Fusion Reactor Technology 2
(459.761, 3 Credits)

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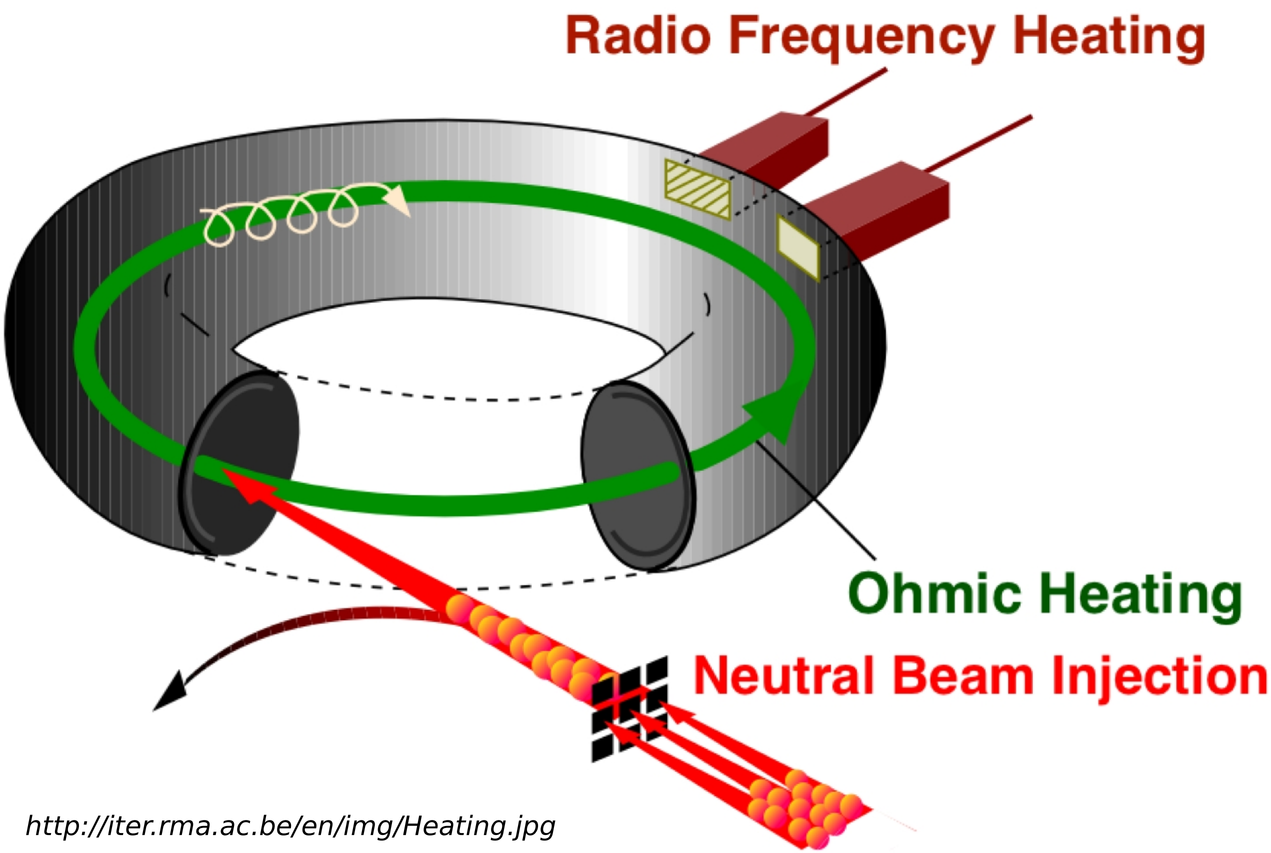
Heating and Current Drive



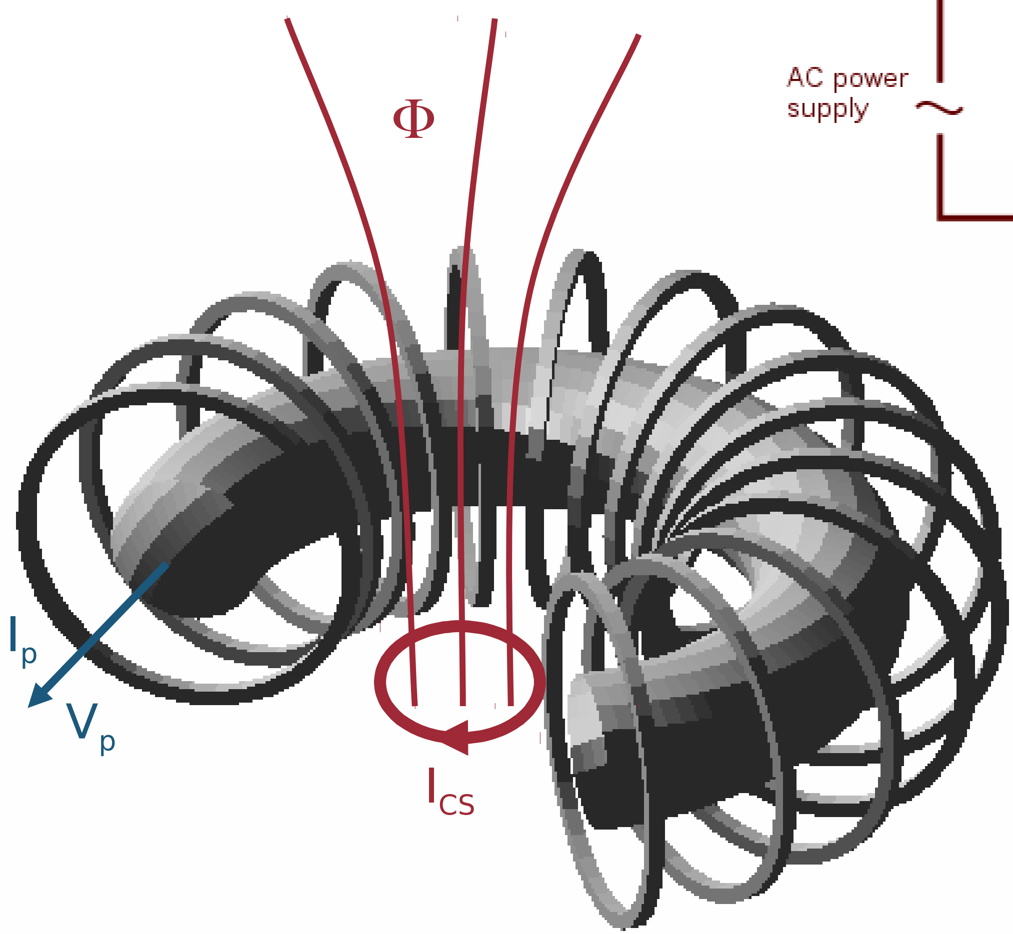
http://iter.rma.ac.be/en/img/Heating.jpg

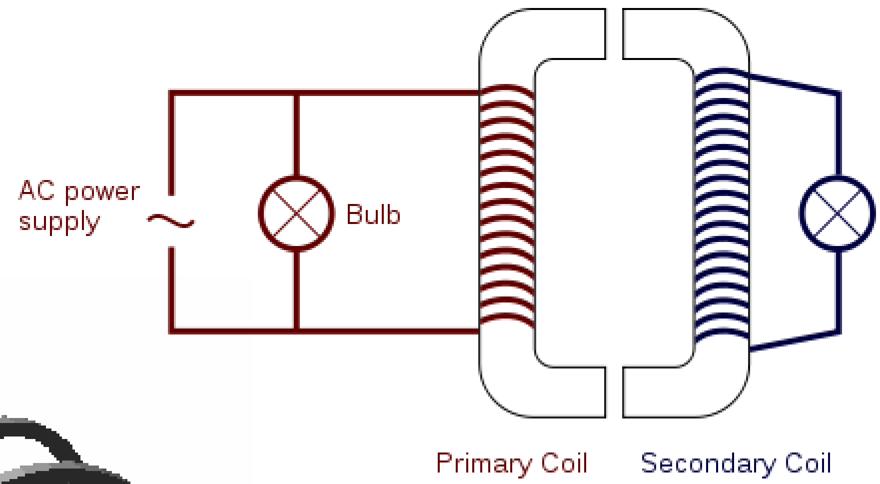
Ohmic Heating

Electric blanket

- Intrinsic primary heating in tokamaks due to Joulian dissipation generated by currents through resistive plasma: thermalisation of kinetic energies of energetic electrons (accelerated by applied **E**) via Coulomb collision with plasma ions
- Primary heating due to lower cost than other auxiliary heatings

Ohmic Heating





I_{CS} $\quad$ I_p

$$L_p \dot{I}_p + I_p R_p = V_p = -\dot{\phi}$$

http://en.wikibooks.org/w/index.php?title=File:Transformer.svg&filetimestamp=20070330220406

Ohmic Heating

$$L_p \dot{I}_p + I_p R_p = V_p = -\dot{\phi}$$

- Total change in magnetic flux needed to induce a final current

$$\Delta\phi_{ind} = \int_0^{t_f} \dot{\phi} dt = L_p I_p^f \approx \mu_0 R_0 \left[\ln\left(\frac{8R_0}{a\sqrt{k}}\right) + \frac{l_i}{2} - 2 \right] I_p^f$$

$$l_i \approx \ln\left[1.65 + 0.89(q_{95} - 1)\right]$$ internal inductance

- Additional magnetic flux needed to overcome resistive losses during start up

$$\Delta\phi_{res} = C_E \mu_0 R_0 I_p^f, \quad C_E \approx 0.4$$ Ejima coefficient

- Further change in magnetic flux needed to maintain I_p after start up

$$\Delta\phi_{burn} = \int_0^{t} I_p^f R_p dt'$$

- Technological limit to the maximum value of B_{OH}

$$\Delta\phi \approx \pi r_v^2 \Delta B_{OH}$$ Tokamak is inherently a pulsed device.

Ohmic Heating

- **Ohmic heating density**

$$P_\Omega = \mathbf{j}\cdot\mathbf{E} = \eta\left\langle j^2 \right\rangle \quad [W/m^2]$$

$$\eta_n = \frac{\eta_s}{\left(1 - \left(\frac{r}{R}\right)^{\frac{1}{2}}\right)^2}$$

: Neoclassical resistivity

η_s: Spitzer resistivity

$$Z_{eff} = \frac{\sum_s n_s Z_s^2}{n_e}, \quad n_e = \sum_s n_s Z_s$$

Z_s: charge number for the s-type ion

$$\eta \approx 8\times 10^{-8} Z_{eff} / T_e^{\frac{3}{2}} \quad (r = a/2,\ R/a = 3)$$

$$j(r) = j_0(1 - (r/a)^2)^\nu$$

$$\left\langle j^2 \right\rangle = j_0^2/(2\nu + 1)$$

$$B_\theta(r) = \frac{\mu_0 a^2 j_0}{2(\nu+1)r}\left[1 - \left(1 - \frac{r^2}{a^2}\right)^{\nu+1}\right]$$

Ampère's law

$$q_a = aB_\phi / RB_\theta, \quad q_a/q_0 = \nu + 1, \quad j_0 = 2B_\phi / Rq_0\mu_0$$

$$\left\langle j^2 \right\rangle = 2\left(\frac{B_\phi}{\mu_0 R}\right)^2 \frac{1}{q_0\left(q_a - \frac{1}{2}q_0\right)}$$

Ohmic Heating

$$P_\Omega = \eta\langle j^2\rangle = 1.0\times 10^5\left(\frac{Z_{eff}}{T^{3/2}}\right)\left[\frac{1}{q_o(q_a - q_o/2)}\right]\left(\frac{B_\phi}{R}\right)^2$$

- Z_{eff} limited by radiation losses
- High T required for enough fusion reactions

q_a limited by instabilities

$$q_a = \frac{aB_\phi}{RB_\theta} = \frac{aB_\phi}{R\frac{\mu_0 I_p}{2\pi a}} = \frac{aB_\phi}{R\frac{\mu_0\langle j\rangle\pi a^2}{2\pi a}} = \frac{2B_\phi}{\mu_0\langle j\rangle R} > 2$$

$$\langle j\rangle < \frac{B_\phi}{\mu_0 R}$$

Magnetic field limited by engineering
→ compact high-field tokamak

ASDEX Upgrade: P_Ω~1 MW

Ohmic Heating

$$P_\Omega = \eta \langle j^2 \rangle = 1.0 \times 10^5 \left(\frac{Z_{eff}}{T^{3/2}} \right) \left[\frac{1}{q_o(q_a - q_o/2)} \right] \left(\frac{B_\phi}{R} \right)^2$$

$$= 3nT/\tau_E = P_L$$

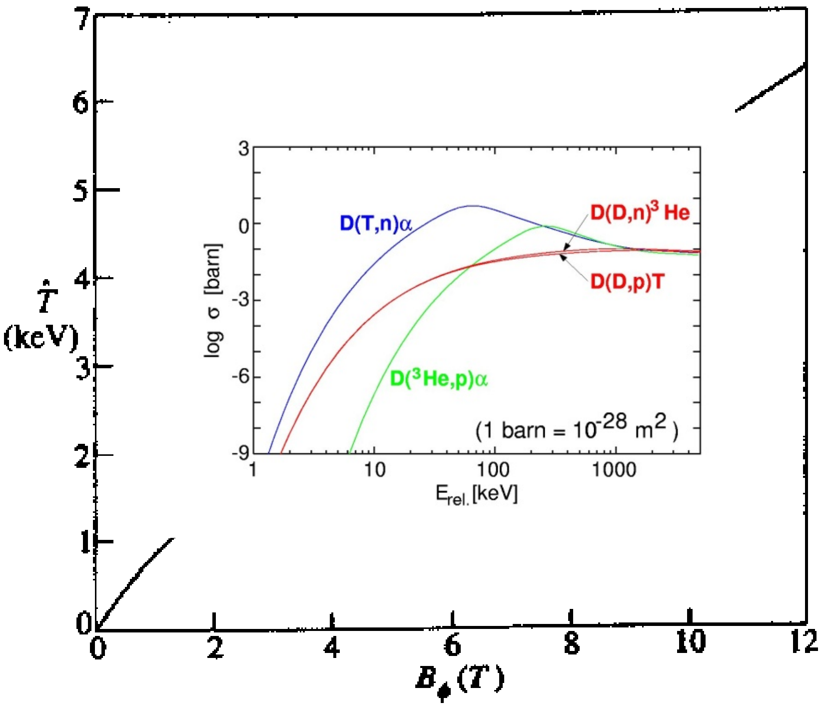
$$T = 2.7 \times 10^8 \left(\frac{Z_{eff}\tau_E}{nq_a q_0} \right)^{\frac{2}{5}} \left(\frac{B_\phi}{R} \right)^{\frac{4}{5}}$$

$$Z_{eff} = 1.5 \qquad q_a q_o = 1.5$$

$$\tau_E = (n/10^{20})a^2/2$$

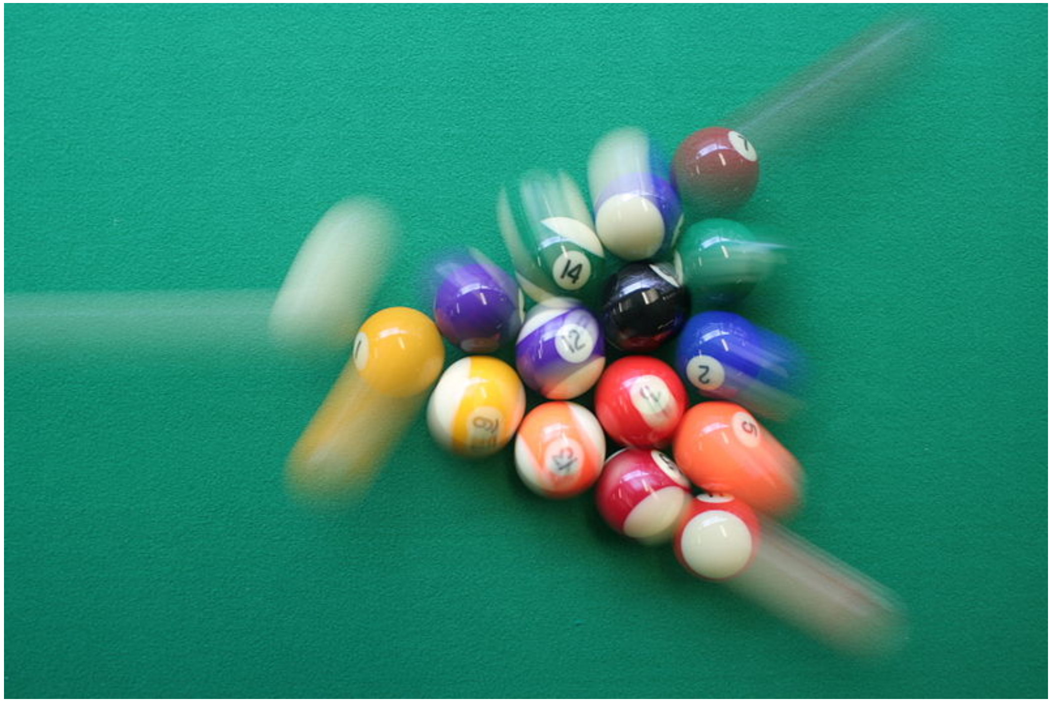
Alcator scaling

$$T = 0.87 B_\phi^{\frac{4}{5}}$$



It seems unlikely that tokamaks that would lead to practical reactors can be heated to thermonuclear temperatures by Ohmic heating!

Neutral Beam Injection



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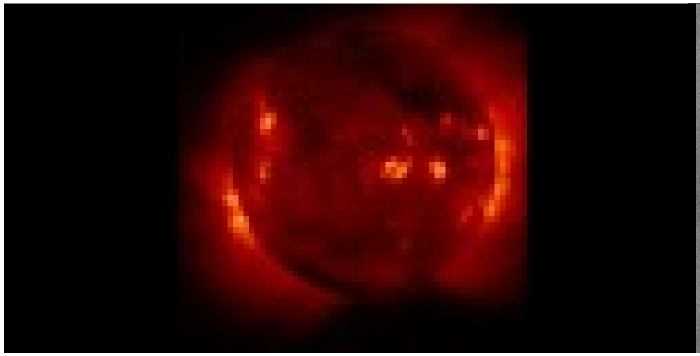
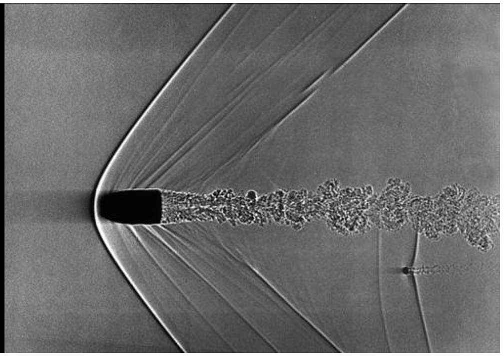
The Free Encyclopedia

Neutral Beam Injection



Plasma



Neutral beam

NBI

Andy Warhol

http://www.nasa.gov/mission_pages/galex/20070815/f.html

- Supplemental heating by energy transfer of neutral beam to the plasma through collisions
- Requirements

- Enough energy for deep penetration
- Enough power for desired heating
- Enough repetition rate and pulse length > τ_E
- Allowable impurity contamination

Neutral Beam Injection

Injection of a beam of neutral
fuel atoms (H, D, T)
at high energies (E_b > 50 keV)*

