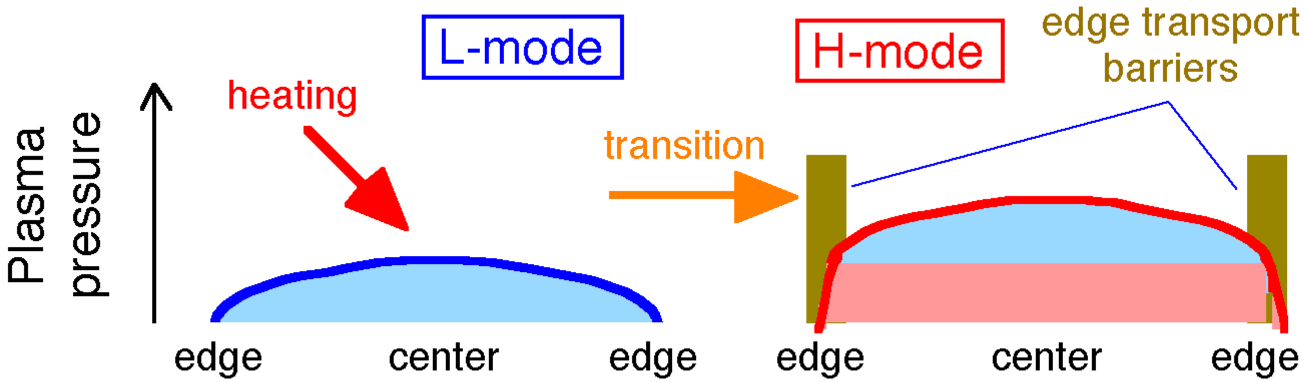
$\tau_{E,th}^{IPB98(y)} = 0.0365 M^{0.2} I_p^{0.97} B_t^{0.08} R^{1.7} a^{0.23} \kappa_a^{0.67} n_{19}^{0.41} P^{-0.63}$

$\propto \tau_B \rho*^{-0.83} \beta^{-0.50} \nu*^{-0.10} M^{0.97} q^{-2.52} \varepsilon^{-0.55} \kappa_a^{2.72}$

$\chi \sim \frac{a^2}{\tau_E} \sim \chi_{Bohm} (\rho *)^{\mu} F(\beta, \nu *)$: close to “gyroBohm” scaling ($\mu \sim 1$, $\tau_E/\tau_B \propto \rho*^{-1}$)

$$\chi \sim \left(\frac{\rho_i}{L}\right)\left(\frac{cT_i}{eB}\right) \qquad \left(\frac{\rho_i}{L}\right) << 1$$

Confinement scaling



$$\Delta x \sim 1/\bar{k}_\perp \sim \sqrt{a\rho_i} \text{ (radially elongated eddy) and } \gamma \sim \omega_{*e} \sim \frac{\bar{k}_\perp \rho_s}{a} c_s$$

