Fusion Reactor Technology 2
(459.760, 3 Credits)

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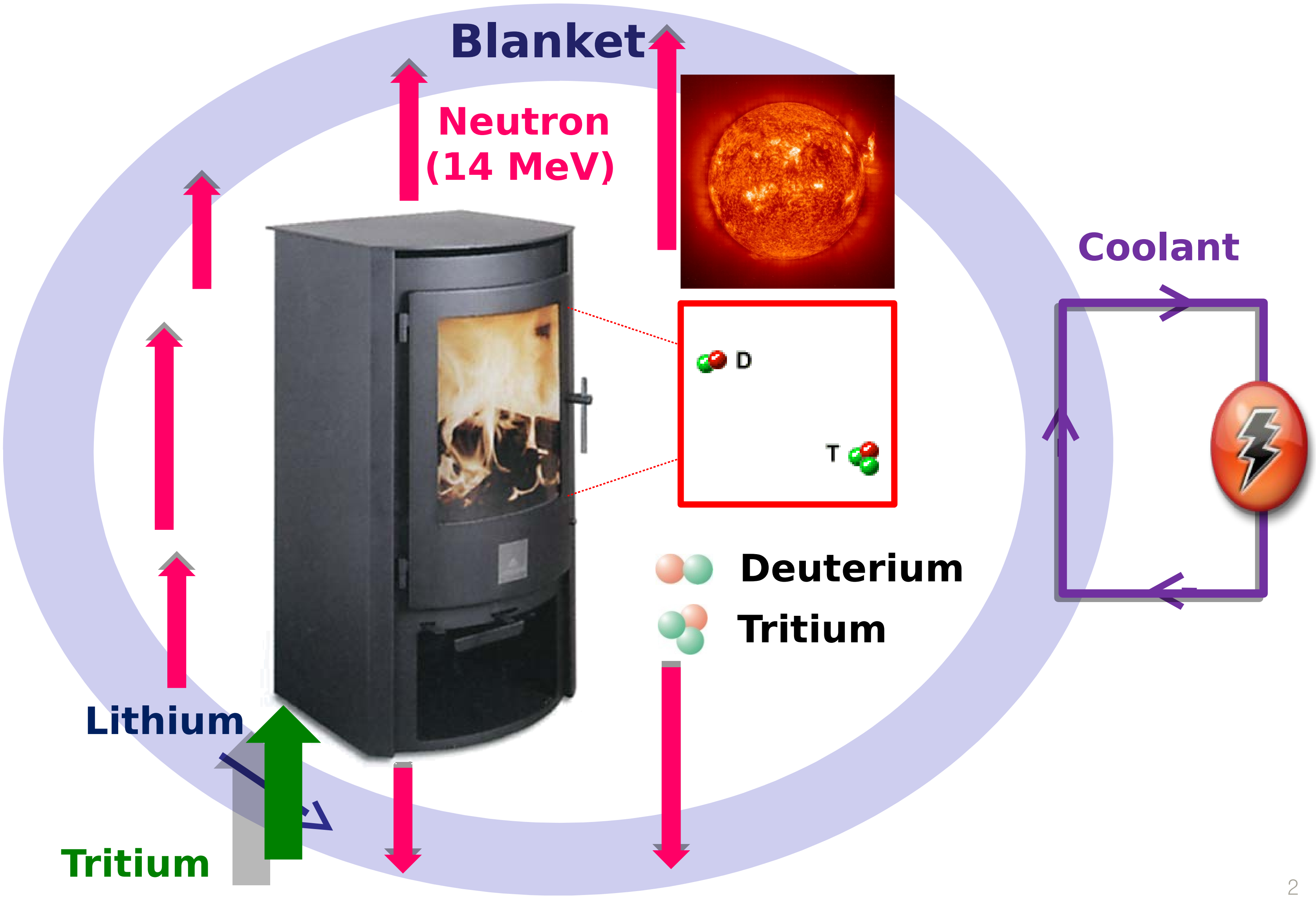
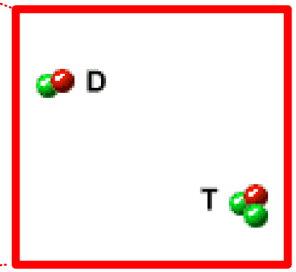
Fusion Power Plant (FPP) System

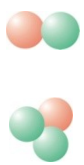


Fusion Reactor Energetics

What is required to light a fire in a stove?



- Fuel: D, T
- Amount/density: n
- Heat insulation: τ
- Ignition temperature: T

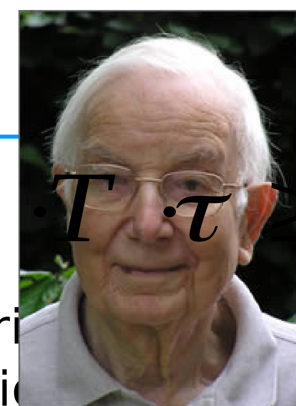


Deuterium

Tritium

$$n \cdot T \cdot \tau \geq ?$$

Cri ed
(fusi duct)



J. D. Lawson

Fusion Reactor Energetics

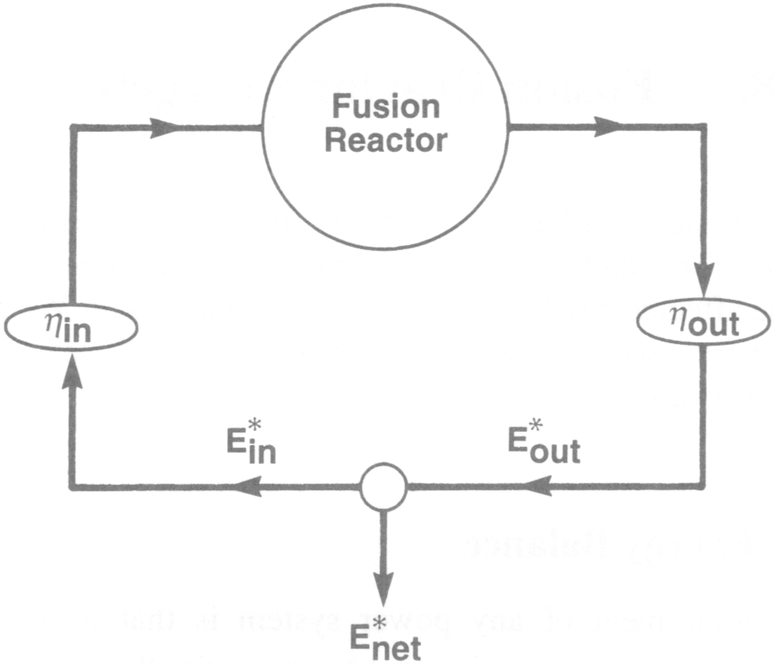
- **Fundamental requirement of a fusion reactor system**

The overall net energy should be larger than the total energy externally supplied to sustain fusion reactions and associated processes subtracted from the total recovered energy

$$E_{net}^{*} = E_{out}^{*} - E_{in}^{*} > 0$$

*: referring to the entire reaction volume

$$\int_V E_{out}(\vec{r},t)\,d^3r = E_{out}^{*}(t)$$



Considering the time variations of power (Particularly for pulsed systems)

$$\int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{net} dt = \int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{out} dt - \int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{in} dt > 0$$

τ_b: burning time

Fusion Reactor Energetics

- **Fusion Plasma Energy Balance**

$$\int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{net} dt = \int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{out} dt - \int_0^{\tau_b}\left(\frac{dE^*}{dt}\right)_{in} dt > 0$$

Thermal energy content in the total plasma volume

$$\int_o^{\tau_b}\frac{dE^*_{th}}{dt}dt = E^*_{aux} + E^*_{fu} - E^*_n - E^*_{rad} - \int_o^{\tau_b}\frac{E^*_{th}}{\tau_{E^*}}dt$$