⇓

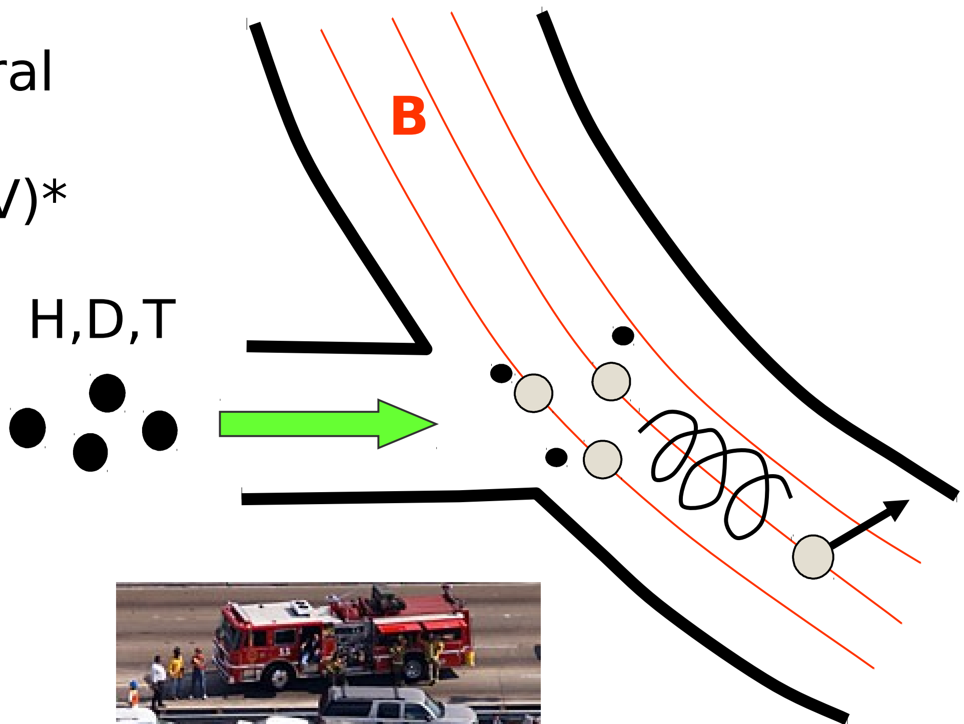
Ionisation in the plasma

⇓

Beam particles confined

⇓

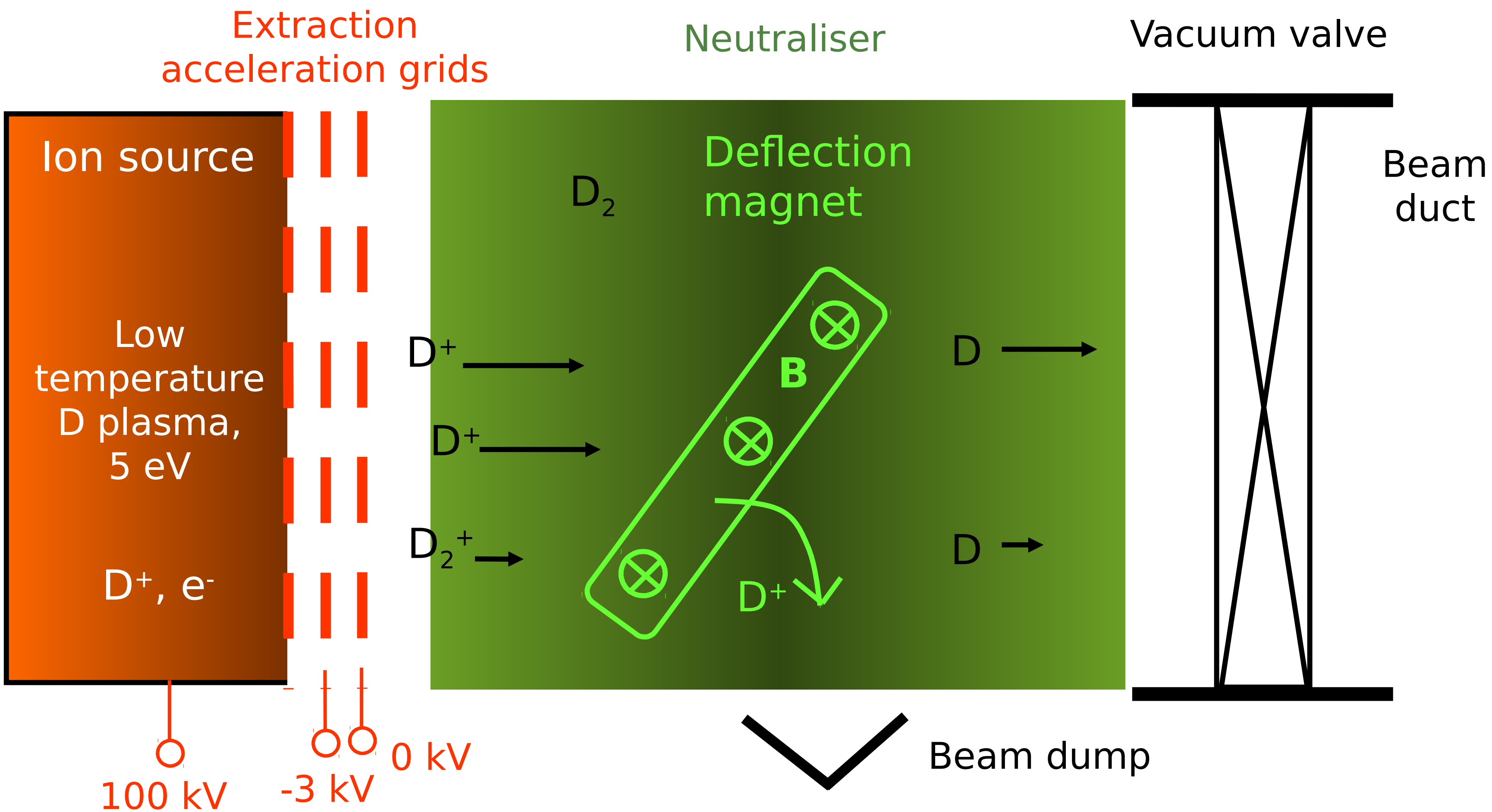
Collisional slowing down



* E_b = 120 keV and 1 MeV for KSTAR and ITER, respectively

Neutral Beam Injection

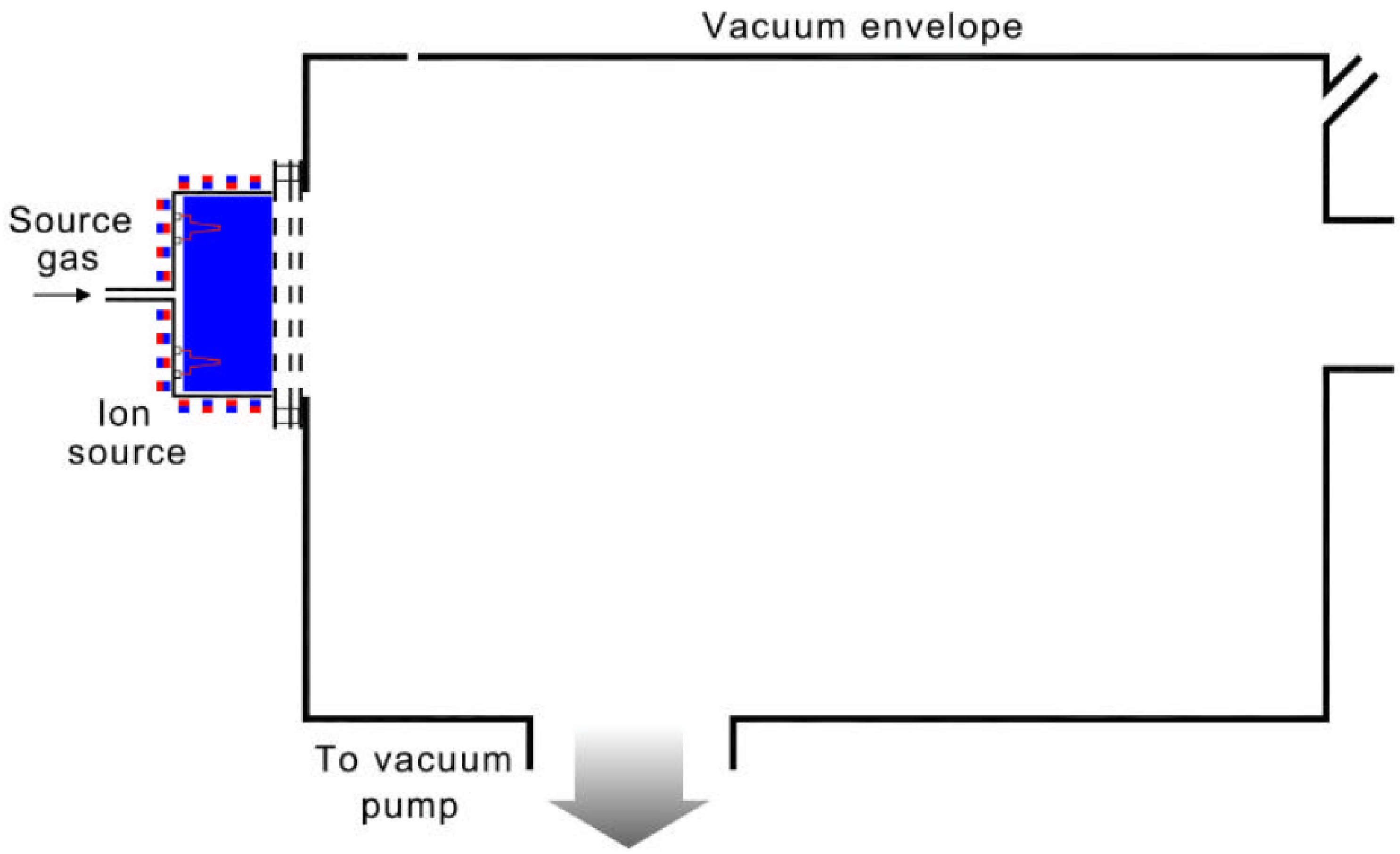
- **Generation of a Neutral Fuel Beam**



Ex) W7-AS: V=50 kV, I=25 A, power deposited in plasma: 0.4 MW

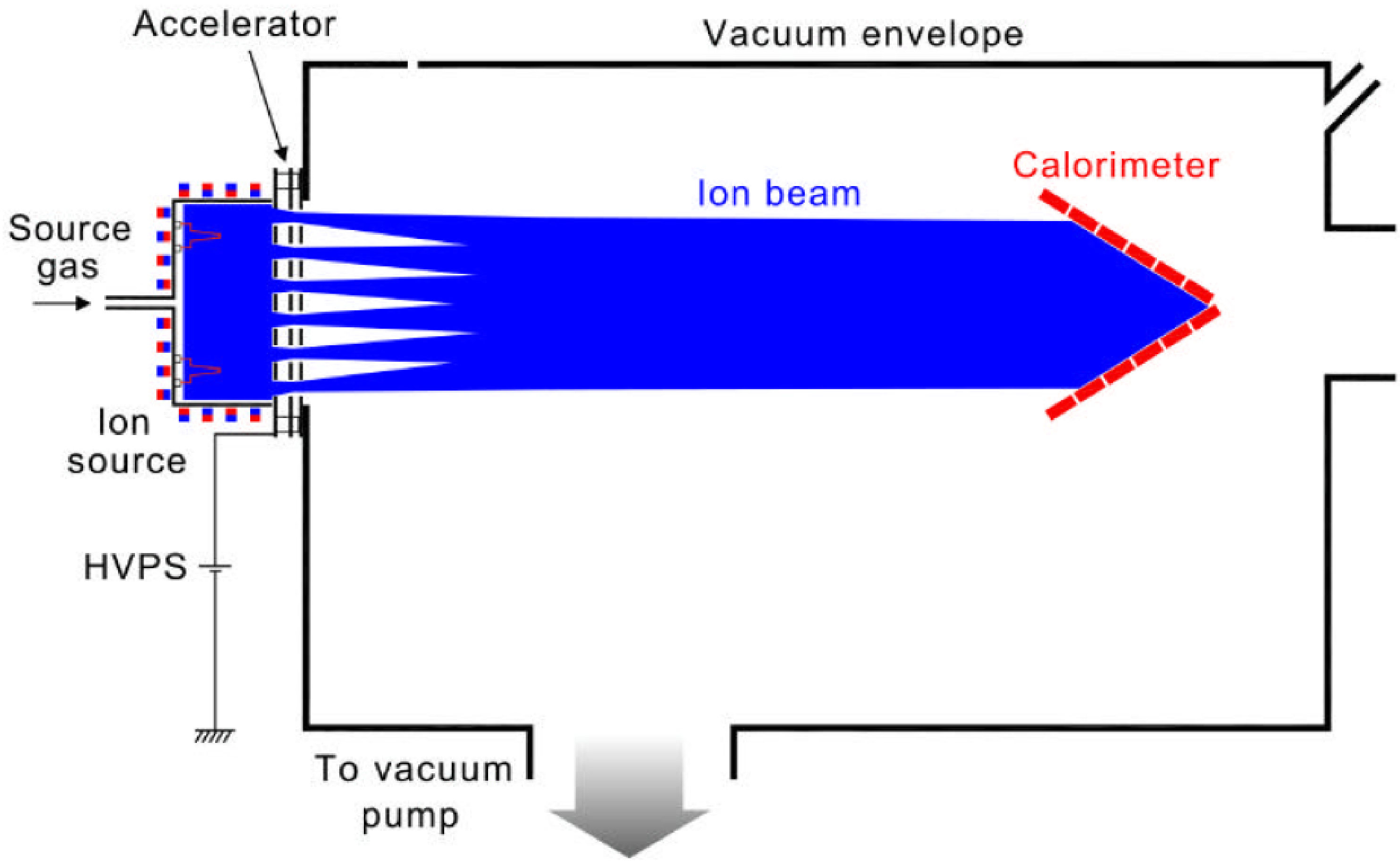
Neutral Beam Injection

- **Ion Source**



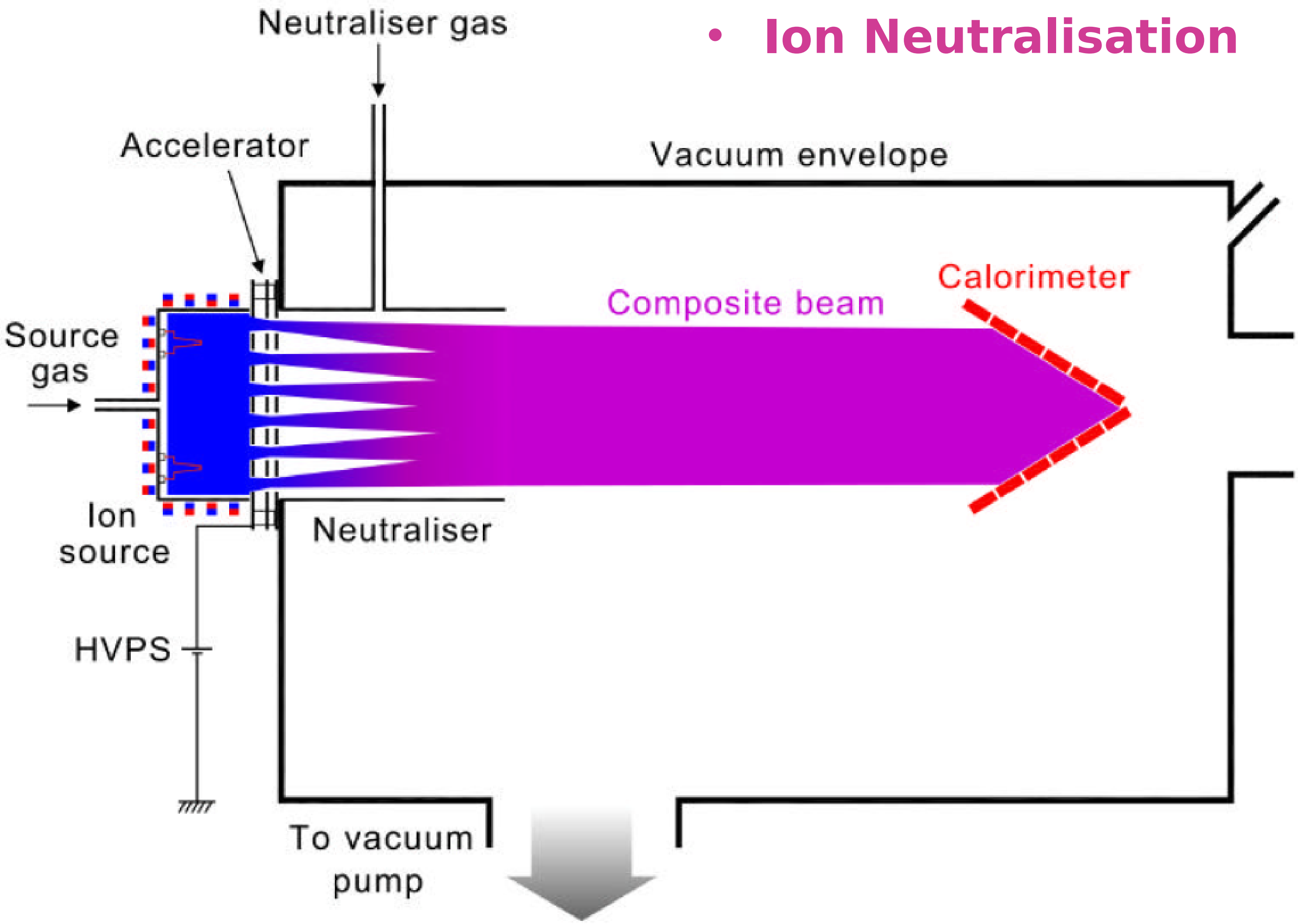
Neutral Beam Injection

- **Ion Acceleration**



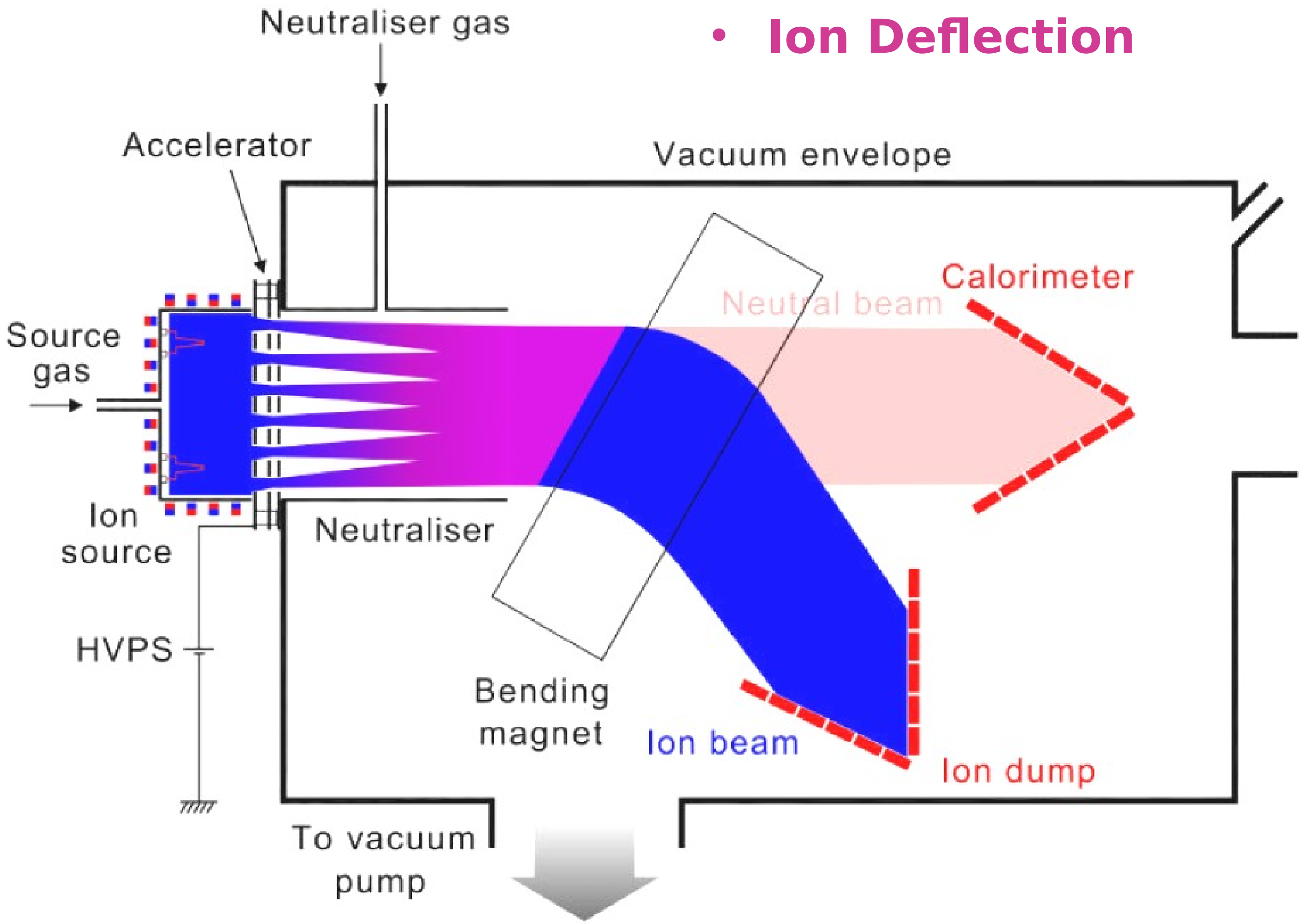
Neutral Beam Injection

- **Ion Neutralisation**



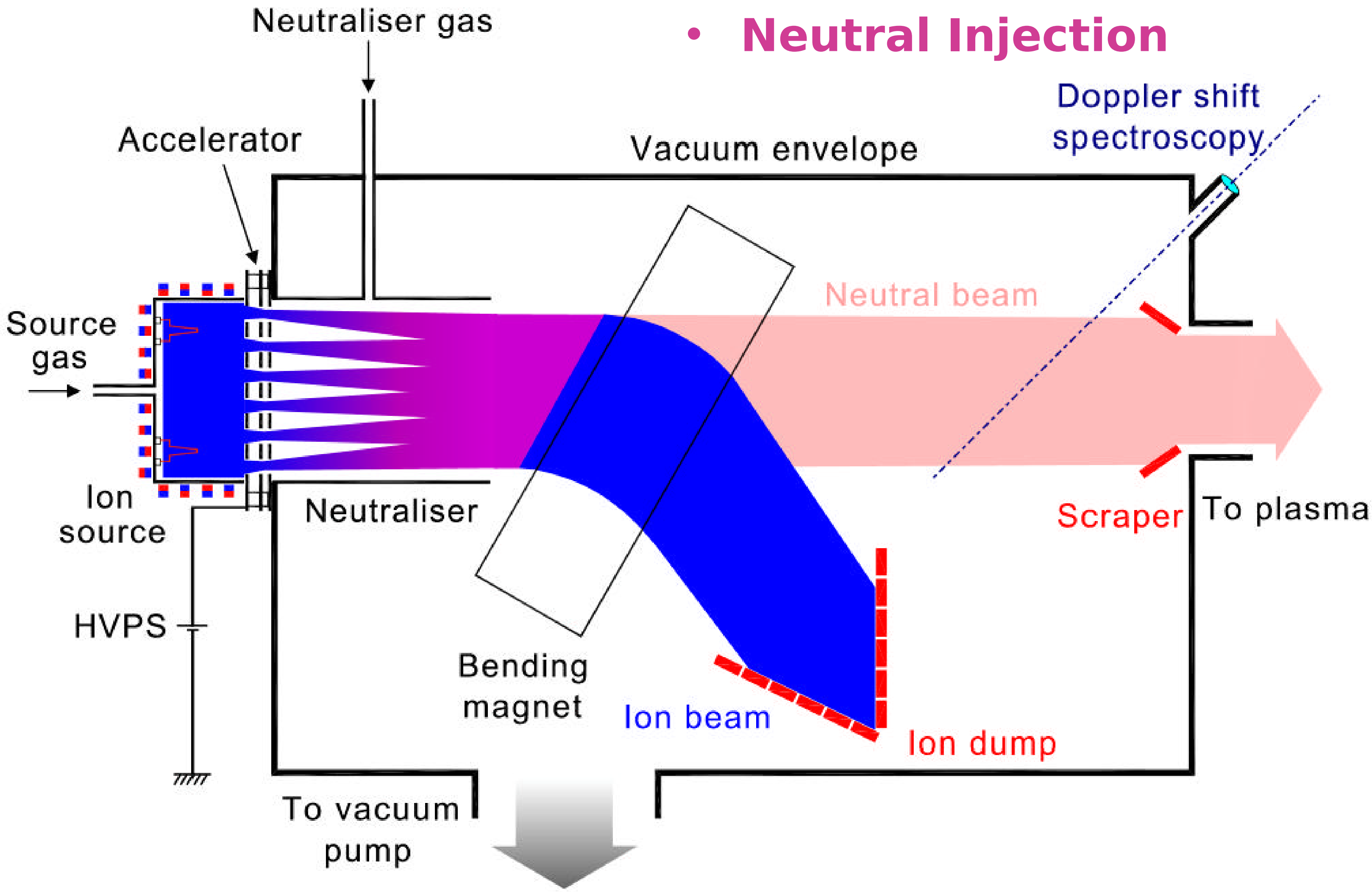
Neutral Beam Injection

- **Ion Deflection**



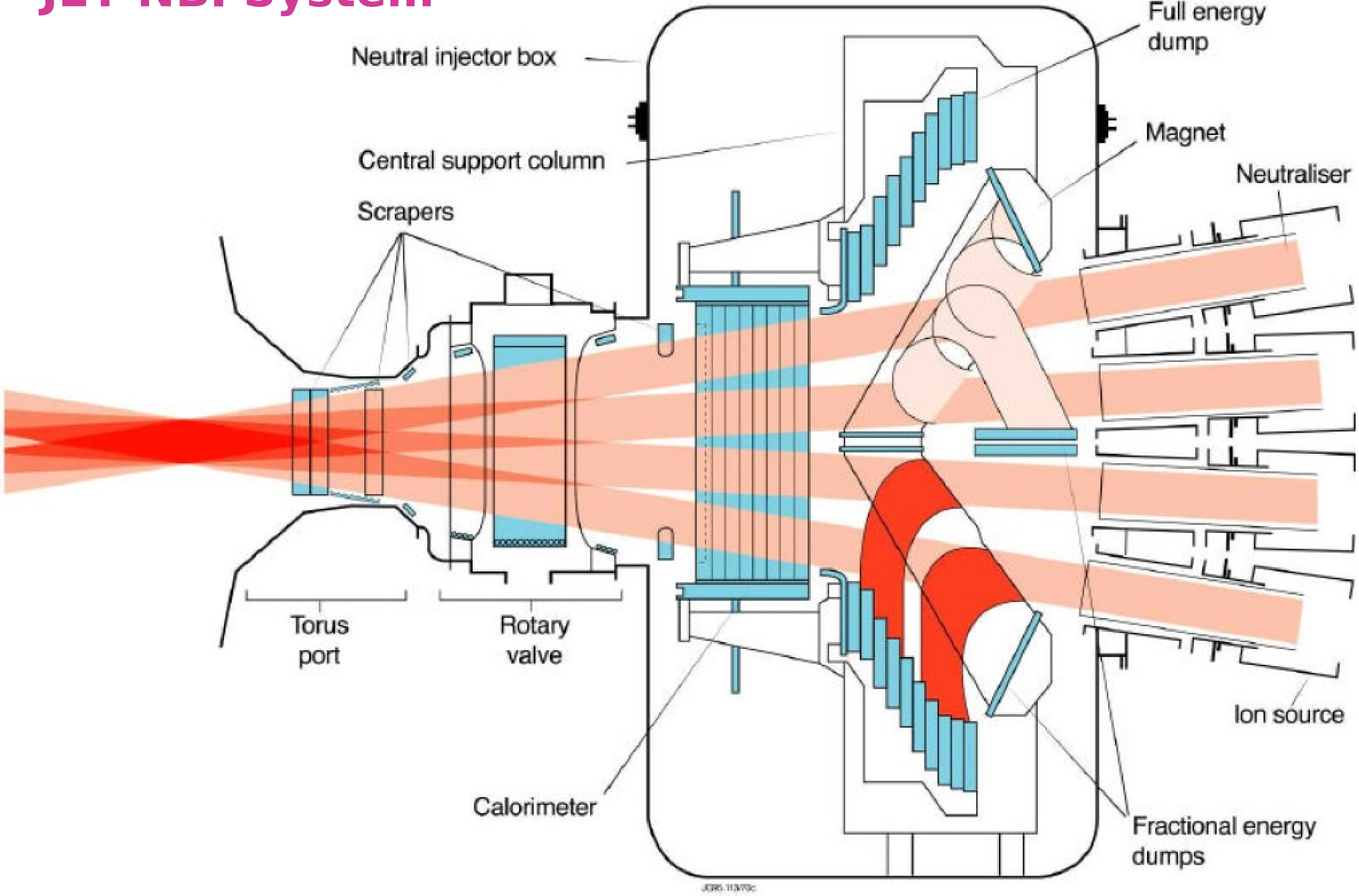
Neutral Beam Injection

- **Neutral Injection**



Neutral Beam Injection

- **JET NBI System**



Neutral Beam Injection

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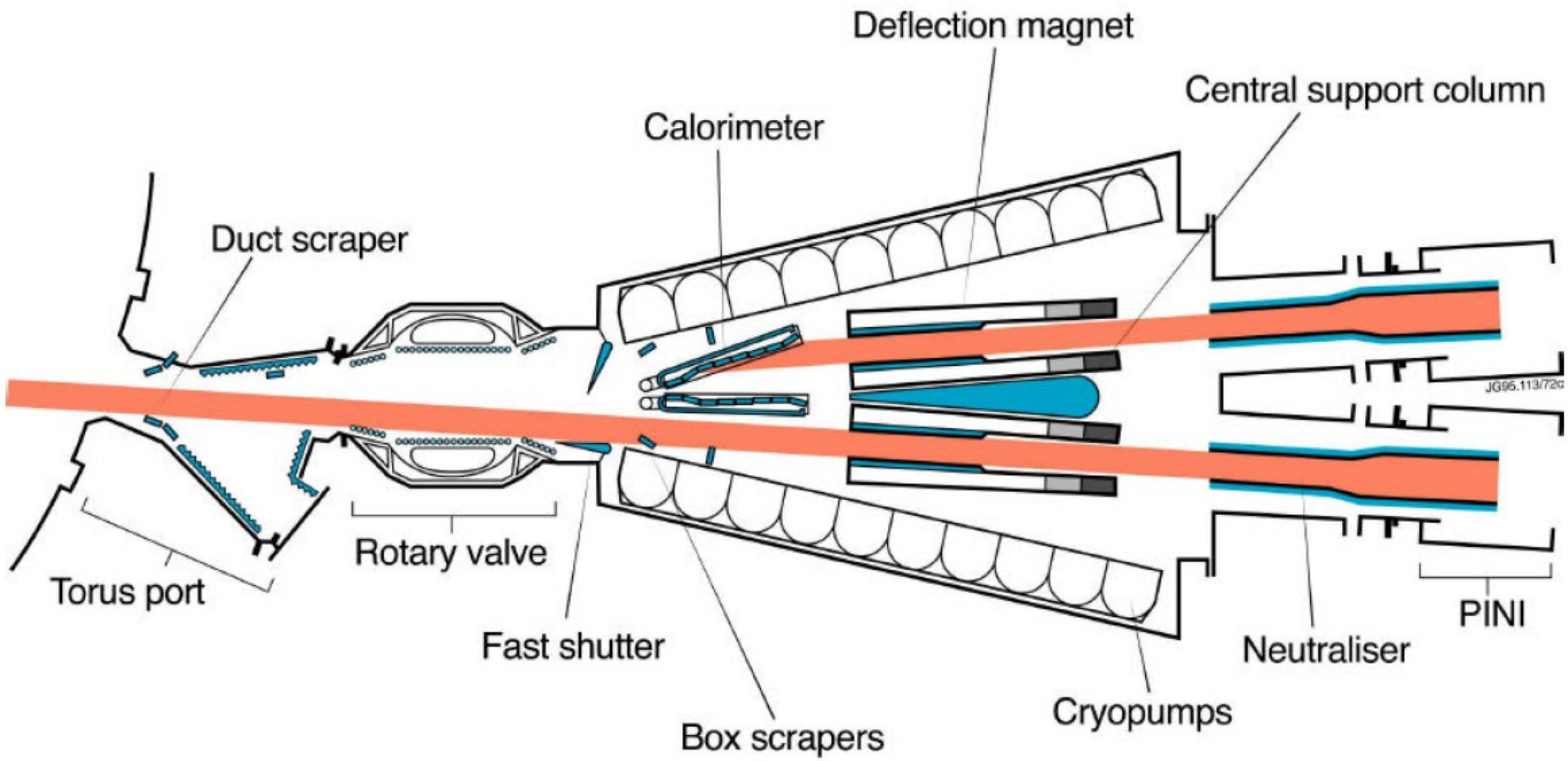

Neutral Beam Injection

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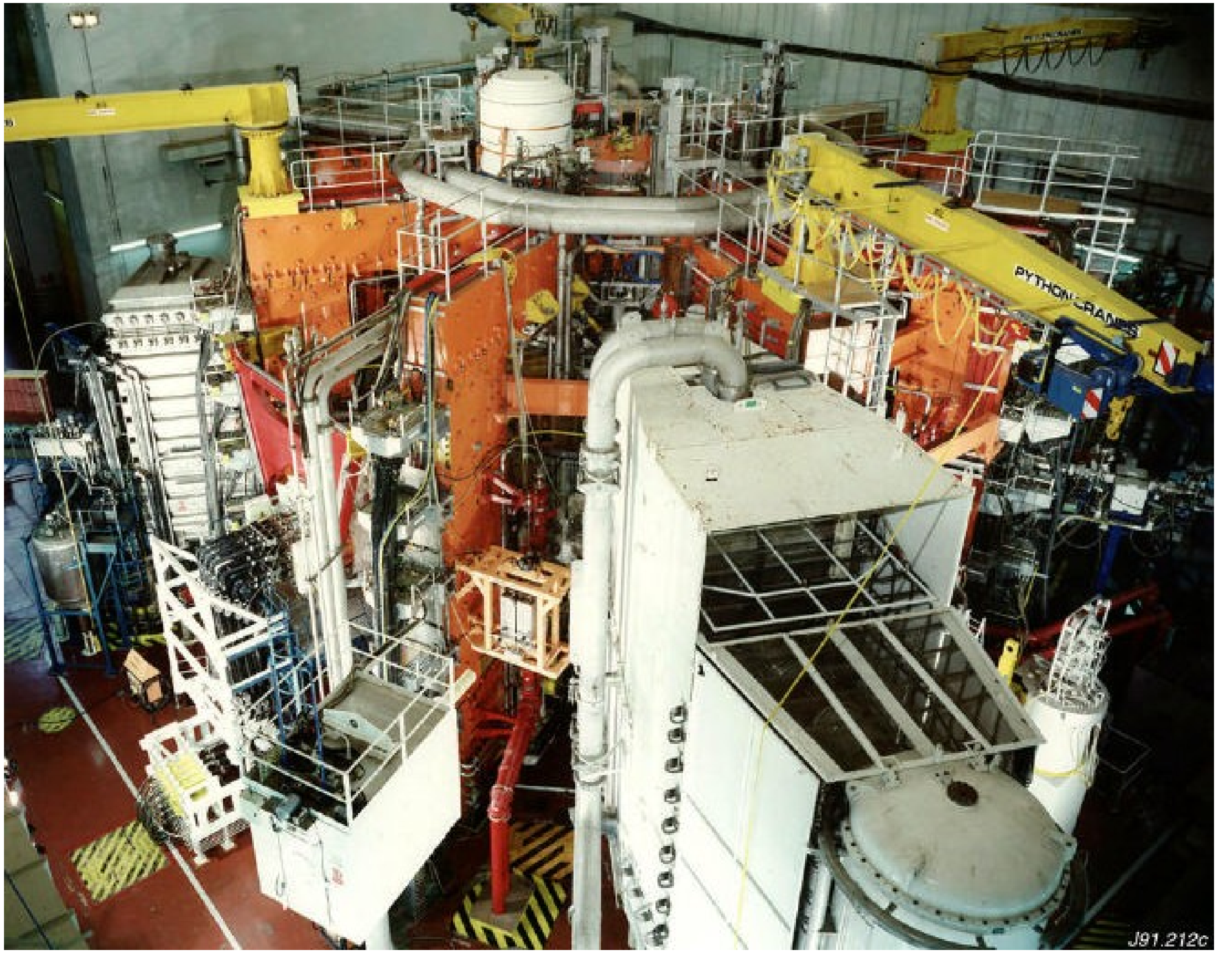
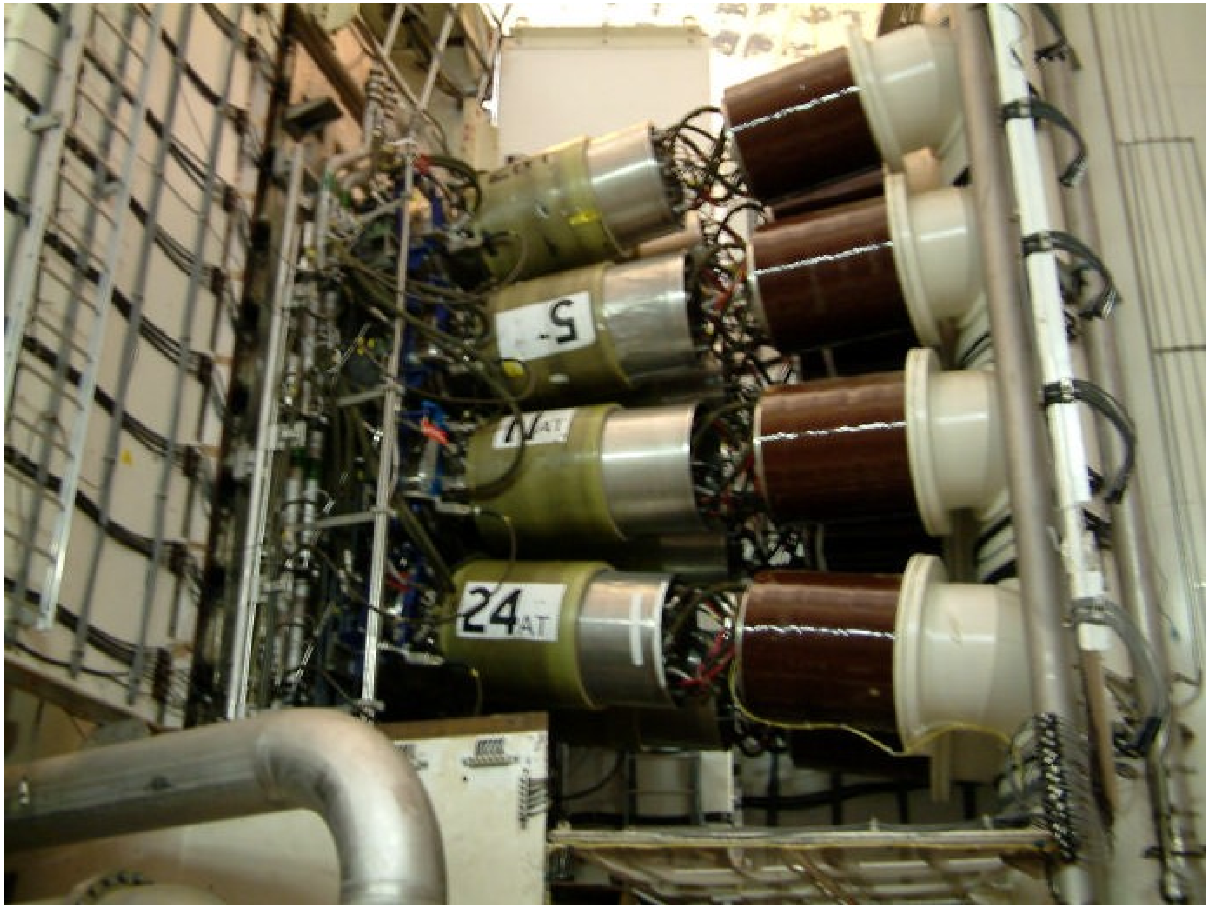


JET with machine and Octant 4 Neutral Injector Box

Neutral Beam Injection

- **JET NBI System**



Octant 4 Neutal Injector Box

Neutral Beam Injection

- **Ion source**

- Requirements
- Large-area uniform quiescent flux of high-current ions
- Large atomic ion fraction (D^+, D^-) > 75 % → adequate penetration
- Low ion temperature (<< 1 eV) to minimize irreducible divergence of extracted ion beams due to random thermal motion of ions

Neutral Beam Injection

- **Ion source**

- Ion generation
  - Positive ion generation by electric discharge

$$D_2 + e \rightarrow D^+ + D + e + e$$

$$D_2 + e \rightarrow D_2^+ + e + e$$

$$D_2 + D_2^+ \rightarrow D_3^+ + D$$

  - Negative ion generation

$D + e \rightarrow D^- + h\nu$ Radiative attachment in high density gas ($E_{binding}$ = 0.75 eV)

$D_2^* + e \rightarrow D^- + D$ Dissociative electron attachment by electric discharge (~eV)

$D^+ + cathode\ surface\ (+Cs) \rightarrow D^-$

$D^0 + cathode\ surface\ (+Cs) \rightarrow D^-$

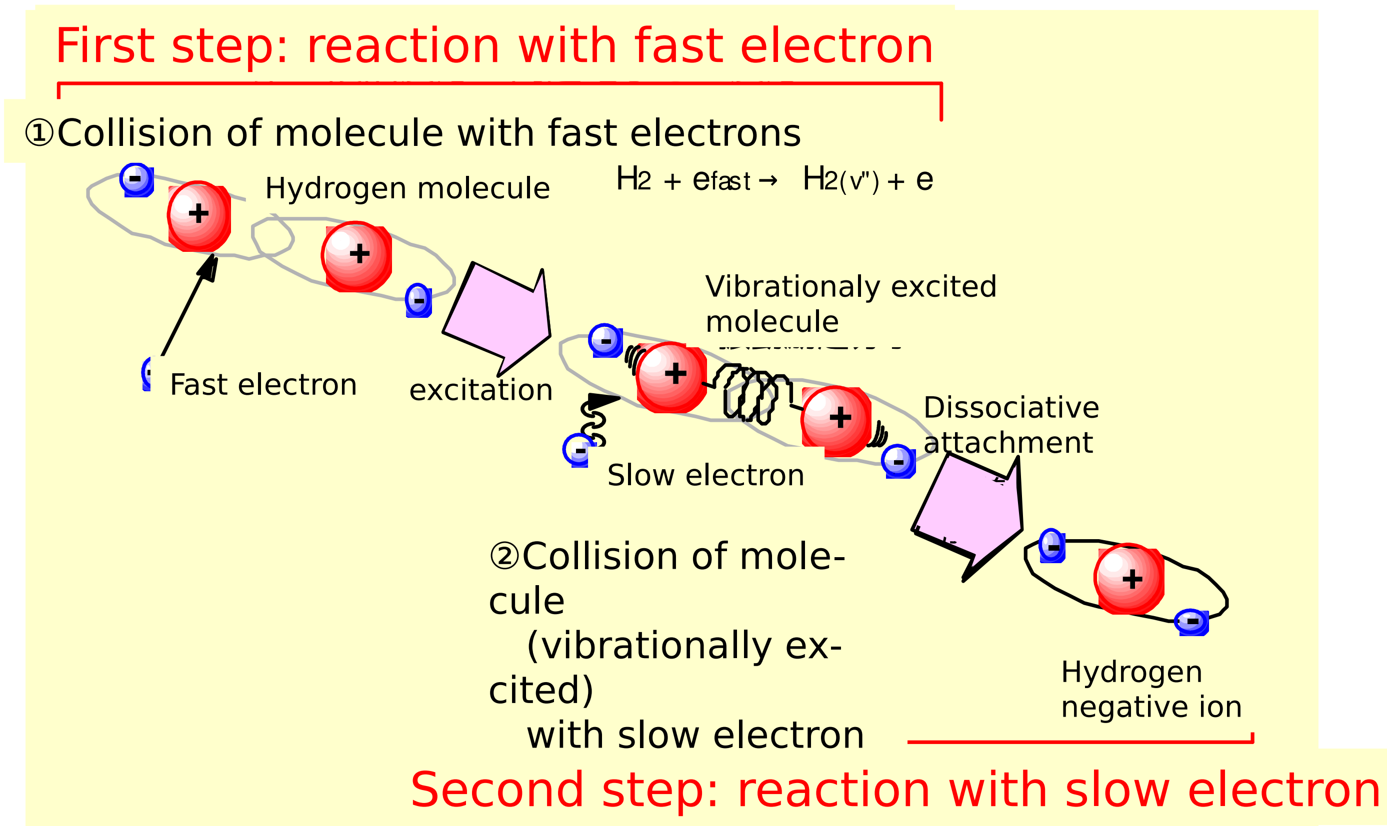
Surface production by electric discharge (~100 eV range)

$D^+ + M^0 \rightarrow D^0 + M^+$

$D^0 + M^0 \rightarrow D^- + M^+$

Electron attachment (Double electron capture)
M: alkali or alkali-earth metal vapor (Cs, Rb, Na, Sr, Mg)

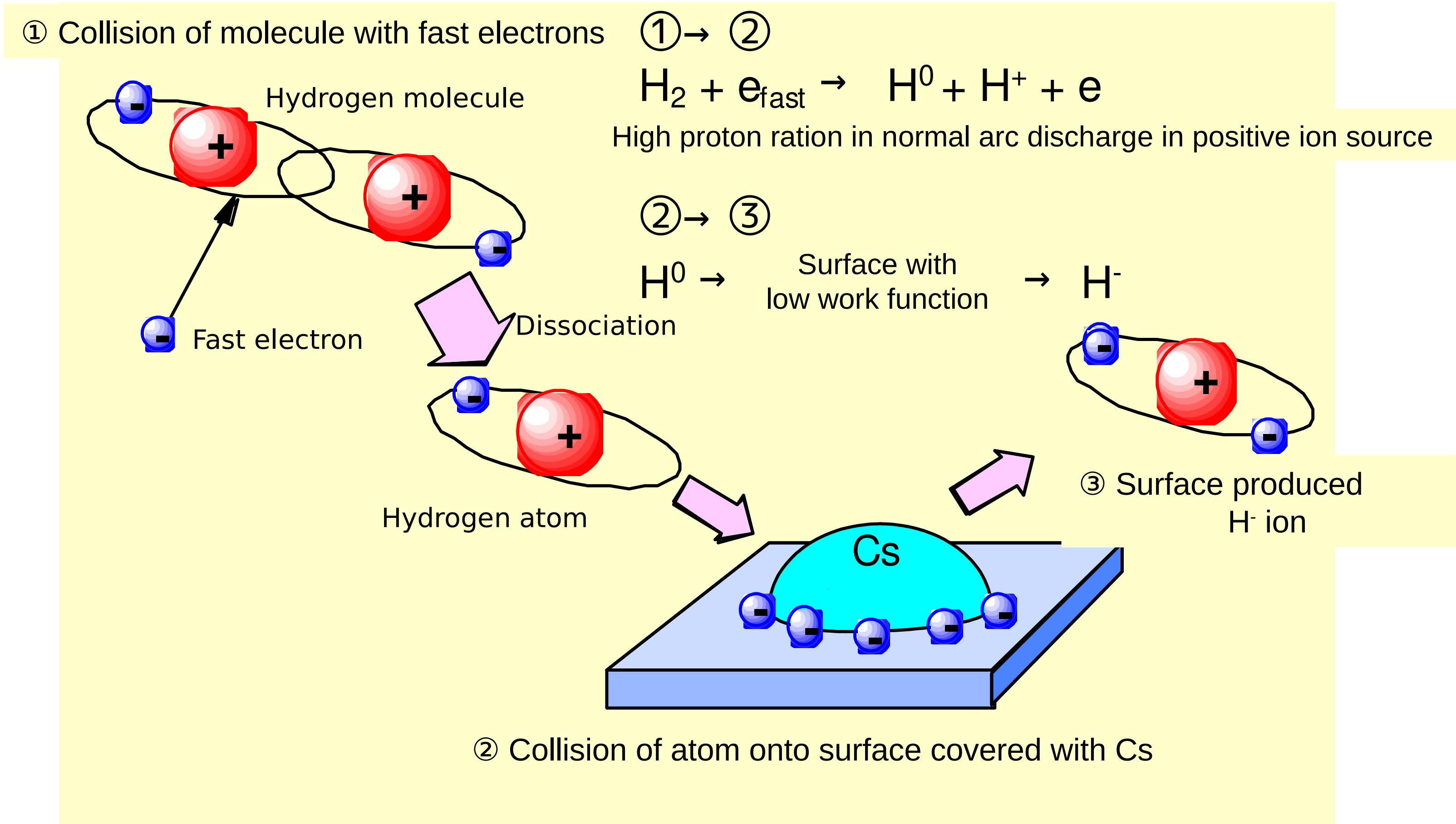
Volume production of negative ions (pure volume production)



Volume production process: two step reaction

- Negative ion from molecule,
- Suitable electron temperature for each reaction.

Surface production of negative ions (Cesium seeded source)

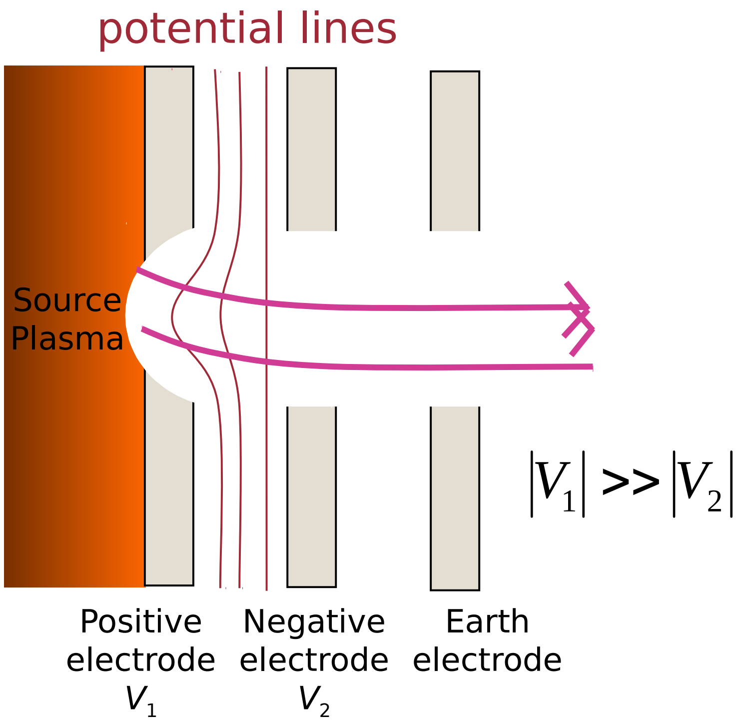


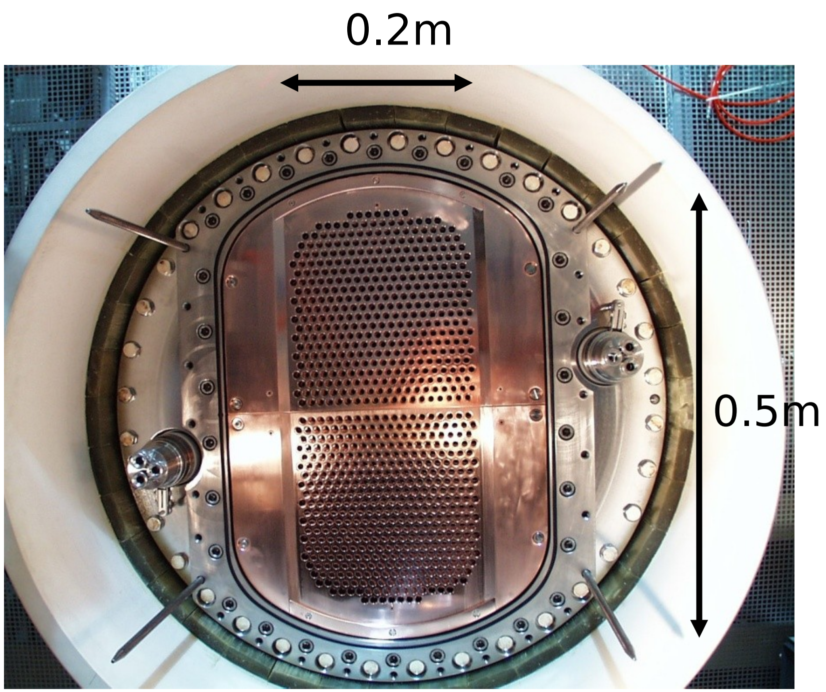
Surface process

Negative ion production from hydrogen atom

Neutral Beam Injection

- **Beam Forming System: Extraction and steering**
  - 3-lens system



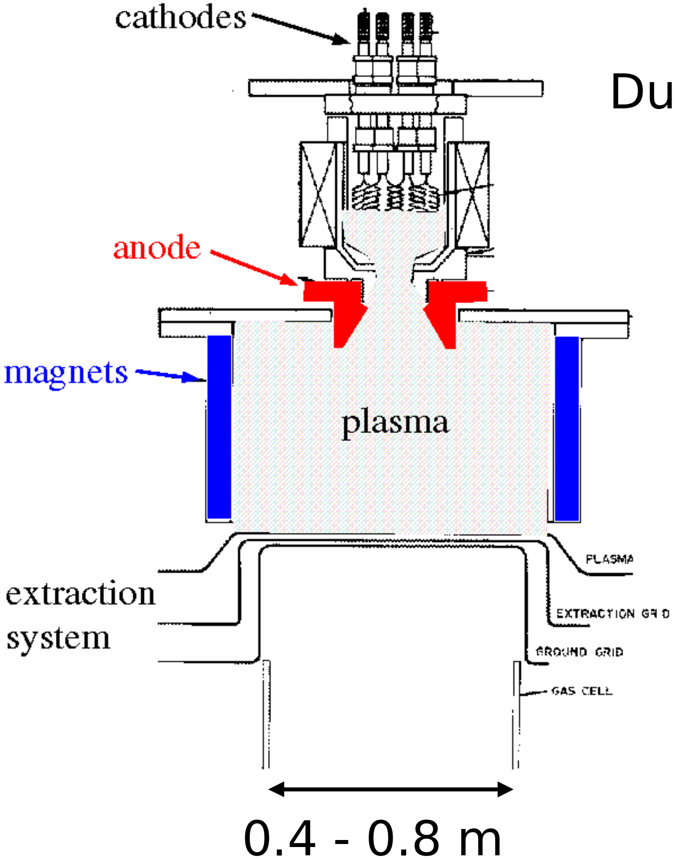


Grid system at ASDEX Upgrade

- Ion extraction + acceleration + minimum beam divergence (≤ 1°)