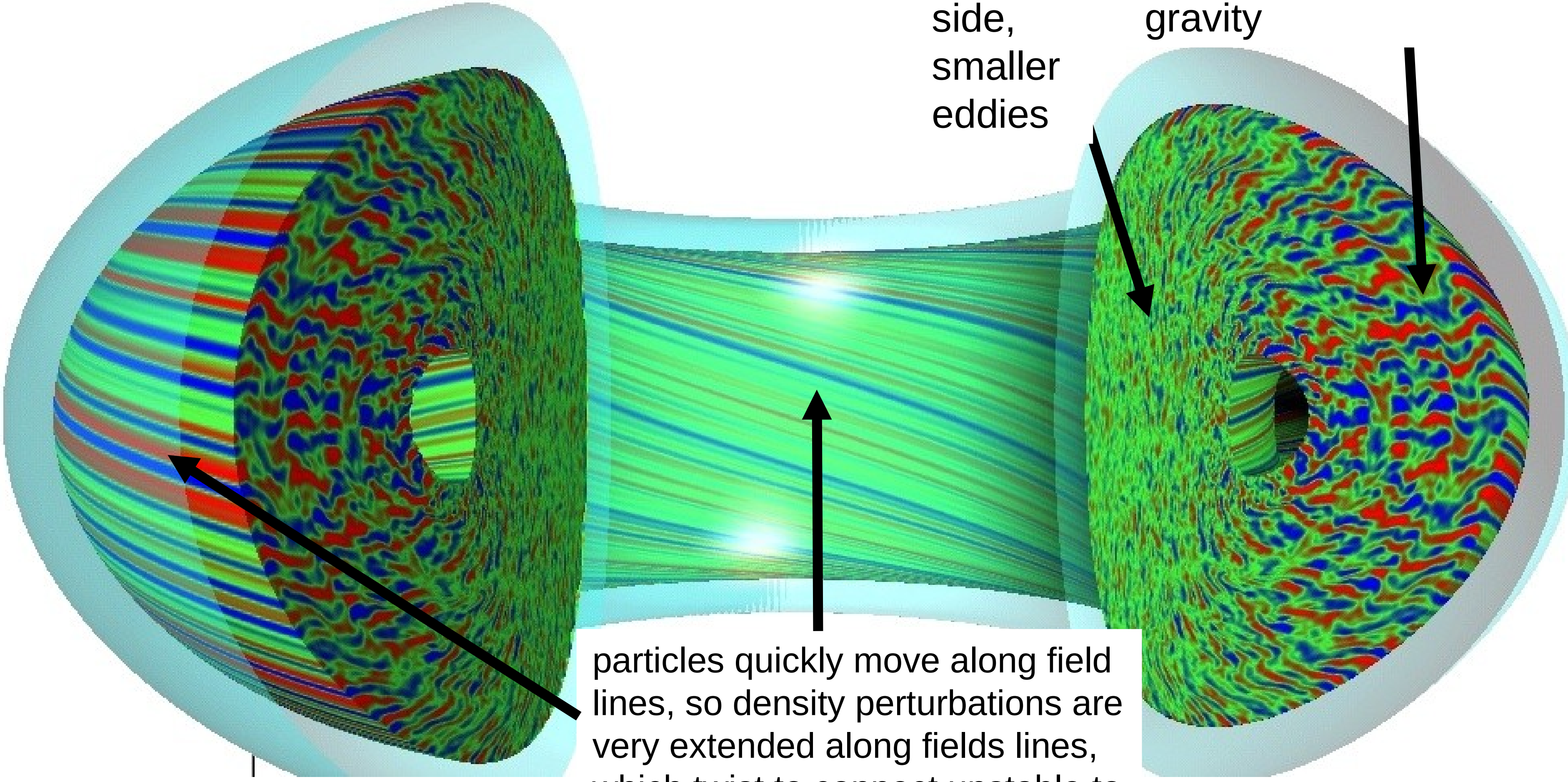
$$\Rightarrow \quad \chi \sim \frac{T_e}{eB} \qquad \text{Bohm scaling}$$

$$\Delta x \sim 1/\bar{k}_\perp \sim \rho_i \text{ (sheared eddy) and } \gamma \sim \omega_{*e} \sim \frac{\bar{k}_\perp \rho_s}{a} c_s$$

$$\Rightarrow \quad \chi \sim \frac{\rho_i}{a}\frac{T_e}{eB} \qquad \text{gyroBohm scaling}$$