$$E^*_{out} = E^*_{aux} + E^*_{alpha} = E^*_{fu} - E^*_n$$

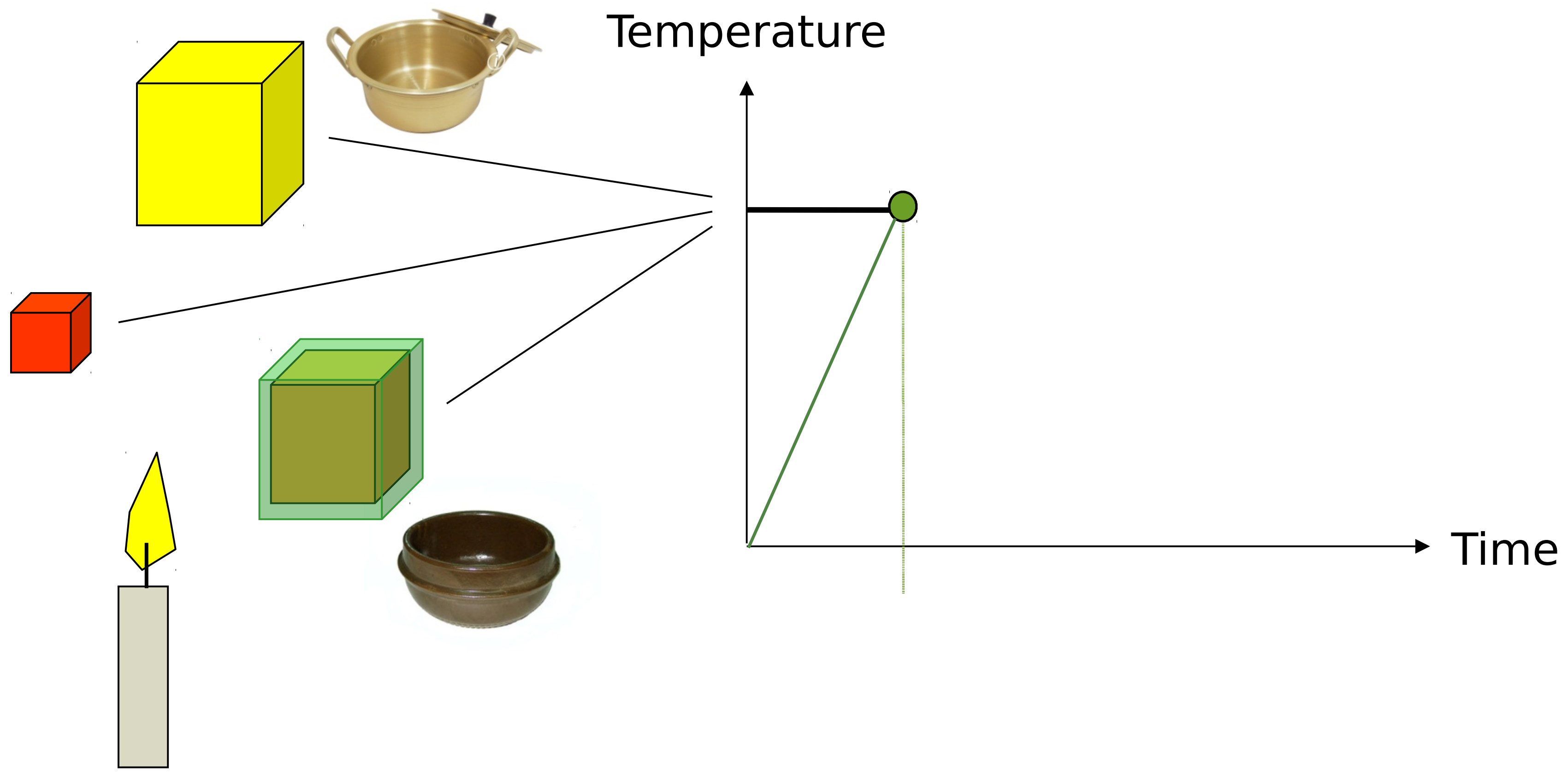
$$E^*_{in} = E^*_{rad} + \int_o^{\tau_b}\frac{E^*_{th}}{\tau_{E^*}}dt$$

$$E^*_{aux} = \eta_{in} E^*_{in}$$

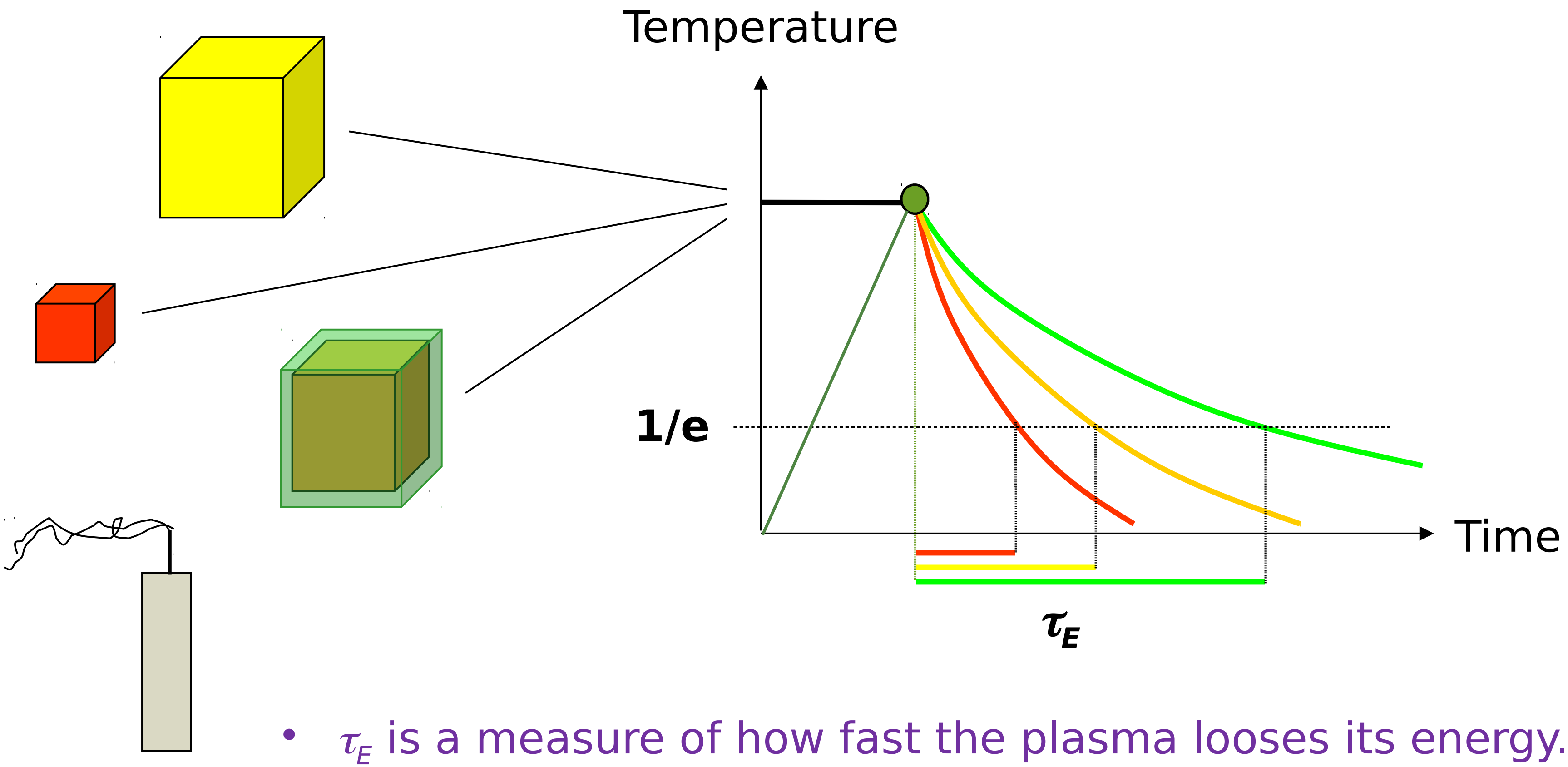
$$\frac{E^*_{alpha}}{E^*_{fu}} = f_c \qquad E^*_n = (1 - f_c)E^*_{fu}$$

f_c: alpha particle fraction of fusion product energy

Energy Confinement Time

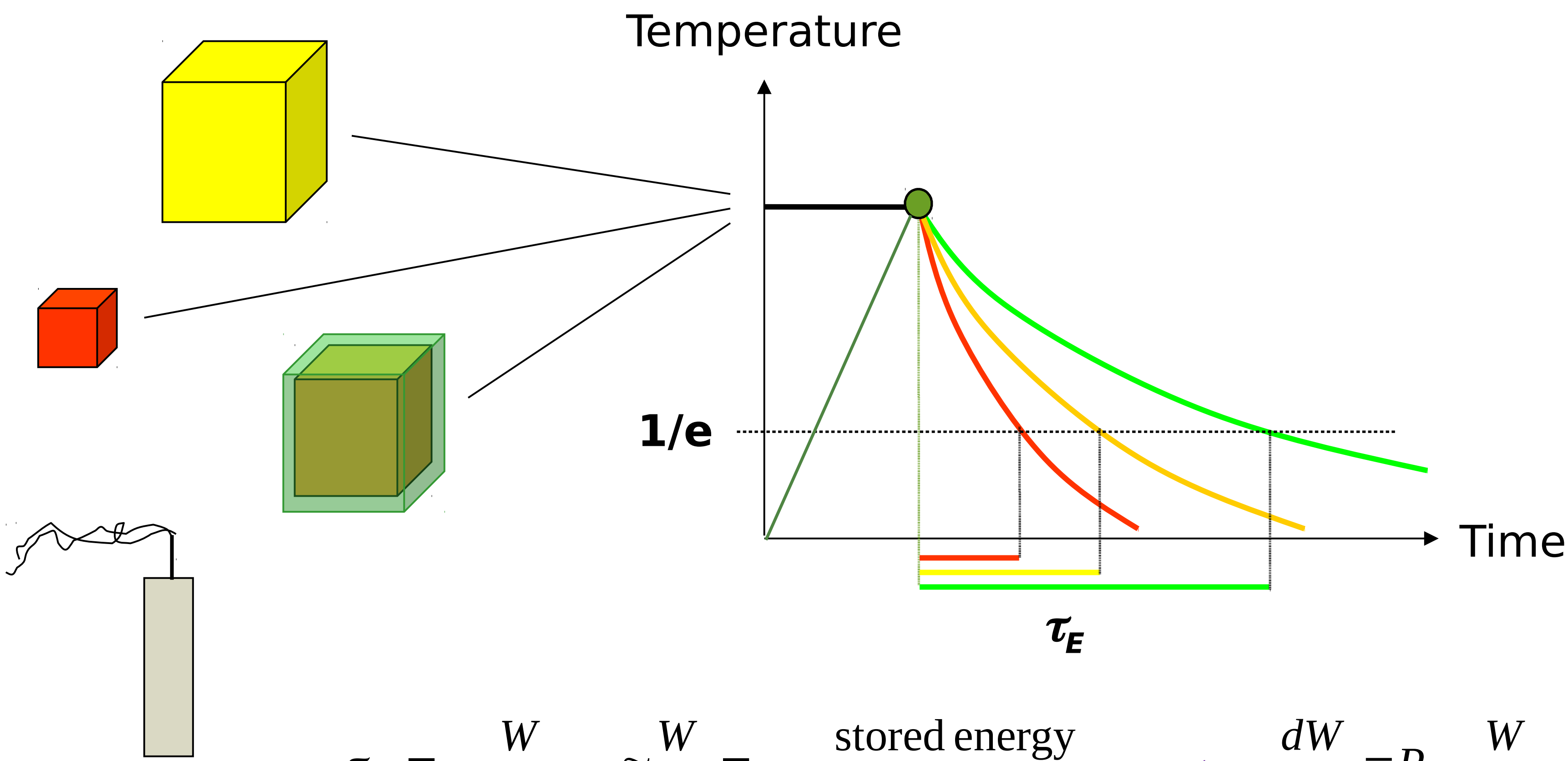


Energy Confinement Time



- τ_E is a measure of how fast the plasma looses its energy.
- The loss rate is smallest, τ_E largest
  if the fusion plasma is big and well insulated.