Neutral Beam Injection

- **Ion sources**



Duopigatron

RF Source

Cathodes: difficult to replace, finite life time

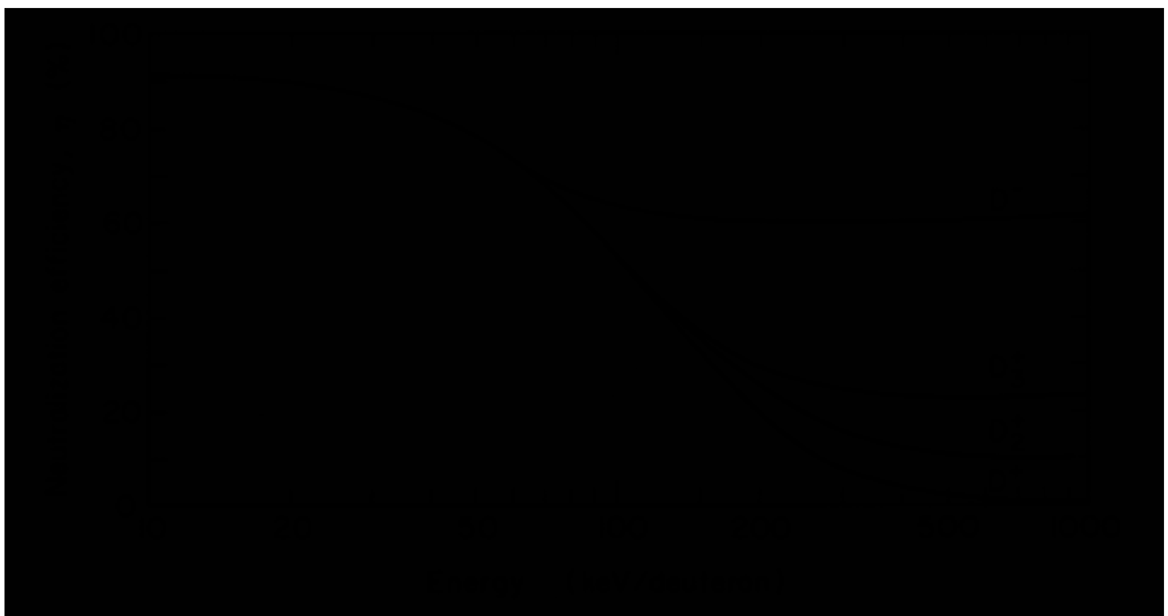
Neutral Beam Injection

- **Neutraliser**

  - Charge exchange: $\underset{fast}{D^{+}} + \underset{gas}{D_2} \rightarrow \underset{fast}{D} + \underset{slow}{D_2^{+}}$

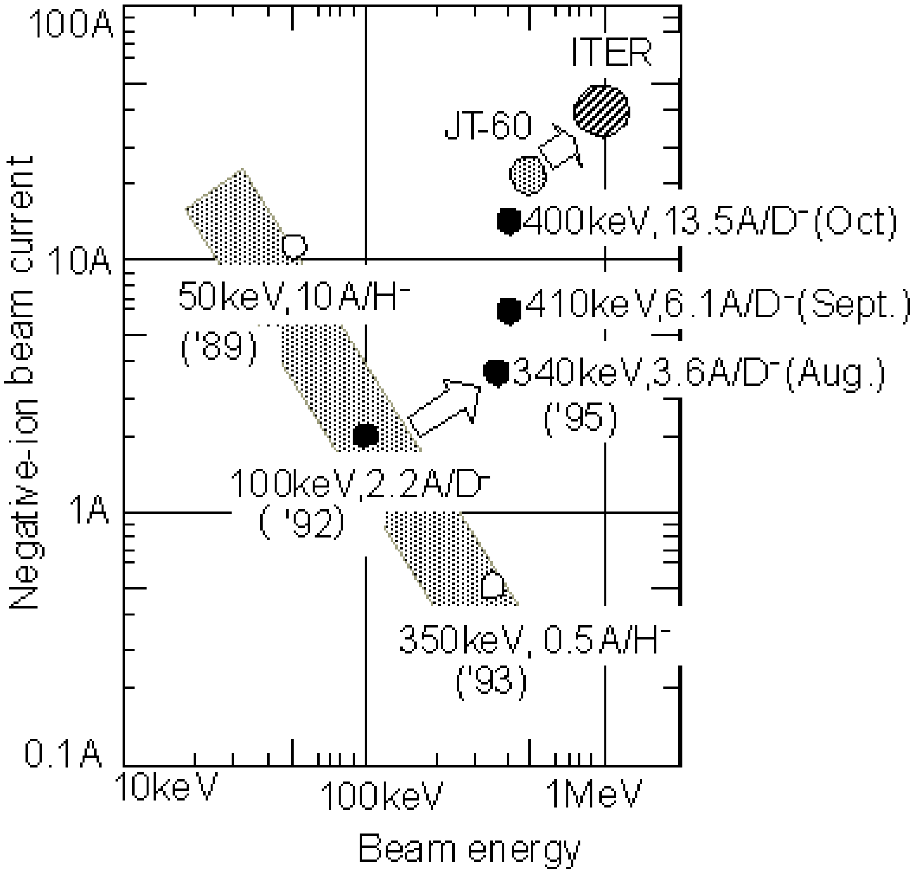
  - Re-ionisation: $\underset{fast}{D} + \underset{gas}{D_2} \rightarrow \underset{fast}{D^{+}} + \underset{gas}{D_2} + e^{-}$

  - Efficiency: (outgoing NB power)/(entering ion beam power)



Neutral Beam Injection

- Negative ion beam development in JT-60U



Neutral Beam Injection

- **Ion Beam Dump and Vacuum Pumps**
  - Beam dump
  - Deflect by analyzing magnet
  - Minimise reionisation losses
  - Prevent local power dump at undesirable place (~kW/m^2)
  - Possible application to direct energy conversion

  - Pumping
  - Minimise reioninsaton losses
  - Prevent cold neutral particles from flowing into reactor plasma
  - Liquid He cryopumps (~10^6 l/s for ~MW system)

Neutral Beam Injection

- **Energy Deposition in a Plasma**

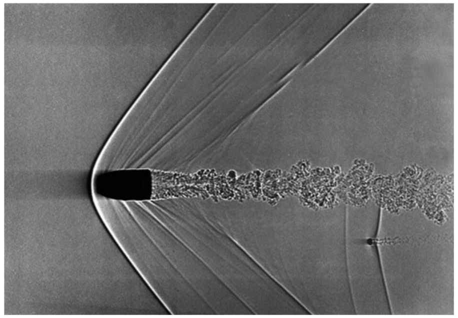
Charge exchange: $D_{fast} + D^{+} \rightarrow D^{+}_{fast} + D$

Ion collision: $D_{fast} + D^{+} \rightarrow D^{+}_{fast} + D^{+} + e$

Electron collision: $D_{fast} + e \rightarrow D^{+}_{fast} + e + e$

Attenuation of a beam of neutral particles in a plasma

n: density
σ: cross section



beam
energy

NBI

Andy Warhol

http://www.nasa.gov/mission_pages/galex/20070815/f.html

Neutral Beam Injection

- **Energy Deposition in a Plasma**

Charge exchange: $D_{fast} + D^+ \to D^+_{fast} + D$

Ion collision: $D_{fast} + D^+ \to D^+_{fast} + D^+ + e$

Electron collision: $D_{fast} + e \to D^+_{fast} + e + e$

Attenuation of a beam of neutral particles in a plasma

$$\frac{dN_b(x)}{dx} = - N_b(x) n(x) \sigma_{tot}$$

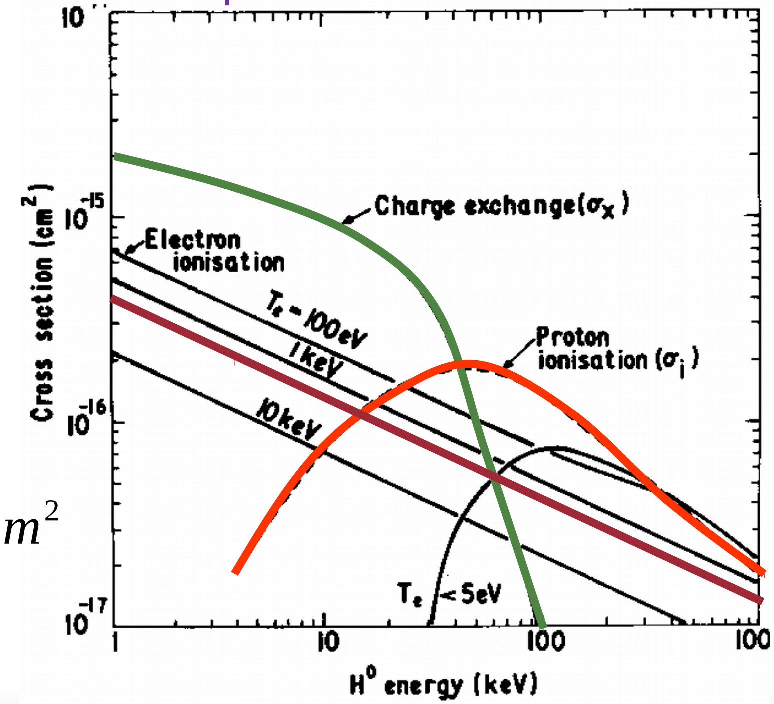
Ex. beam intensity: $I(x) = N_b(x) v_b$

$$= I_0 \cdot \exp(- x/\lambda)$$

$$\lambda = \frac{1}{n\sigma_{tot}} \approx 0.4m$$

Penetration (attenuation) length

$$n = 5 \cdot 10^{20} m^{-3} \quad E_{b0} = 70keV \quad \sigma_{tot} = 5 \cdot 10^{-20} m^2$$



In large reactor plasmas, beam cannot reach core!

Neutral Beam Injection

- **Energy Deposition in a Plasma**

Charge exchange: $D_{fast} + D^{+} \rightarrow D^{+}_{fast} + D$

Ion collision: $D_{fast} + D^{+} \rightarrow D^{+}_{fast} + D^{+} + e$

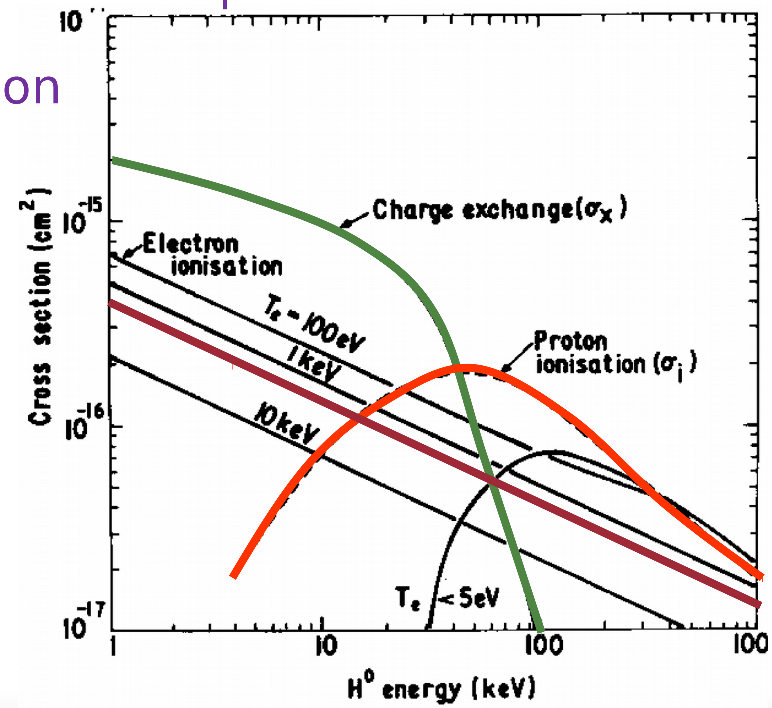
Electron collision: $D_{fast} + e \rightarrow D^{+}_{fast} + e + e$

Attenuation of a beam of neutral particles in a plasma

General criterion for adequate penetration

$$\lambda \equiv \frac{1}{n\sigma_{tot} Z_{eff}^{\gamma}} = \frac{5.5\times 10^{17} E_b(keV)}{A(amu)n(m^{-3})Z_{eff}^{\gamma}} \geq a/4$$

$$E_b \geq 4.5\times 10^{-19} AnaZ_{eff}^{\gamma}$$



Neutral Beam Injection

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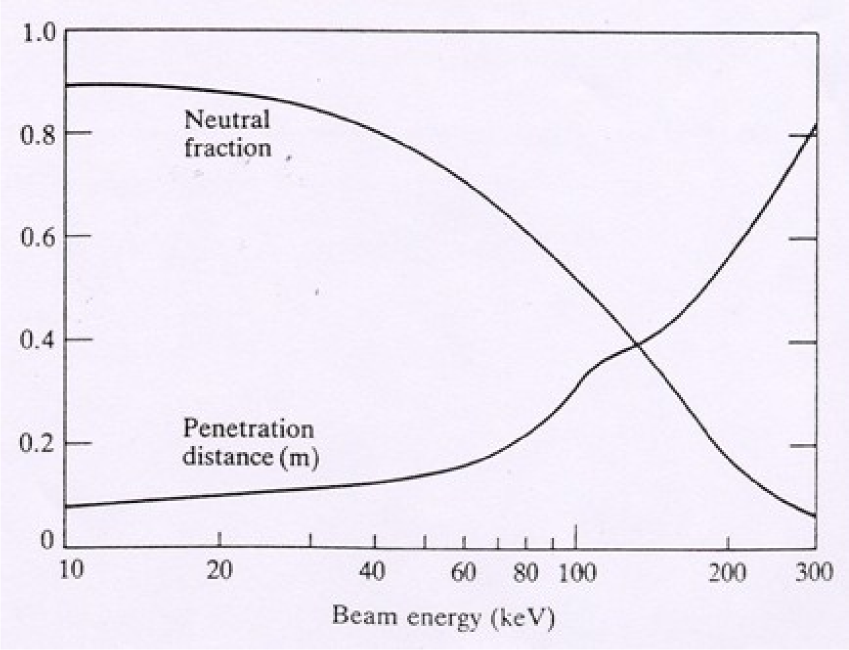
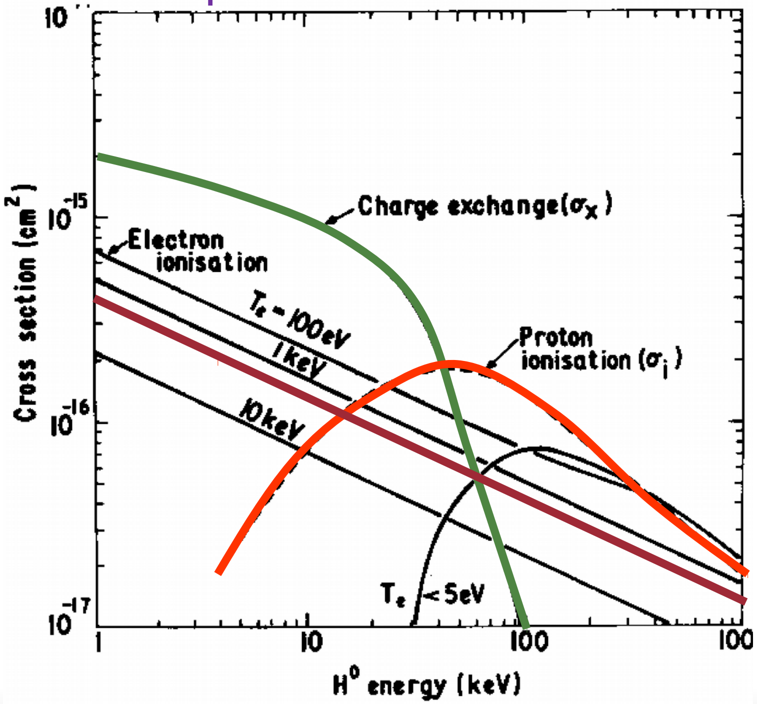
Attenuation of a beam of neutral particles in a plasma





Neutral Beam Injection

- **Slowing down**

$$-\frac{d\xi_b}{dt} = P = P_e + P_i \qquad \xi_b = \frac{1}{2} m_b v_b^2$$

$$= \frac{2^{\frac{1}{2}} n_e Z_b^2 e^4 m_e^{\frac{1}{2}} \ln \Lambda}{6\pi^{\frac{3}{2}} \varepsilon_0^2 A_b} \left(\frac{\xi_b}{T_e^{\frac{3}{2}}} + \frac{C}{\xi_b^{\frac{1}{2}}} \right), \quad C = 3\pi^{\frac{1}{2}} Z^2 A_b^{\frac{3}{2}} / 4 m_e^{\frac{1}{2}} m_i \approx 81$$

$$P = 1.71 \times 10^{-18} \frac{n_e \xi_b}{A_b \hat{T}_e^{3/2}} \left(1 + \left(\frac{\xi_c}{\xi_b} \right)^{3/2} \right) \quad [\mathrm{keVs}^{-1}]$$

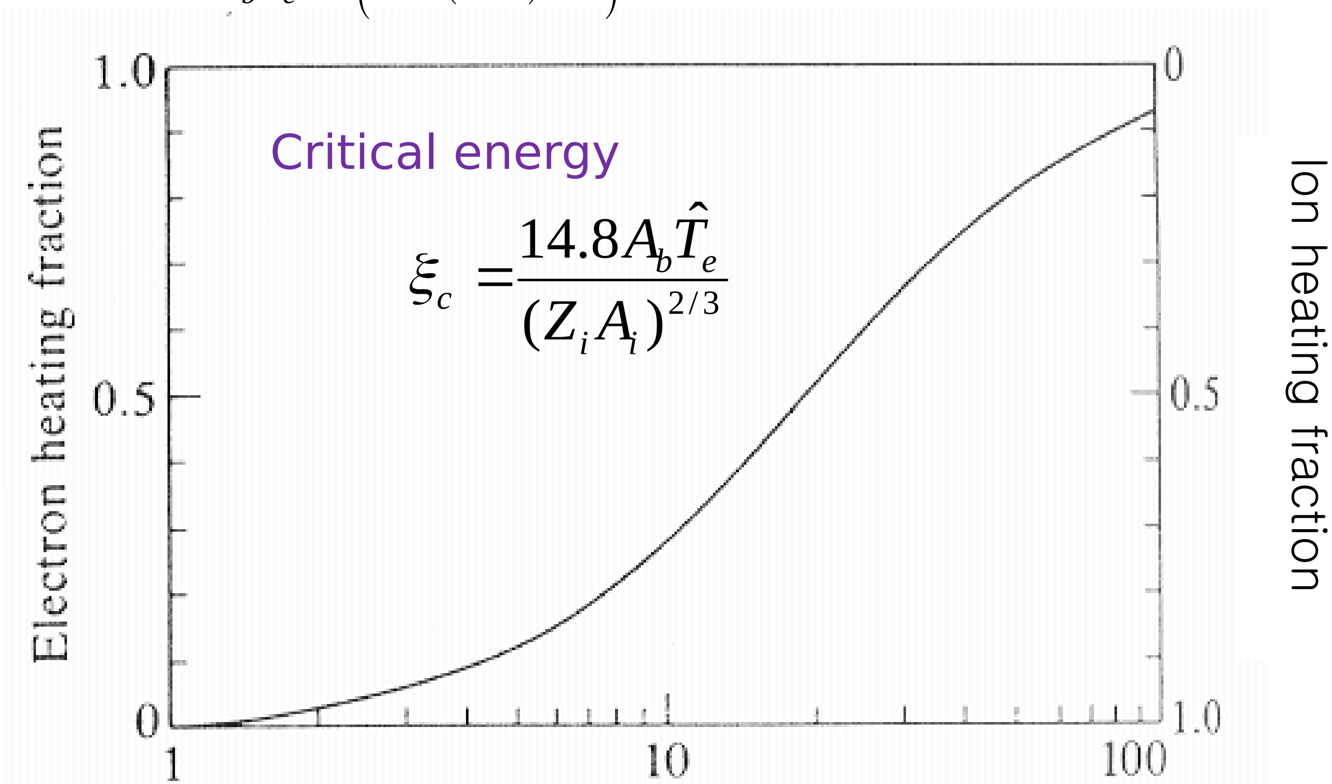
- Critical energy: The electron and ion heating rates are equal

$$\xi_c = \frac{14.8 A_b \hat{T}_e}{(Z_i A_i)^{2/3}}$$

Neutral Beam Injection

- **Slowing down**

$$P = 1.71 \times 10^{-18} \frac{n_e \xi_b}{A_b \hat{T}_e^{3/2}} \left(1 + \left(\frac{\xi_c}{\xi_b} \right)^{3/2} \right) \quad [\mathrm{keVs}^{-1}]$$



$$\xi_c = \frac{14.8 A_b \hat{T}_e}{(Z_i A_i)^{2/3}}$$

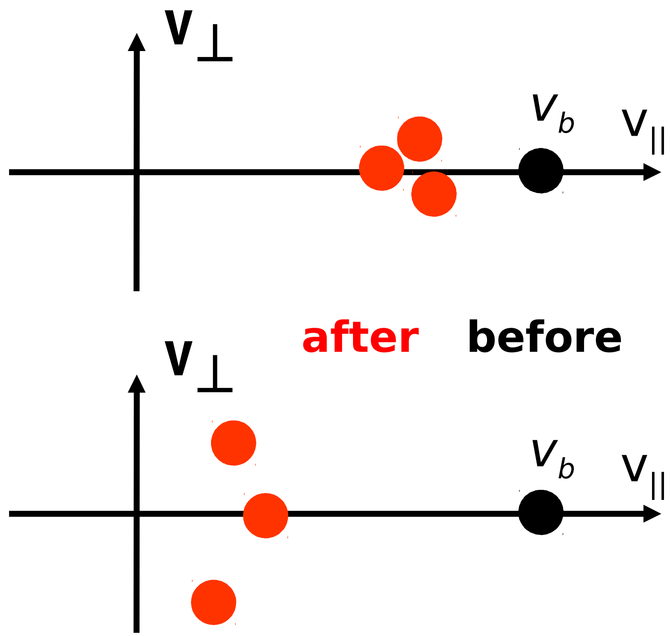
$\xi_b / \hat{T}_e$

Neutral Beam Injection

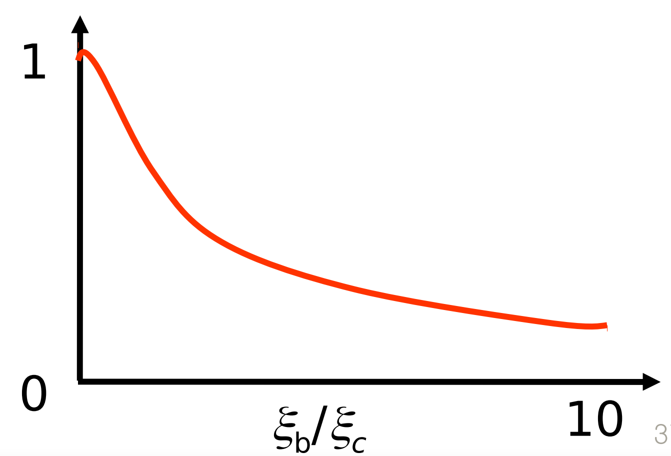
- **Slowing down**

1. $\xi_b > \xi_c$: Slowing down on electrons
   no scatter

2. $\xi_b < \xi_c$: Slowing down on ions
   scattering of beams

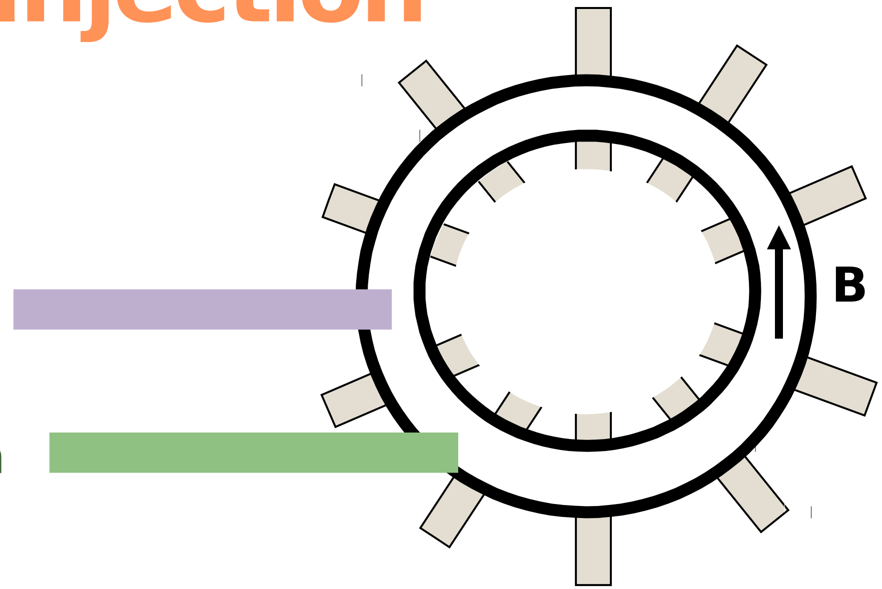


Fraction of initial beam energy going to ions



Neutral Beam Injection

- **Injection Angle**

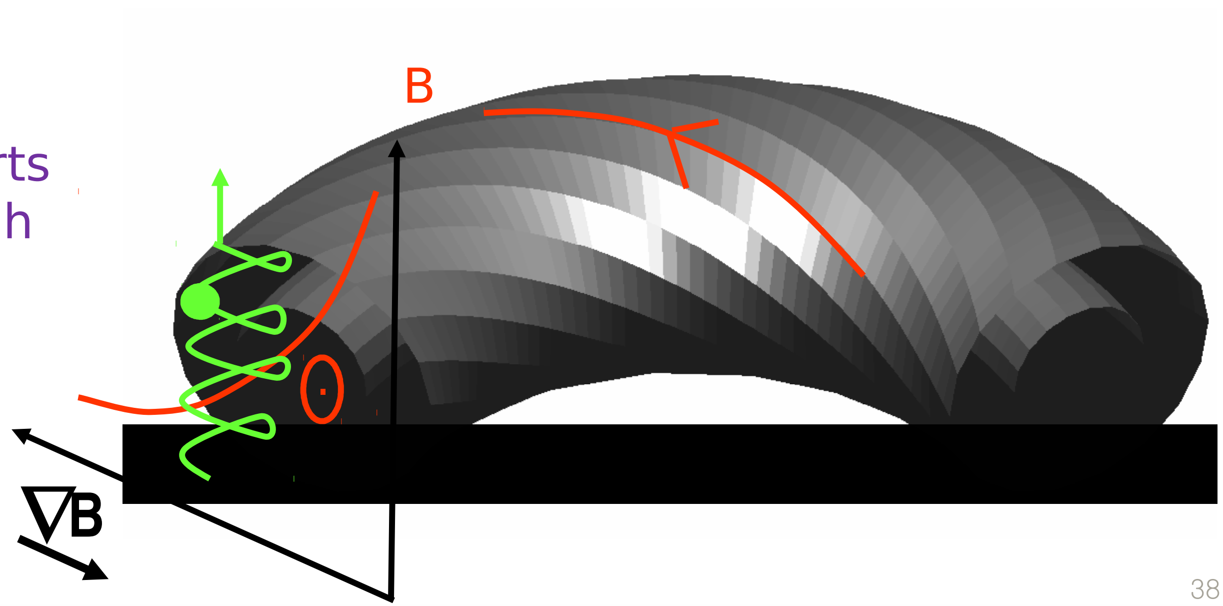


Radial (perpendicular, normal) injection

Tangential injection

Radial injection:

- standard ports
- shine-through
- particle loss

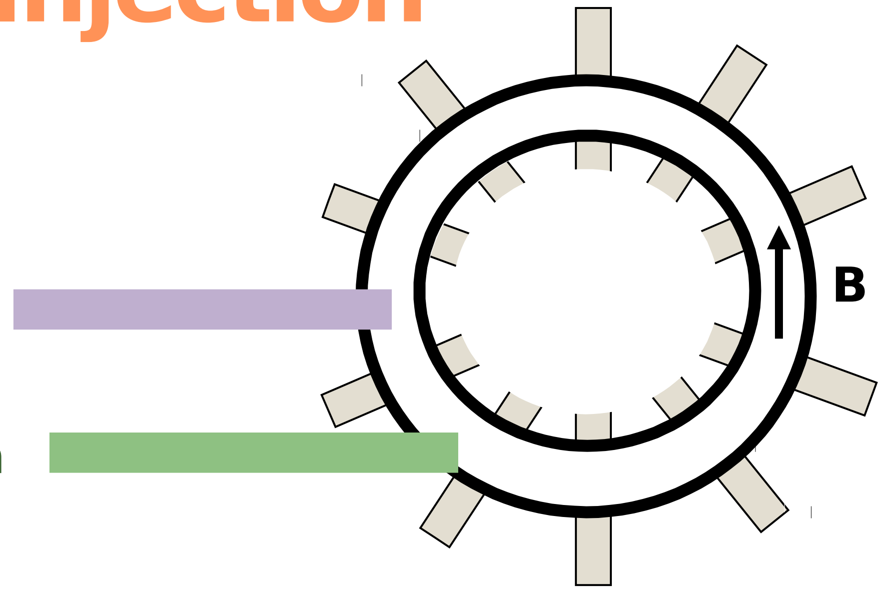


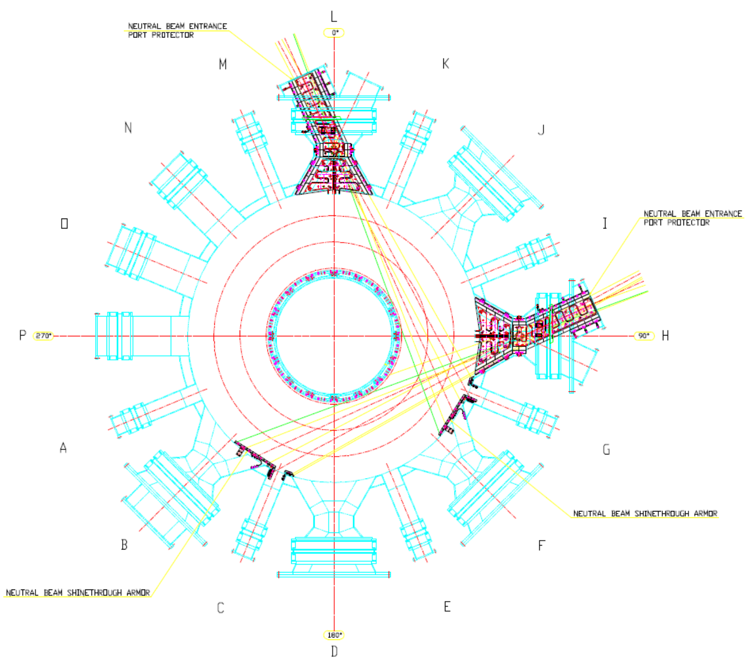
Neutral Beam Injection

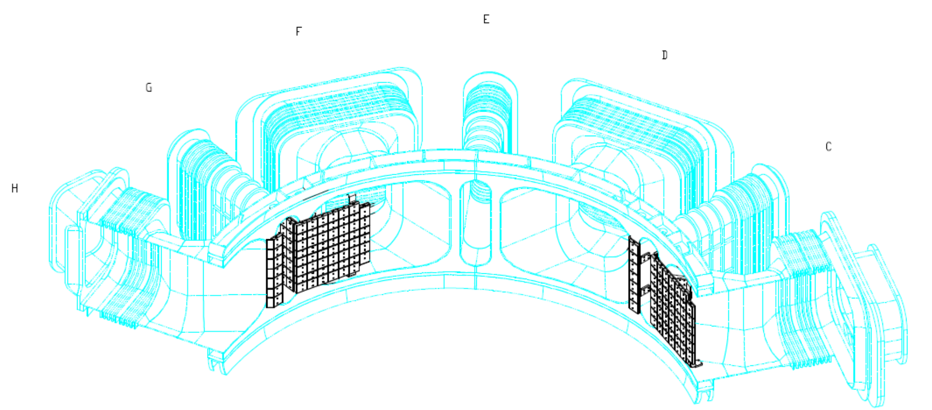
- **Injection Angle**

Radial (perpendicular, normal) injection

Tangential injection



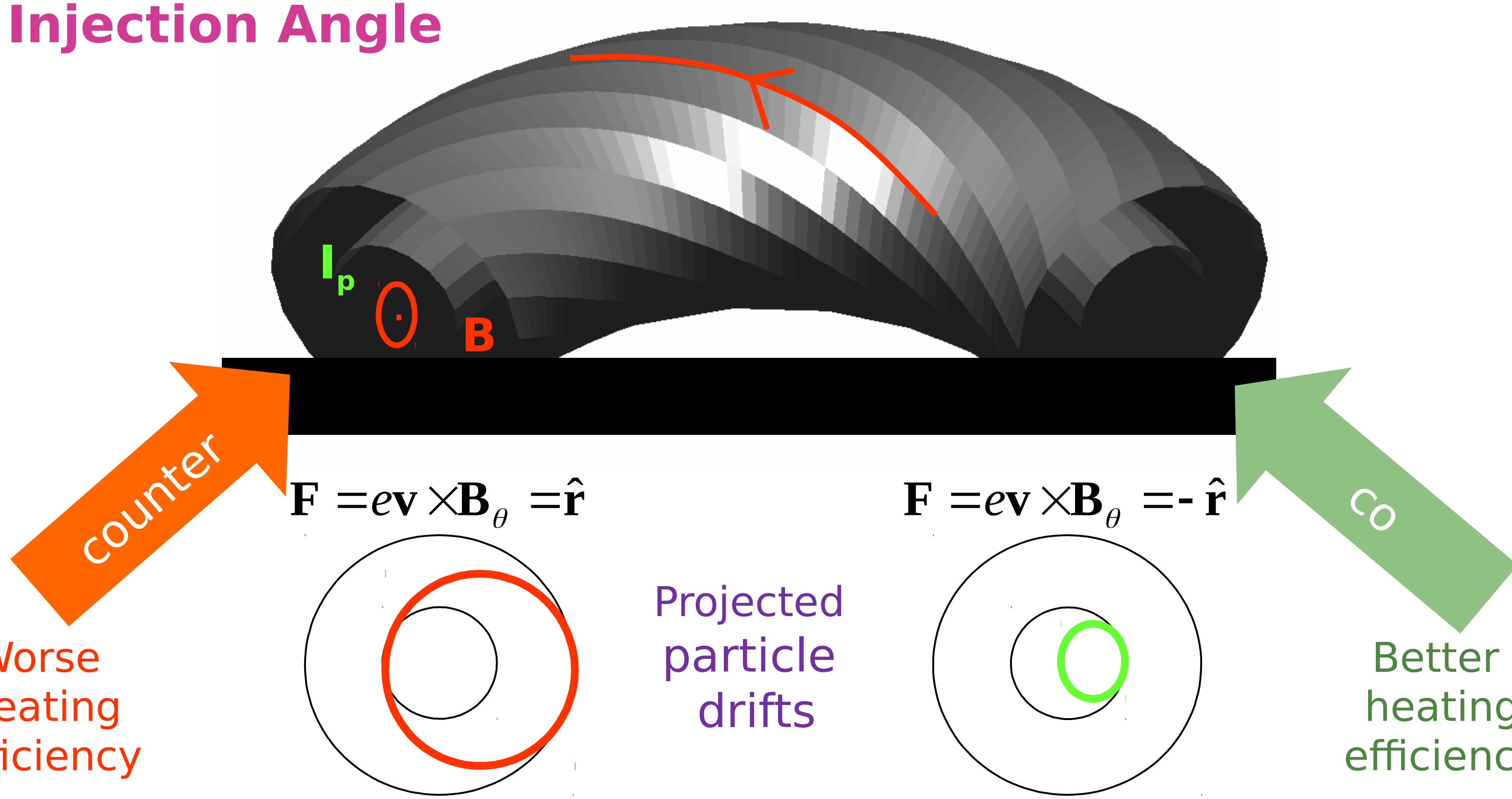




KSTAR NB shine-through armor

Neutral Beam Injection

- **Injection Angle**



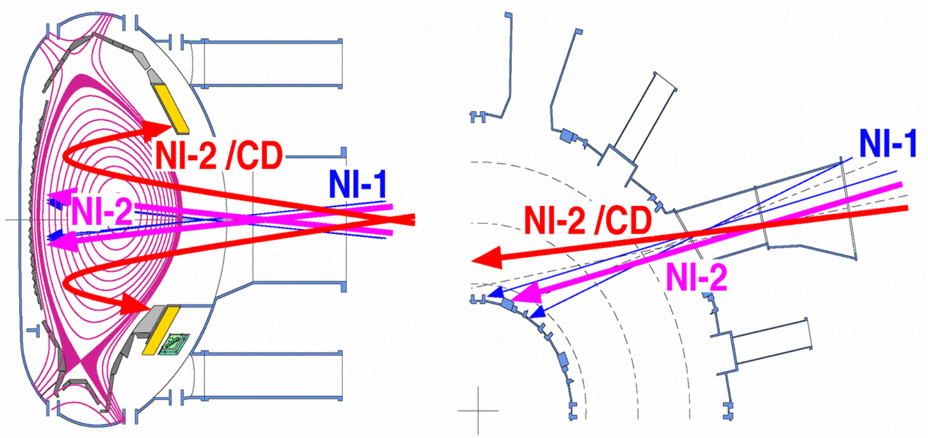
inward shift → bad drift orbits
→ energetic ion loss to wall/limiters

outward shift of orbit center

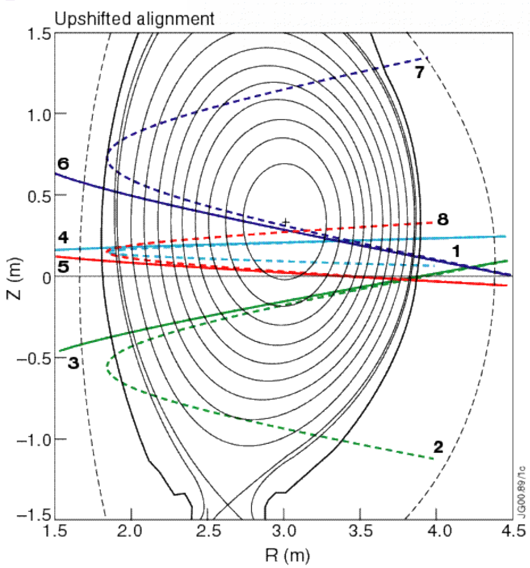
- At low magnetic fields heating efficiency depends on NBI direction.
- Best injection angle for maximum penetration and minimum orbital excursion = 10-20° off perpendicular in co-injection direction

Neutral Beam Injection

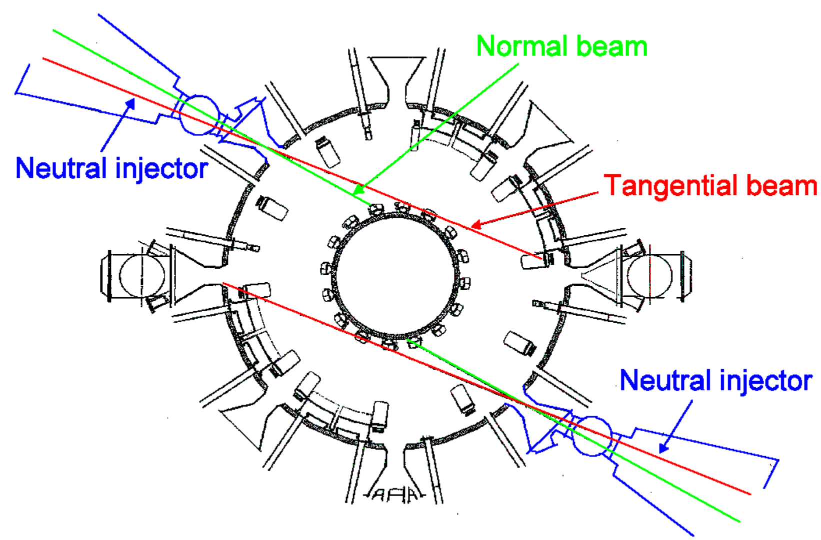
- **ASDEX Upgrade**



- **JET**



Solid = Normal, Dashed = Tangential



Neutral Beam Injection

- **ITER NBI System**

ITER NBI requirements .VS. achieved parameters from existing facilities

		ITER HNB (rf)		IPP (prototype source at BATMAN(short pulses) and MANITU (long pulses))				IPP (ELISE)				MVTF	LHD	JT-60U	
Source height	m	1.95		0.58				1					1.45	1.22	
Source width	m	1		0.31				0.87					0.35	0.64	
No. of aper-tures		1280, ø 14 mm		BATMAN: 126, ø 8 mm MANITU: 262 or 406, ø 8 mm				640, ø 14 mm					770	1080	
Energy	keV	870 (H$^-$)	1000 (D$^-$)	23				60				979	190	400	
Species		H$^-$	D$^-$	H$^-$		D$^-$		H$^-$		D$^-$		H-	H-	H-	D-
Source power	kW	800		90	47	76	43	200	120	200	80		180	350	
Ex-tracted current density	A/m^2	329	286	339	159	319	98.0	256 (53kW per driver)	138 (32kW per driver)	176 473kW per driver)	57.3 (21kW per driver)	190	250	126	144
Pulse length	s	1000	3600	4.0	1000	4.0	3600	9.5	1000 (pulsed)	9.5	3600 (pulsed)	60	2	2	

RF waves in Fusion Plasmas

Seminar at SNU

26th. Sep. 2013

S. H. Kim, KAERI

CONTENTS

The role of RF Heating and CD

Increase of temperature

- Nuclear fusion requires **high temperature more than 10 keV.**
- Ohmic heating is limited by the low resistance in high temperature.
- Alternatives : NB heating / **RF heating**
- NB heating is effective but requires high technology to increase the beam energy up to 1 MeV. (negative ion generation/acceleration/cooling)
- RF wave can heat up selectively ion and electrons and is deposited locally or globally depending on the driving schemes (magnetic field/driving frequency/plasma density)
- But, there are coupling problems related with ICRF and LHRF and power transmission and power source limitations regarding ECRF power.