XGC1 turbulence simulation

These physical mechanisms can be seen in gyrokinetic simulations and movies

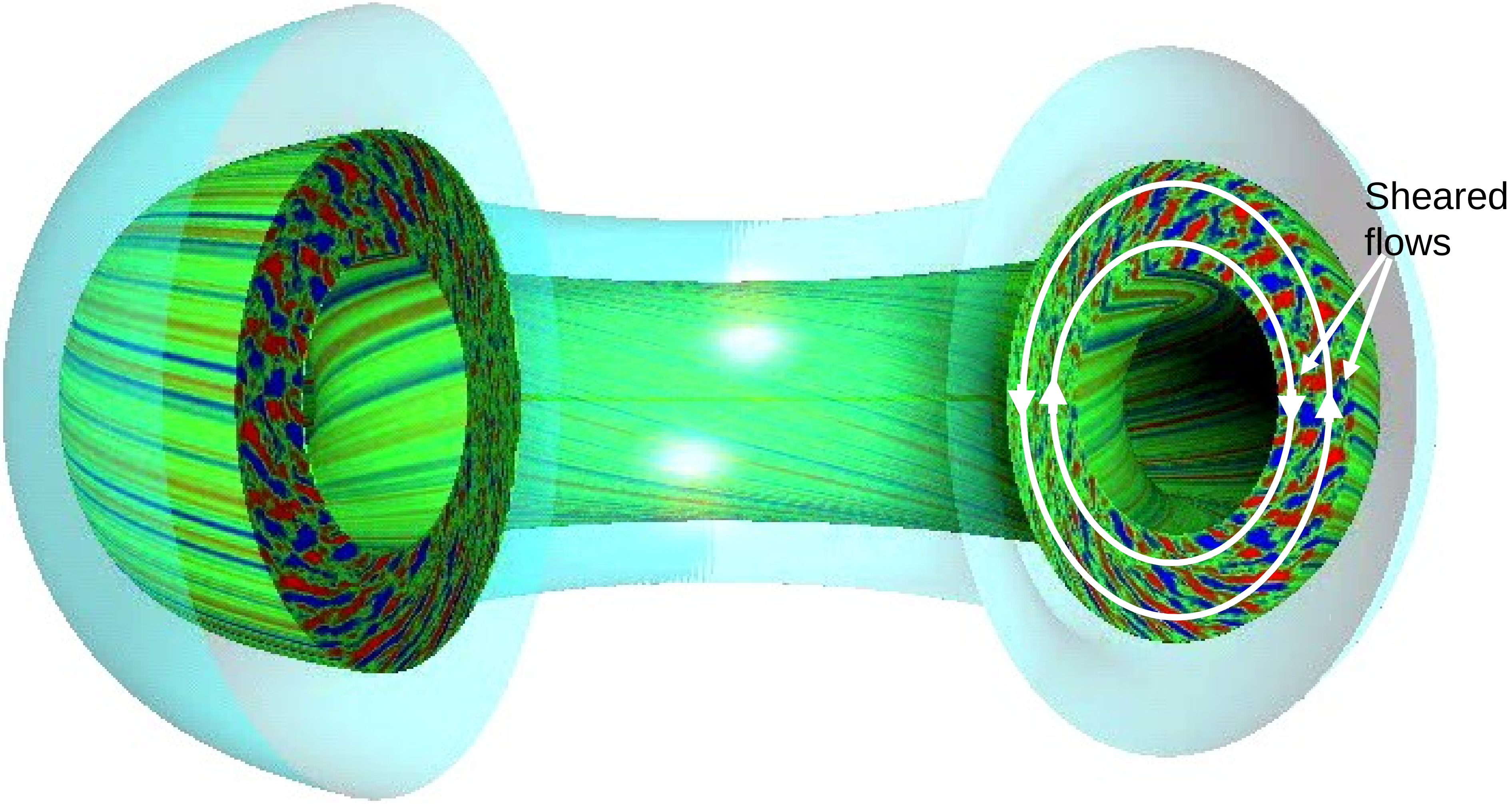


Stable side, smaller eddies

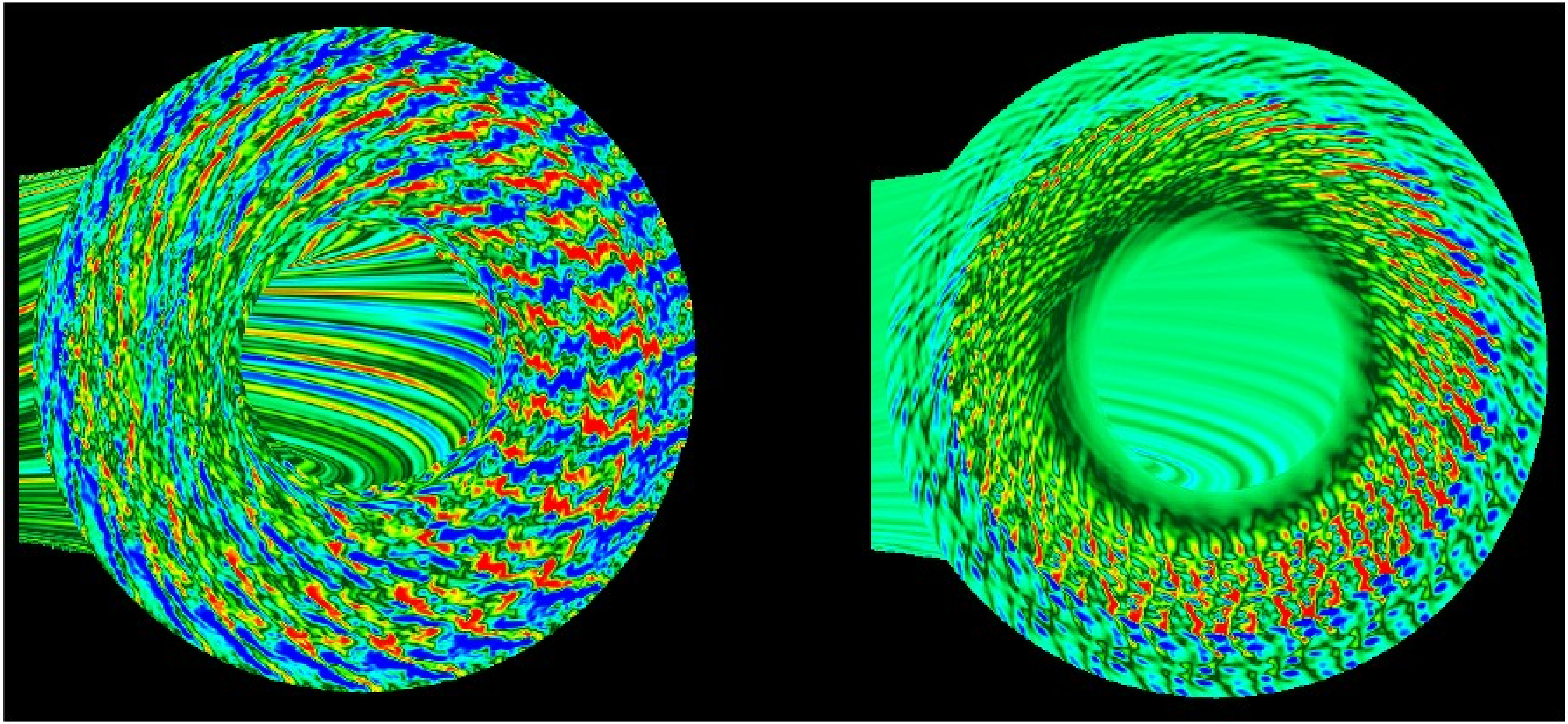
Unstable bad-curvature side, eddies point out, direction of effective gravity

particles quickly move along field lines, so density perturbations are very extended along fields lines, which twist to connect unstable to stable side

Movie http://fusion.gat.com/THEORY/images/3/35/D3d.n16.2x_0.6_fly.mpg from http://fusion.gat.com/theory/Gyromovies shows contour plots of density fluctuations in a cut-away view of a GYRO simulation (Candy & Waltz, GA). This movie illustrates the physical mechanisms described in the last few slides. It also illustrates the important effect of sheared flows in breaking up and limiting the turbulent eddies. Long-wavelength equilibrium sheared flows in this case are driven primarily by external toroidal beam injection. (The movie is made in the frame of reference rotating with the plasma in the middle of the simulation. Barber pole effect makes the dominantly-toroidal rotation appear poloidal..) Short-wavelength, turbulent-driven flows also play important role in nonlinear saturation.



Sheared ExB Flows can regulate or completely suppress turbulence (analogous to twisting honey on a fork)



Dominant nonlinear interaction between turbulent eddies and ±θ-directed zonal flows.

Additional large scale sheared zonal flow (driven by beams, neoclassical) can completely suppress turbulence

Waltz, Kerbel, Phys. Plasmas 1994 w/ Hammett, Beer, Dorland, Waltz Gyrofluid Eqs., Numerical Tokamak Project, DoE Computational Grand Challenge