Energy Confinement Time



$$\tau_E = \frac{W}{P_{in} - \frac{dW}{dt}} \approx \frac{W}{P_{in}} = \frac{\text{stored energy}}{\text{applied heating power}} \leftarrow \frac{dW}{dt} = P_{in} - \frac{W}{\tau_E}$$

Fusion Reactor Energetics

$$Q_p = \frac{E^*_{fu}}{E^*_{aux}} = \frac{E^*_{fu}}{\eta_{in} E^*_{in}}$$

Plasma *Q*-value (fusion multiplication factor): measure for how efficiently an energy input to the plasma is converted into fusion energy

$$\int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = E^*_{aux} + E^*_{fu} - E^*_n - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt \longrightarrow \int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = \left(\frac{1}{Q_p} + f_c \right) E^*_{fu} - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt$$

$$E^*_n = (1 - f_c) E^*_{fu}$$

If, steady state

$$E^*_{fu} = \frac{E^*_{rad} + \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt}{f_c + \frac{1}{Q_p}}$$

- if, $Q_p \rightarrow \infty$, the fusion energy delivered to the plasma via the charged reaction products is seen to balance the total energy loss from the plasma.

→ What are requirements of a fusion reactor?

Fusion Reactor Energetics

- **Ignition**

  Energy viability of the fusion *plasma*:

  actual self-sustaining engineering reactor condition with no heating power

$$\frac{E^*_{fu}}{\eta_{in} E^*_{in}} = Q_p \rightarrow \infty$$

Considering a D-T plasma with $Q_p \rightarrow \infty$,

$$\int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = \left(\frac{1}{Q_p} + f_c \right) E^*_{fu} - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt > 0$$

$$f_c E^*_{fu} > E^*_{rad} + \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt$$

Fusion Reactor Energetics

$$f_c E_{fu}^* > E_{rad}^* + \int_o^{\tau_b} \frac{E_{th}^*}{\tau_{E^*}} dt$$

$$f_{c,dt} \int_V d^3r \int_o^{\tau_b} R_{dt}(\vec{r},t) Q_{dt} dt > \int_V d^3r \left[\int_o^{\tau_b} (P_{br} + P_{cyc}^{net}) dt + \int_o^{\tau_b} \frac{E_{th}(\vec{r},t)}{\tau_E(r,t)} dt \right]$$

$$P_{br} = A_{br} n_i n_e Z^2 \sqrt{kT_e} \qquad A_{br} \approx 1.6 \times 10^{-38} \left[\frac{m^3 J}{\sqrt{eV}s} \right]$$

$$P_{cyc}^{net} = A_{cyc} n_e B^2 kT_e \psi \qquad A_{cyc} \approx 6.3 \times 10^{-20} \left[JeV^{-1} T^{-2} s^{-1} \right]$$

$$\int_V d^3r \frac{E_{th}(\vec{r},t)}{\tau_E(r,t)} = \frac{E_{th}^*(t)}{\tau_{E*}(t)}$$

volume integrated

Fusion Reactor Energetics

- In a homogeneous plasma, local D-T fusion ignition condition:
  Charged particle self-heating power > loss powers
  (radiation + plasma transport)

$$f_{c,dt}\int_V d^3r\int_o^{\tau_b} R_{dt}(\vec{r},t)Q_{dt}\,dt > \int_V d^3r\left[\int_o^{\tau_b}(P_{br}+P_{cyc}^{net})dt+\int_o^{\tau_b}\frac{E_{th}(\vec{r},t)}{\tau_E(\vec{r},t)}dt\right]$$

$$f_{c,dt}P_{dt}(n_i,T_i) > P_{br}(n_i,n_e,T_e)+P_{cyc}^{net}(n_e,T_e)+\frac{3}{2}\frac{(n_iT_i+n_eT_e)}{\tau_{E^*}} \qquad E_{th.j}=\frac{3}{2}n_jT_j\,,\quad j=i,e$$

$$f_{c,dt}P_{dt}\tau_{E*}\geq(P_{br}+P_{cyc}^{net})\tau_{E*}+3nT$$ ⟵ $n_i = n_e = n$, $T_i = T_e = T$

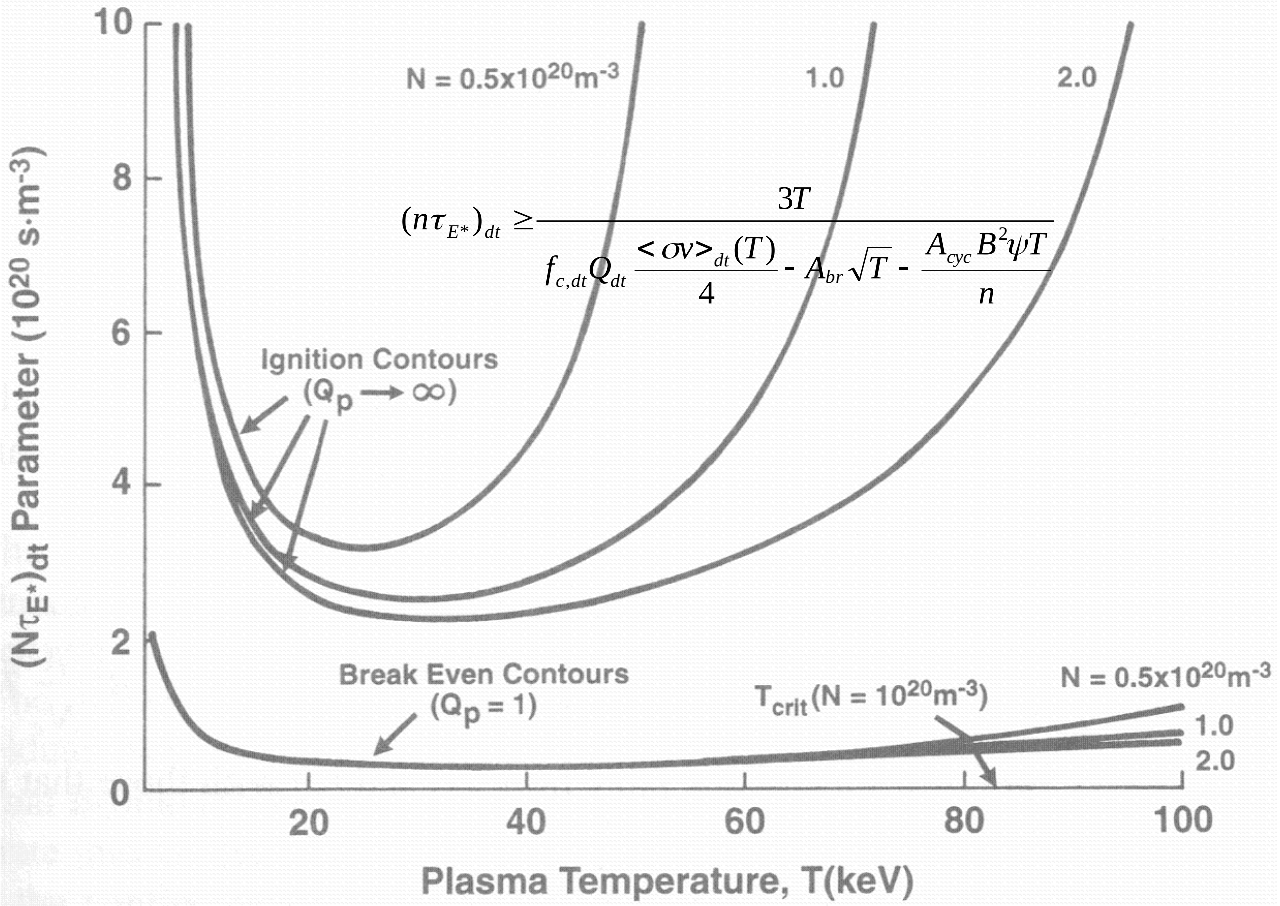
$$(n\tau_{E*})_{dt}\geq\frac{3T}{f_{c,dt}Q_{dt}\dfrac{<\sigma v>_{dt}(T)}{4}-A_{br}\sqrt{T}-\dfrac{A_{cyc}B^2\psi T}{n}}$$

no energy conversion efficiency contained

- complex interrelation between the plasma density and its temperature as required for ignition

Plot?