- ICRF : Ion heating
- LHRF : Current drive
- ECRF : local current drive and MHD control / pre-ionization and start-up

The role of RF waves

Non inductive current drive

- Tokamak requires current drive to confine the plasmas. Otherwise, the particles is lost outward by EXB drift due to charge separation of non-uniform magnetic field.
- Most efficient current drive is Ohmic inductive current drive. However, it is limited by Ohmic swing flux.
- Therefore, the **non-inductive current drive is an indispensable element** for the success of fusion reactor.

- NB current drive/RF current drive/Helicity injection
- **LHRF current drive is proven to be most efficient non-inductive current drive scheme** ever tried and experimentally, 2 hours 20 kA in TRIAM and 2 minute 0.8 MA in Tore supra. 3.6 MA and 3 MA in JT-60U and JET are achieved respectively.
- However, there is a coupling problem.

RF waves in plasmas

- To utilize RF waves for the heating and current drive of tokamak plasmas, we should answer the two questions?
- What kind of RF waves can exist in plasmas? (Identity of RF plasma waves)
- How do RF waves propagate and are mode converted, and absorbed in plasmas? (Characteristics of RF plasma waves)

RF waves in plasmas

- What kind of RF waves can exist in plasmas? (Identity of RF plasma waves)
- Wave Equation in vacuum?
- Governing Equation: Maxwell equation with vacuum medium property.

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

- Wave Equation in Plasmas?
- Governing Equation: Maxwell equation with plasma medium property.

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 \left(\varepsilon_0 \frac{\partial E}{\partial t} + J_{rf} \right) = \mu_0 \left(\varepsilon_0 \frac{\partial E}{\partial t} + \vec{\sigma} E \right) = \mu_0 \varepsilon_0 \vec{\varepsilon}_r \frac{\partial E}{\partial t}, \quad \vec{\varepsilon}_r \equiv I + \frac{i \vec{\sigma}}{\varepsilon_0 \omega} (\chi_s)$$

- The plasma waves can be described by above Maxwell equation. One can obtain information of linear plasma waves from this governing equation.
- The remaining problem is how to obtain the conductivity or dielectric tensor.

RF waves in plasmas

- How to obtain the dielectric tensor?
- Governing Equation: Vlasov equation : Equation of evolution of particle distribution in phase space

$$\frac{df_s}{dt} = \frac{\partial f_s}{\partial t} + v \cdot \nabla f_s + a \cdot \nabla_v f_s = 0$$

$$a = \frac{eZ_s}{m_s}\left[E + v \times \left(B + B_0 \right) \right]$$

- By linearization, one can obtain linearized Vlasov equation $f_s = F_s(r,v) + \tilde{f}_s(r,v,t)$

$$\frac{d\tilde{f}_s}{dt} = \frac{\partial \tilde{f}_s}{\partial t} + v \cdot \nabla \tilde{f}_s + \frac{eZ_s}{m_s}\left[v \times B_0 \right] \cdot \nabla_v \tilde{f}_s = - \frac{eZ_s}{m_s}\left[E + v \times B_0 \right] \cdot \nabla_v F_s$$

- The solution is as follows.

$$\tilde{f}_s = - \frac{eZ_s}{m_s} \int_{-\infty}^{t} \left[E(r',t') + v' \times B(r',t') \right] \cdot \nabla_{v'} F_s dt'$$

$$J_{rf} = \sum_s n_s e Z_s \int_v \tilde{f}_s v \, dv$$

RF waves in plasmas

- Dielectric(Conductivity) tensor

$$\overleftrightarrow{\varepsilon}_r = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}, \quad \overleftrightarrow{\sigma} = \frac{\varepsilon_0 \omega}{i}(\overleftrightarrow{\varepsilon}_r - \overleftrightarrow{I}) = -i\varepsilon_0 \omega \chi_s$$

- The detailed expression of dielectric tensor elements for F_s of Maxwellian distribution function are as follows.

$$\varepsilon_{xx} = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{n=\infty} \frac{n^2}{\lambda_s} I_n(\lambda_s) e^{-\lambda_s} \left[-\varsigma_{0s} Z(\varsigma_{ns}) \right]$$

$$\varepsilon_{xy} = -i \sum_s \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{n=\infty} n \left[I_n'(\lambda_s) - I_n(\lambda_s) \right] e^{-\lambda_s} \left[-\varsigma_{0s} Z(\varsigma_{ns}) \right]$$

$$\varepsilon_{xz} = -\frac{1}{2} N_\perp N_\parallel \sum_s \frac{\omega_{ps}^2}{\omega \Omega_{cs}} \frac{v_{th\,s}^2}{c^2} \sum_{n=-\infty}^{n=\infty} \frac{n}{\lambda_s} I_n(\lambda_s) e^{-\lambda_s} \left[\varsigma_{0s}^2 Z'(\varsigma_{ns}) \right]$$

$$\varepsilon_{yy} = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{n=\infty} \left| \frac{n^2}{\lambda_s} I_n(\lambda_s) - 2\lambda_s \left[I_n'(\lambda_s) - I_n(\lambda_s) \right] \right| e^{-\lambda_s} \left[-\varsigma_{0s} Z(\varsigma_{ns}) \right]$$

$$\varepsilon_{yz} = \frac{i}{2} N_\perp N_\parallel \sum_s \frac{\omega_{ps}^2}{\omega \Omega_{cs}} \frac{v_{th\,s}^2}{c^2} \sum_{n=-\infty}^{n=\infty} \left[I_n'(\lambda_s) - I_n(\lambda_s) \right] e^{-\lambda_s} \left[\varsigma_{0s}^2 Z'(\varsigma_{ns}) \right]$$

$$\varepsilon_{zz} = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} \sum_{n=-\infty}^{n=\infty} I_n(\lambda_s) e^{-\lambda_s} \left[-\varsigma_{0s} \varsigma_{ns} Z'(\varsigma_{ns}) \right]$$

$$\lambda_s = \frac{k_\perp^2 v_{th\,s}^2}{2\Omega_{cs}^2}$$

$$\varsigma_{ns} = \frac{\omega - n\Omega_{cs}}{k_\parallel v_{th\,s}}$$

$$\varepsilon_{yx} = -\varepsilon_{xy}$$

$$\varepsilon_{zx} = \varepsilon_{xz}$$

$$\varepsilon_{zy} = -\varepsilon_{yz}$$

RF waves in plasmas

- Cold dielectric tensor

$$\lim_{v_{ths}\to 0} \vec{\varepsilon}_r = \lim_{v_{ths}\to 0} \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}$$

- The detailed expression of cold dielectric tensor elements are as follows.

$$S = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2 - \Omega_{cs}^2}$$

$$D = \sum_s \frac{\Omega_{cs}}{\omega} \frac{\omega_{ps}^2}{\omega^2 - \Omega_{cs}^2}$$

$$P = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}$$

- If the plasma density goes to zero, the cold dielectric tensor becomes unity tensor.
- It is a vacuum relative permittivity.

RF waves in plasmas

- It is easier to approach from cold plasma dielectric tensor for RF wave exploration.

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 \varepsilon_0 \overleftrightarrow{\varepsilon}_c \frac{\partial E}{\partial t}$$

$$\overleftrightarrow{\varepsilon}_{cold} = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}$$

- By manipulating the Maxwell equation with cold plasma dielectric response, one can obtain the wave equation.

$$\nabla \times \nabla \times E = -\mu_0 \varepsilon_0 \overleftrightarrow{\varepsilon}_c \frac{\partial^2 E}{\partial t^2}$$

- For spatially uniform plasmas

$$N \times N \times E_0 = \overleftrightarrow{\varepsilon}_c E_0, \quad E = E_0 e^{i(k_0 N \vec{r} - wt)}$$

$$(\overleftrightarrow{N}^2 - \overleftrightarrow{\varepsilon}_c) E_0 = 0$$

$$\overleftrightarrow{N}^2 = \begin{bmatrix} N_\parallel^2 & 0 & -N_\parallel N_\perp \\ 0 & N^2 & 0 \\ -N_\parallel N_\perp & 0 & N_\perp^2 \end{bmatrix}$$

$$\det(\overleftrightarrow{N}^2 - \overleftrightarrow{\varepsilon}_c) = 0 : \textit{dispersion relation}$$

RF waves in plasmas

- Dispersion relation

$$H \equiv \det(\vec{N}^2 - \vec{\varepsilon}_c) = 0 : \textit{dispersion relation}$$

- Several forms of dispersion relations

$$AN^4 + BN^2 + C = 0$$
$$A = S\sin^2\theta + P\cos^2\theta,\ \ \theta = \angle(\vec{B}, \vec{N})$$
$$B = -RL\sin^2\theta - SP(1+\cos^2\theta),\ R = S + D, L = S - D$$
$$C = PRL$$

$$AN_\perp^4 + BN_\perp^2 + C = 0$$
$$A = S$$
$$B = N_\parallel^2(S+P) - (SP + RL)$$
$$C = P(N_\parallel^2 - R)(N_\parallel^2 - L)$$

$$AN_\parallel^4 + BN_\parallel^2 + C = 0$$
$$A = P$$
$$B = N_\perp^2(S+P) - 2SP$$
$$C = (N_\perp^2 - P)(SN_\perp^2 - RL)$$

RF waves in plasmas

- Polarization

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S}$$

$$\frac{E_z}{E_x} = \frac{N_{\|} N_{\perp}}{N_{\perp}^2 - P}$$

$$\frac{E_+}{E_-} = \frac{R - N^2}{L - N^2}, \quad E_+ = \frac{E_x + iE_y}{\sqrt{2}}, E_- = \frac{E_x - iE_y}{\sqrt{2}}$$

- Group velocity

$$v_g = -\frac{\partial H / \partial k}{\partial H / \partial \omega}$$

$$\frac{\tan\theta_g}{\tan\theta} = \frac{v_{g\perp}}{v_{g\|}} \left(\frac{v_{p\perp}}{v_{p\|}} \right)^{-1} = \frac{SN_{\perp}^2 + P(N_{\|}^2 - R)(N_{\|}^2 - L)}{PN_{\|}^2 + (N_{\|}^2 - P)(SN_{\|}^2 - RL)}$$

RF waves in plasmas

- Cut-off /Resonance

$$cutoff : N = 0, \lambda = \infty \; ; wave \; is \; evanescent.$$

$$resonance : N = \infty, \lambda = 0 ; wave \; is \; locally \; piled \; up.$$

- Cut-off

$$N, C = 0$$

$$P = 0 : O \; wave \; cutoff$$

$$R = 0 : R \; wave \; cutoff$$

$$L = 0 : L \; wave \; cutoff$$

$$AN^4 + BN^2 + C = 0$$

$$A = S\sin^2\theta + P\cos^2\theta, \;\; \theta = \angle(\vec{B}, \vec{N})$$

$$B = -RL\sin^2\theta - SP(1+\cos^2\theta), \; R = S + D, L = S - D$$

$$C = PRL$$

- Resonance

$$N = \infty,$$

$$\theta = (0, \frac{\pi}{2}) \Rightarrow \; A = S\sin^2\theta + P\cos^2\theta = 0; \; resonance \; cone \; wave$$

$$\theta = 0(parallel) \Rightarrow \; R, L = \infty \; ; \; cyclotron \; resonance$$

$$\theta = \frac{\pi}{2}(perpendicular) \Rightarrow S = 0; \; UHR, \; LHR$$

RF waves in plasmas

- Perpendicular / Parallel propagation

$$AN_{\perp}^4 + BN_{\perp}^2 + C = 0$$
$$A = S$$
$$B = N_{\parallel}^2(S+P) - (SP+RL)$$
$$C = P(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)$$
$$N_{\parallel} = 0,$$
$$N_{\perp}^2 = \frac{RL}{S} \quad (X\ wave)$$
$$N_{\perp}^2 = P \quad (O\ wave)$$

$$AN_{\parallel}^4 + BN_{\parallel}^2 + C = 0$$
$$A = P$$
$$B = N_{\perp}^2(S+P) - 2SP$$
$$C = (N_{\perp}^2 - P)(SN_{\perp}^2 - RL)$$
$$N_{\perp} = 0,$$
$$N_{\parallel}^2 = R \quad (R\ wave),$$
$$N_{\parallel}^2 = L \quad (L\ wave)$$

- Perpendicular / Parallel Cut-off

$$P = 0 : O\ wave\ cutoff$$
$$R = 0 : X\ wave\ cutoff$$
$$L = 0 : X\ wave\ cutoff$$

$$R = 0 : R\ wave\ cutoff$$
$$L = 0 : L\ wave\ cutoff$$

- Perpendicular / Parallel Resonance

$$S = 0 : X\ wave\ resonance\ (UHR,\ LHR)$$

$$R = \infty : R\ wave\ resonance$$
$$L = \infty : L\ wave\ resonance$$

RF waves in plasmas

- Cut-off

$$P = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2} = 0 \Rightarrow X \cong 1 \quad : O \text{ wave cutoff} \qquad X = \frac{\omega_{pe}^2}{\omega^2}, Y = \frac{\omega_{ce}}{\omega}, \delta = \frac{m_e}{m_i}$$

$$R = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}\frac{\omega}{\omega + \Omega_{cs}} = 0 \Rightarrow Y \cong -X + 1 \; : R(X) \text{ wave cutoff}$$

$$L = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}\frac{\omega}{\omega - \Omega_{cs}} = 0 \Rightarrow -\delta Y^2 + Y \cong X - 1 \quad : L(X) \text{ wave cutoff}$$

- Resonance

$$S = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2 - \Omega_{cs}^2} = 0 \Rightarrow Y = (-X + 1)^{1/2} \; : \text{Upper Hybrid resonance}$$

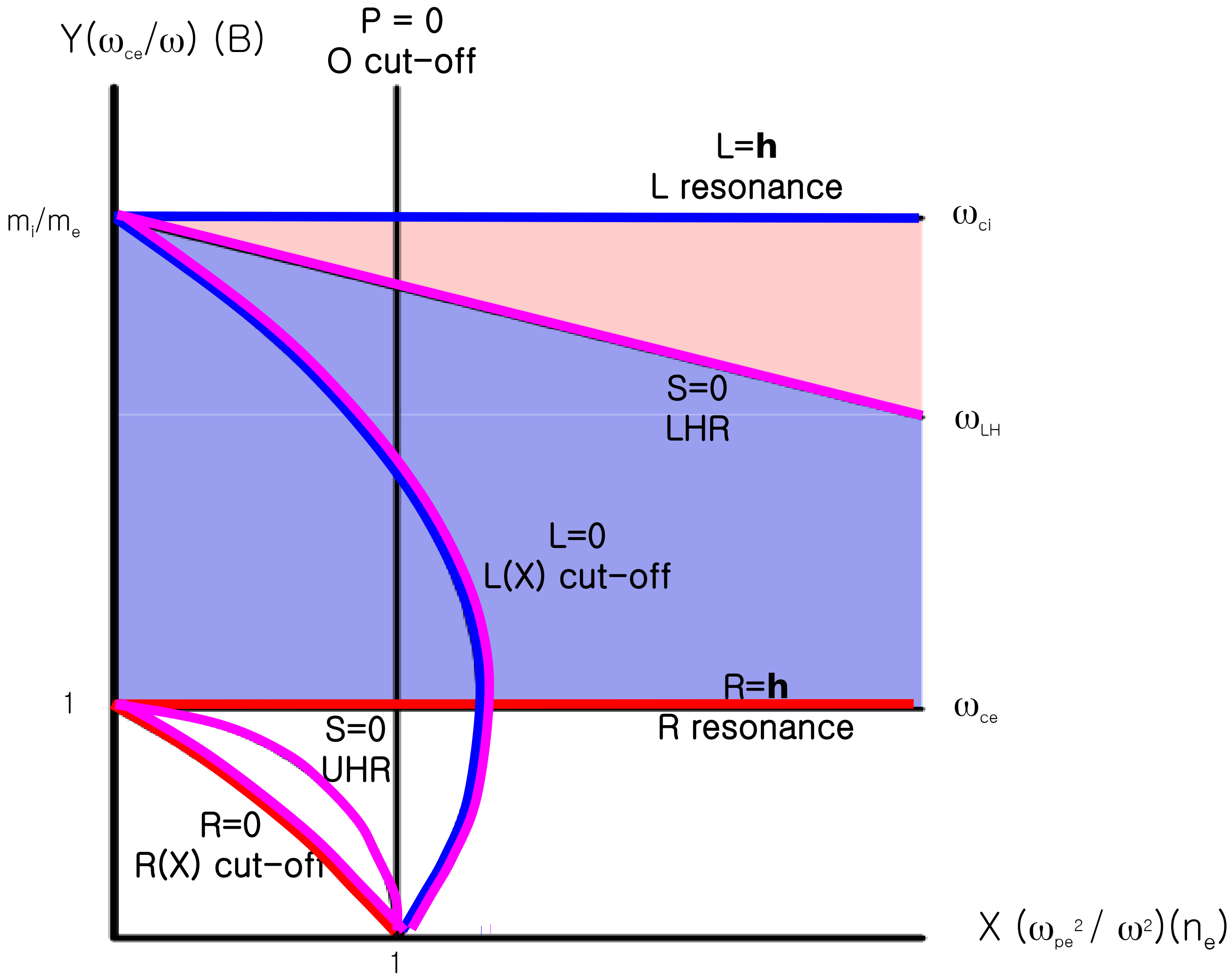
$$\Rightarrow Y^3 - \delta XY^2 + X = 0 \; : \text{Lower Hybrid resonance}$$

$$R = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}\frac{\omega}{\omega + \Omega_{cs}} = \infty \Rightarrow \omega = \omega_{ce} \Rightarrow Y = 1 \; : \text{electron cyclotron resonance}$$

$$L = 1 - \sum_s \frac{\omega_{ps}^2}{\omega^2}\frac{\omega}{\omega - \Omega_{cs}} = \infty \Rightarrow \omega = \omega_{ci} \Rightarrow Y = \delta^{-1} ; \text{ ion cyclotron resonance}$$

RF waves in plasmas

- CMA diagram



RF waves in plasmas

- Polarization of 4 wave branches

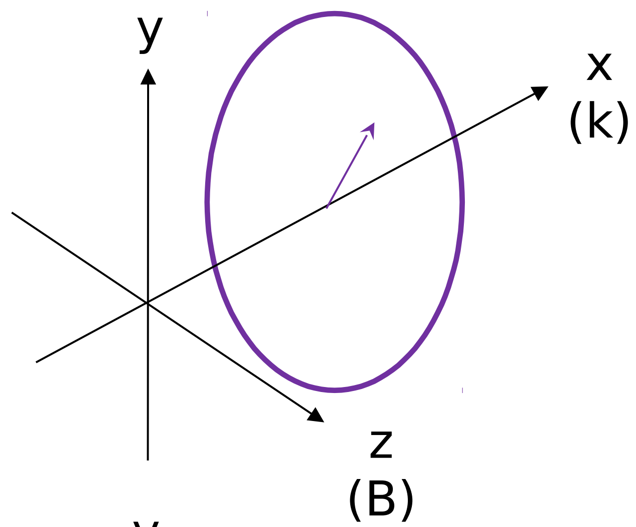
$$N_\perp^2 = \frac{RL}{S} \quad (X\ wave)$$

$$N_\perp^2 = P \quad (O\ wave)$$

X wave

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iD}{N_\perp^2 - S} = \frac{iSD}{RL - S^2} = -i\frac{S}{D}$$

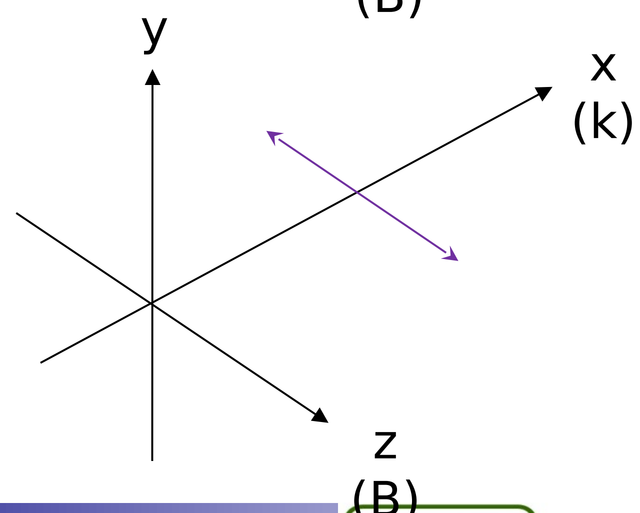
$$\frac{E_z}{E_x} = \frac{N_\parallel N_\perp}{N_\perp^2 - P} = 0 \qquad \frac{\tan\theta_g}{\tan\theta} = \frac{1+P}{P} \sim 1$$



O wave

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iD}{N_\perp^2 - S} = \frac{iD}{P - S}$$

$$\frac{E_z}{E_x} = \frac{N_\parallel N_\perp}{N_\perp^2 - P} = \infty \qquad \frac{\tan\theta_g}{\tan\theta} = \frac{S + RL}{RL}$$



RF waves in plasmas

- Polarization of 4 wave branches

$$N_{\parallel}^2 = R \quad (R\ wave),$$
$$N_{\parallel}^2 = L \quad (L\ wave)$$

R wave

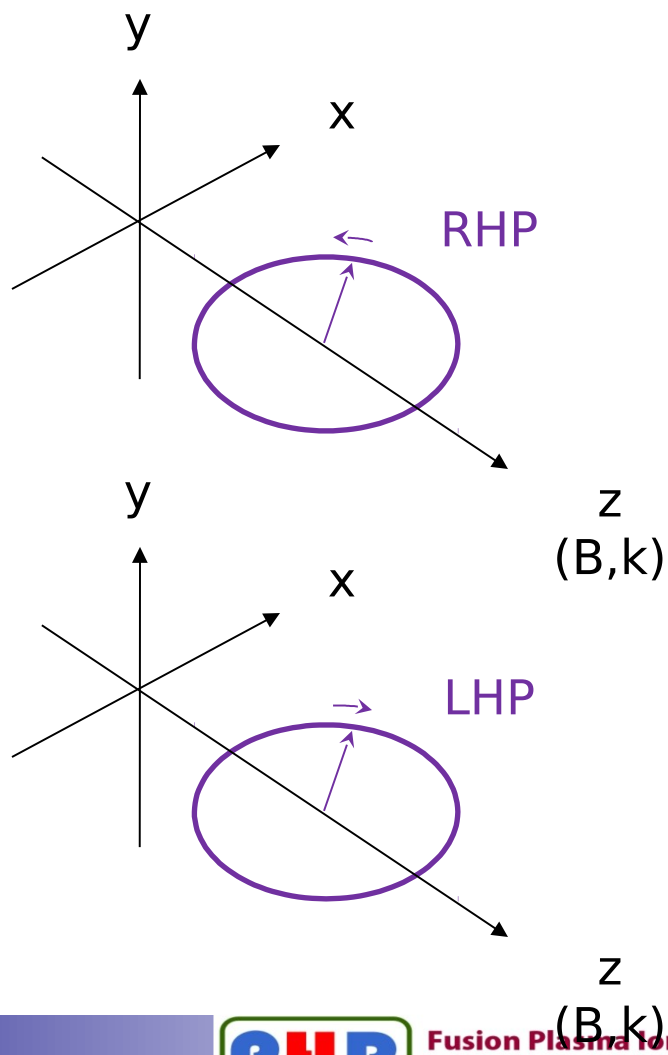
$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iD}{N_{\parallel}^2 - S} = i \qquad \frac{E_+}{E_-} = \frac{R - N^2}{L - N^2} = \frac{R - N_{\parallel}^2}{L - N_{\parallel}^2} = 0$$

$$\frac{E_z}{E_x} = \frac{N_{\parallel} N_{\perp}}{N_{\perp}^2 - P} = 0 \qquad \frac{\tan\theta_g}{\tan\theta} = \frac{P(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)}{PR + (R - P)(SR - RL)} = 0$$

L wave

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iD}{N_{\parallel}^2 - S} = -i \qquad \frac{E_+}{E_-} = \frac{R - N^2}{L - N^2} = \frac{R - N_{\parallel}^2}{L - N_{\parallel}^2} = \infty$$

$$\frac{E_z}{E_x} = \frac{N_{\parallel} N_{\perp}}{N_{\perp}^2 - P} = 0 \qquad \frac{\tan\theta_g}{\tan\theta} = \frac{P(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)}{PR + (R - P)(SR - RL)} = 0$$



RF waves in plasmas

- Electrostatic waves

$$E = -\nabla\varphi = -ik\varphi \Rightarrow E \parallel k$$

$$\vec{N} \times \vec{N} \times \vec{E}_0 = \overleftrightarrow{\varepsilon}_c E_0$$

$$\vec{N} \cdot (N \times N \times E_0) = \vec{N} \cdot \overleftrightarrow{\varepsilon}_c E_0 \;\; : Wave\ equation\ parallel\ to\ propagation$$

$$\vec{N} \cdot \overleftrightarrow{\varepsilon}_c E_0 = 0$$

$$N \cdot \overleftrightarrow{\varepsilon}_c (\vec{E}_{\parallel} + \vec{E}_{\perp}) = 0\ ,\ \angle(\vec{N}, E_{\parallel}) = 0,\ \angle(\vec{N}, E_{\perp}) = 90^{\circ}$$

$$\Rightarrow (N \cdot \overleftrightarrow{\varepsilon}_c \cdot N) E_{\parallel} = 0$$

$$\Rightarrow SN_{\perp}^2 + PN_{\parallel}^2 = 0 : Dispersion\ relation\ of\ cold\ electrostatic\ waves$$

- For X waves

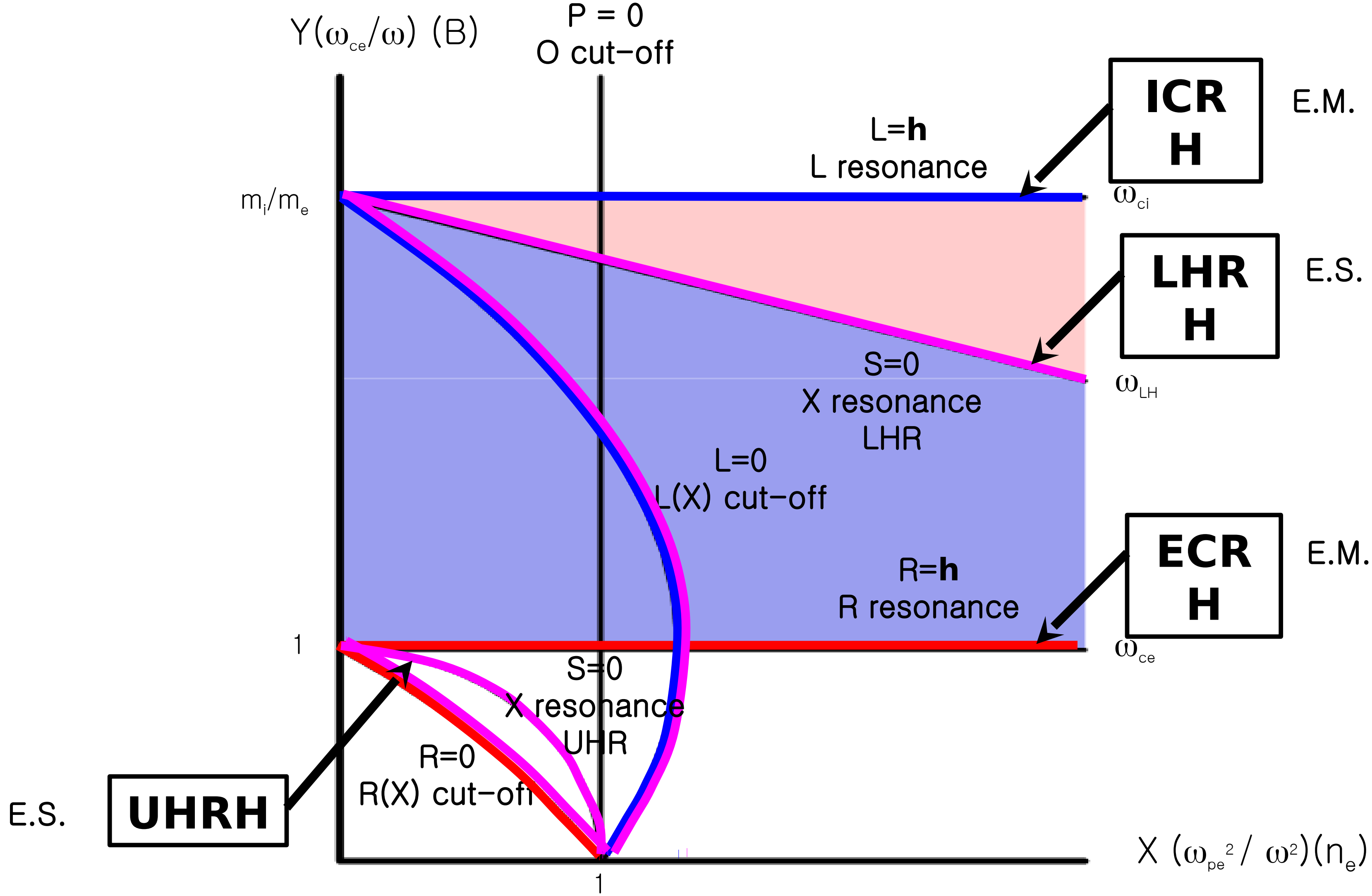
$$if\ S = 0\ at\ UHR, LHR\ in\ X\ wave \qquad N_{\perp}^2 = \frac{RL}{S}\ (X\ wave) \rightarrow \infty\ at\ UHR, LHR$$

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iD}{N_{\perp}^2 - S} = \frac{iSD}{RL - S^2} = -i\frac{S}{D} \rightarrow 0$$

$$\Rightarrow Purely\ x\ polarization \Rightarrow k = k_x \parallel E_x \ \therefore X\ wave\ becomes\ e.s.\ at\ UHR, LHR$$

RF waves in plasmas

- What kinds of waves can be used? We should use **resonances.**



RF waves in plasmas

- Oblique injection

RF wave is launched obliquely but almost perpendicular to magnetic field in tokamak with fixed parallel refractive index which just changes in the major radius direction.

$$AN_\perp^4 + BN_\perp^2 + C = 0$$

$$A = S$$

$$B = N_\parallel^2 (S+P) - (SP + RL)$$

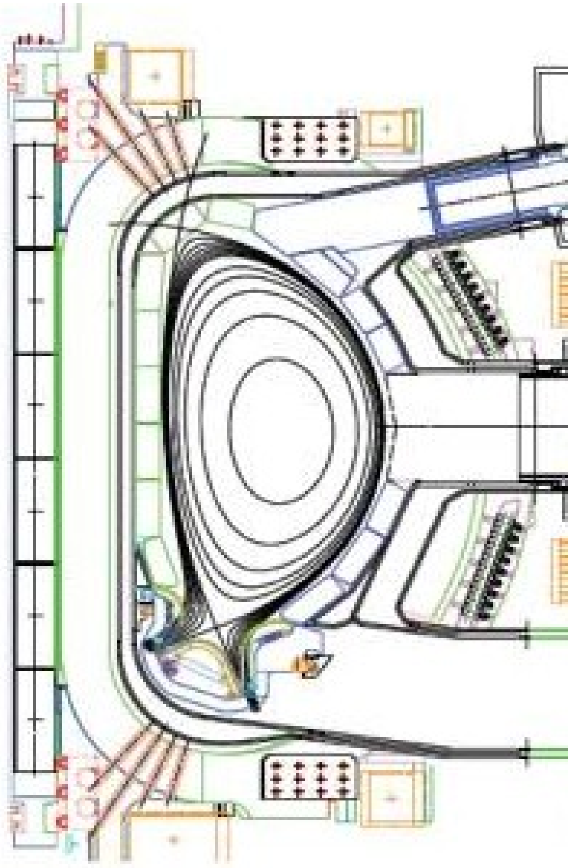
$$C = P(N_\parallel^2 - R)(N_\parallel^2 - L)$$

$$N_\perp^2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\cong -\frac{P(N_\parallel^2 - S)}{S} + \frac{PD^2 N_\parallel^2}{S\left[(N_\parallel^2 - S)(S-P) + D^2\right]},$$

$$-\frac{S(N_\parallel^2 - S) + D^2}{S} - \frac{PD^2 N_\parallel^2}{S\left[(N_\parallel^2 - S)(S-P) + D^2\right]}$$

$$\cong -\frac{P(N_\parallel^2 - S)}{S}, -\frac{(N_\parallel^2 - R)(N_\parallel^2 - L)}{(N_\parallel^2 - S)} : Low\ frequency\ range$$



RF waves in plasmas

- Polarization of oblique injection

$$N_\perp^2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\cong -\frac{P(N_\parallel^2 - S)}{S} + \frac{PD^2N_\parallel^2}{S\left[(N_\parallel^2 - S)(S-P) + D^2\right]}, -\frac{S(N_\parallel^2 - S) + D^2}{S} - \frac{PD^2N_\parallel^2}{S\left[(N_\parallel^2 - S)(S-P) + D^2\right]}$$

$$\cong -\frac{P(N_\parallel^2 - S)}{S}, -\frac{(N_\parallel^2 - R)(N_\parallel^2 - L)}{(N_\parallel^2 - S)} \; : Low\ frequency\ range$$

- Slow wave and Fast wave (Low frequency range)

Slow wave

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iSD}{(N_\parallel^2 - S)(S-P)}$$

$$\frac{E_z}{E_x} = \frac{N_\parallel N_\perp}{N_\perp^2 - P} = -\frac{SN_\perp}{PN_\parallel}$$

Fast wave

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{i(N_\parallel^2 - S)}{D}$$

$$\frac{E_z}{E_x} = \frac{N_\parallel N_\perp}{N_\perp^2 - P} \qquad \frac{E_+}{E_-} = \frac{R - N^2}{L - N^2} = -\frac{R - N_\parallel^2}{L - N_\parallel^2}$$

RF waves in plasmas

- Polarization near ion cyclotron resonance of oblique injection
- Slow wave and Fast wave

Slow wave

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iSD}{(N_\parallel^2 - S)(S - P)} \rightarrow -\frac{iD}{S} \rightarrow i\ ,\ E_+ \rightarrow 0\ :\ RHP,\ near\ \ ICR$$

$$\frac{E_z}{E_x} = \frac{N_\parallel N_\perp}{N_\perp^2 - P} = -\frac{SN_\perp}{PN_\parallel} \rightarrow \infty$$

Fast wave

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{i(N_\parallel^2 - S)}{D} \rightarrow -\frac{iS}{D} \rightarrow i,\ E_+ \rightarrow 0\ :\ RHP,\ near\ \ ICR$$

$$\frac{E_z}{E_x} = \frac{N_\parallel N_\perp}{N_\perp^2 - P} \rightarrow 0$$

- ***There are no cyclotron absorption near fundamental ion cyclotron resonances for obliquely injected cold slow or fast waves (This result is similar for electron cyclotron resonance).***

RF waves in plasmas

- ICRF fast wave has not favorable LHP near fundamental ion cyclotron resonance.

$$N_{\perp}^2 \cong -\frac{(N_{\parallel}^2 - R)(N_{\parallel}^2 - L)}{(N_{\parallel}^2 - S)}$$

$$\frac{E_+}{E_-} = \frac{R - N^2}{L - N^2} = -\frac{R - N_{\parallel}^2}{L - N_{\parallel}^2} = -\frac{D + S - N_{\parallel}^2}{-D + S - N_{\parallel}^2}$$

- If $(N_{\parallel}^2 - S) = 0$, then $N_{\perp}^2 \to \infty,$

$$\frac{E_+}{E_-} = \frac{R - N^2}{L - N^2} = -\frac{R - N_{\parallel}^2}{L - N_{\parallel}^2} = -\frac{D + S - N_{\parallel}^2}{-D + S - N_{\parallel}^2} = 1$$

- More rigorous absorption can be obtained from the hot dielectric tensor.
- $(N_{\parallel}^2 - S) = 0$ can be achieved with multi-species (major and minority ion species).
- There are two regimes with respect to the minority fraction, $\eta = n_m/n_M$.

$$\eta < \eta_c \; : Minority\ heating\ regime$$

$$\eta > \eta_c \; : Ion - Ion\ hybrid\ resonance\ regime$$

RF waves in plasmas (Summary I)

- One obtain 4 wave branches from cold dielectric tensors.
- And there are four wave resonances.
- We should use the resonance for plasma heating.
- However, there is very weak collision in fusion plasmas.
- Therefore, there is only weak power absorption even in resonances.
- In addition, there is no cyclotron resonance heating for obliquely injected waves.