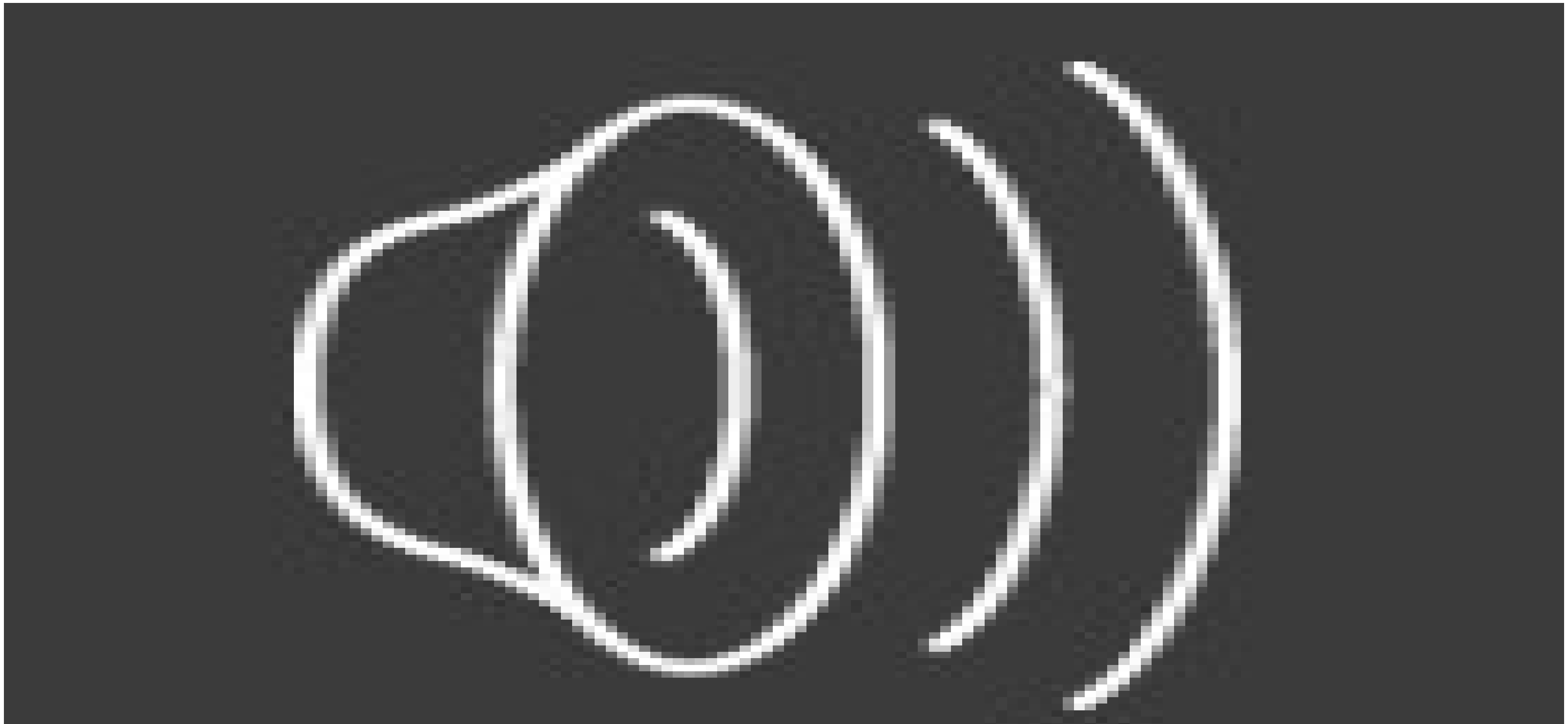
Tokamak Transport

- **Anomalous Transport**

- Ion density fluctuations in the DIII-D tokamak for discharge 121717

Tokamak Transport

- **Anomalous Transport**

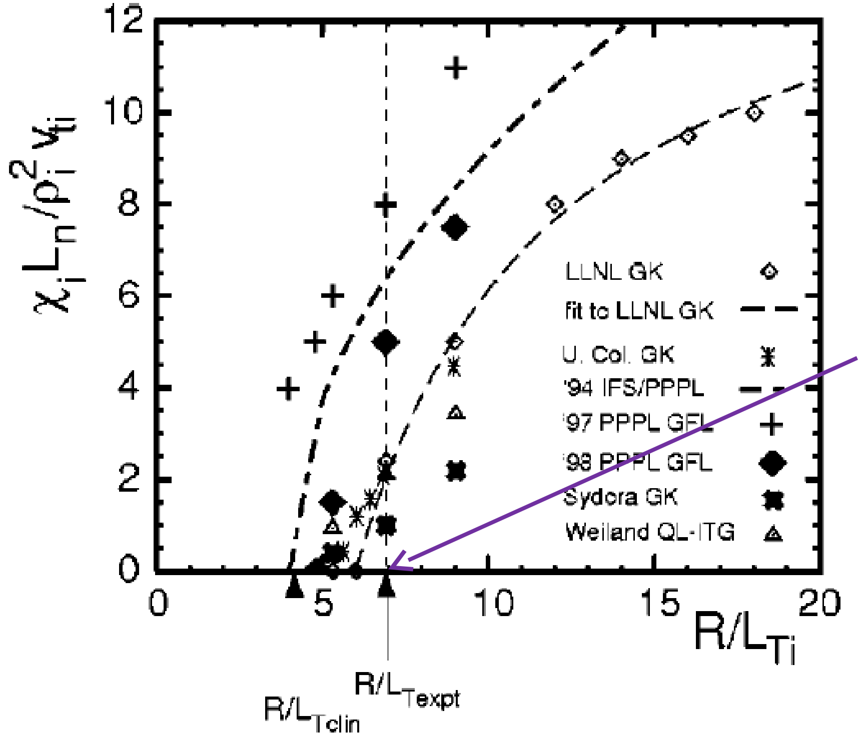


- Evolution of potential fluctuations in a plasma very similar to DIII-D 101381/101391. Simulation is centered at *r*/*a* = 0.6. Note the strong equilibrium