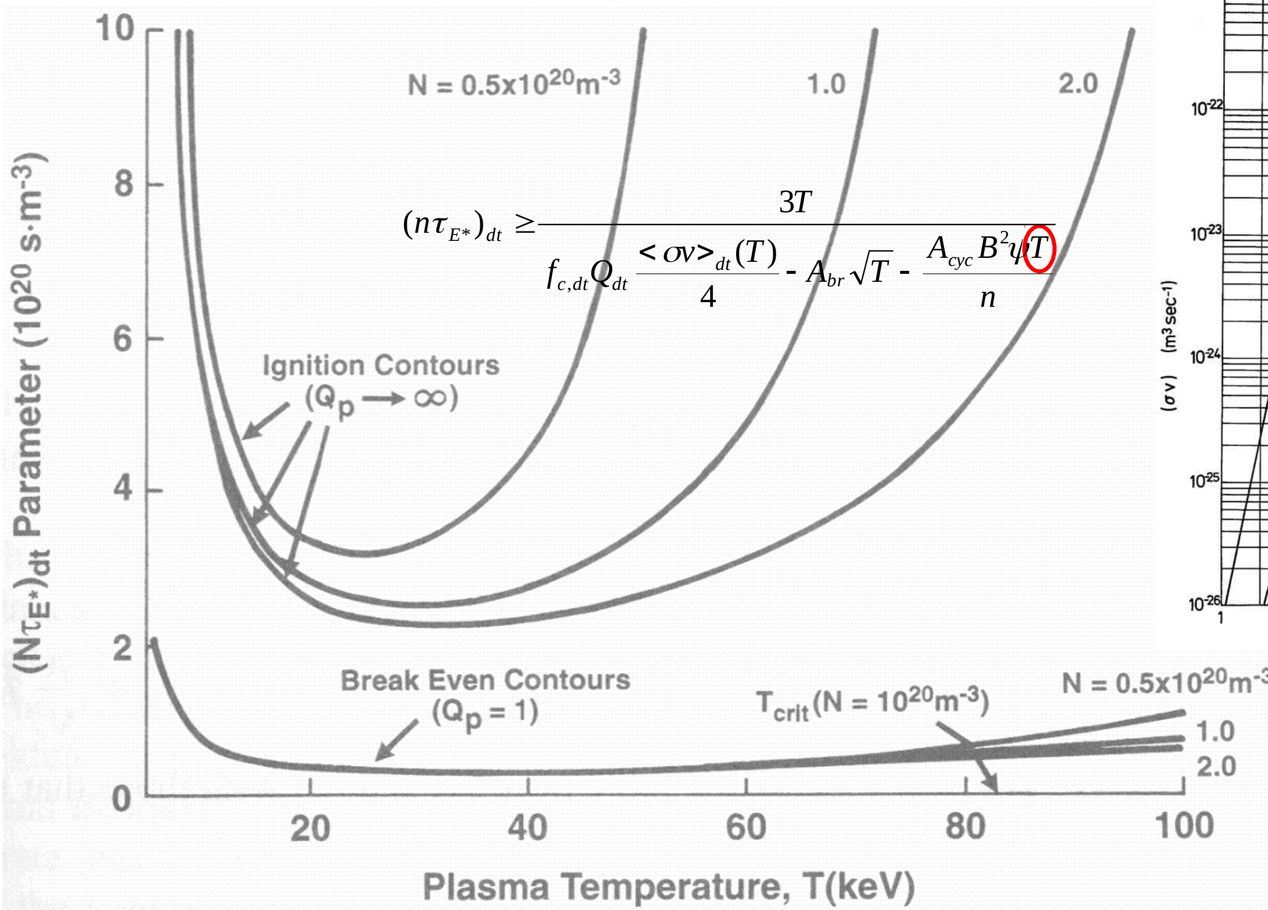
Fusion Reactor Energetics



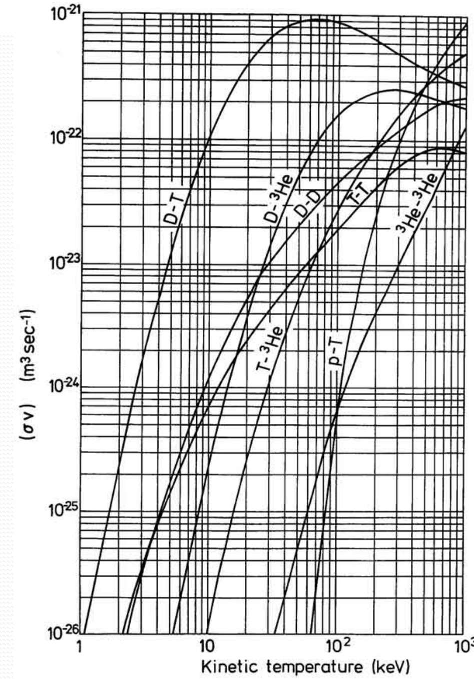
- n = 10^{20} m^{-3}: T ~ 30 keV, $n\tau_{E*}$ ~ 2.7x10^{20} m^{-3}s, τ_{E*} ~ 2.7 s
- Ignition contours tend towards infinity as T approaches T_{crit}.

Why?

Fusion Reactor Energetics



$$(n\tau_{E*})_{dt} \geq \frac{3T}{f_{c,dt}Q_{dt}\dfrac{<\sigma v>_{dt}(T)}{4} - A_{br}\sqrt{T} - \dfrac{A_{cyc}B^2\psi T}{n}}$$



$<\sigma v>_{dt} \propto T^2$ at 10-20 keV but ~constant above 50 keV

- n = 10^{20} m^{-3}: T ~ 30 keV, $n\tau_{E*}$ ~ 2.7x10^{20} m^{-3}s, τ_{E*} ~ 2.7 s
- Ignition contours tend towards infinity as T approaches T_{crit} due to the high Bremmstrahlung and the cyclotron radiation.

Fusion Reactor Energetics

- **Break-even (scientific)**

The total fusion energy production amounts to a magnitude equal to the effective plasma energy input.

$$\frac{E^*_{fu}}{\eta_{in} E^*_{in}} = Q_p = 1$$

$$\int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = \left(\frac{1}{Q_p} + f_c \right) E^*_{fu} - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau_{E^*}} dt > 0$$

$$(n\tau_{E*})_{dt} \geq ?$$

Plot?

Fusion Reactor Energetics

- **Lawson criterion *from the original paper***
  - reactor criterion: energy viability of the entire plant

Some Criteria for a Power Producing Thermonuclear Reactor

By J. D. LAWSON

Atomic Energy Research Establishment, Harwell, Berks.

Communicated by D. W. Fry; MS. received 2nd November 1956

Abstract. Calculations of the power balance in thermonuclear reactors operating under various idealized conditions are given. Two classes of reactor are considered: first, self-sustaining systems in which the charged reaction products are trapped and, secondly, pulsed systems in which all the reaction products escape so that energy must be supplied continuously during the pulse. It is found that not only must the temperature be sufficiently high, but also the reaction must be sustained long enough for a definite fraction of the fuel to be burnt.

Proceedings of the Physical Society (London), ***B70*** *6 (1957)*

Fusion Reactor Energetics

- **Lawson criterion *from the original paper***
  - Deriving some criteria which have to be satisfied in a power producing system by considering power balance for systems in which the reaction products escape, defined as “pulsed systems” by Lawson.
  - The gas is heated instantaneously to a temperature *T*, this temperature is maintained for a time *t*, after which the gas is allowed to cool.
  - The energy released by the reaction appears as heat generated in the walls of the apparatus (*blanket*), and thus has to be converted to electrical, mechanical or chemical energy before it can be fed back into the gas with efficiency *η*.
  - Assumptions:

$$P_B = 1.4 \times 10^{-34} n^2 T^{1/2} \text{ watts cm}^{-3}$$

considering Bremsstrahlung radiation only (*Spitzer 1956*)

(cyclotron radiation neglected)

neglecting conduction loss entirely

$$E_{aux}^* = E_{rad}^* + E_{th}^*$$

energy used to heat the gas and supply the radiation loss regained as useful heat

$$R = E_{fu}^* / E_{aux}^*$$

  - introducing *R*: ratio of the energy released in the hot gas to the energy supplied

Fusion Reactor Energetics

- **Lawson criterion *from the original paper***
  - Condition for a system with net power gain

$$E^*_{out} > E^*_{in} = \frac{E^*_{aux}}{\eta_{in}}$$

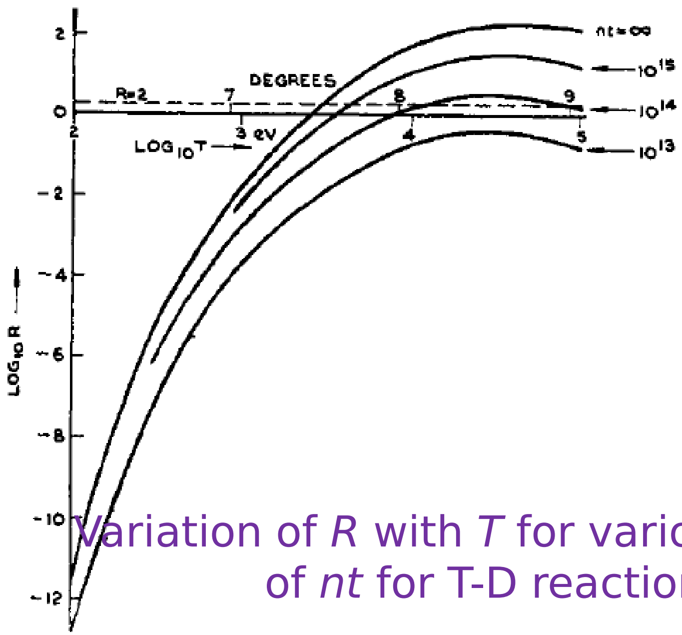
$$E^*_{out} = E^*_{aux} + E^*_{fu}$$

$$E^*_{aux} = E^*_{rad} + E^*_{th}$$

$$\eta_{in} \frac{\left(E^*_{aux} + E^*_{fu}\right)}{E^*_{aux}} > 1 \qquad \eta\left(R+1\right) > 1$$

$$R = \frac{E^*_{fu}}{E^*_{aux}} = \frac{tP_R}{tP_B + \frac{3}{2}(n_i T_i + n_e T_i)k} \approx \frac{tP_R}{tP_B + 3nkT} = \frac{P_R / 3n^2 kT}{P_B / 3n^2 kT + 1/nt}$$

Energy required to heat the gas to a temperature *T*



Variation of *R* with *T* for various values of *nt* for T-D reaction

$$\frac{tP_R + tP_B + 3nkT}{tP_B + 3nkT} > \frac{1}{\eta}$$

$$tP_R + tP_B + 3nkT > \frac{tP_B + 3nkT}{\eta}$$

Total output energy after a pulse > input energy for heating and compensating loss

Fusion Reactor Energetics

- **Lawson criterion *from the original paper***
  – Conclusion:
    For a successful thermonuclear reactor not only has the temperature to be
    sufficiently high, but also the reaction has to be sustained for a sufficient time. The reason for this is that the organized energy used to heat the gas is ultimately degraded to the temperature of the walls of the apparatus and, consequently, sufficient thermonuclear energy must be released during each heating cycle to compensate for this degradation.