- As a result, we should analyze the power absorption with a hot dielectric tensor.
- It means that wave power absorption in fusion plasmas is possible via kinetic effect.

RF waves in plasmas (Power absorption)

- Power absorption can be represented as follows.

$$P_{abs} = \frac{1}{2}\mathrm{Re}[J \cdot E^*] = \frac{1}{2}\mathrm{Re}[E^* \cdot \overset{\leftrightarrow}{\sigma} \cdot E] = \frac{1}{2}\mathrm{Re}[(-i\varepsilon_0\omega)E^* \cdot (\overset{\leftrightarrow}{\varepsilon}_r - \overset{\leftrightarrow}{I}) \cdot E] = \frac{1}{2}\varepsilon_0\omega[E^* \cdot \overset{\leftrightarrow}{\varepsilon}_A \cdot E]$$

$$\overset{\leftrightarrow}{\varepsilon}_r = \overset{\leftrightarrow}{\varepsilon}_H + i\overset{\leftrightarrow}{\varepsilon}_A$$

$$\overset{\leftrightarrow}{\varepsilon}_H = \frac{1}{2}\left[\overset{\leftrightarrow}{\varepsilon}_r + \overset{\leftrightarrow}{\varepsilon}_r^{T*}\right]$$

$$\overset{\leftrightarrow}{\varepsilon}_A = \frac{1}{2i}\left[\overset{\leftrightarrow}{\varepsilon}_r - \overset{\leftrightarrow}{\varepsilon}_r^{T*}\right]$$

- For Maxwellian plasmas

$$P_{abs} = \frac{1}{2}\varepsilon_0\omega\sum_{i,j} E_i^* \cdot \overset{\leftrightarrow}{\varepsilon}_{Aij} \cdot E_j$$

$$\lim_{k_\parallel \to 0} \frac{\omega}{k_\parallel v_{ths}} e^{-\frac{(\omega - n\Omega_{cs})^2}{k_\parallel^2 v_{ths}^2}} = \sqrt{\pi}\,\delta\left(\frac{\omega - n\Omega_{cs}}{\omega}\right)$$

$$P_{LD} \cong \frac{1}{2}\varepsilon_0\omega\frac{\omega_{ps}^2}{\omega^2}2\sqrt{\pi}\frac{\omega^3}{k_\parallel^3 v_{ths}^3} e^{-\frac{\omega^2}{k_\parallel^2 v_{ths}^2}} |E_z|^2 \quad (n=0)$$

$$P_{MP} \cong \frac{1}{2}\varepsilon_0\omega\frac{\omega_{ps}^2}{\omega^2}\frac{k_\perp^2 v_{ths}^2}{\Omega_{cs}^2}\sqrt{\pi}\frac{\omega}{k_\parallel v_{ths}} e^{-\frac{\omega^2}{k_\parallel^2 v_{ths}^2}} |E_y|^2 \quad (n=0)$$

$$P_{\Omega} \cong \frac{1}{2}\varepsilon_0\omega\frac{\omega_{ps}^2}{\omega^2}\frac{k_\parallel^2 v_{ths}^2}{\Omega_{cs}^2} g\sqrt{\pi}\frac{\omega}{k_\parallel v_{ths}} e^{-\frac{(\omega-\Omega_{cs})^2}{k_\parallel^2 v_{ths}^2}} |E|^2 \quad (n=1)$$

$$P_{n\Omega} \cong \frac{1}{2}\varepsilon_0\omega\frac{\omega_{ps}^2}{\omega^2}\left(\frac{k_\perp^2 v_{ths}^2}{2\Omega_{cs}^2}\right)^{n-1}\sqrt{\pi}\frac{\omega}{k_\parallel v_{ths}} e^{-\frac{(\omega-n\Omega_{cs})^2}{k_\parallel^2 v_{ths}^2}} |E_\pm|^2 \quad (n \geq 2)$$

RF waves in plasmas (Landau damping)

- Landau damping

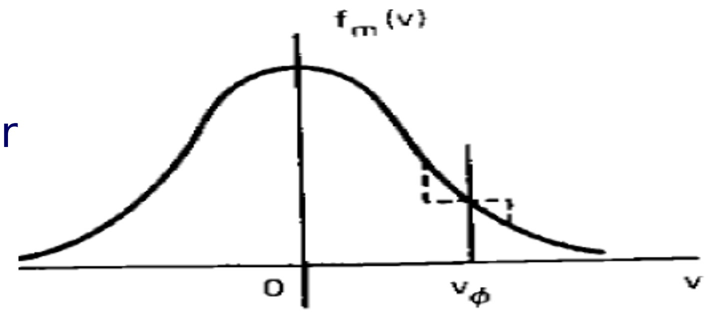
$$P_{LD} \cong \frac{1}{2}\varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} 2\sqrt{\pi} \frac{\omega^3}{k_{\parallel}^3 v_{ths}^3} e^{-\frac{\omega^2}{k_{\parallel}^2 v_{ths}^2}} |E_z|^2$$

- Optimum phase velocity $\frac{\omega}{k_{\parallel}} \sim v_{ths}$
- Electric field parallel to magnetic field is required.
- Low frequency is better for given E_z field.
- Slow wave has large E_z electric field.

- General form(non-Maxwellian plasmas) & Pictur

$$P_{LD} \cong \frac{1}{2}\varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \pi \frac{\omega}{|k_{\parallel}|} \int_0^{\infty} \left(-v_{\parallel} \frac{\partial F_s}{\partial v_{\parallel}} \right)_{v_{\parallel} = \omega / k_{\parallel}} v_{\perp} dv_{\perp} |E_z|^2$$

- It requires negative particle distribution near particle phase velocity.



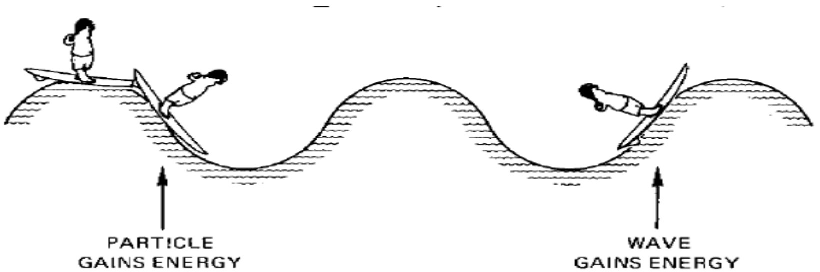


FIGURE 7-17 Customary physical picture of Landau damping.

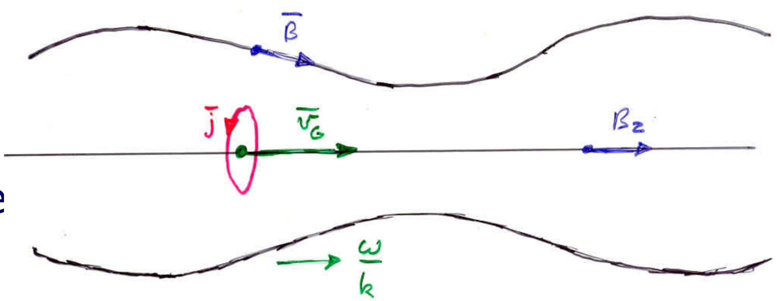
RF waves in plasmas (TTMP)

- TTMP : Transit Time Magnetic Pumping

$$P_{MP} \cong \frac{1}{2}\varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2}\frac{k_\perp^2 v_{ths}^2}{\Omega_{cs}^2}\sqrt{\pi}\frac{\omega}{k_\parallel v_{ths}} e^{-\frac{\omega^2}{k_\parallel^2 v_{ths}^2}} \left|E_y\right|^2$$

  - Optimum phase velocity : $\frac{\omega}{k_\parallel} \sim v_{ths}$
  - E_y perpendicular to magnetic field is required.
  - Low frequency is better for given Ey.
  - Fast wave has large Ey electric field (Bz).

- Picture
  - Driving force comes from the gradient of wave magnetic field which gyrating particles by external magnetic field feel during paralle motion in phase of phase velocity.



R. Koch, "Summer school in KAIST" 2009

$$F_{MP} \cong -\mu \nabla B_z$$

  - It is similar to Landau damping in view that it gain energy from wave during motion in phase of wave phase velocity except that it just gain energy from wave magnetic field instead of electric field

RF waves in plasmas (Cyclotron damping)

- Fundamental cyclotron damping

$$P_{\Omega} \cong \frac{1}{2}\varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \frac{k_{\|}^2 v_{ths}^2}{\Omega_{cs}^2} g\sqrt{\pi}\frac{\omega}{k_{\|} v_{ths}} e^{-\frac{(\omega-\Omega_{cs})^2}{k_{\|}^2 v_{ths}^2}} |E|^2 \Leftarrow \quad \frac{1}{2}\varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2}\sqrt{\pi}\frac{\omega}{k_{\|} v_{ths}} e^{-\frac{(\omega-\Omega_{cs})^2}{k_{\|}^2 v_{ths}^2}} |E_{\pm}|^2$$

- There is no power absorption without parallel wave number.

- It is because the field polarization is RHP.

- Harmonic cyclotron damping

$$P_{n\Omega} \cong \frac{1}{2}\varepsilon_0 \omega \frac{\omega_{ps}^2}{\omega^2} \left(\frac{k_{\perp}^2 v_{ths}^2}{2\Omega_{cs}^2}\right)^{n-1} \sqrt{\pi}\frac{\omega}{k_{\|} v_{ths}} e^{-\frac{(\omega-n\Omega_{cs})^2}{k_{\|}^2 v_{ths}^2}} |E_{\pm}|^2$$

- Harmonic cyclotron damping is possible due to FLR(Finite Larmor Radius) effect.

- If Larmor radius is comparable to wavelength, the gyrating particles feel the non-uniform electric field during one gyration period.

- As a result, it is accelerated in average by the LH or RH circulating wave electric field with harmonic frequency.

- Power absorption decreases as the harmonic number increases if $k_{\perp} r_L < 1$. There-fore, Landau damping or TTMP becomes important for high harmonic heating in HHFW heating on ST.

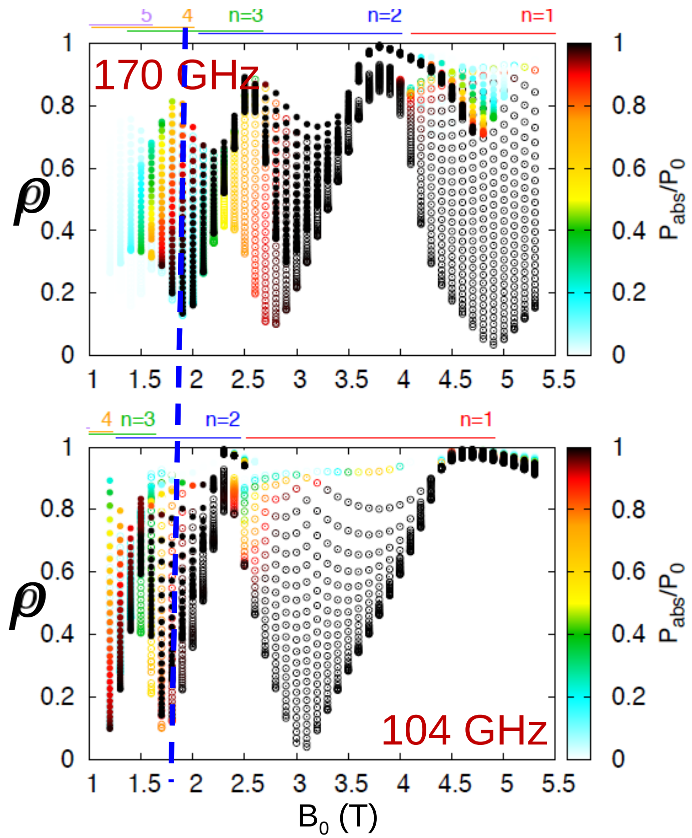
ECH modelling

- 2 options:
  1) 24 gyrotrons 170 GHz (20 MW)
  2) Some of them at 104 GHz, the rest at 170 GHz

- Key questions:
  - Is EC absorption efficient enough with pure 3rd harmonic (170 GHz) or is 2nd harmonic (104 GHz) needed to pre-heat the plasma up to a temperature where 3rd harmonic becomes efficient?
  - Ohmic and EC-assisted breakdown capabilities (modelling or analysis of present-day devices)

- Involved codes:
  - EC: **GRAY**, **OGRAY**, **REMA**
  - Transport: **ASTRA**, **CRONOS**, **JINTRAC**, **TASK**, **TRANSP**
  - Breakdown analysis: **DINA**

D. Farina, L. Figini



RF waves in plasmas (Current Drive)

- One can calculate RF heating from a hot dielectric tensor of Maxwellian plasmas. However, one cannot obtain current drive by the power absorption since the Maxwell distribution Function is symmetric in velocity space. In addition, the power absorption can be different for non-Maxwellian plasmas.
- Therefore, we should know the changed asymmetric particle distribution by the heating.
- It can be obtained from Vlasov equation with collision(Fokker-Planck equation) in longer time scale than the wave period.

$$\frac{df_s}{dt} = \frac{\partial f_s}{\partial t} + v \cdot \nabla f_s + a \cdot \nabla_v f_s = C(f_s),\ a = \frac{eZ_s}{m_s}\left[E + v \times (B + B_0) \right]$$

$$f_s = F_s(t, r, v) + \tilde{f}_s(t, r, v)$$

$$\frac{dF_s}{dt} = \frac{\partial F_s}{\partial t} + v \cdot \nabla F_s + \frac{eZ_s}{m_s}\left[v \times B_0 \right] \cdot \nabla_v F_s = -\frac{eZ_s}{m_s}\left[E + v \times B \right] \cdot \nabla_v \tilde{f}_s + C(F_s)$$

$$= Q(F_s) + C(F_s)$$

Quasi-linear term by waves

RF waves in plasmas (Current Drive)

- Quasi-linear operator can be represented as follows.

$$Q(F_s) = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left[v_\perp\left(D_{v_\perp v_\perp}\frac{\partial F_s}{\partial v_\perp} + D_{v_\perp v_\parallel}\frac{\partial F_s}{\partial v_\parallel}\right)\right] + \frac{\partial}{\partial v_\parallel}\left(D_{v_\parallel v_\perp}\frac{\partial F_s}{\partial v_\perp} + D_{v_\parallel v_\parallel}\frac{\partial F_s}{\partial v_\parallel}\right)$$

$$D_{v_\perp v_\perp} = \frac{\pi}{2\omega}\left(\frac{Ze}{m_s}\right)^2 \sum_n \delta\left(\frac{\omega - n\Omega_{cs} - k_\parallel v_\parallel}{\omega}\right)\left|d_\perp^{(n)}E\right|^2$$

$$D_{v_\perp v_\parallel} = D_{v_\parallel v_\perp} = \frac{\pi}{2\omega}\left(\frac{Ze}{m_s}\right)^2 \sum_n \delta\left(\frac{\omega - n\Omega_{cs} - k_\parallel v_\parallel}{\omega}\right)\mathrm{Re}\left[d_\perp^{(n)*}E \cdot d_\parallel^{(n)}E\right]$$

$$D_{v_\parallel v_\parallel} = \frac{\pi}{2\omega}\left(\frac{Ze}{m_s}\right)^2 \sum_n \delta\left(\frac{\omega - n\Omega_{cs} - k_\parallel v_\parallel}{\omega}\right)\left|d_\parallel^{(n)}E\right|^2$$

$$d_\perp^{(n)}E = \frac{1}{\sqrt{2}}\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)\left[J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right)E_+ e^{-i\psi} + J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right)E_- e^{i\psi}\right] + \frac{v_\parallel}{v_\perp}\frac{n\Omega_{cs}}{\omega}J_n\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right)E_z$$

$$d_\parallel^{(n)}E = \frac{1}{\sqrt{2}}\frac{k_\parallel v_\perp}{\omega}\left[J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right)E_+ e^{-i\psi} + J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right)E_- e^{i\psi}\right] + \left(1 - \frac{n\Omega_{cs}}{\omega}\right)J_n\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right)E_z$$

RF waves in plasmas (Current Drive)

- Quasi-linear operator can be represented as follows.

$$Q(F_s) = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left[v_\perp \left(D_{v_\perp v_\perp}\frac{\partial F_s}{\partial v_\perp} + D_{v_\perp v_\parallel}\frac{\partial F_s}{\partial v_\parallel}\right)\right] + \frac{\partial}{\partial v_\parallel}\left(D_{v_\parallel v_\perp}\frac{\partial F_s}{\partial v_\perp} + D_{v_\parallel v_\parallel}\frac{\partial F_s}{\partial v_\parallel}\right)$$

$$D_{v_\perp v_\perp} = \frac{\pi}{2\omega}\left(\frac{Ze}{m_s}\right)^2 \sum_n \delta\left(\frac{\omega - n\Omega_{cs} - k_\parallel v_\parallel}{\omega}\right)\left| d_\perp^{(n)} E\right|^2$$

$$D_{v_\perp v_\parallel} = D_{v_\parallel v_\perp} = \frac{\pi}{2\omega}\left(\frac{Ze}{m_s}\right)^2 \sum_n \delta\left(\frac{\omega - n\Omega_{cs} - k_\parallel v_\parallel}{\omega}\right)\mathrm{Re}\left[d_\perp^{(n)*} E \cdot d_\parallel^{(n)} E\right]$$

$$D_{v_\parallel v_\parallel} = \frac{\pi}{2\omega}\left(\frac{Ze}{m_s}\right)^2 \sum_n \delta\left(\frac{\omega - n\Omega_{cs} - k_\parallel v_\parallel}{\omega}\right)\left| d_\parallel^{(n)} E\right|^2$$

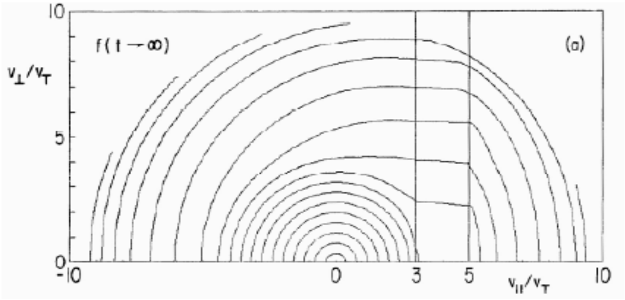
$$d_\perp^{(n)} E = \frac{1}{\sqrt{2}}\left(1 - \frac{k_\parallel v_\parallel}{\omega}\right)\left[J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right) E_+ e^{-i\psi} + J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right) E_- e^{i\psi}\right] + \frac{v_\parallel}{v_\perp}\frac{n\Omega_{cs}}{\omega} J_n\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right) E_z$$

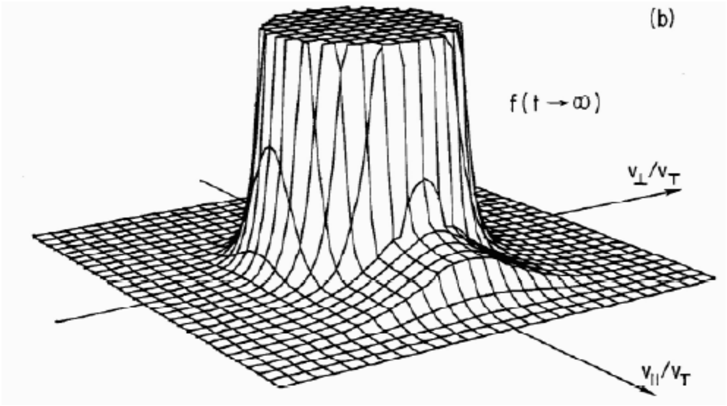
$$d_\parallel^{(n)} E = \frac{1}{\sqrt{2}}\frac{k_\parallel v_\perp}{\omega}\left[J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right) E_+ e^{-i\psi} + J_{n-1}\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right) E_- e^{i\psi}\right] + \left(1 - \frac{n\Omega_{cs}}{\omega}\right) J_n\left(\frac{k_\perp v_\perp}{\Omega_{cs}}\right) E_z$$

RF waves in plasmas (Current Drive)

- Fokker Plank Equation for current drive by Landau damping can be represented as follows. $\frac{\omega}{k_{\parallel}} \sim v_{\parallel}$
  - n = 0,

$$\frac{\partial F_e}{\partial t} - \frac{e}{m_s} E_0 \frac{\partial F_e}{\partial v_{\parallel}} = \frac{\partial}{\partial v_{\parallel}} \left(D_{v_{\parallel} v_{\parallel}} \frac{\partial F_e}{\partial v_{\parallel}} \right) + C(F_e)$$





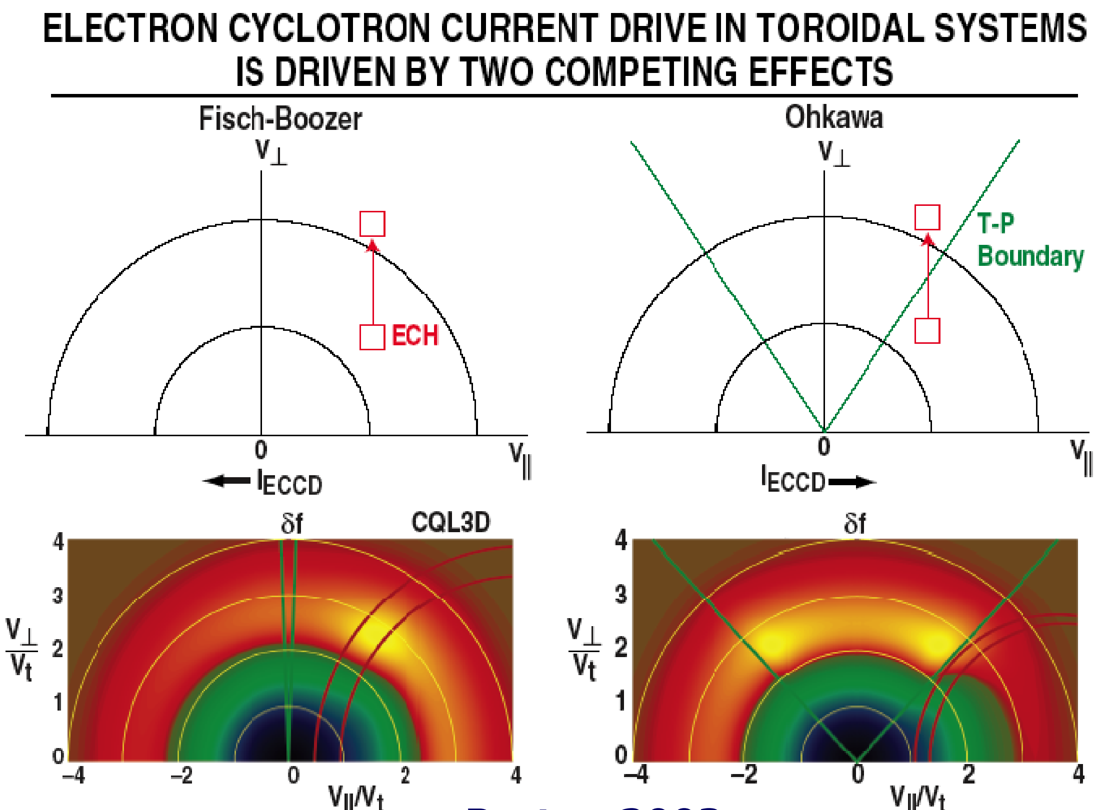
Karney & Fisch, 1979

RF waves in plasmas (Current Drive)

- Generally, current drive is possible if the distribution function is asymmetry in phase space.

  - Minority heating current drive / NB current drive

  - Ohkawa/Fisch-Boozer current drive (ECRF range)



Prater, 2003

RF waves in plasmas (Current Drive)

- Current drive efficiency (rough estimation)

$$\left.\begin{aligned} \Delta E &= n_e m_e v_\parallel \Delta v_\parallel \\ j &= n_e e \Delta v_\parallel \\ p_d &= \Delta E \nu \end{aligned}\right\} \therefore \frac{j}{p_d} = \frac{e}{m_e \nu v_\parallel}$$

$$\frac{j}{p_d} \sim \begin{cases} 1/v_\parallel & : \nu \sim const. \quad for\ \ low\ phase\ velocity\ : ICRF\ range \\ v_\parallel^2 & : \nu \sim v_\parallel^{-3} \quad for\ \ high\ phase\ velocity : LHRF\ range \end{cases}$$

- Current drive efficiency (rigorous estimation)

$$\frac{j}{p_d} = \frac{e}{m_e \nu_0 v_{Te}^3} \frac{2}{(5+Z_{eff})} \frac{\hat{s}\cdot(\partial/\partial \mathbf{v})(v_\parallel v^3)}{\hat{s}\cdot(\partial/\partial \mathbf{v}) v^2}$$

$$= \frac{e}{m_e \nu_0 v_{Te}^3} \frac{2}{(5+Z_{eff})} \frac{v^3 + 3 v v_\parallel^2}{2 v_\parallel} \quad for\ \ parallel\ acceleration$$

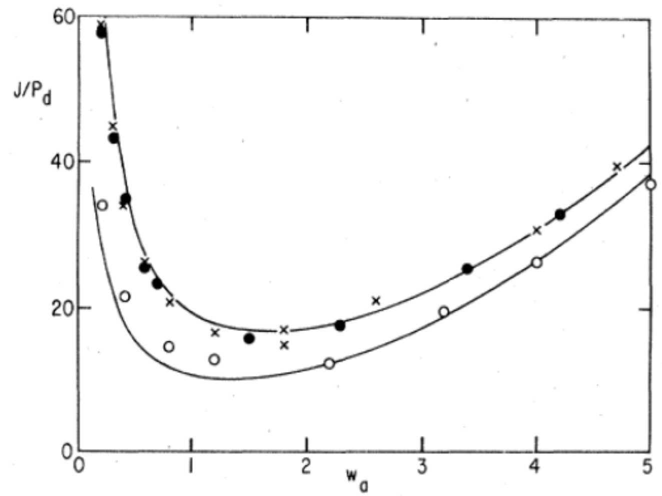


FIG. 21. Normalized J/P_d vs average normalized parallel-phase velocity w_a: ○, Landau damping; ×, magnetic pumping; ●, Alfvén waves in the limit $D_{QL}\to 0$. The solid curves are rough semianalytic fits to the data (Fisch and Karney, 1981).

- Current drive efficiency in practical units and Figure of merit

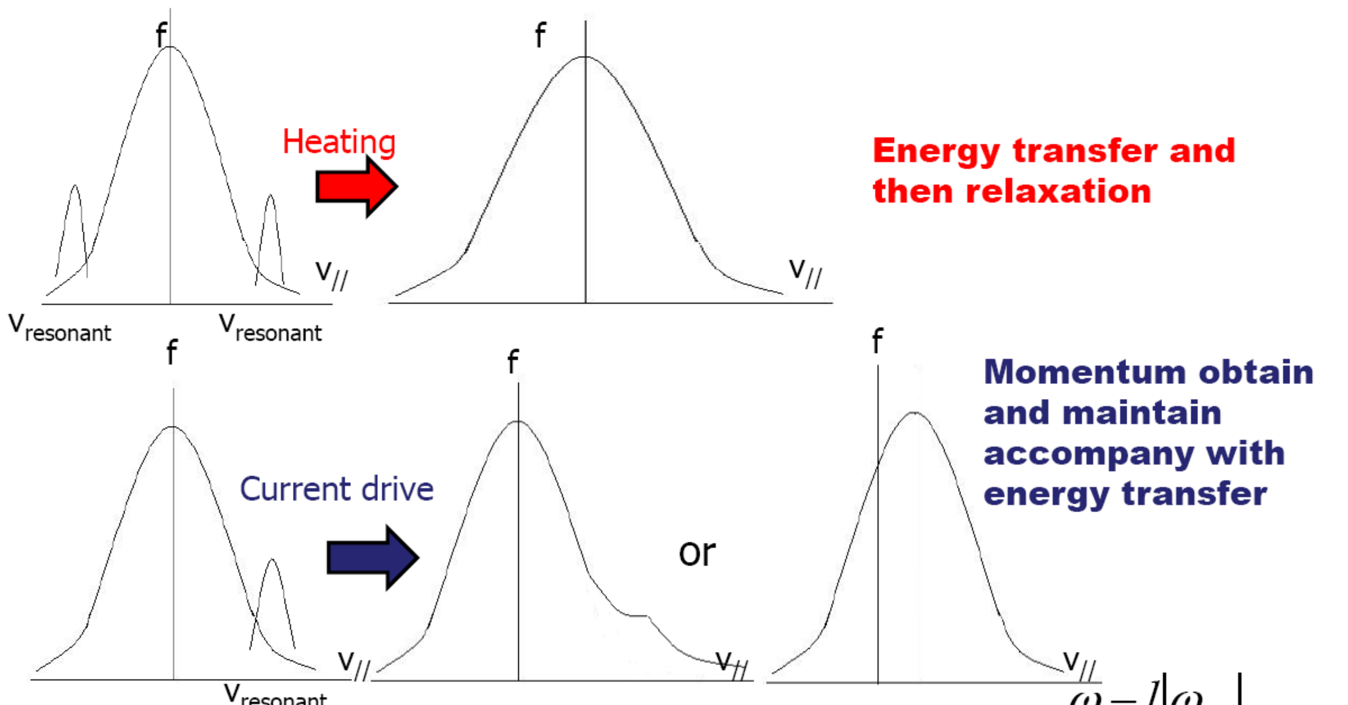
$$\frac{I}{P} = \frac{Aj}{2\pi R A p_d} = 0.061 \frac{T_e}{R n_e^{20} \ln\Lambda} \left(\frac{J}{P_d}\right) [A/W], \qquad \frac{J}{P_d} = \frac{\hat{s}\cdot(\partial/\partial \mathbf{u})(u_\parallel u^3)}{\hat{s}\cdot(\partial/\partial \mathbf{u}) u^2} \qquad u = v/v_{th}$$

$$\eta = \frac{I}{P} R n_e^{20} [A/W/m^2]$$

RF waves in plasmas (Heating)

- What is the difference between Current drive and Heating?

Heating and current drive



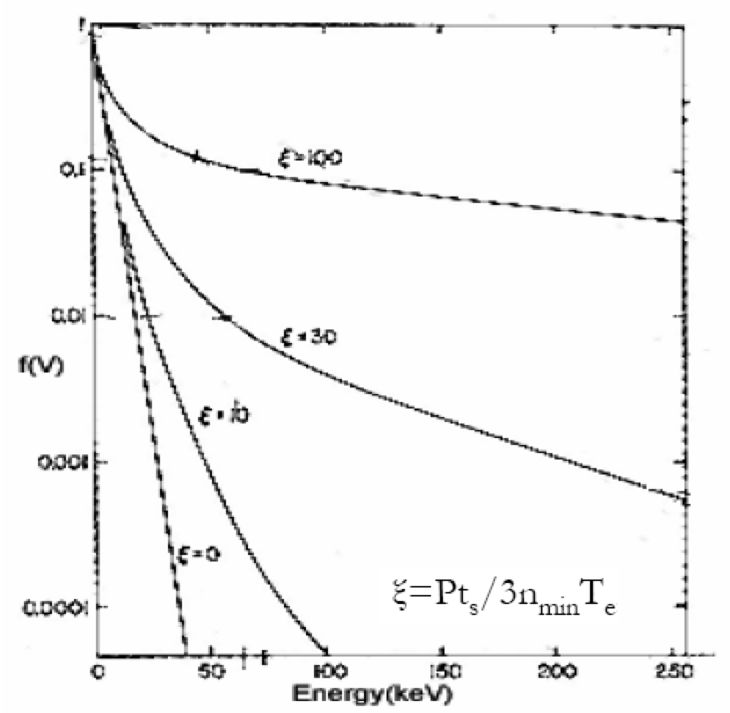
- For current drive, asymmetric N// required

$$v_{//\,resonant} = \frac{\omega - l|\omega_{c\alpha}|}{k_{//}}$$

Z. Gao, "Summer school in KAIST" 2009

RF waves in plasmas (Heating)

- ICRF Harmonic or minority cyclotron heating



$$\ln f(v) = -\frac{E}{T_e(1+\xi)}\left[1+\frac{R_f(T_e - T_f + \xi T_e)}{T_f(1+R_f+\xi)}K(E/E_f)\right]$$

Stix, "Waves in Plasmas" 1992

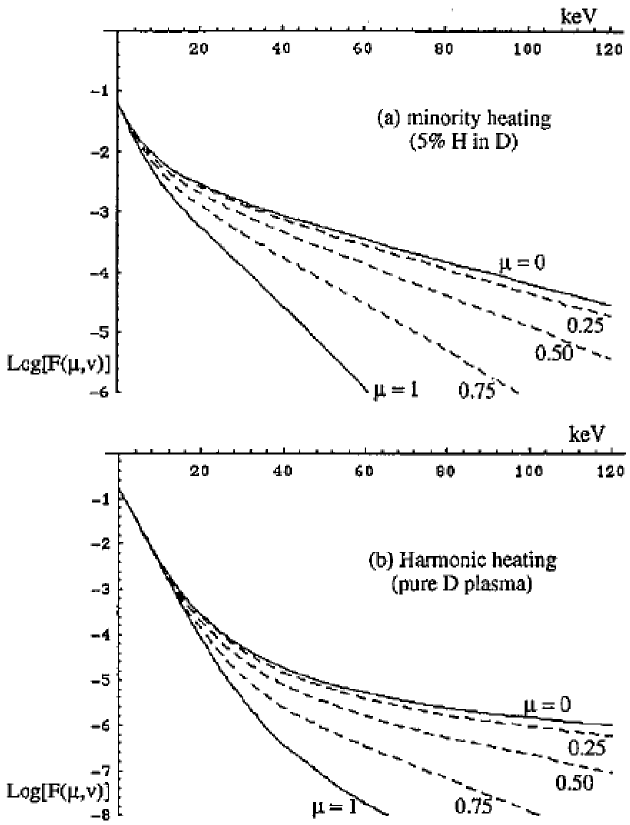


FIG. 43.11 Ion distribution function during ion cyclotron heating, $n_e = 8 \times 10^{13}$ cm^{-3}, $B_0 = 5$ T, 'background' temperature 5 keV, 'linear' power density 0.5 W/cm^3.