sheared rotation, which leads to a strong reduction in transport. This landmark simulation from 2002 includes kinetic electrons at finite-beta, along with the equilibrium **E**x**B** variation.

Tokamak Transport

- **Anomalous Transport**

*A. M. Dimits et al, PHP **7** 969 (2000)*



Dimits shift due to nonlinear effect

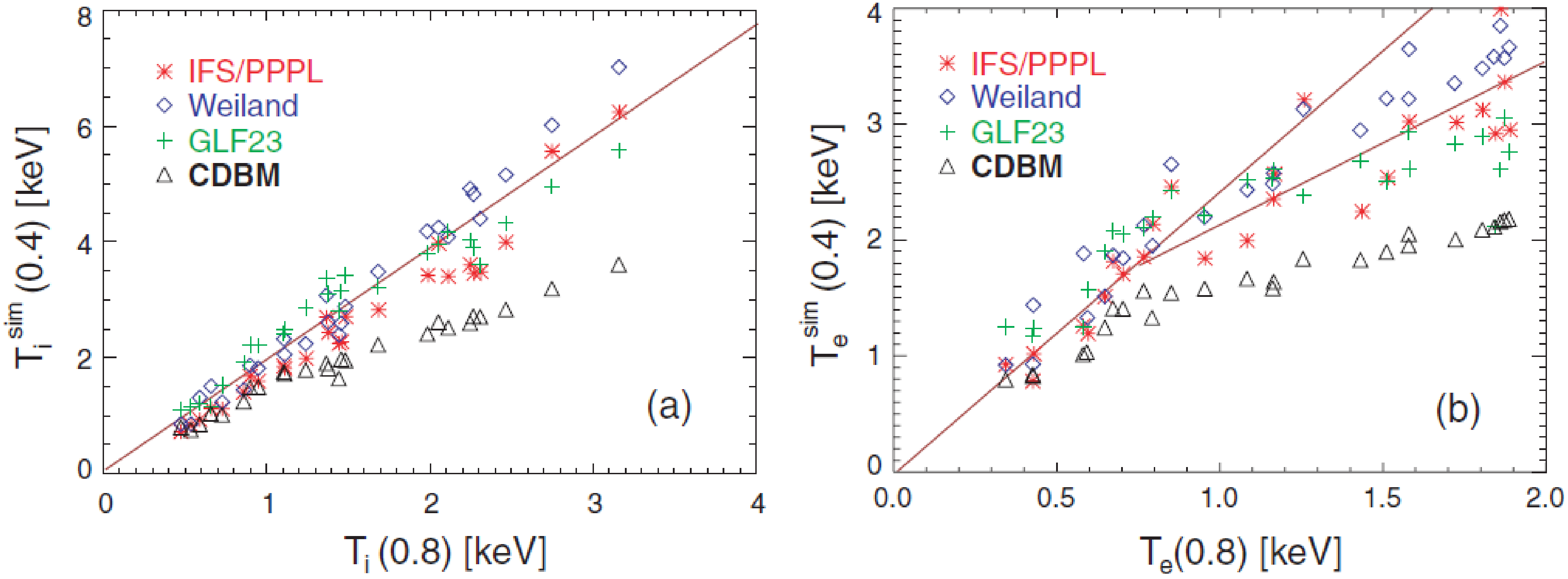
(turbulence stabilisation by zonal flows)