Fusion Reactor Energetics

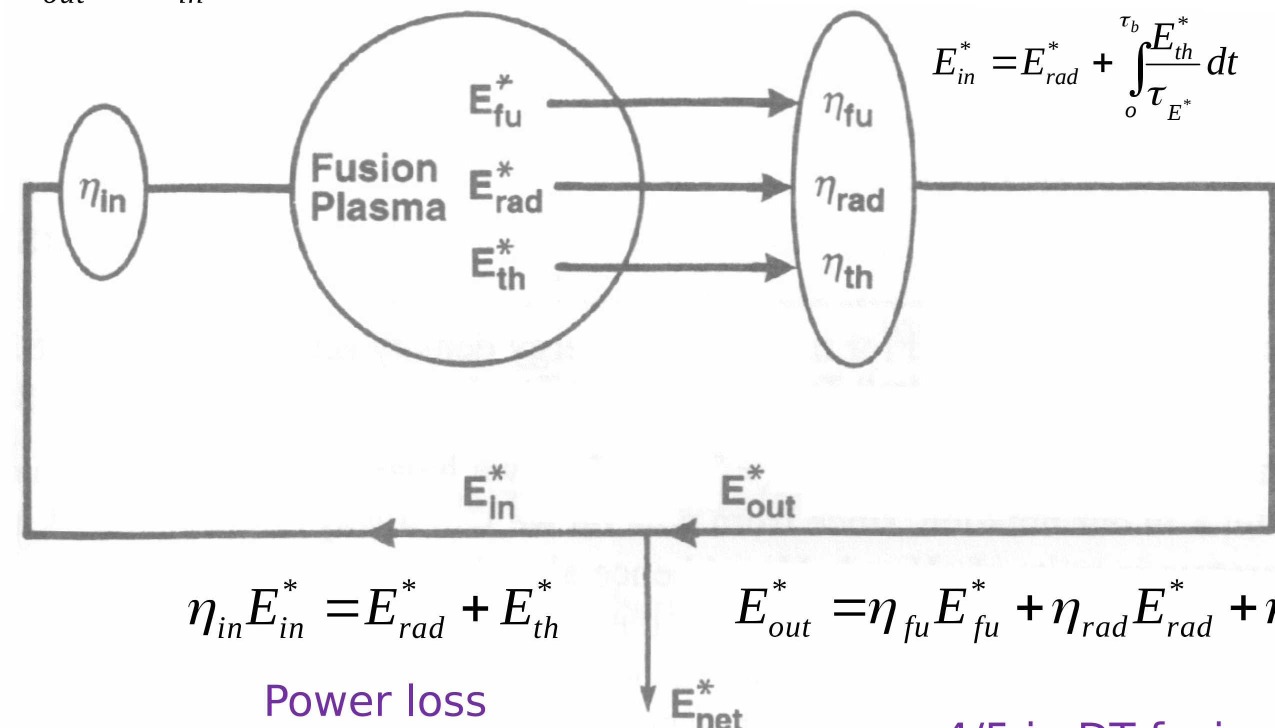
- **Lawson criterion**

  - The recoverable energy from a fusion reactor must exceed the energy which is supplied to sustain the fusion reaction.

$$E_{out}^{*} > E_{in}^{*}$$

$$E_{out}^{*} = E_{aux}^{*} + E_{fu}^{*} = \eta_{in} E_{in}^{*} + E_{alpha}^{*} + E_{n}^{*}$$

$$E_{in}^{*} = E_{rad}^{*} + \int_{o}^{\tau_b} \frac{E_{th}^{*}}{\tau_{E^*}} dt$$



$$\eta_{in} E_{in}^{*} = E_{rad}^{*} + E_{th}^{*}$$

$$E_{out}^{*} = \eta_{fu} E_{fu}^{*} + \eta_{rad} E_{rad}^{*} + \eta_{th} E_{th}^{*}$$

Power loss

conduction loss

η_{fu} < 4/5 in DT fusion?

Fusion Reactor Energetics

- **Lawson criterion**

- output electric energy (recoverable energy) > required input energy

$$E^*_{out} > E^*_{in}$$

$$E^*_{out} = \eta_{fu} E^*_{fu} + \eta_{rad} E^*_{rad} + \eta_{th} E^*_{th}$$

$$\eta_{in} E^*_{in} = E^*_{rad} + E^*_{th}$$

$$\eta_{fu} E^*_{fu} + \eta_{rad} E^*_{rad} + \eta_{th} E^*_{th} > \frac{E^*_{rad} + E^*_{th}}{\eta_{in}}$$

$$\eta_{in} \eta_{out} (E^*_{fu} + E^*_{rad} + E^*_{th}) > E^*_{rad} + E^*_{th}$$

$$\eta_{out} = \frac{\sum_l \eta_l E_l}{\sum_l E_l}, \; l = fu, rad, th$$

average conversion efficiency

$$E^*_l = \tau_{E*} \int_V P_l(\vec{r}) d^3 r$$

global energy terms

Fusion Reactor Energetics

Assuming, Bremsstrahlung only

$$\eta_{in}\eta_{out}\int_V d^3r(\tau_{E*}P_{fu}+\tau_{E*}P_{br}+3nT) > \int_V d^3r(\tau_{E*}P_{br}+3nT)$$

$$E_{th}(\vec{r}) = \frac{3}{2}(n_iT_i+n_eT_e) = 3nT$$

Assuming, homogeneity throughout the plasma volume *V*

$$\eta_{in}\eta_{out}\left(\frac{n_a n_b}{1+\delta_{ab}} <\sigma v>_{ab} Q_{ab}\tau_{E*} + A_{br}n^2\sqrt{T}\tau_{E*} + 3nT\right) > A_{br}n^2\sqrt{T}\tau_{E*} + 3nT$$

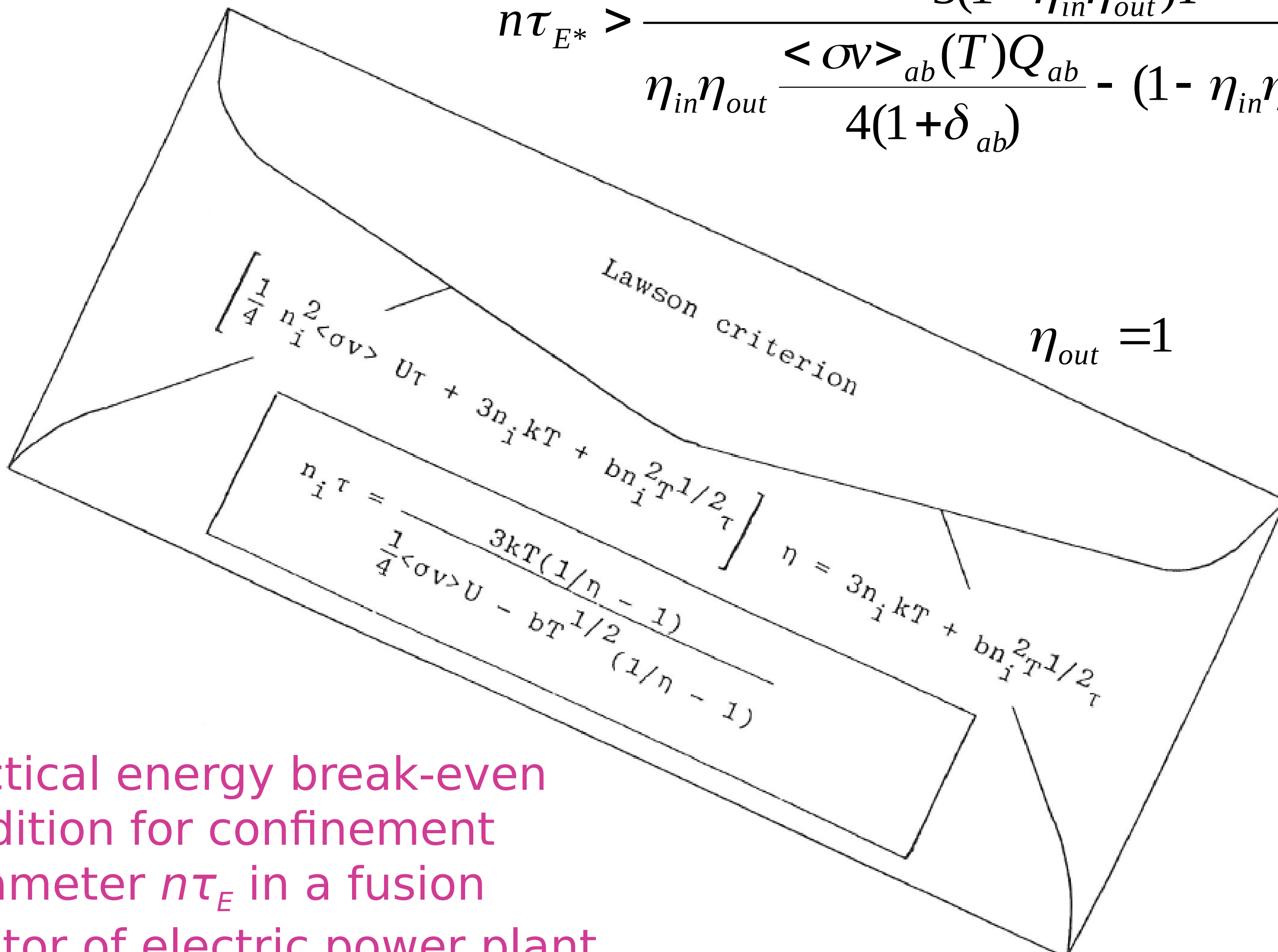
Kronecker-δ introduced to account for the case of indistinguishable reactants

$$n\tau_{E*} > \frac{3(1-\eta_{in}\eta_{out})T}{\eta_{in}\eta_{out}\frac{<\sigma v>_{ab}(T)Q_{ab}}{4(1+\delta_{ab})} - (1-\eta_{in}\eta_{out})A_{br}\sqrt{T}} \qquad \eta_{in}\eta_{out} \approx 1/3$$