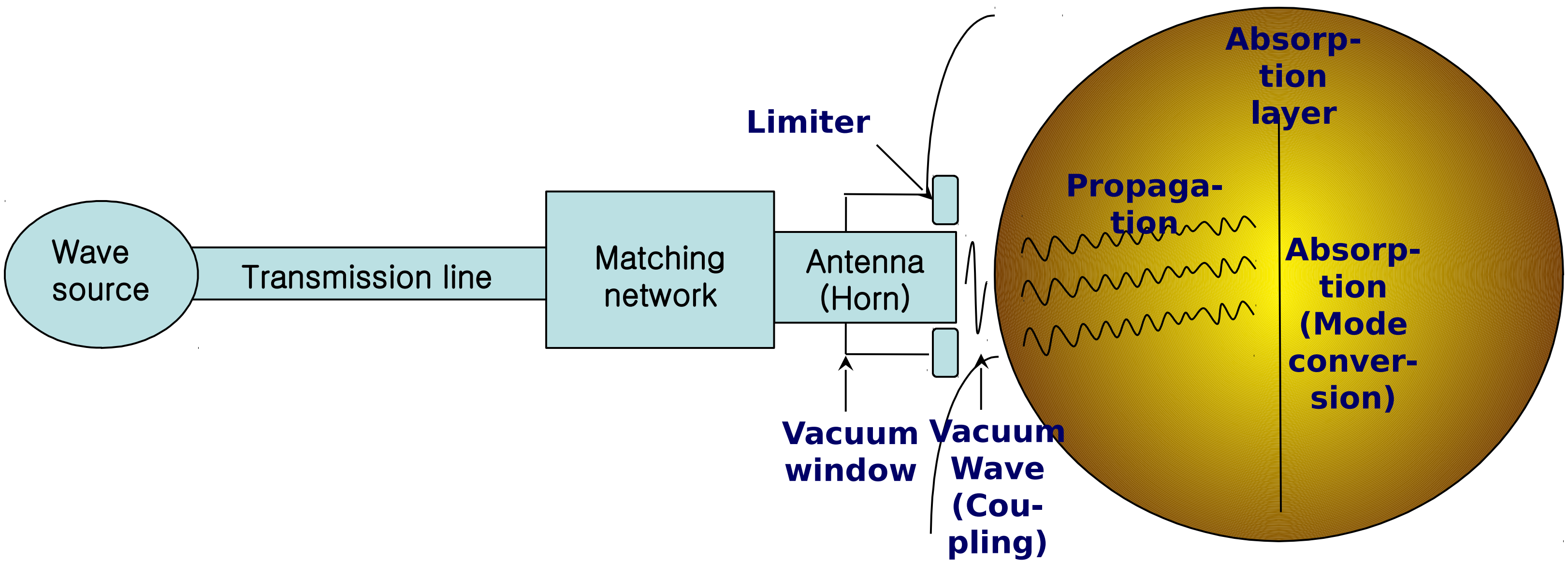
Brambila, "Kinetic theory of plasma waves" 1995

RF waves in plasmas (Summary II)

- General RF heating and current drive can be obtained through quasilinear Fokker-Planck equation.
- Heating and current drive is the result of the increase of high energy population in phase space.

Wave launching, propagation, absorption in fusion plasmas

- RF waves in fusion plasmas is usually launched from LFS(Low Field Side) with different launching structure for each frequency range.
- And it propagates through non-uniform plasmas.
- Finally, the wave power is absorbed near cyclotron resonance layer (harmonic cyclotron damping) and bulk plasmas (Landau damping or TTMP).
- Sometimes, the wave is mode converted into hot electrostatic wave branches(Ion or electron Bernstein waves) and finally absorbed through cyclotron resonance or Landau damping.



Wave launching, propagation, absorption in fusion plasmas (ICRF/LHRF/ECRF)

- RF waves in fusion plasmas is usually launched from LFS(Low Field Side) with different launching structure for each frequency range.

	Sources	Transmission	Coupling	Objectives
ICRF	Tube 25-100MHz 2 MW	Coaxial Line	Antenna (Current Strap)	Localised ion heating. Central CD Sawtooth control
LH	Klystron 1~5GHz 1MW	Waveguide	Waveguide grill	Off-axis CD for SS regimes. AT scenarios. Assisted ramp-up.
ECRF	Gyrotron 50~200GHz 1MW	Waveguide	Horn	Heating. Central CD. MHD control (NTM). Plasma start-up

Wave launching, propagation, absorption in fusion plasmas (ICRF/LHRF/ECRF)

- Full wave and WKB approach

Full Wave Approach

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times B = \mu_0 \left(\varepsilon_0 \frac{\partial E}{\partial t} + J_{rf} \right)$$

1D analytic Approach (Mode Conversion Study)

2D/3D Numerical Simulation (TORIC/AORSA/ …)

WKB Approach (Spatially slowly varying medium)

$$E = E_0 e^{i\Psi},$$

$$B = B_0 e^{i\Psi},$$

$$\Psi = k(\vec{r},t) \cdot \vec{r} - \omega(\vec{r},t)t$$

$$\left(\nabla\Psi = k(\vec{r},t), \ \frac{\partial \Psi}{\partial t} = -\omega(\vec{r},t) \right)$$

$$\frac{d\vec{r}}{dt} = -\frac{\partial H / \partial k}{\partial H / \partial \omega}$$

$$\frac{d\vec{k}}{dt} = -\frac{\partial H / \partial r}{\partial H / \partial \omega}$$

Ray Tracing Equation (TORAY/GENRAY/ …)

Uniform Plasmas

$$E = E_0 e^{i\Psi},$$

$$B = B_0 e^{i\Psi},$$

$$\Psi = \vec{k} \cdot r - \omega t$$

$$N \times N \times E_0 = \overleftrightarrow{\varepsilon}_c E_0$$

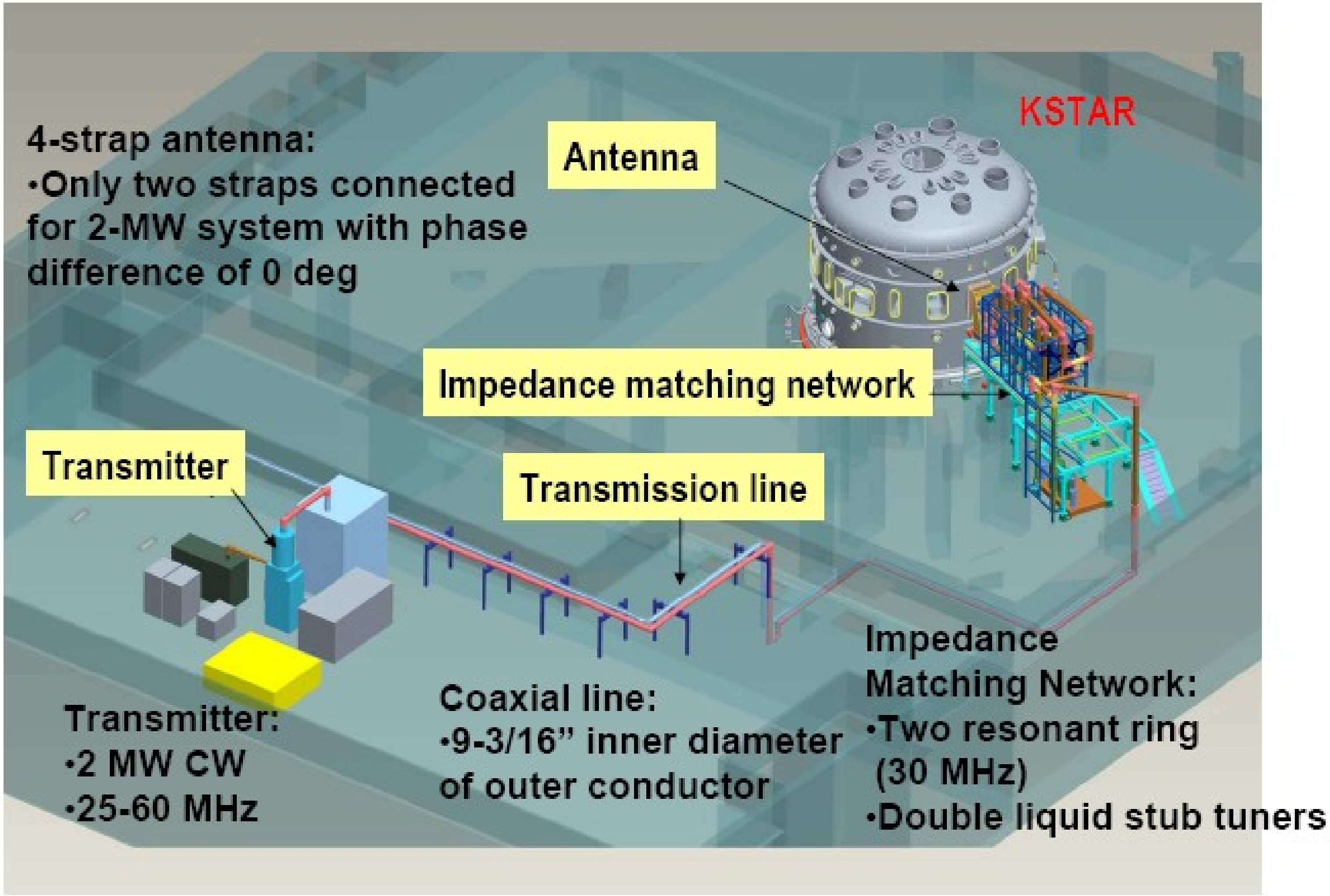
$$(\overleftrightarrow{N}^2 - \overleftrightarrow{\varepsilon}_c) E_0 = 0$$

$$H \equiv \det(\overleftrightarrow{N}^2 - \overleftrightarrow{\varepsilon}_c) = 0$$

Dispersion Relation

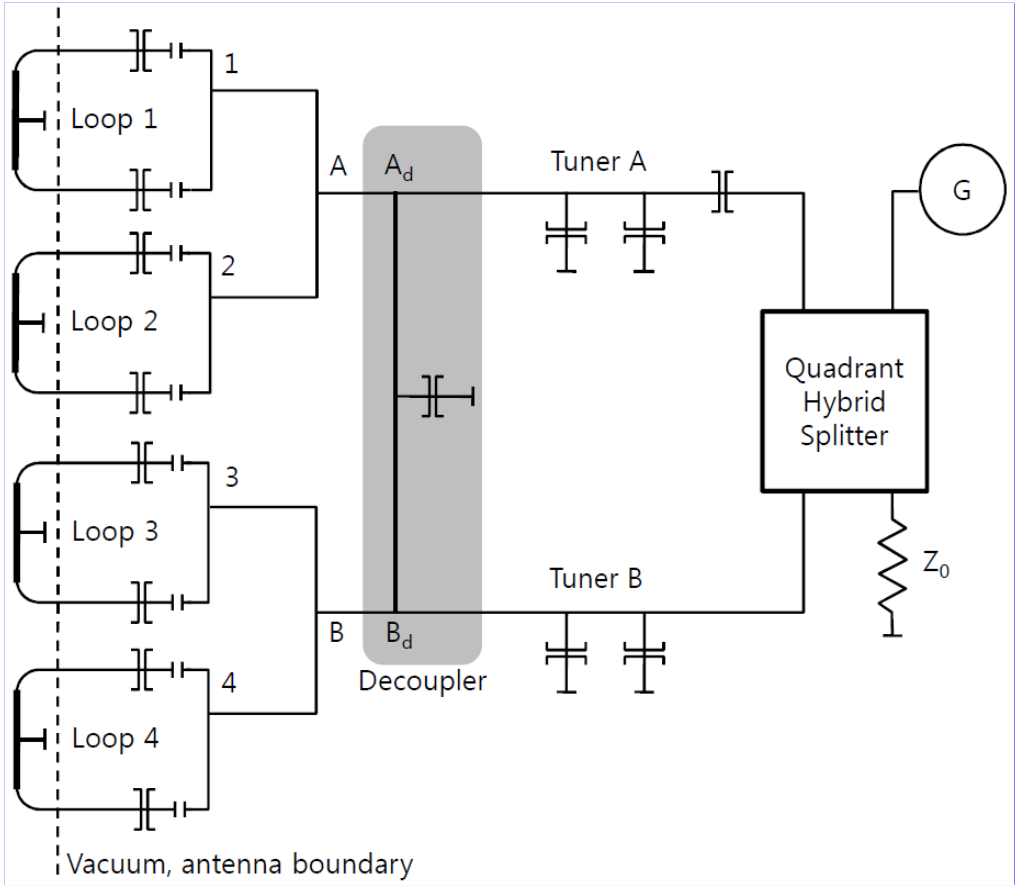
Wave launching, propagation, absorption in fusion plasmas (ICRF)

- ICRF launching and Transmission Coupling System in KSTAR



Wave launching, propagation, absorption in fusion plasmas (ICRF)

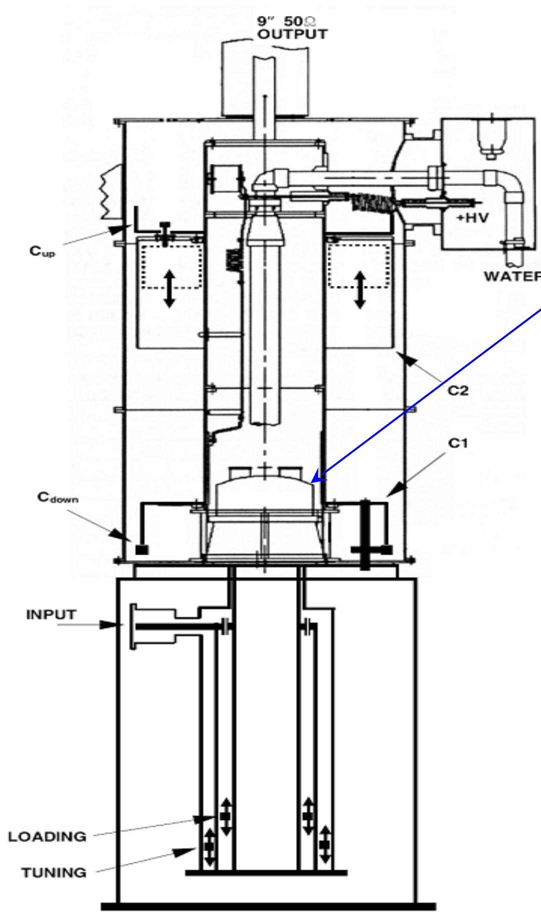
- ICRF launching and Transmission Coupling System in KSTAR



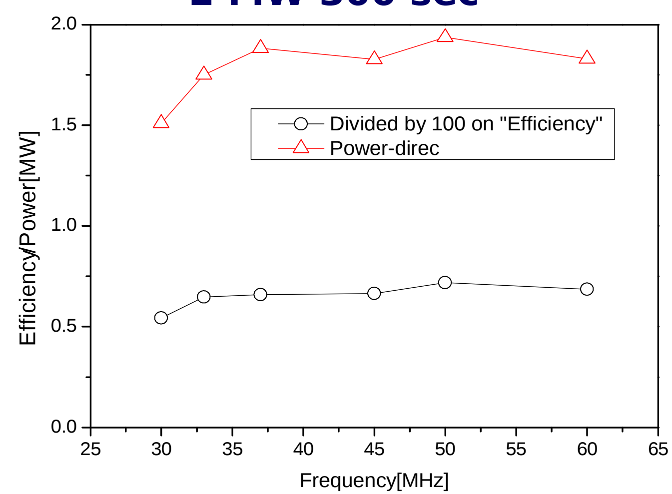
Schematic Resonant loop/matching system

Wave launching, propagation, absorption in fusion plasmas (ICRF)

- ICRF wave generator: Transmitter (Tetrode tube)



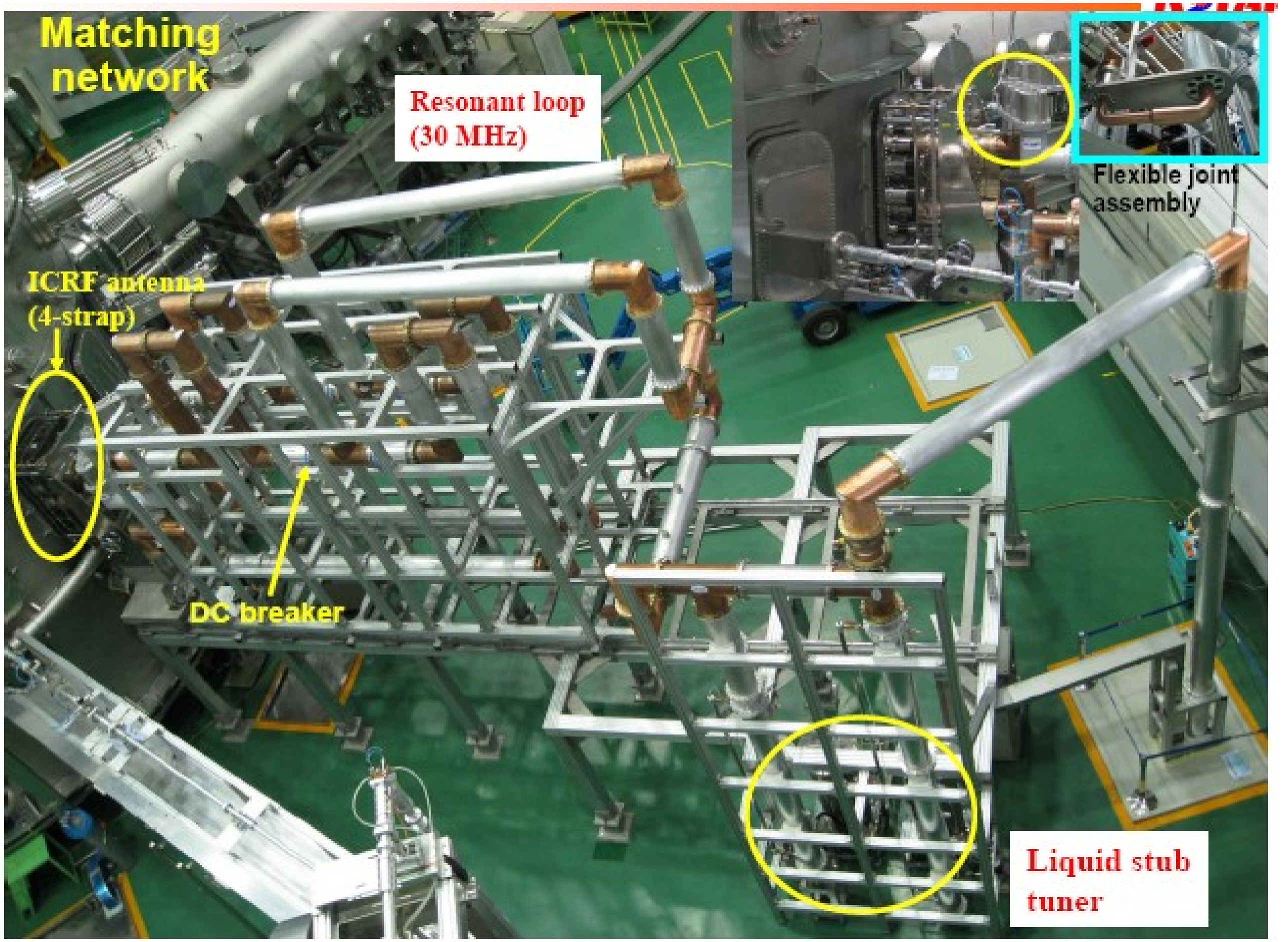
- **Tetrode (4CM2500KG)**
- **20~60 MHz**
- **2 MW 300 sec**



Transmitter FPA(Final Power Amplifier)

Wave launching, propagation, absorption in fusion plasmas (ICRF)

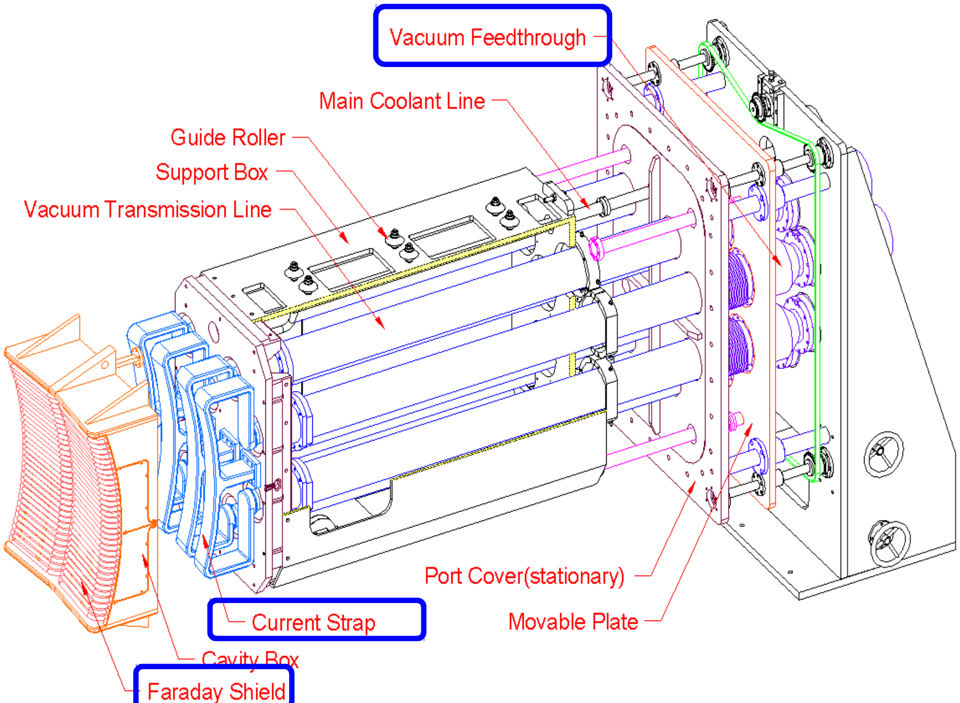
- ICRF Resonant loop and Matching System



KSTAR Resonant loop and matching system

Wave launching, propagation, absorption in fusion plasmas (ICRF)

- ICRF launcher: Antenna



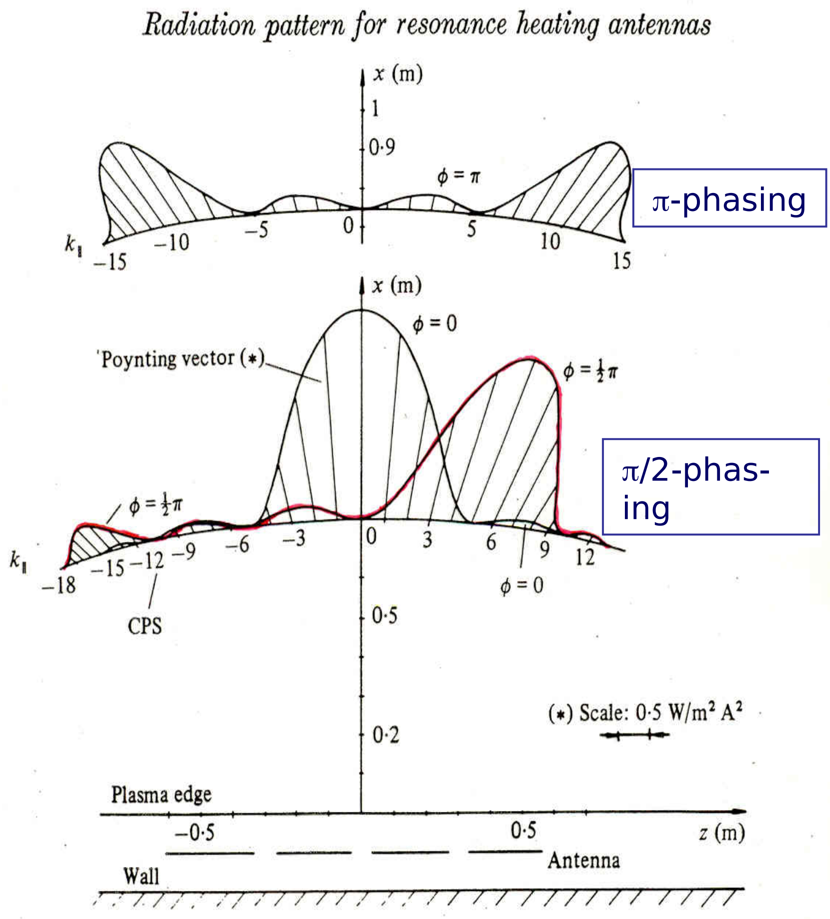
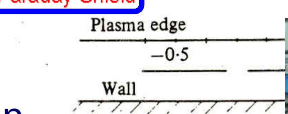
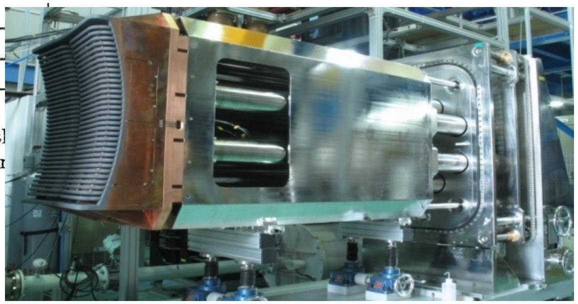


FIGURE 4. Distribution of the Poynting vector on a CPS in uniform plasma for an antenna array



- 4 strap
- 0-0-π-π : Heating
- 0-0-π/2-π/2 : Current



Wave launching, propagation, absorption in fusion plasmas (ICRF)

- ICRF Antenna
  - Electric field is perpendicular to magnetic field in ICRF fast wave.
  - Stray Ez field is screened by Faraday shield.

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{i(N_\parallel^2 - S)}{D} \rightarrow -\frac{iS}{D} \qquad N_\perp^2 = -\frac{(N_\parallel^2 - R)(N_\parallel^2 - L)}{(N_\parallel^2 - S)}$$

$$\frac{E_z}{E_x} = \frac{N_\parallel N_\perp}{N_\perp^2 - P} \rightarrow 0$$

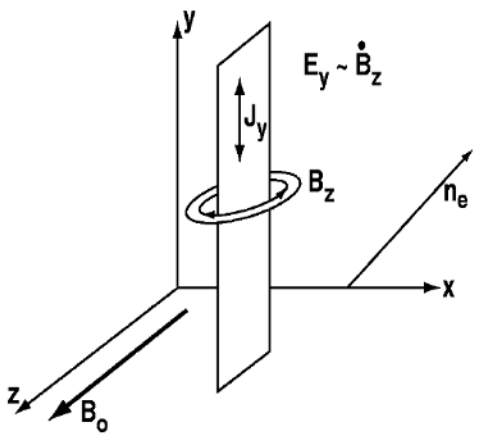
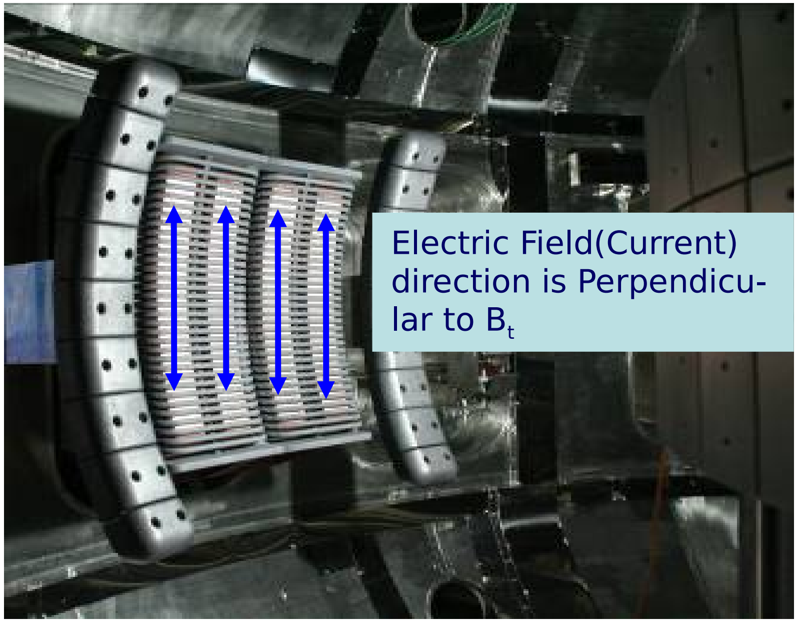


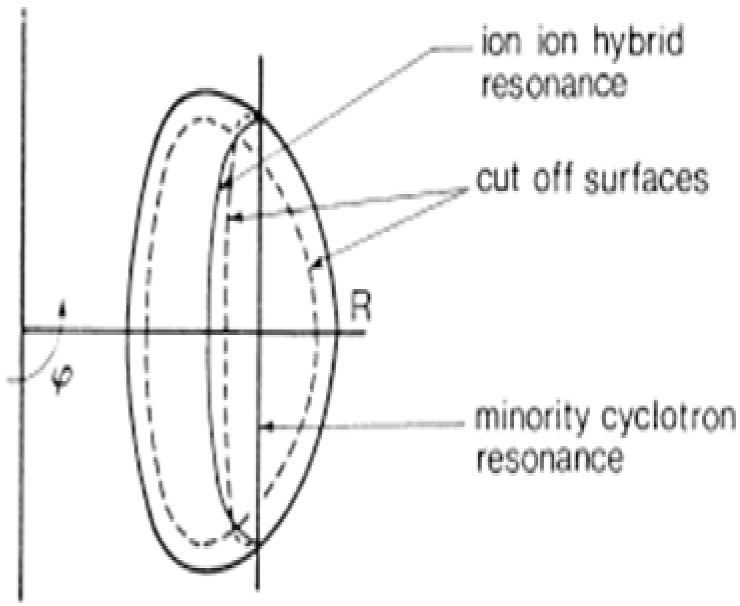
FIG. 3. Geometry of an inductive coupling element (''loop antenna'') used for exciting the fast wave in the ICRF.



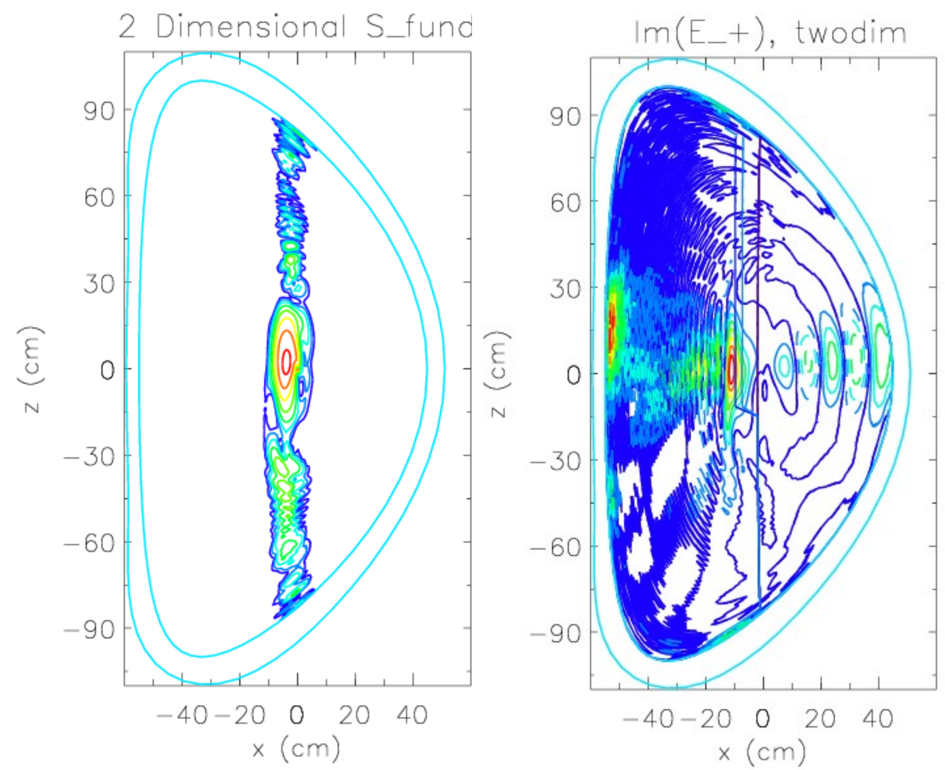
Wave launching, propagation, absorption in fusion plasmas (ICRF)

- ICRF FW propagation and absorption **(Fundamental Minority Heating**)

  - ICRF fast wave wavelength is comparable to system size. Therefore, full wave approach is required.



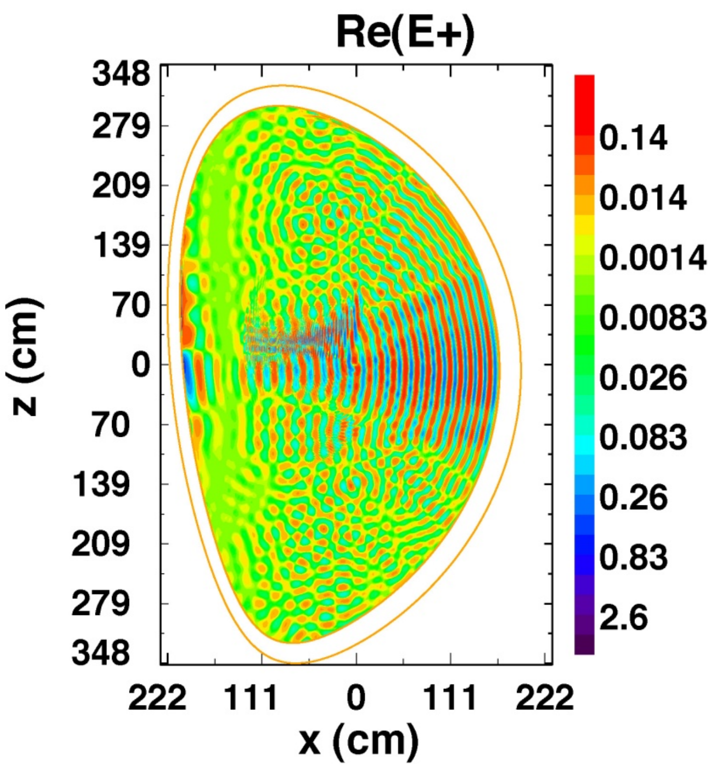
Cut-off/Resonances in minority heating scheme



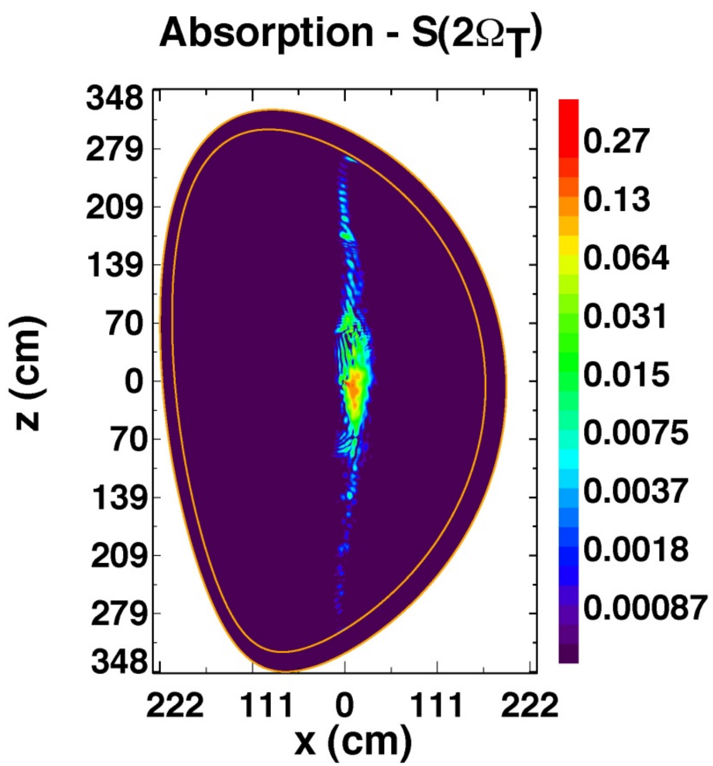
D(H) Minority Heating Scheme in KSTAR
Wang, 2009

Wave launching, propagation, absorption in fusion plasmas (ICRF)

- ICRF FW propagation and absorption (**Second Harmonic Heating**)



LHP wave field of T 2nd Harmonic Heating in ITER



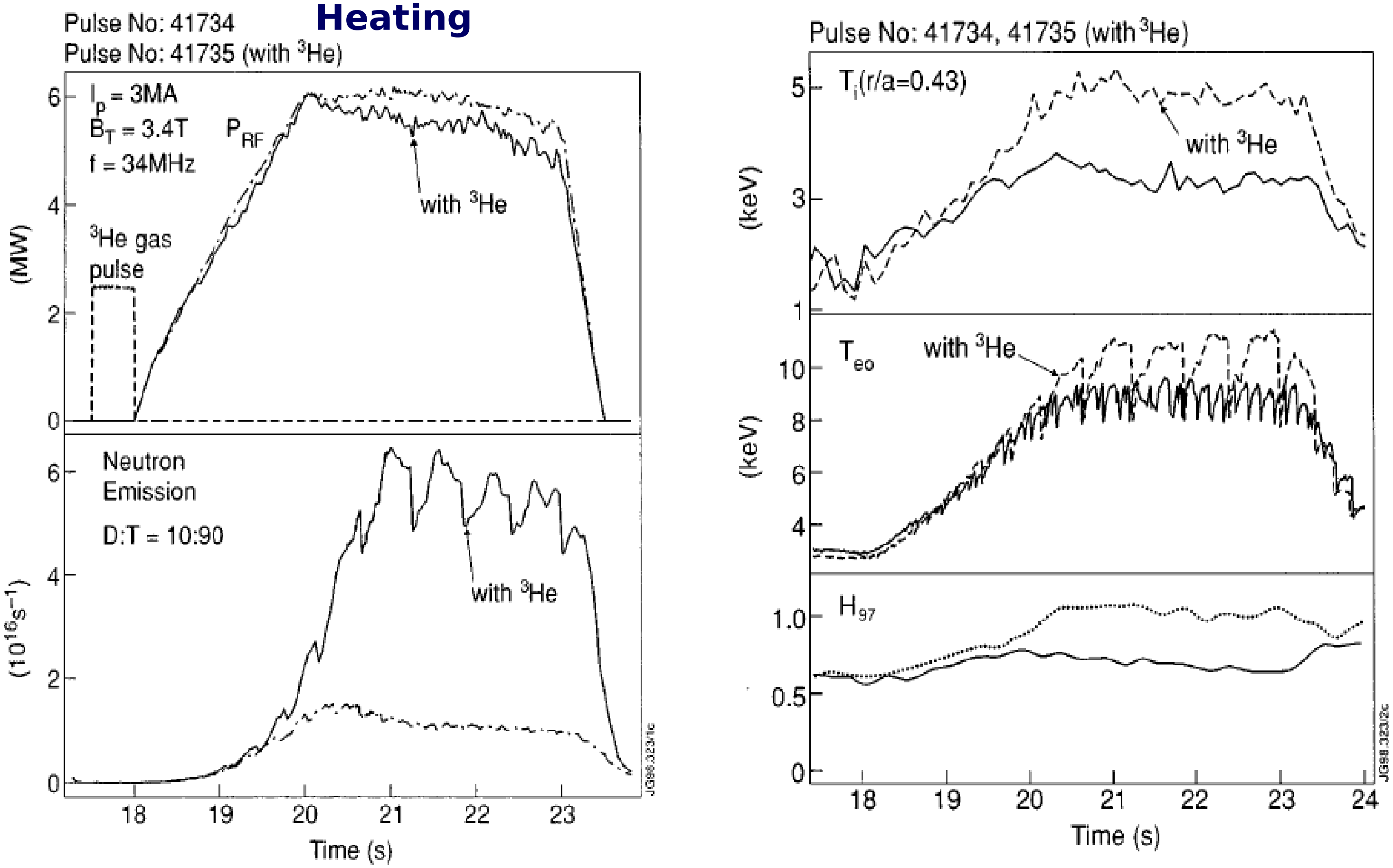
T power absorption profile

D. B. Batchelor, PAC, 2005

Wave launching, propagation, absorption in fusion plasmas (ICRF)