- χ_i vs R/L_T from the gyrofluid code using the 1994 "thesis closure",
  an improved 1998 gyrofluid closure, the 1994 IFS-PPPL model, the LLNL and U. Colorado flux-tube and UCLA (Sydora) global gyrokinetic codes, and the MMM model for the DIII-D base case.

Tokamak Transport

- **Anomalous Transport**

*G. Tardini et al, NF **42** 258 (2002)*



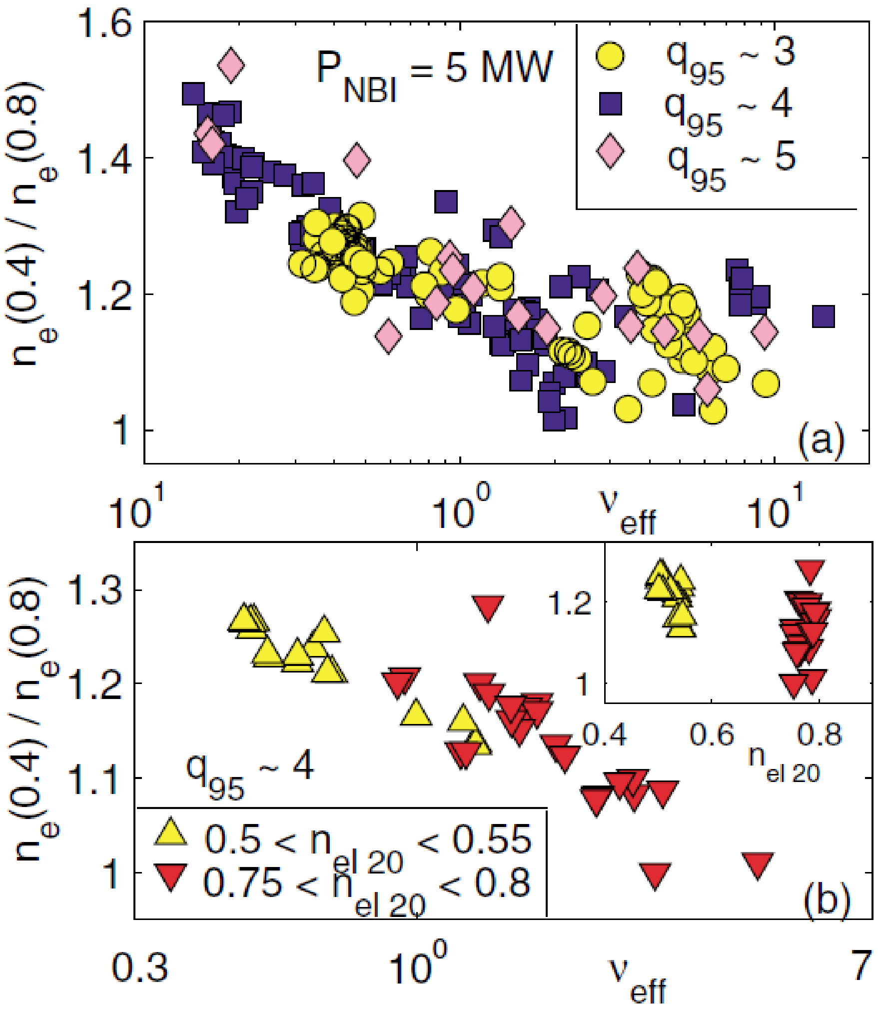
- Transport modelling e.g. CDBM, Weiland, GLF23, TGLF
- Simplified version is a critical gradient model

$$\gamma_{lin} = \underset{\substack{\downarrow \\ \text{Stiffness number}}}{\chi_s} \frac{c_s}{R}\left(\frac{- R\partial_r T}{T} - \kappa_c \right)$$