Fusion Reactor Energetics

- **Lawson criterion**

$$n\tau_{E*} > \frac{3(1-\eta_{in}\eta_{out})T}{\eta_{in}\eta_{out}\dfrac{<\sigma v>_{ab}(T)Q_{ab}}{4(1+\delta_{ab})} - (1-\eta_{in}\eta_{out})A_{br}\sqrt{T}}$$

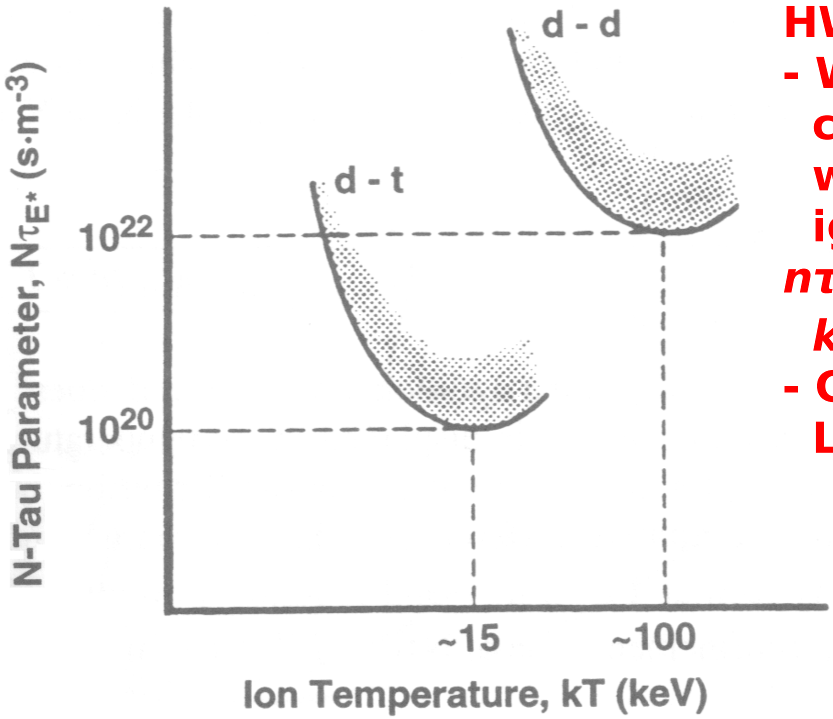


$$\eta_{out} = 1$$

- Practical energy break-even condition for confinement parameter $n\tau_E$ in a fusion reactor of electric power plant

Fusion Reactor Energetics

- **Lawson criterion**



HW

- Where is the Lawson criterion located compared with the breakeven and the ignition condition in the $n\tau_E$, kT plot?

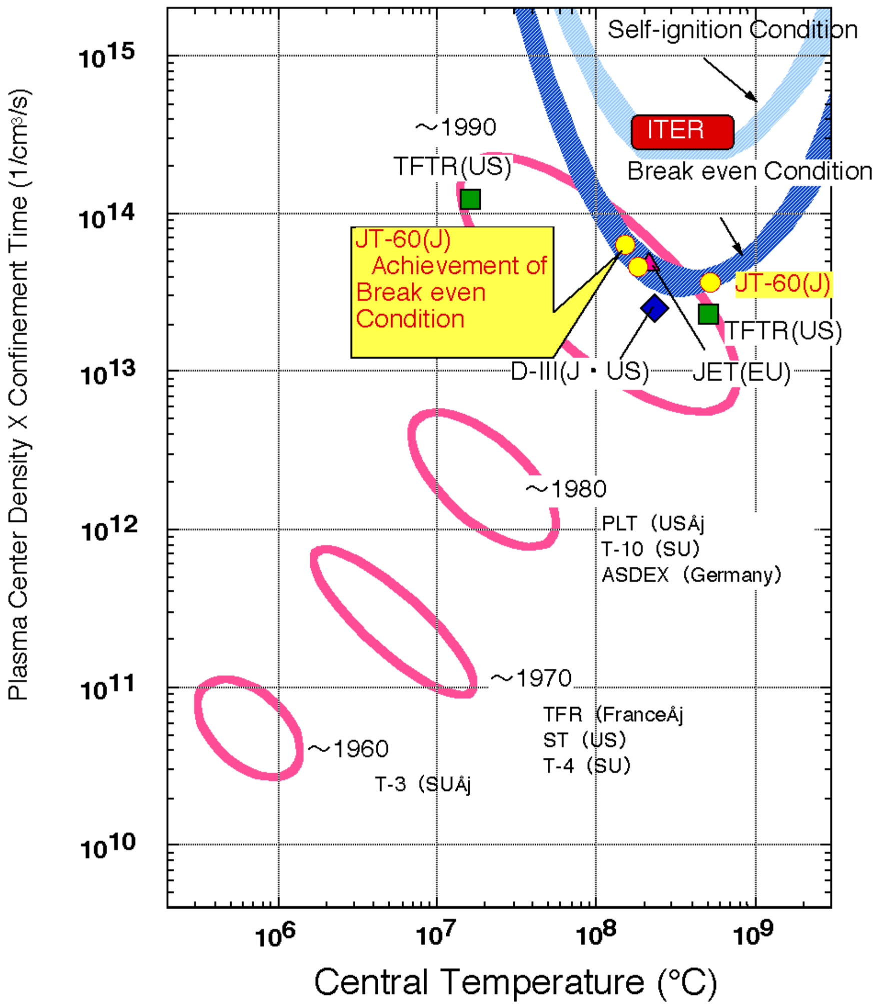
- Can one express the Lawson criterion with Q?

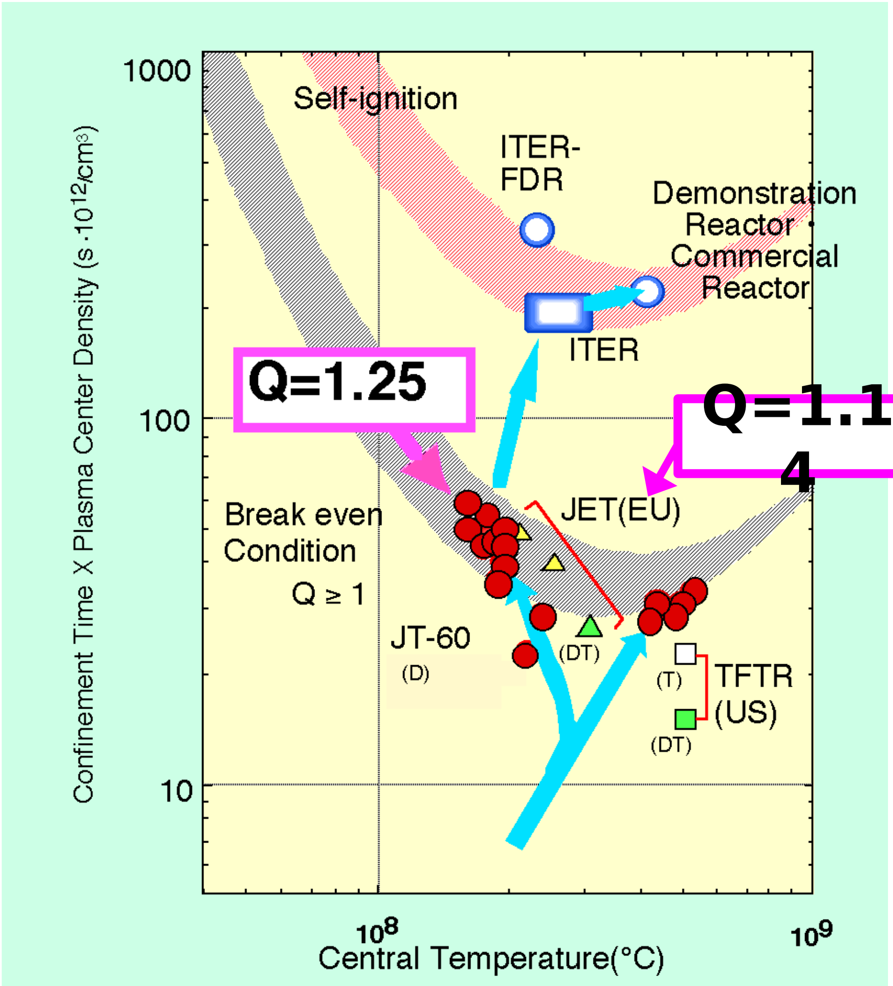
HW: What is the difference in P_{in}, P_{aux}, P_{out} among the criteria?

- No particular fusion design was necessary in the derivation of this criterion.
- Although it does not contain all relevant processes such as cyclotron radiation, it is a useful and widely employed criterion.
- For commercial power applications, it would be necessary to exceed the minimum Lawson limit by perhaps a factor of ten or better.