- Experimental results

T 2nd Harmonic + He3 minority Heating

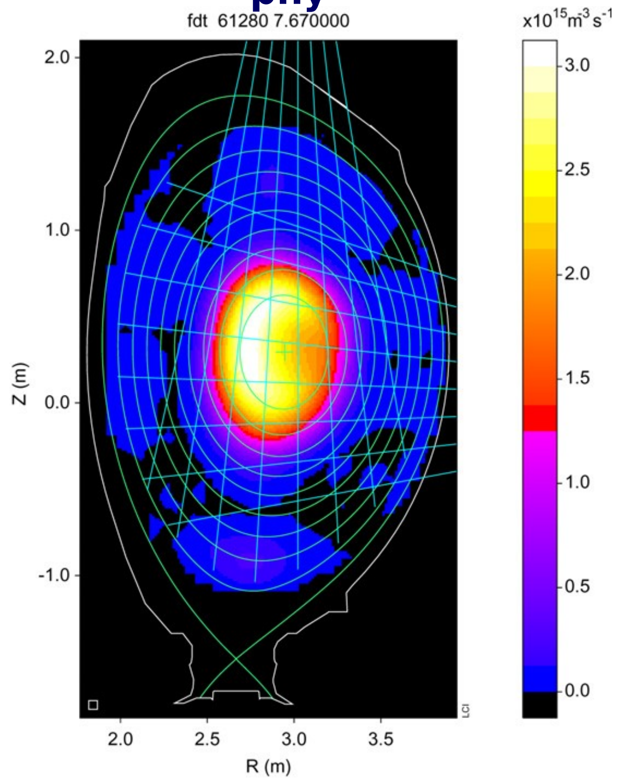


JET[Start et al. 1999]

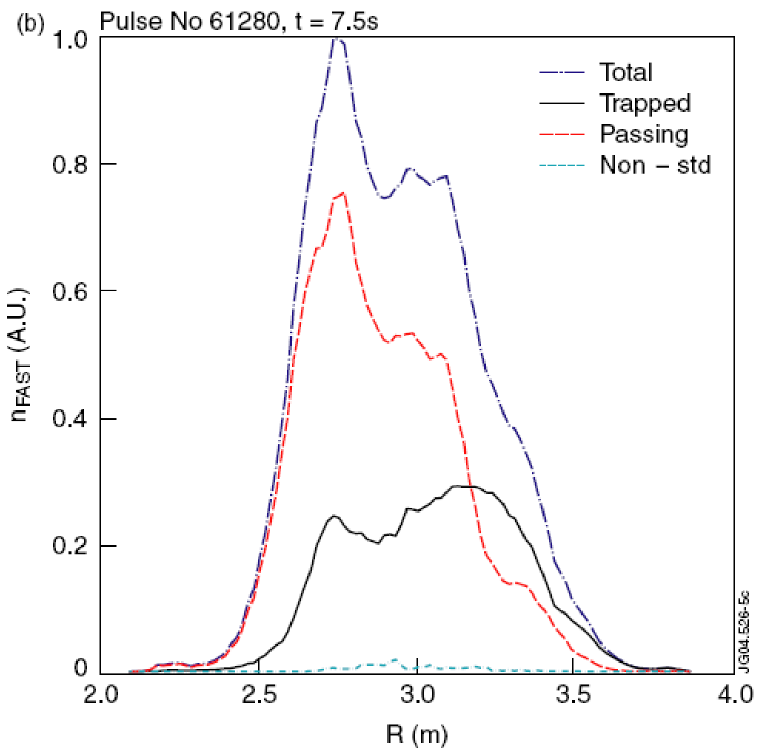
Wave launching, propagation, absorption in fusion plasmas (ICRF)

- Experimental results

Neutron Thomography



SELFO simulation

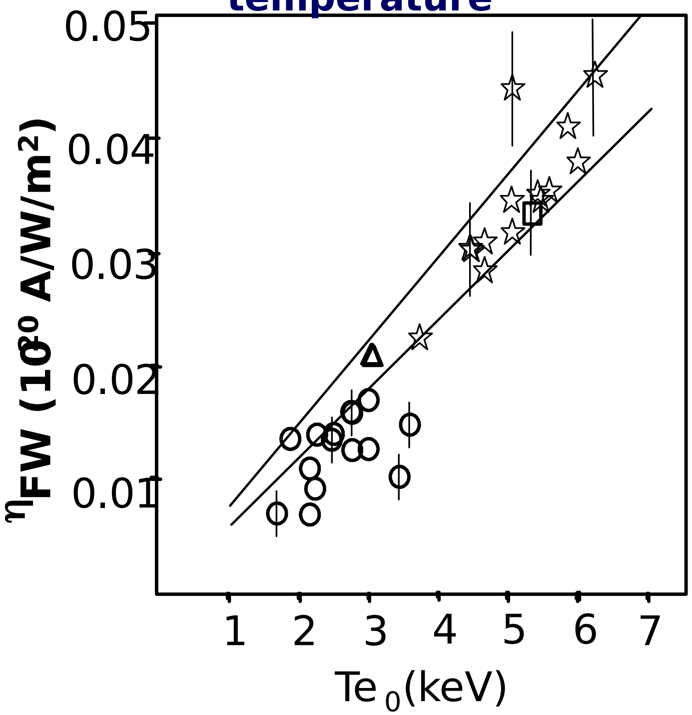


JET [Lamalle et al. 2006]

Wave launching, propagation, absorption in fusion plasmas (ICRF)

- Experimental results (Current drive)

Figure of merit of fast wave current drive versus central temperature



ITER Physics Basis 1999

o : L-mode in DIII-D.

Δ : L-mode in Tore Supra.

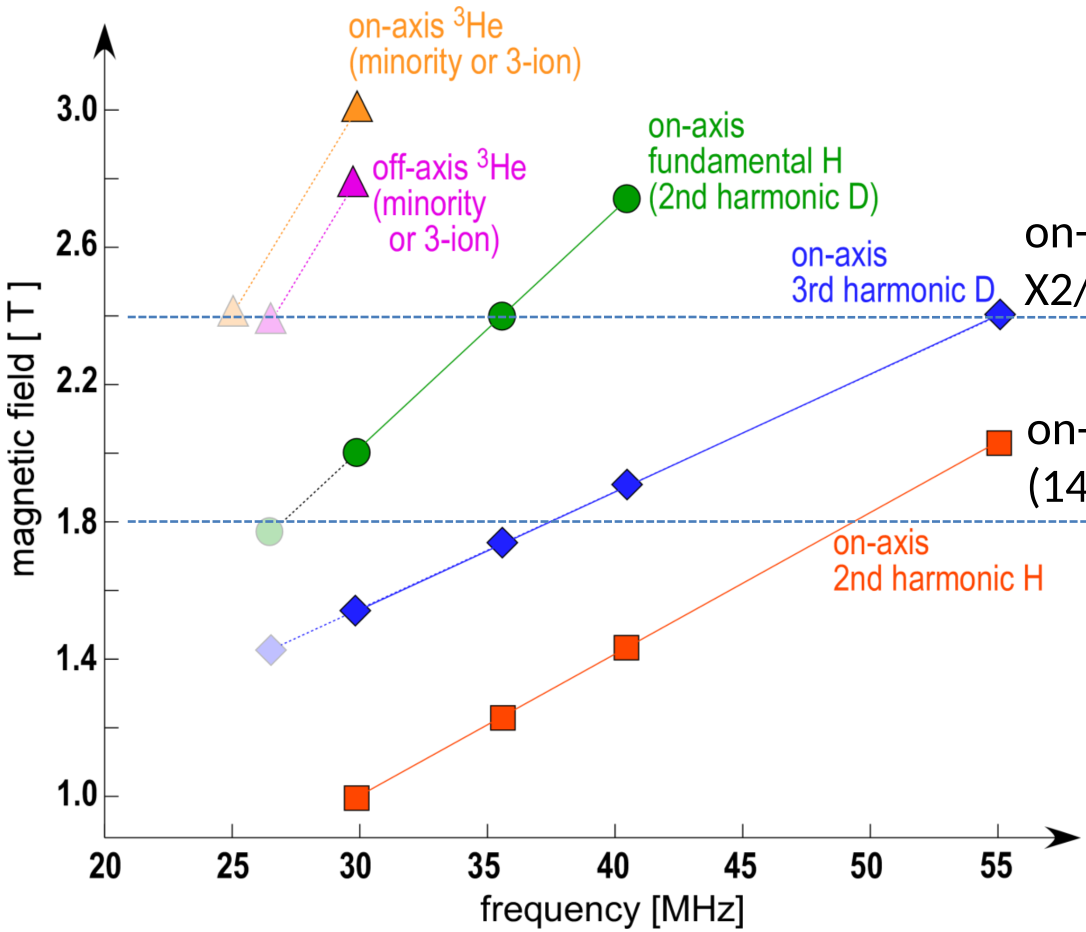
• : VH-mode in DIII-D.

✶ : NCS L-mode in DIII-D.

- : lower and upper bounds of the simulations

(RT code CURRAY/ FW code ALCYON)

ICRF frequencies / heating schemes

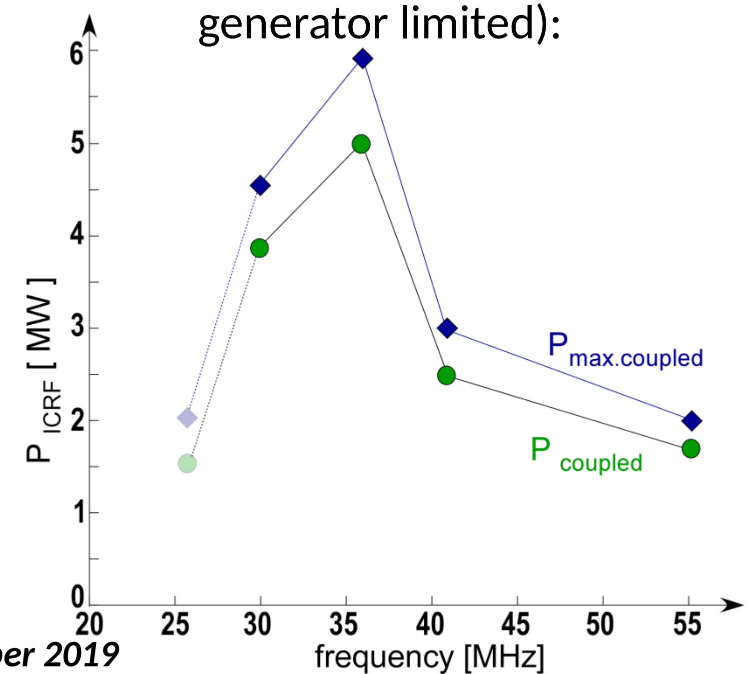


on-axis ECRH X2/O2 (140 GHz)

on-axis ECRH X3 (140 GHz)

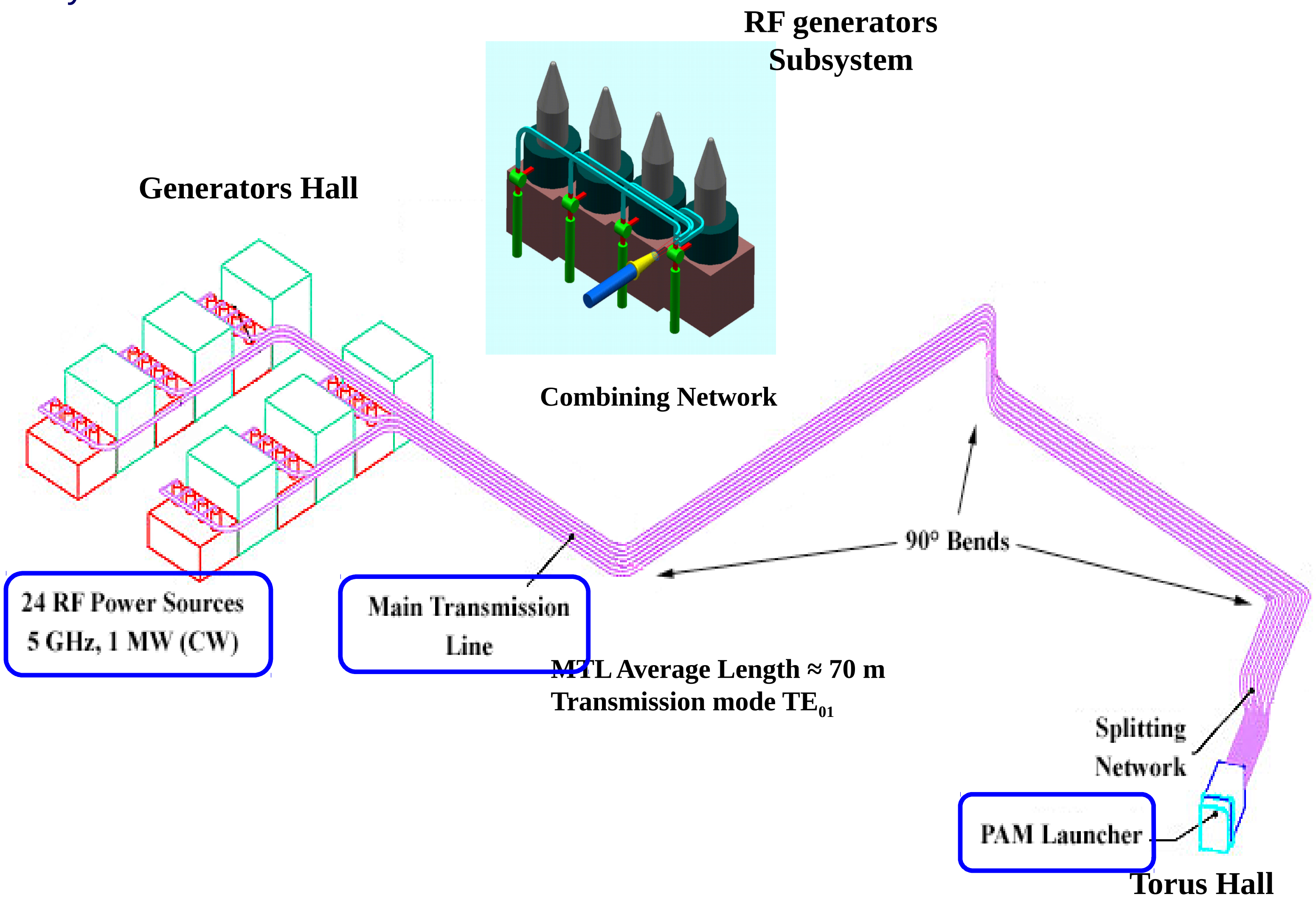
currently available frequencies

Currently available power (if generator limited):



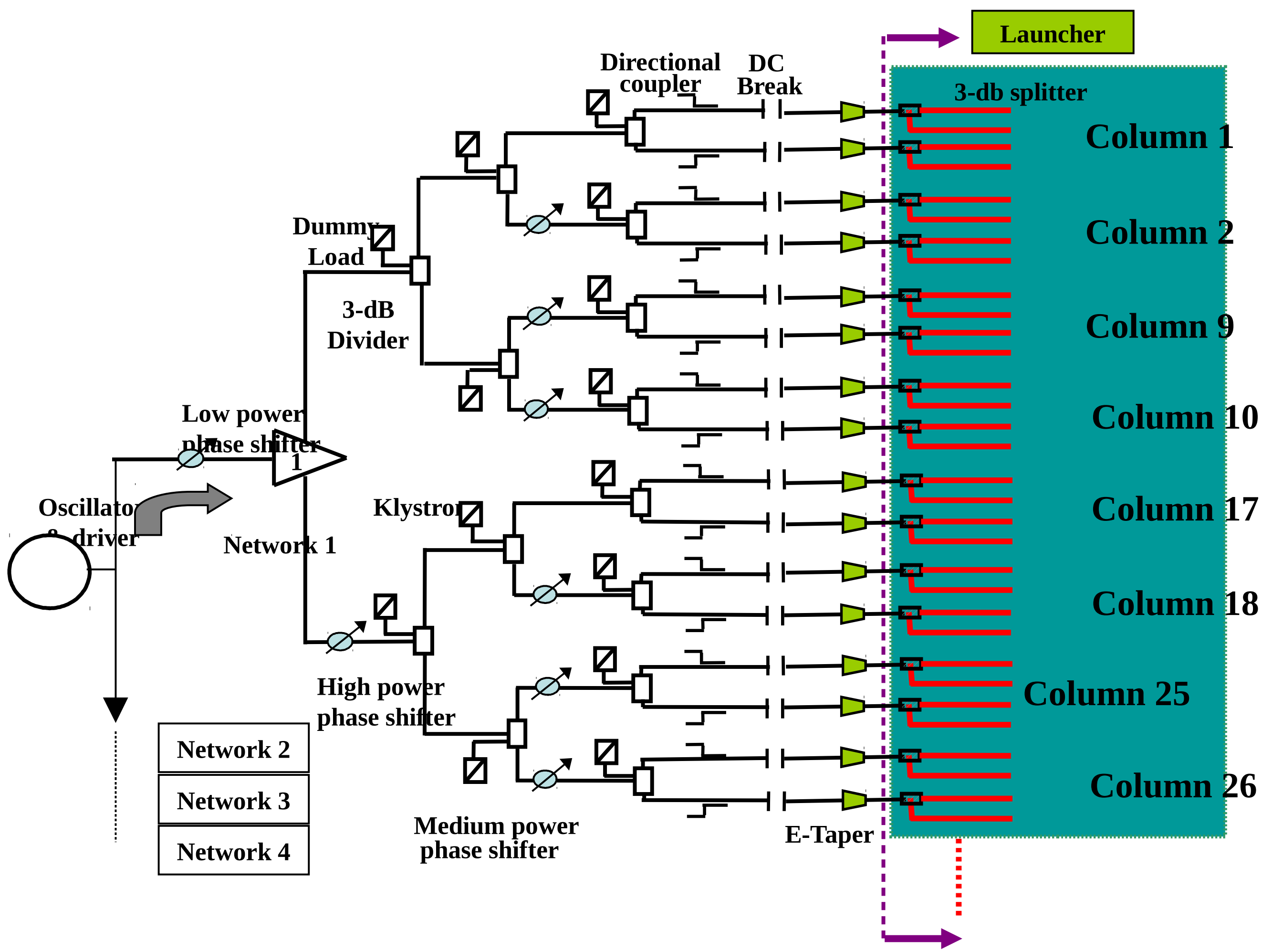
Wave launching, propagation, absorption in fusion plasmas (LHRF)

- LHRF System for ITER



Wave launching, propagation, absorption in fusion plasmas (LHRF)

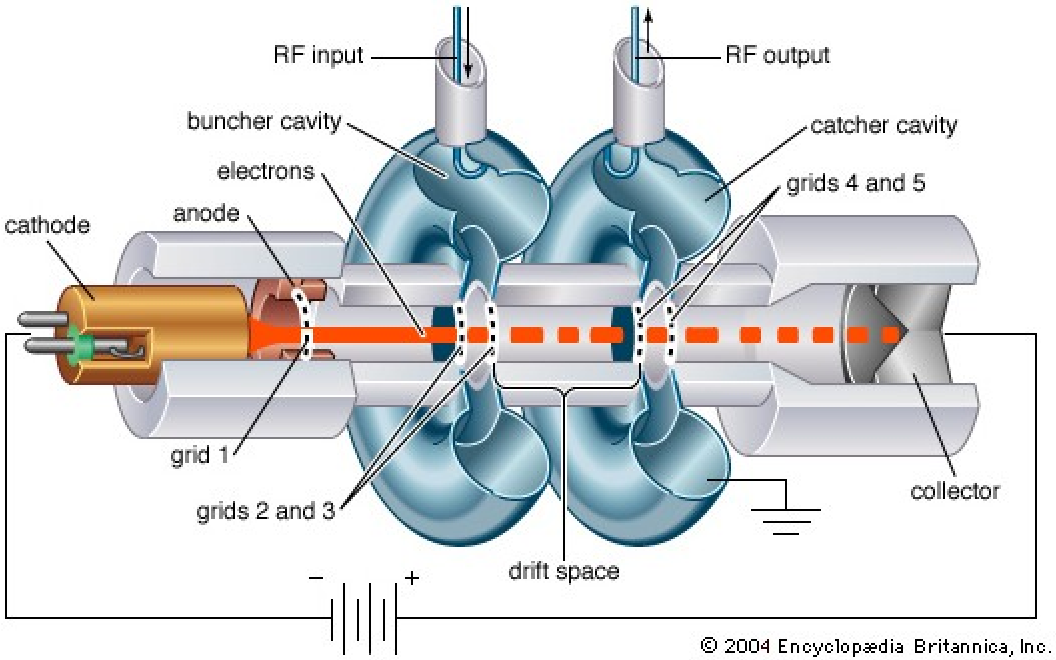
- LHRF System Schematic



Wave launching, propagation, absorption in fusion plasmas (LHRF)

- LHRF Sources (Klystron)

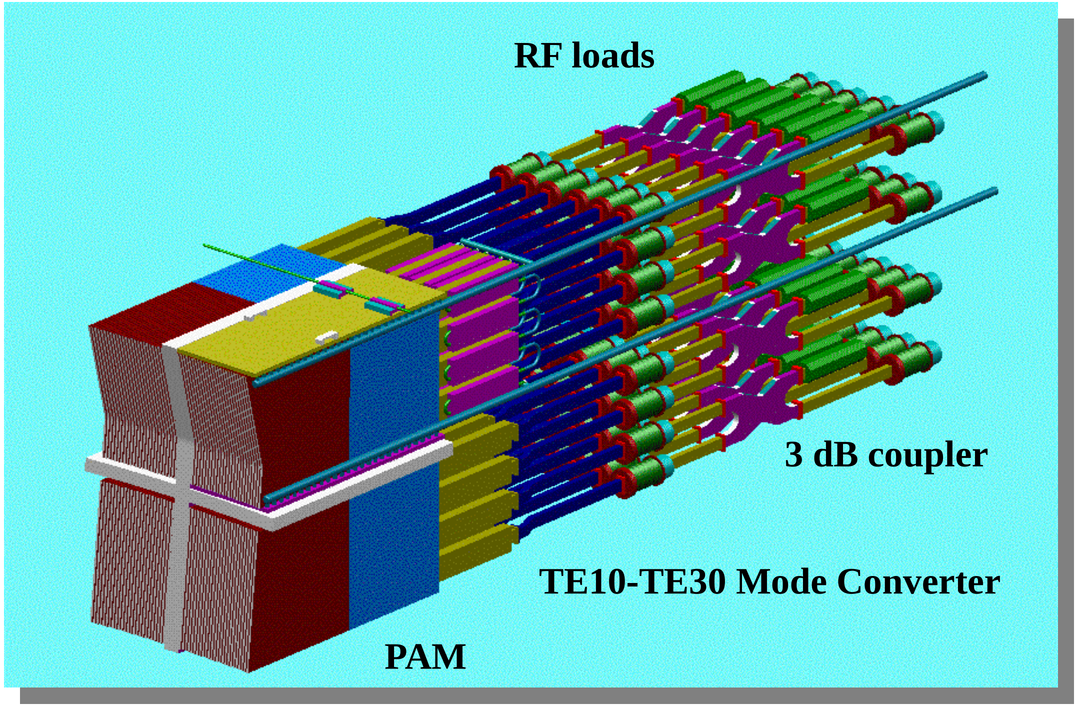
500 kW klystron for ITER

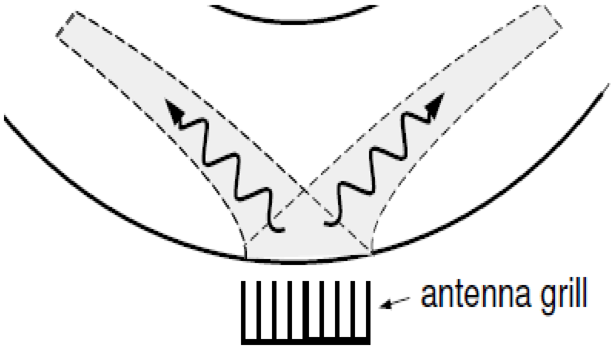


Schematic of klystron structure

Wave launching, propagation, absorption in fusion plasmas (LHRF)

- LHRF Launcher: Waveguide grill

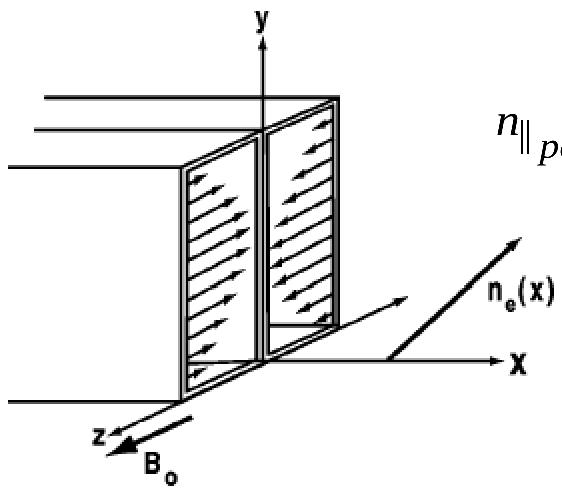




LHRF launcher for ITER

Wave launching, propagation, absorption in fusion plasmas (LHRF)

- LHRF SW Launcher & Accessibility condition



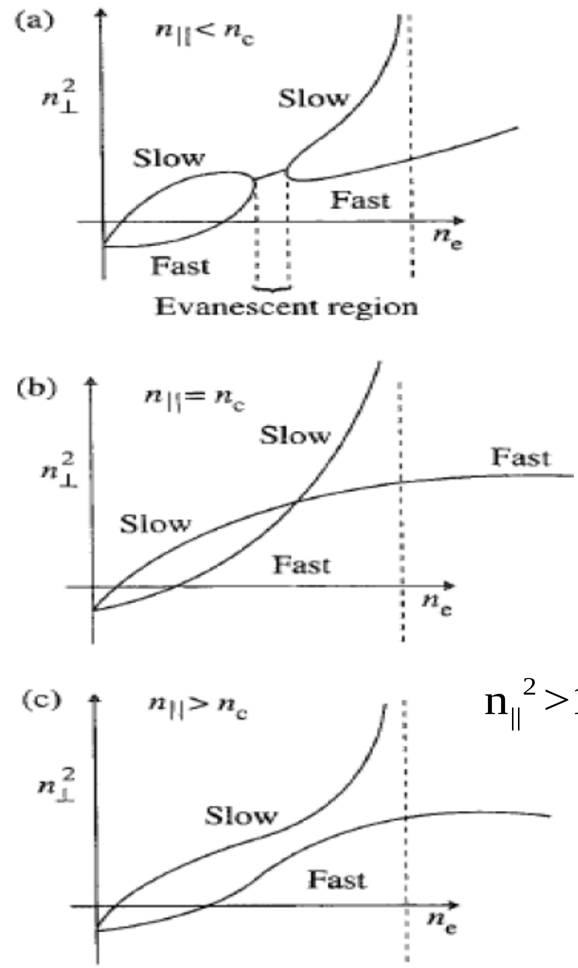
$$n_{\parallel\,peak} \approx \frac{c}{\omega \Delta z}\Delta\phi$$

FIG. 6. Geometry of a phased array of open-ended waveguides used to excite lower hybrid waves in the LHRF.

$$\frac{E_y}{E_x} = \frac{iD}{N^2 - S} = \frac{iSD}{(N_\parallel^2 - S)(S - P)} \to -\frac{iD}{S}$$

$$\frac{E_z}{E_x} = \frac{N_\parallel N_\perp}{N_\perp^2 - P} = -\frac{SN_\perp}{PN_\parallel} \to \pm\infty$$

$$N_\perp^2 = -\frac{P(N_\parallel^2 - S)}{S}$$



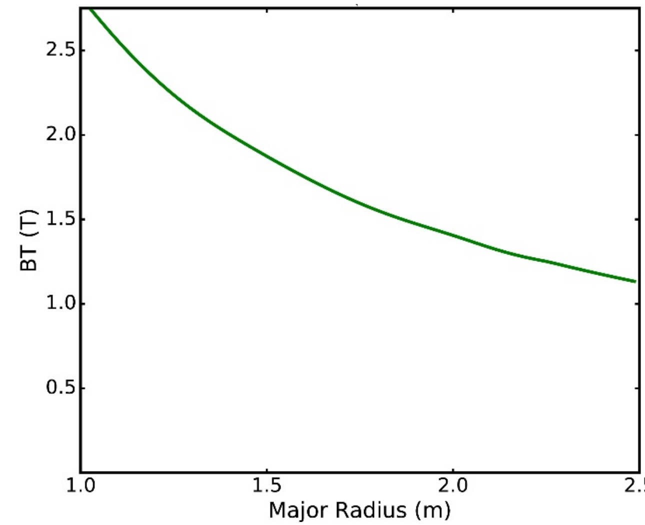
$$n_\parallel^2 > 1 + \left.\frac{\omega_{pe}^2}{\omega_{ce}^2}\right|_{LH}$$

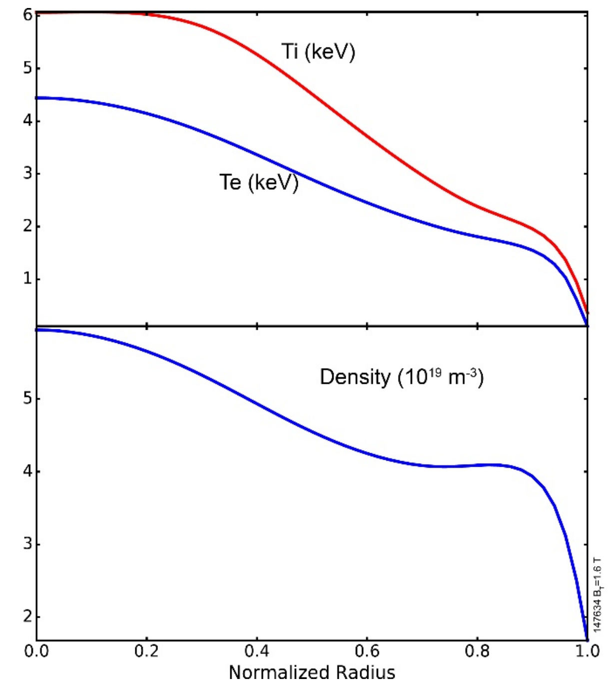
Wesson, Tokamaks, 2007

HFS Launch Improves RF Core Physics

- **Higher local B field allows wave accessibility at smaller n||**
  - Wave accessibility: $n_{||acc} \sim \sqrt{n_e}/B$
- **Smaller n|| → more absorption in the core**
  - Wave absorption: $n_{||abs} \sim \sqrt{30/T_e}$
- **Smaller n|| → higher current drive efficiency**
  - Current drive efficiency $\propto 1/n_{||}^2$

Higher local B field allows wave accessibility at smaller n||
Wave accessibility:
Smaller n|| ⊠ more absorption in the core
Wave absorption:
Smaller n|| ⊠ higher current drive efficiency
Current drive efficiency





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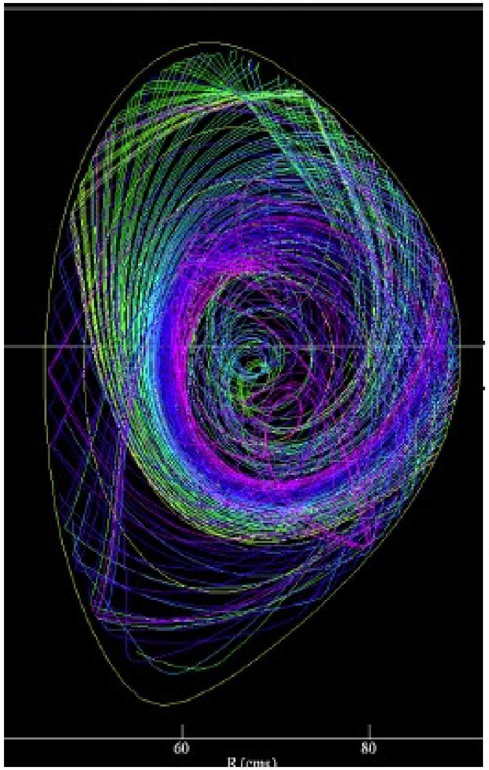
Wave launching, propagation, absorption in fusion plasmas (LHRF)

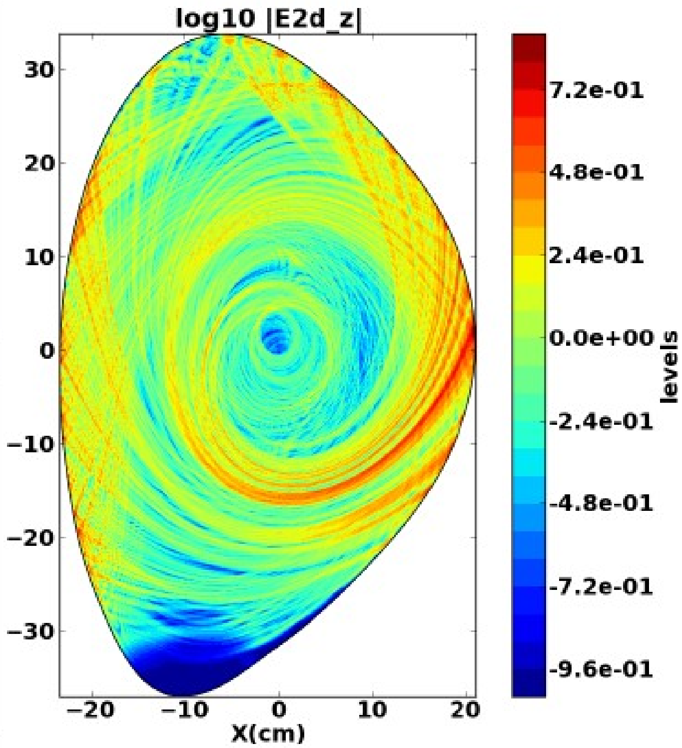
- Propagation & Absorption

GENRAY

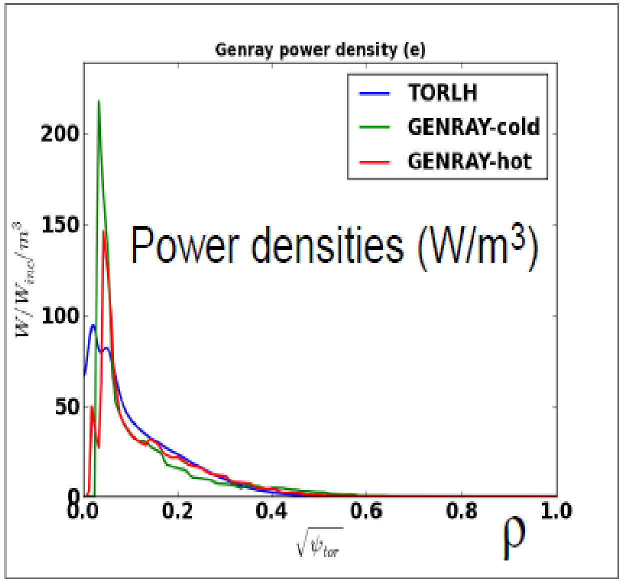


Petrov, CompX

TORLH



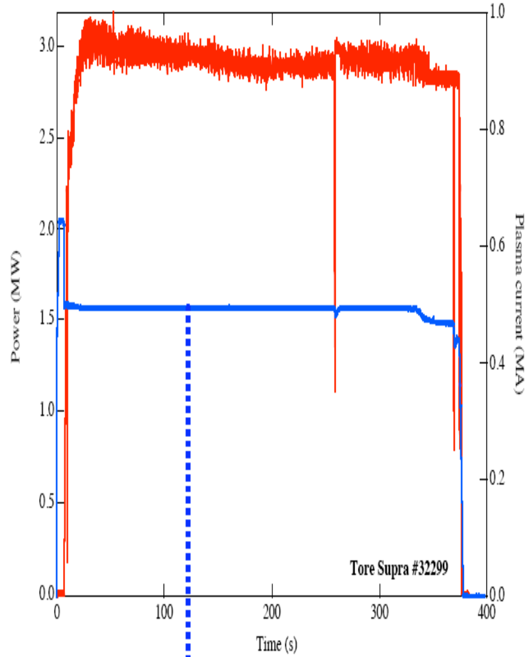
J. C. Wright, POP, 2009



Petrov, CompX

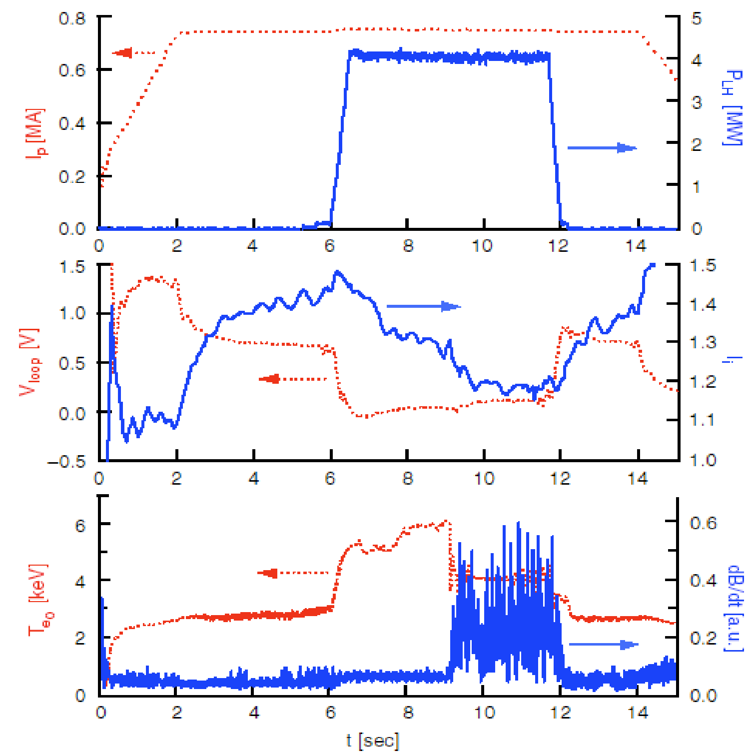
Wave launching, propagation, absorption in fusion plasmas (LHRF)