Tokamak Transport

- **Density Peaking**

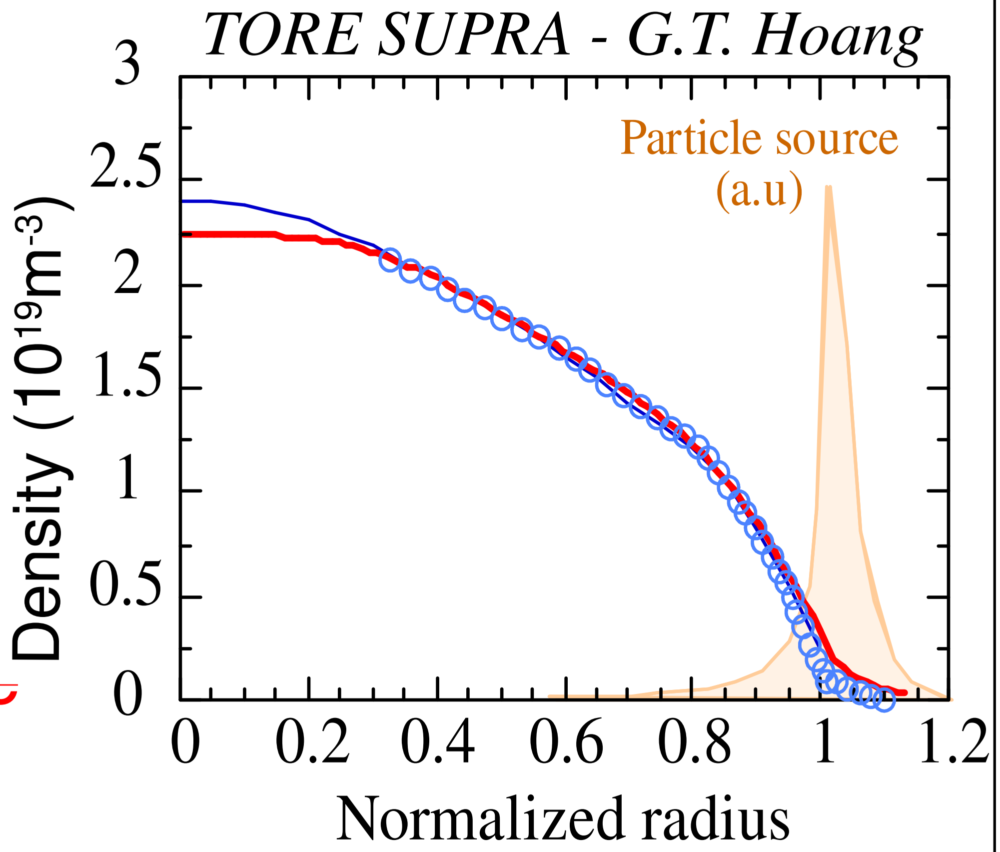
*C. Angioni et al, PRL **90** 205003 (2003)*



Diffusion and pinch velocity

- Particle flux

$$\Gamma_e = - D \frac{dn_e}{dr} + V n_e$$



- Diffusion is turbulent

$$D = D_{turb}$$

- pinch velocity= collisions + turbulence

•$V = V_{coll} + V_{turb}$

- In a reactor:

- ionisation source localised in the edge → Γ_e=0

- V_{coll}~V_{Ware}=0. Turbulent pinch V_{turb} → density peaking?

References

- *T. S. Hahm, “Turbulent Transport in Tokamaks”, Lecture at NFRI and SNU (2009)*
- *X. Garbet, “Physics of Transport in Tokamaks”, EPS (2004)*
- *Greg Hammett (PPPL), ”Status of Research on Fusion Energy and Plasma Turbulence”, University of Ottawa, Physics Dept. Seminar (Nov. 29, 2007)*
- *https://fusion.gat.com/theory/Gyromovies*

Density and safety factor profiles are correlated in L-mode

- Combined heating and current drive :

\- consistent with curva re pinch

\- no indication of therm diffusion:

e/ion-mode transition?

→ density and q profile are correlated in JET, DIIID, TCV, TS…

JET- H. Weisen, A.Zabolotsky