Status of the Tokamak Research

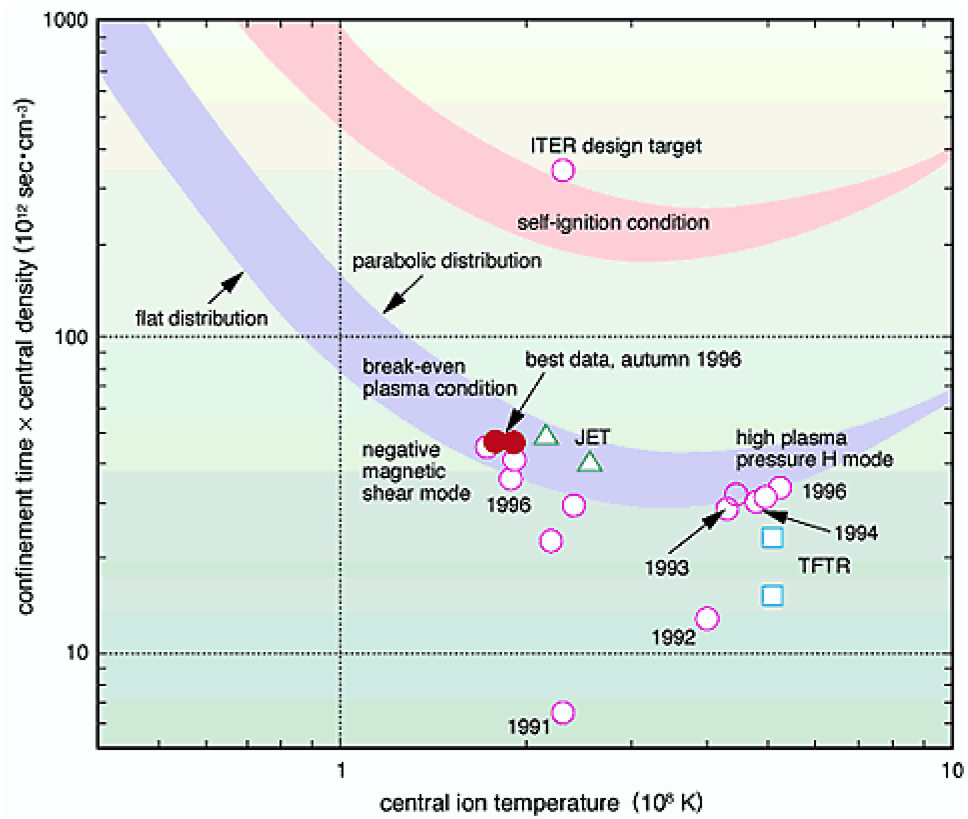
Q = 1 **Q = ∞**





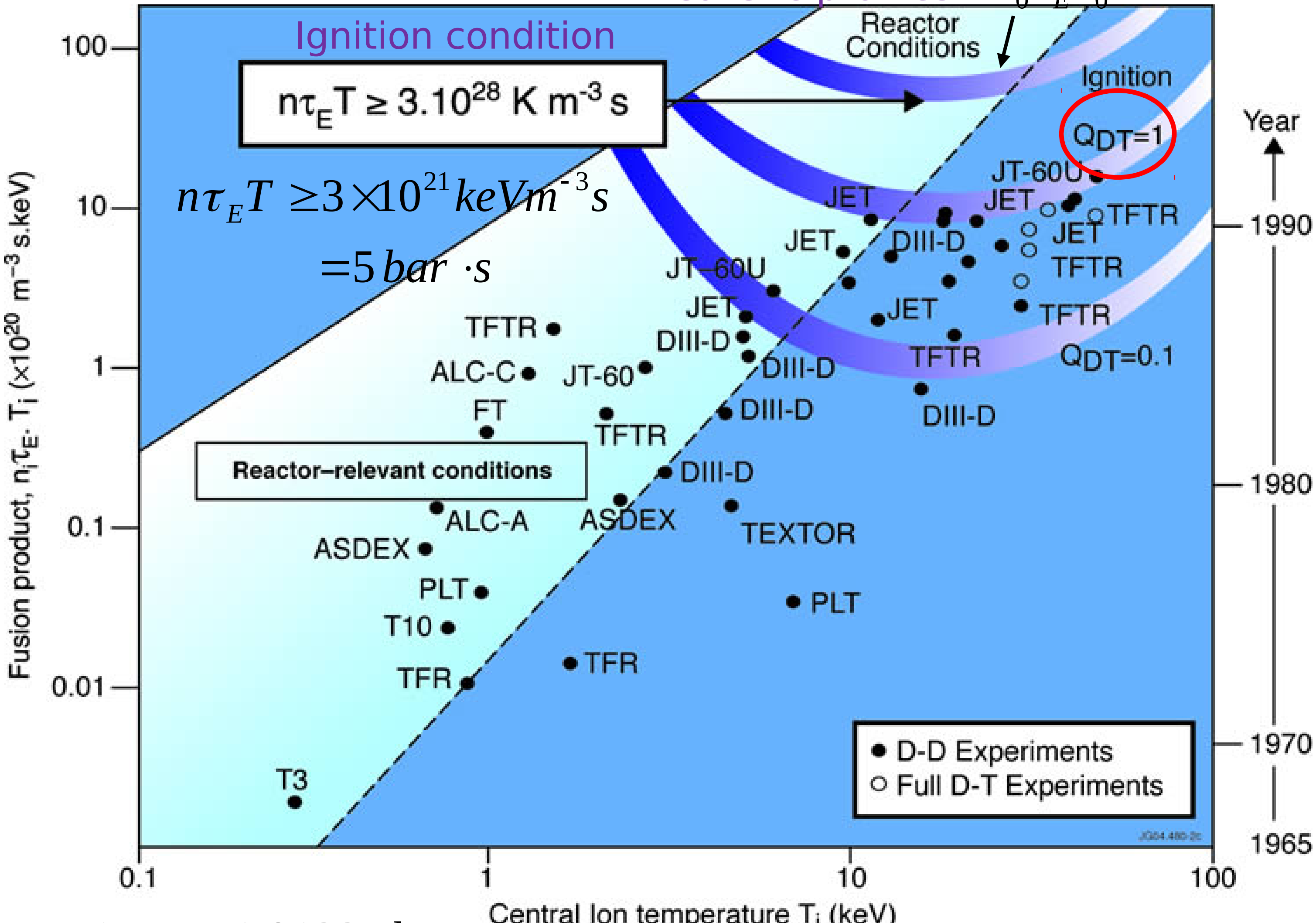
HW. What would be typical Q_p in KSTAR?

Status of the Tokamak Research



Status of the Tokamak Research

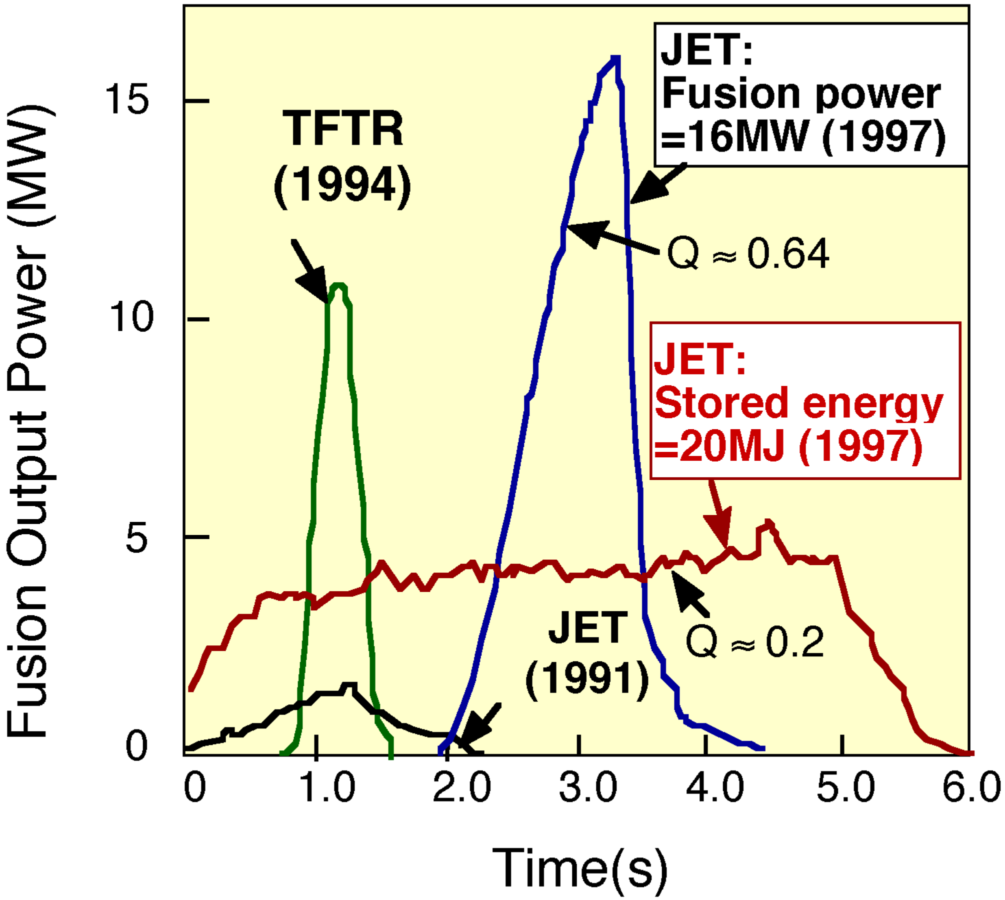
with realistic profiles: $n_0 \tau_E T_0 \geq 5 \times 10^{21} keV m^{-3} s$