- Experimental results (Full non-inductive current drive)



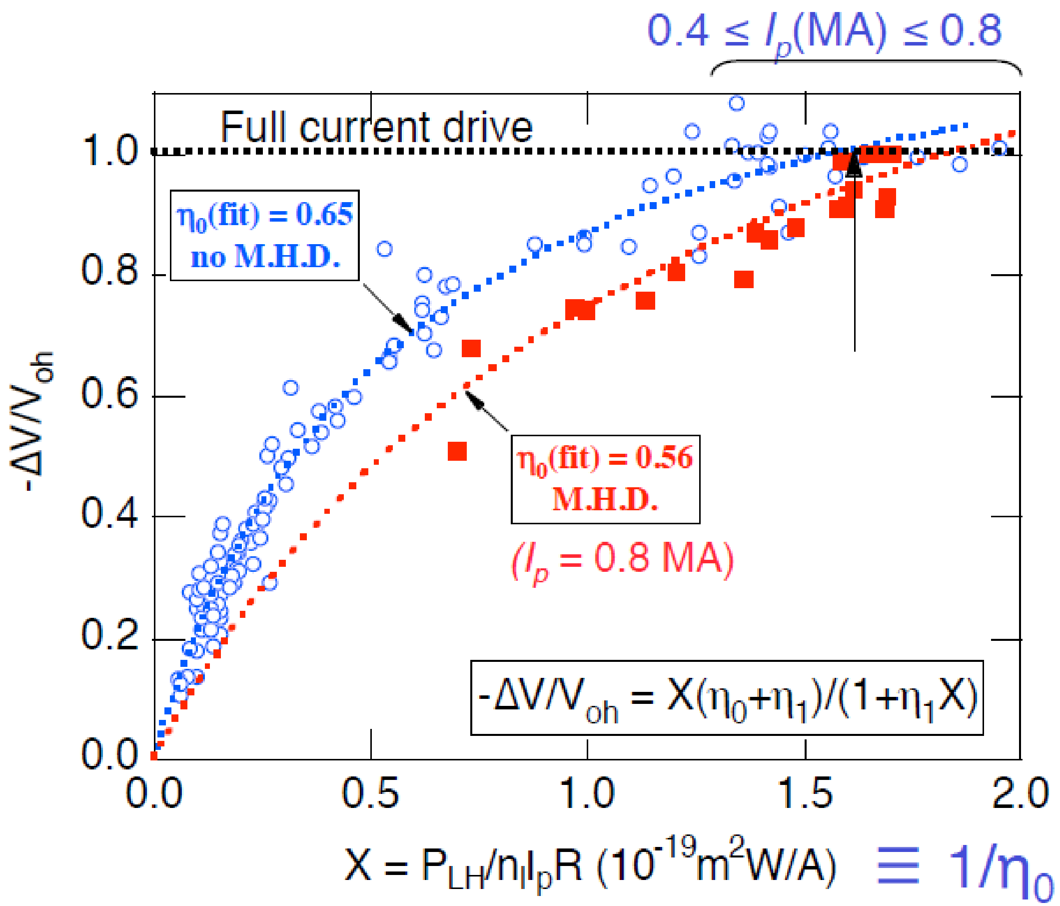
- Full LHCD (6 min.)
- 2 antennas
- $n_{\|0} = 1.7 \pm 0.2$
- $P_{lh} = 3$ MW
- directivity: 0.6 & 0.7

$I_p \approx 500$ kA



Y. Peysson, Fusion summer school in KAIST, 2009

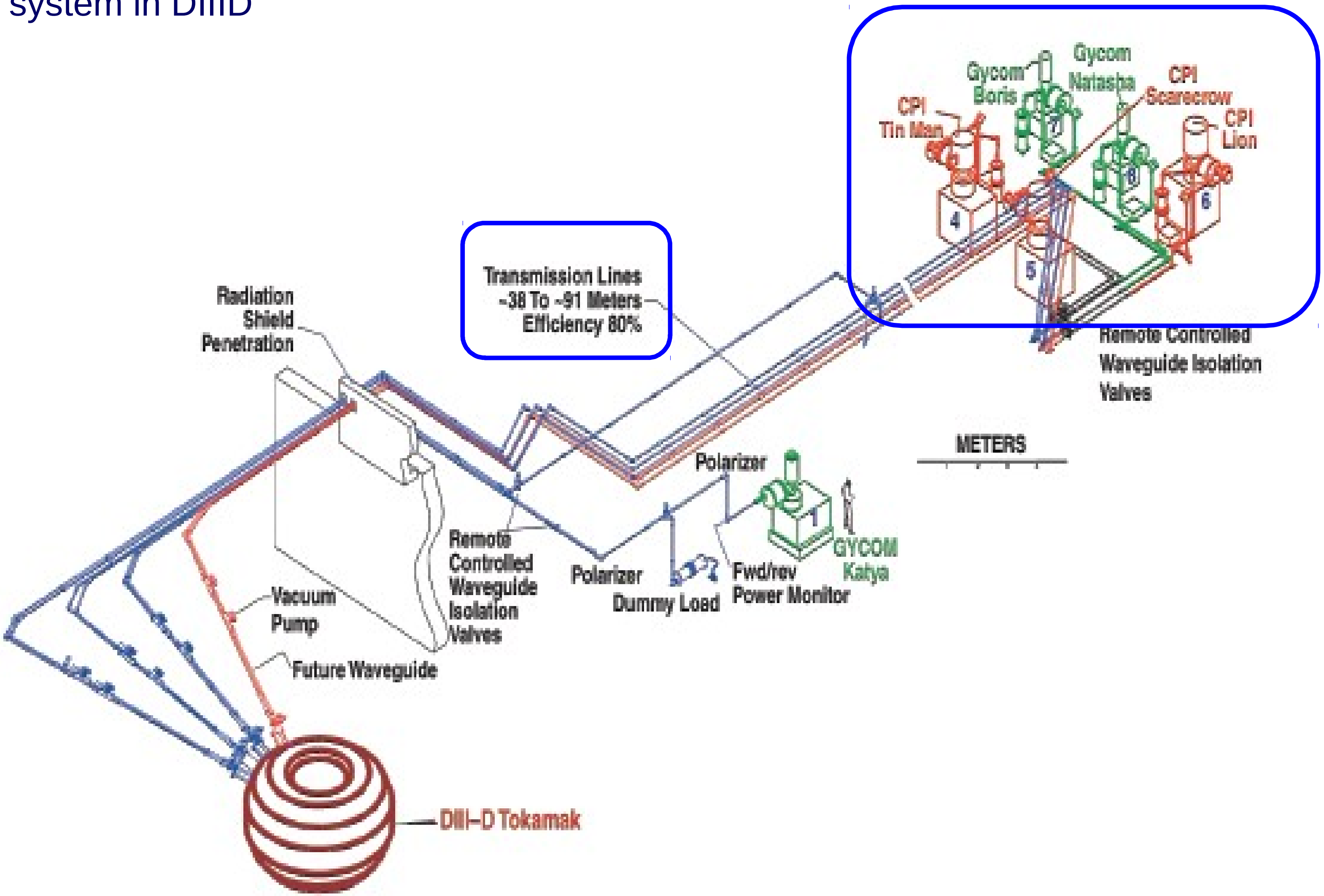
- Experimental results (Current drive efficiency)



Y. Peysson, Fusion summer school in KAIST, 2009

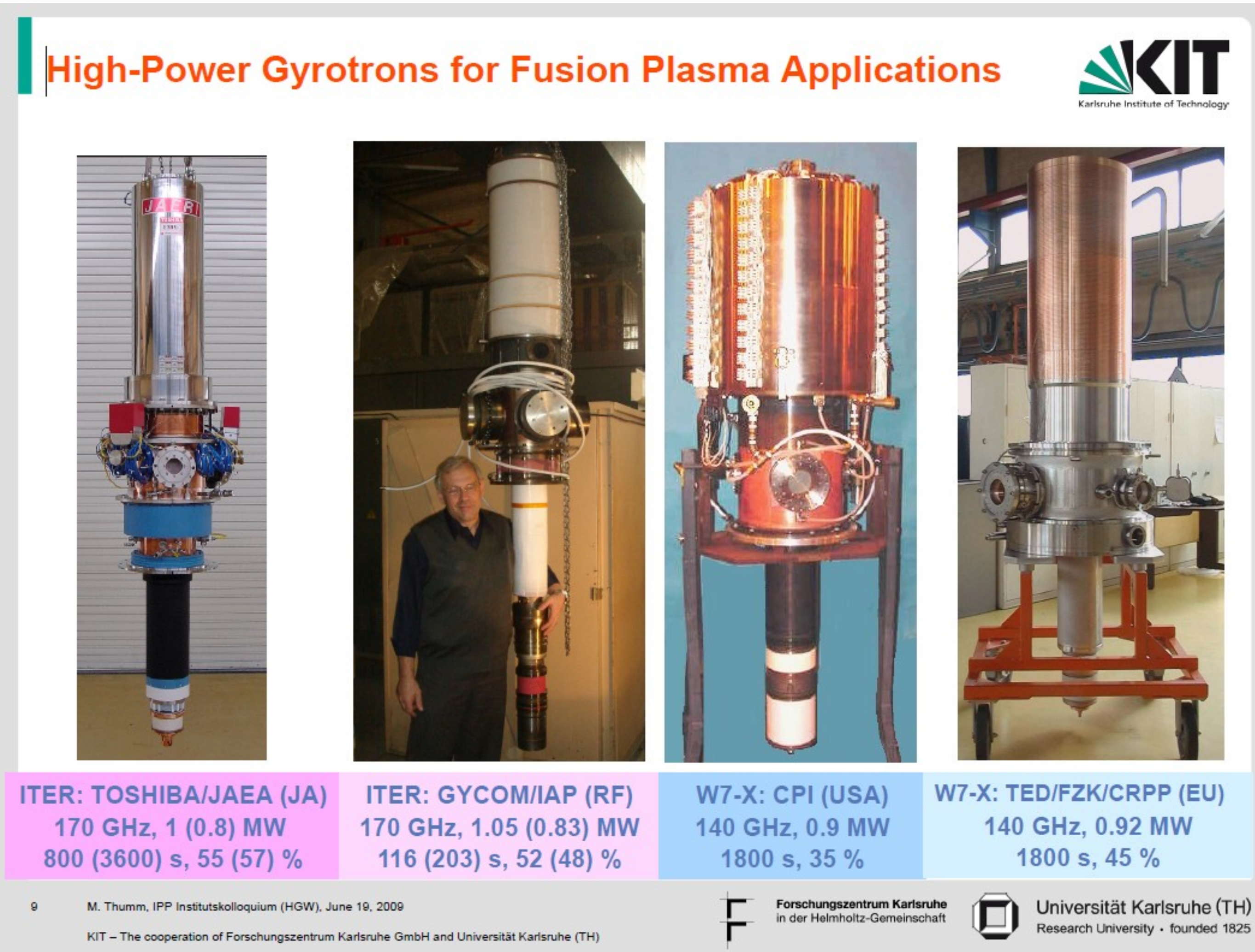
Wave launching, propagation, absorption in fusion plasmas (ECRF)

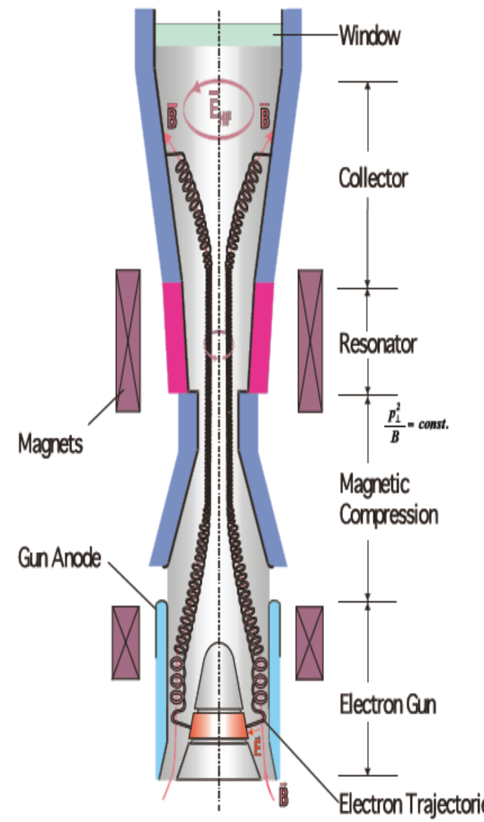
- ECRF system in DIIID



Wave launching, propagation, absorption in fusion plasmas (ECRF)

- ECRF source: Gyrotron

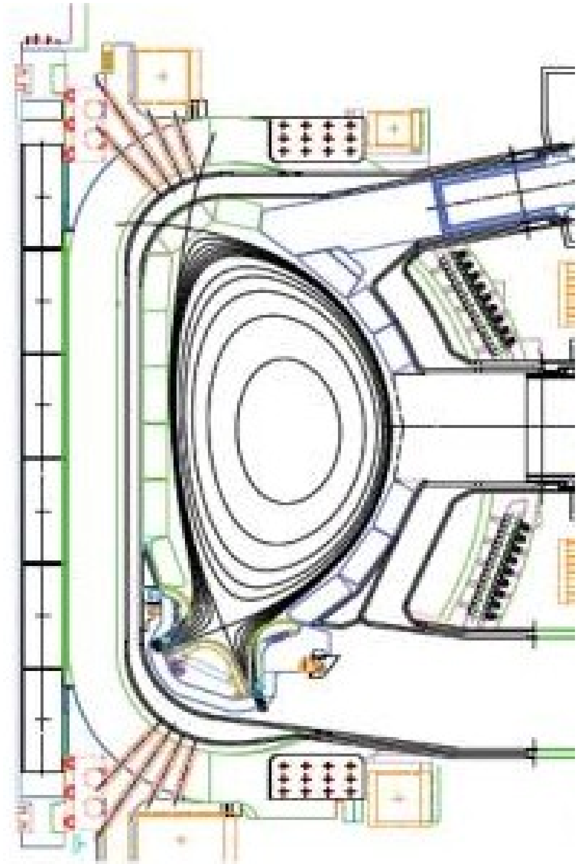


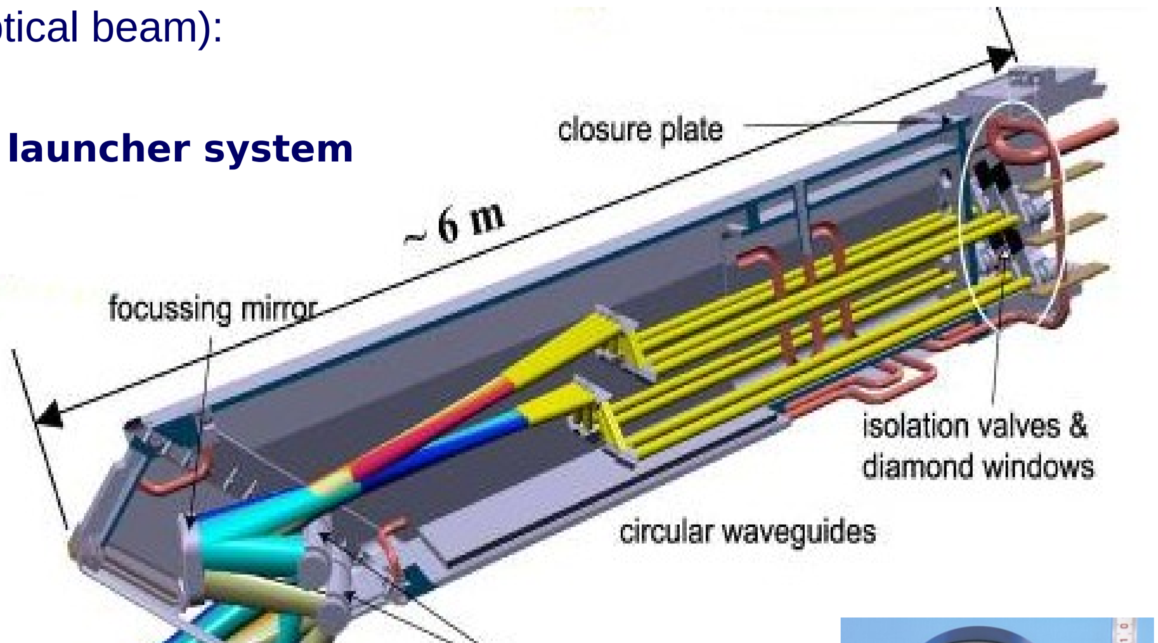


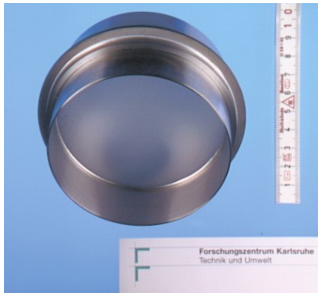
Wave launching, propagation, absorption in fusion plasmas (ECRF)

- ECRF launcher (Mirror: quasi-optical beam):

ITER ECRF upper launcher system



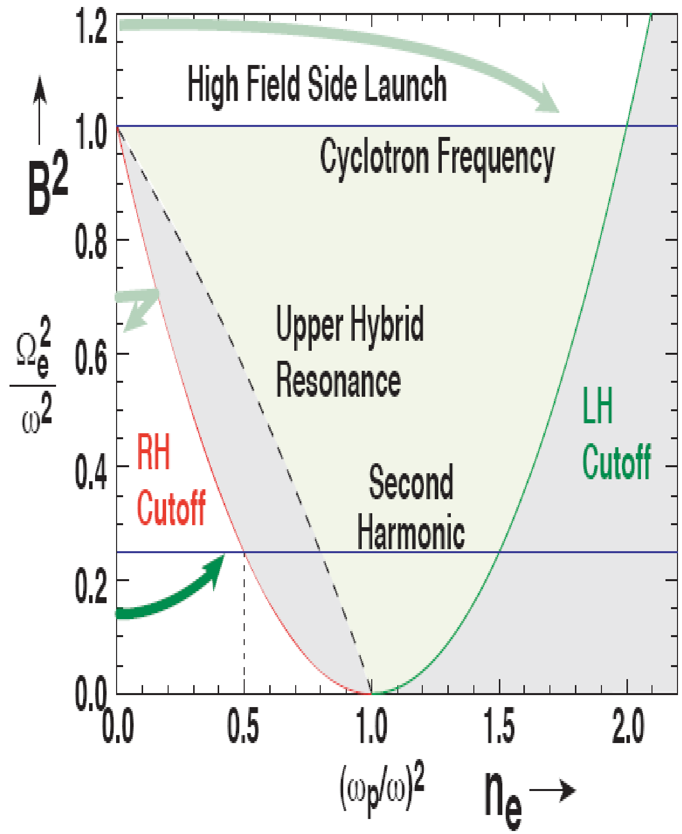


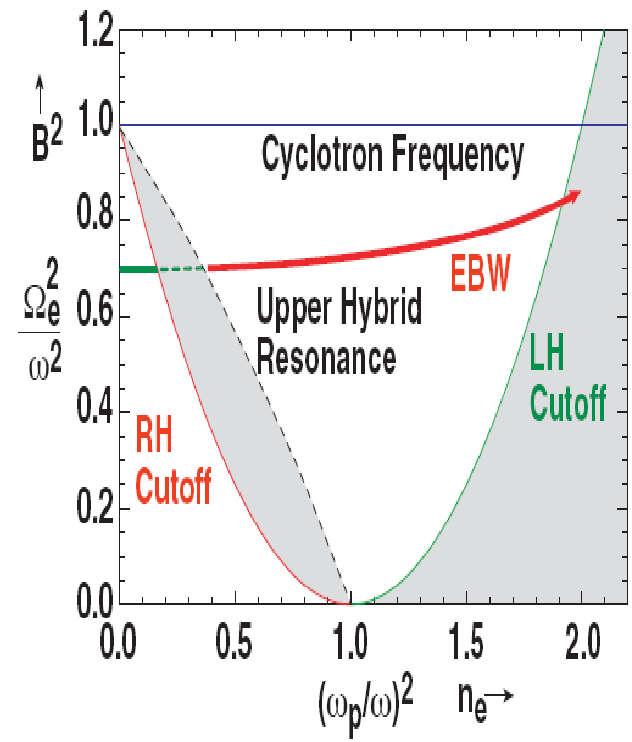


R. Prater, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (ECRF)

- O1, X2, X3 cyclotron heating and CD in tokamak
- XB, OXB EBW heating and CD in high beta ST

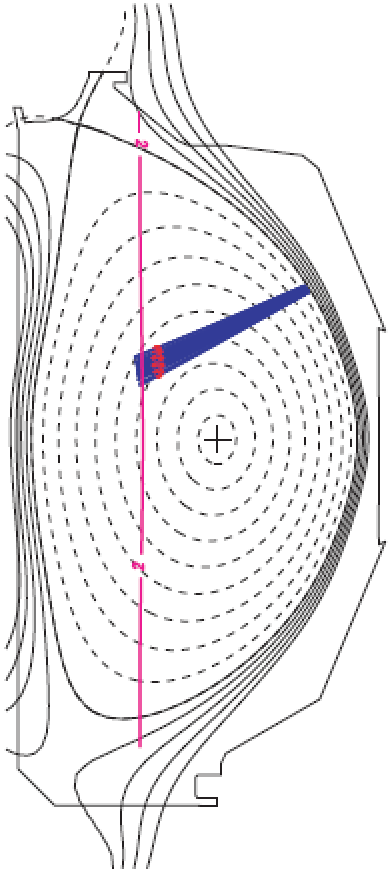




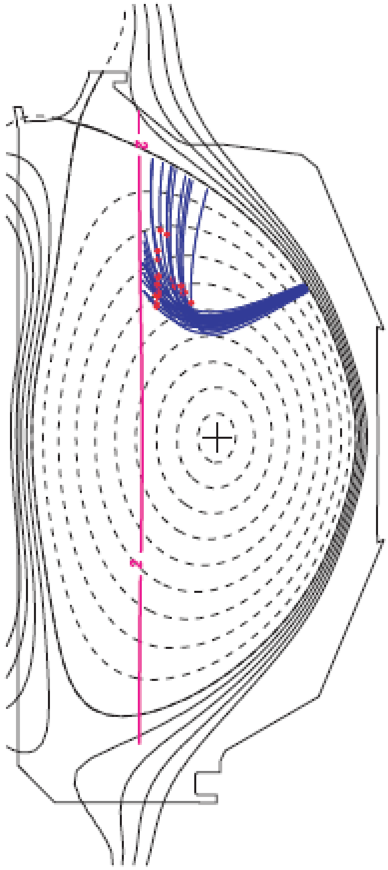
R. Prater, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (ECRF)

- Wave propagation



Low density under R(X) cut-off

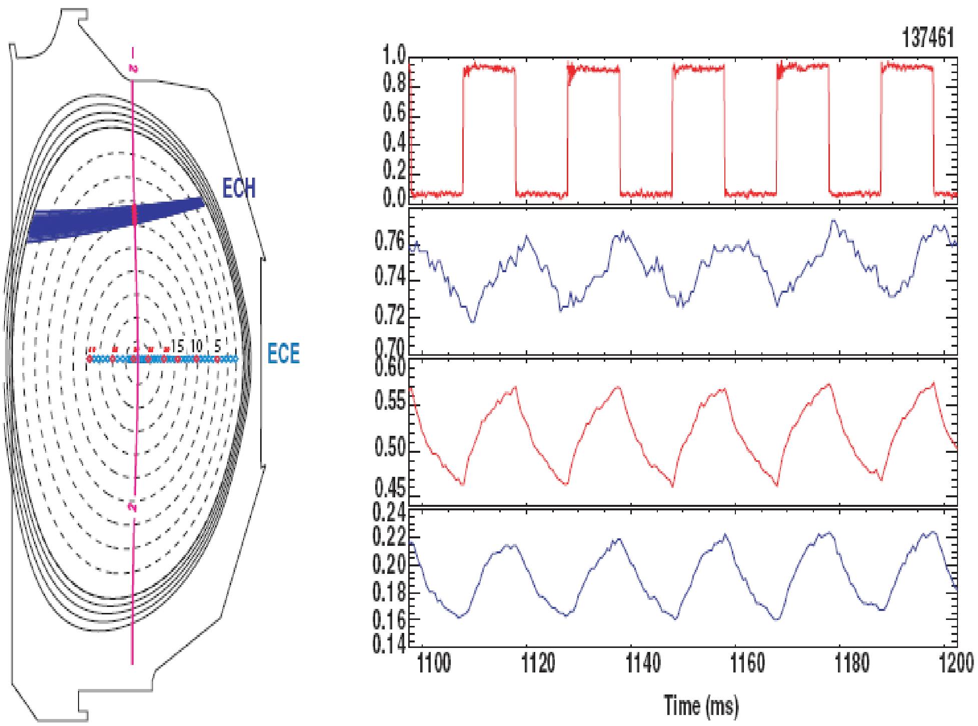


high density above R(X) cut-off

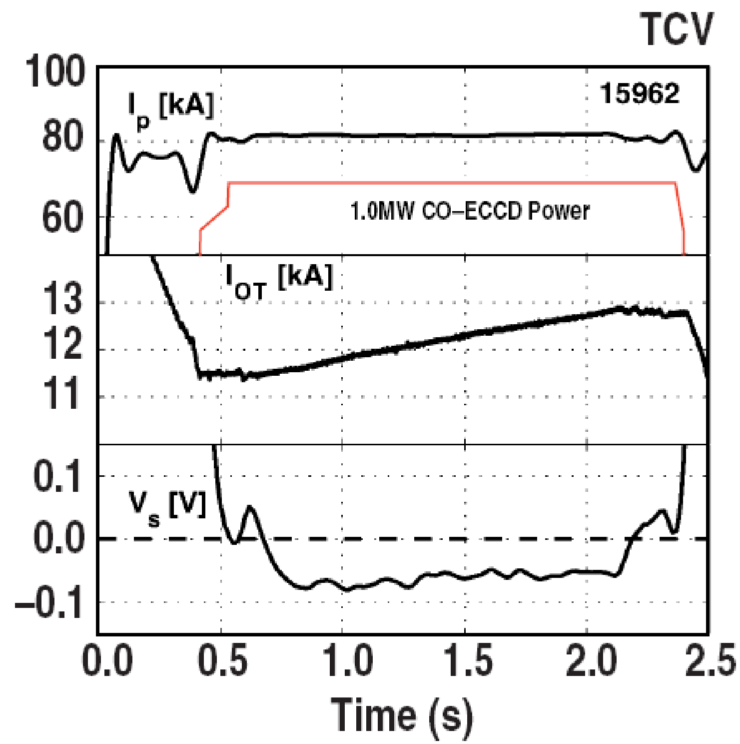
R. Prater, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (ECRF)

❑ Experiments (heating and current drive)



X2 heating in DIID

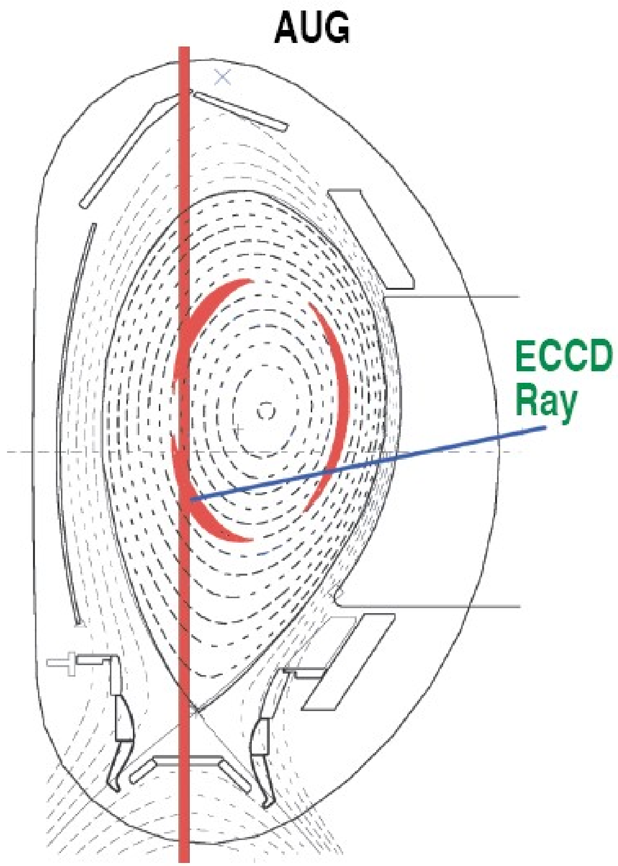


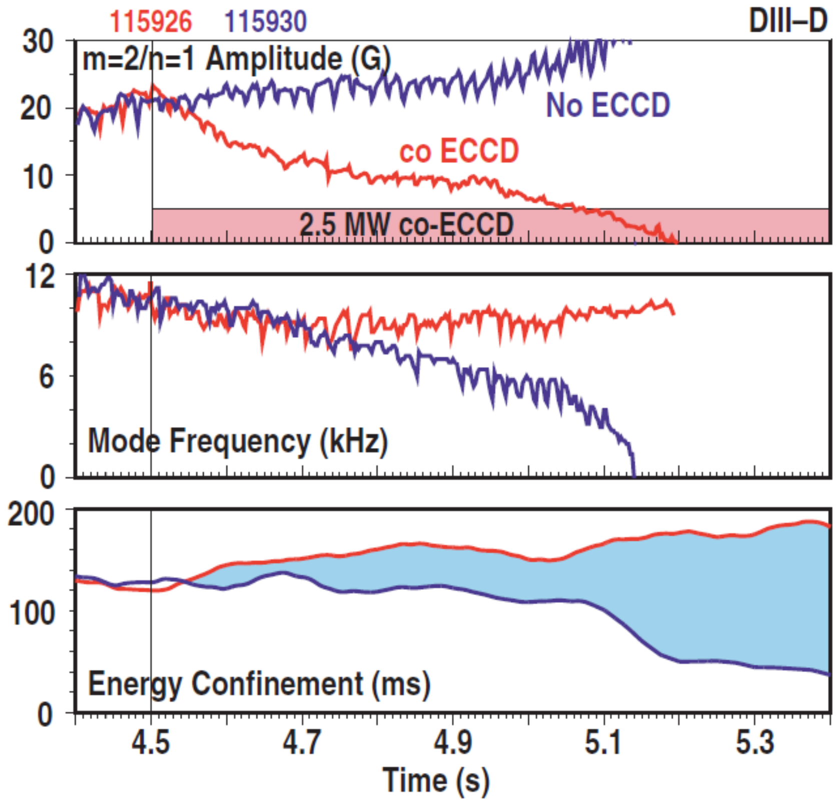
Full non-inductive CD in TCV

R. Prater, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (ECRF)

- NTM stabilization



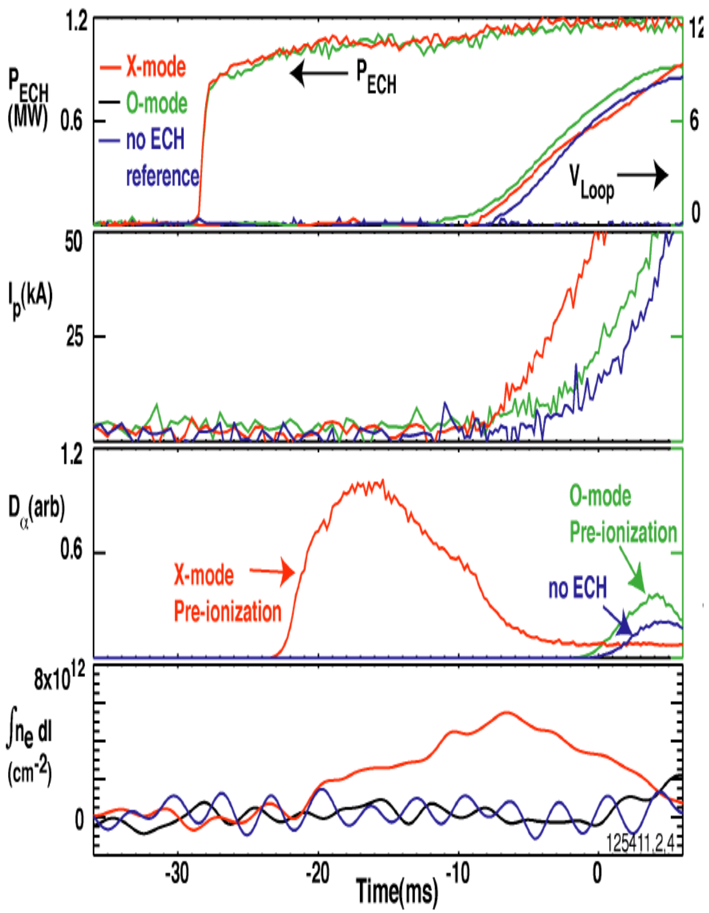


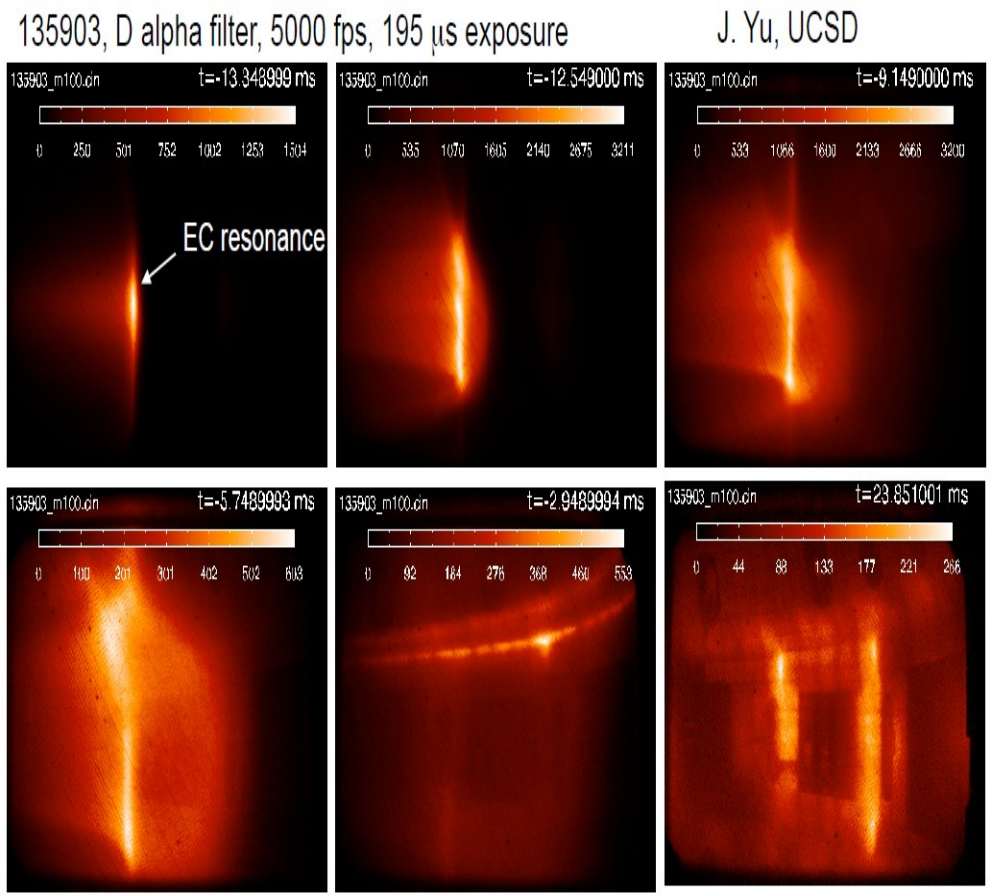
Suppression of 2/1 NTM by ECCD

R. Prater, Fusion summer school in KAIST, 2009

Wave launching, propagation, absorption in fusion plasmas (ECRF)

- Start up





R. Prater, Fusion summer school in KAIST, 2009

Summary

- RF waves have been successfully proven in tokamak experiments.
  - ICRF: Ion heating (Minority / 2nd Harmonic heating)
  - LHRF: Current drive (Landau damping)
  - ECRF: Pre-ionization and startup, NTM stabilization (Cyclotron damping of O1, X2, X3)

- There are still critical issues in RF systems to be solved (ICRF/LHRF).
  - Stable power transmission (arcing)
  - Power coupling

Reference

- T. Stix, "Waves in plasmas", 1992
- M. Brambilla, "Kinetic theory of plasma waves", 1998
- D. Swanson, "Plasma waves", 2003
- Presentations on RF waves "Fusion Summer School in KAIST", 2009