$1\, atm = 1.01325\, bar$

$<\sigma v>_{dt} \propto T^2$ at 10-20 keV → $n\tau_{E*}T$

Status of the Tokamak Research

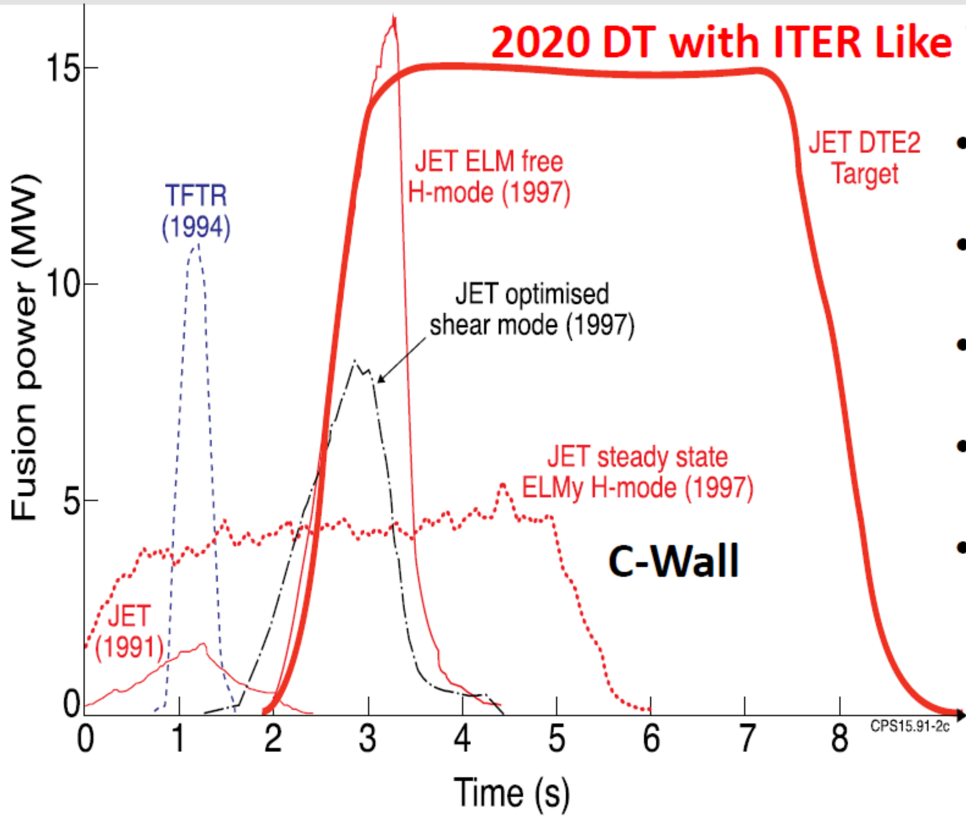


- DT-Experiments only in
- JET
- TFTR

- Present machines produced significant fusion power:
  - TFTR (USA) ~10 MW in 1994
  - JET (EU) 16 MW (Q = 0.64) in 1997

Status of the Tokamak Research

Objectives of 2020 JET D-T operation:
15MW fusion power for 5 sec stationary state



- ITER scenarios
- Isotope effect
- T-cycle
- α-particle physics
- Fusion technology (Neutronics , remote-handling, …)

Previous DT was 20 years before and next DT will be 20 years later in ITER.
2020 JET DT is also Important to transfer the knowledge to the ITER generation.

JET

Status of the Tokamak Research

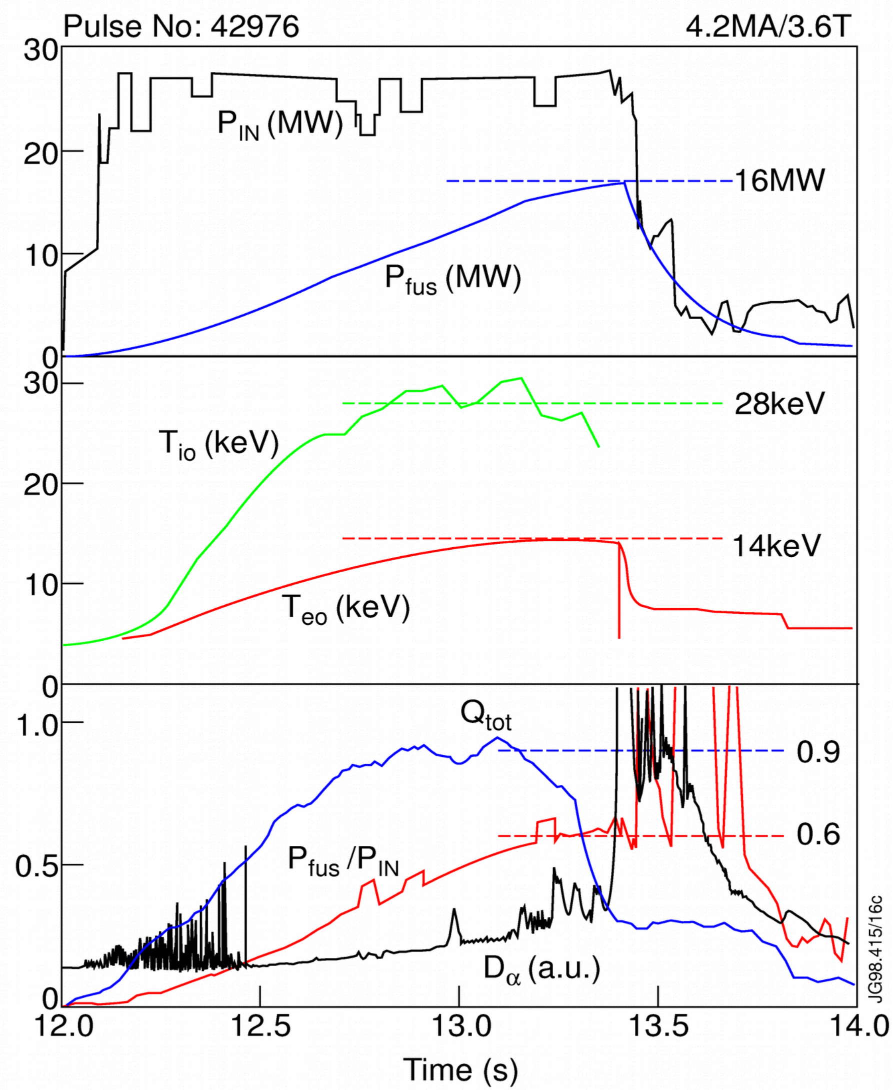
- DT-Experiments only in

- JET
- TFTR

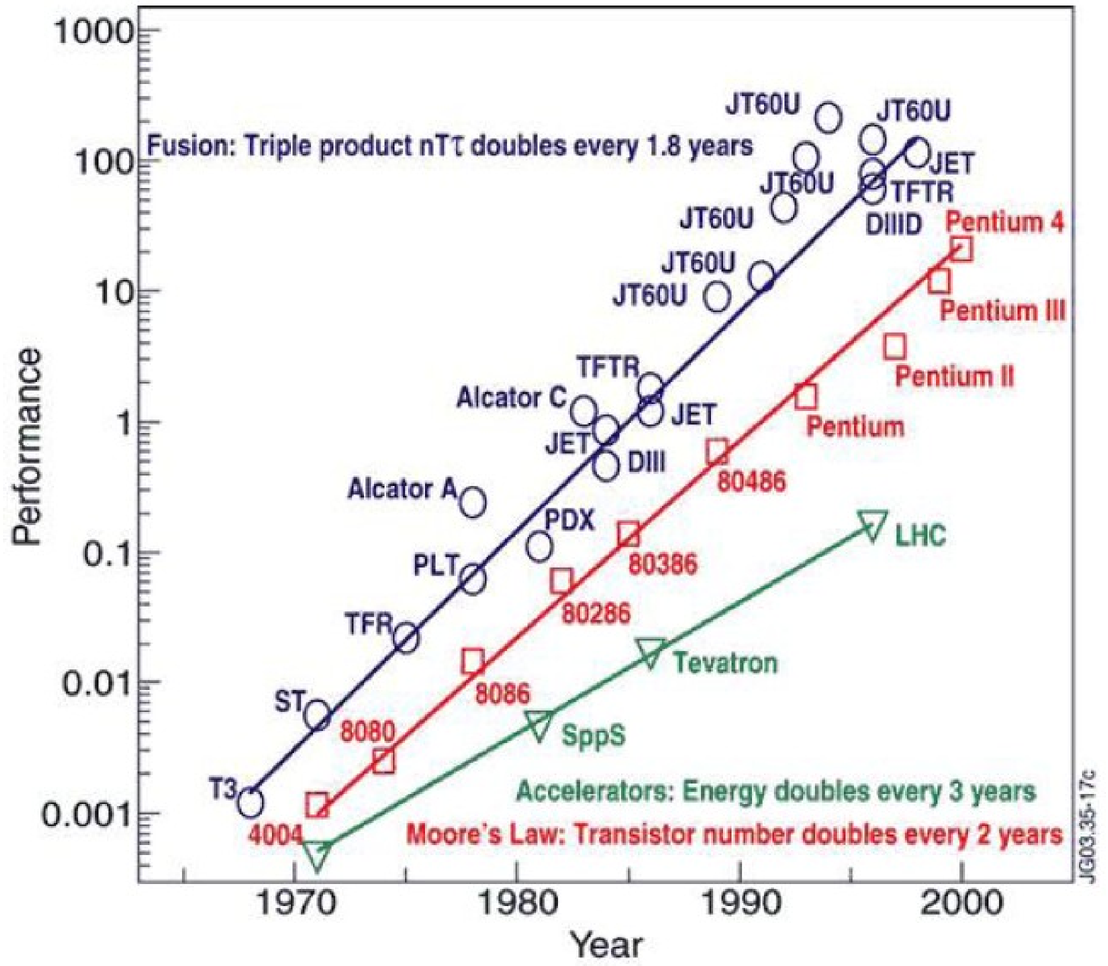
- with world records in JET:

- P_{fusion} = 16 MW
- Q = 0.65

HW. Q_{tot} ?



Status of the Tokamak Research



- Progress in fusion can be compared with the computing power and particle physics accelerator energy.

Homework

- Problems 8.4

(submission until the next Thursday)

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