Fusion Reactor Technology 2
(459.761, 3 Credits)

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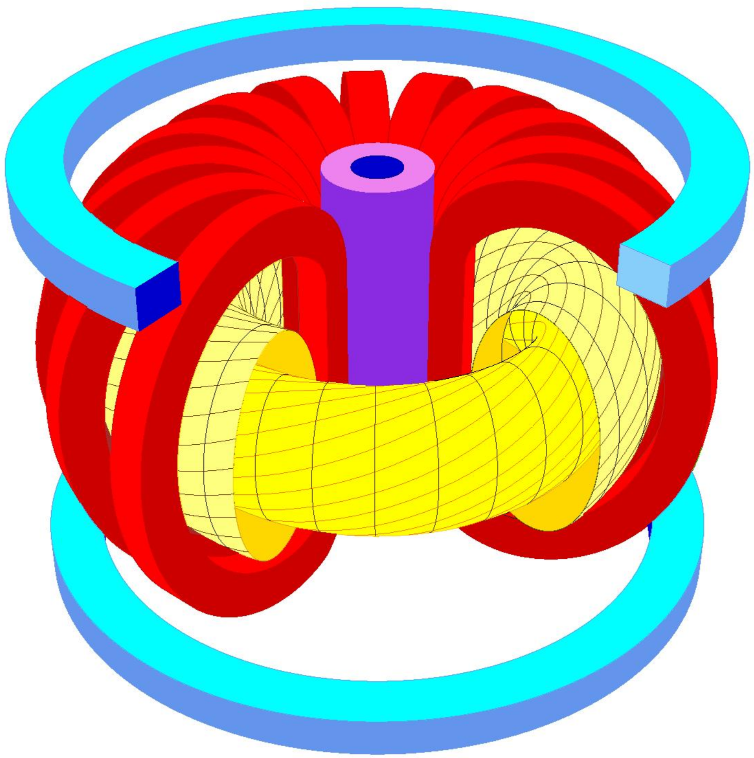
Objectives of the Tokamak Operation

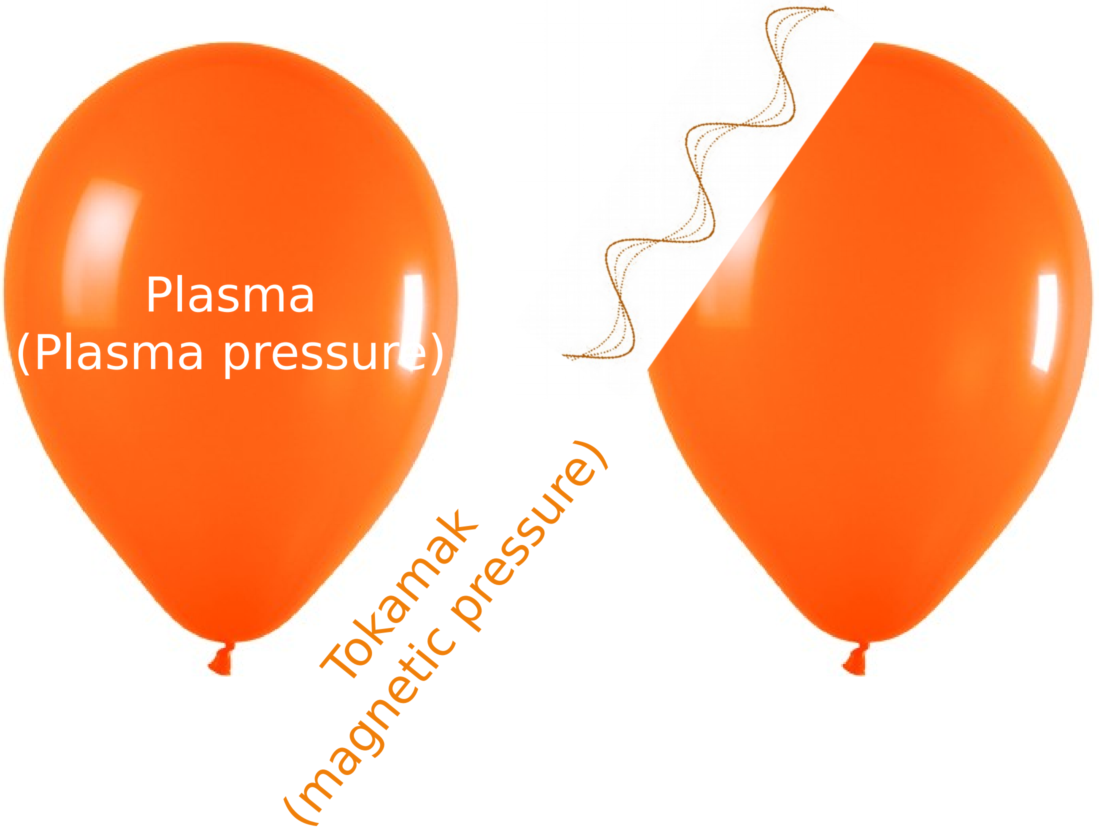
$$n\tau_E T \geq 3 \times 10^{21} m^{-3} keVs = 5\, bar \cdot s$$



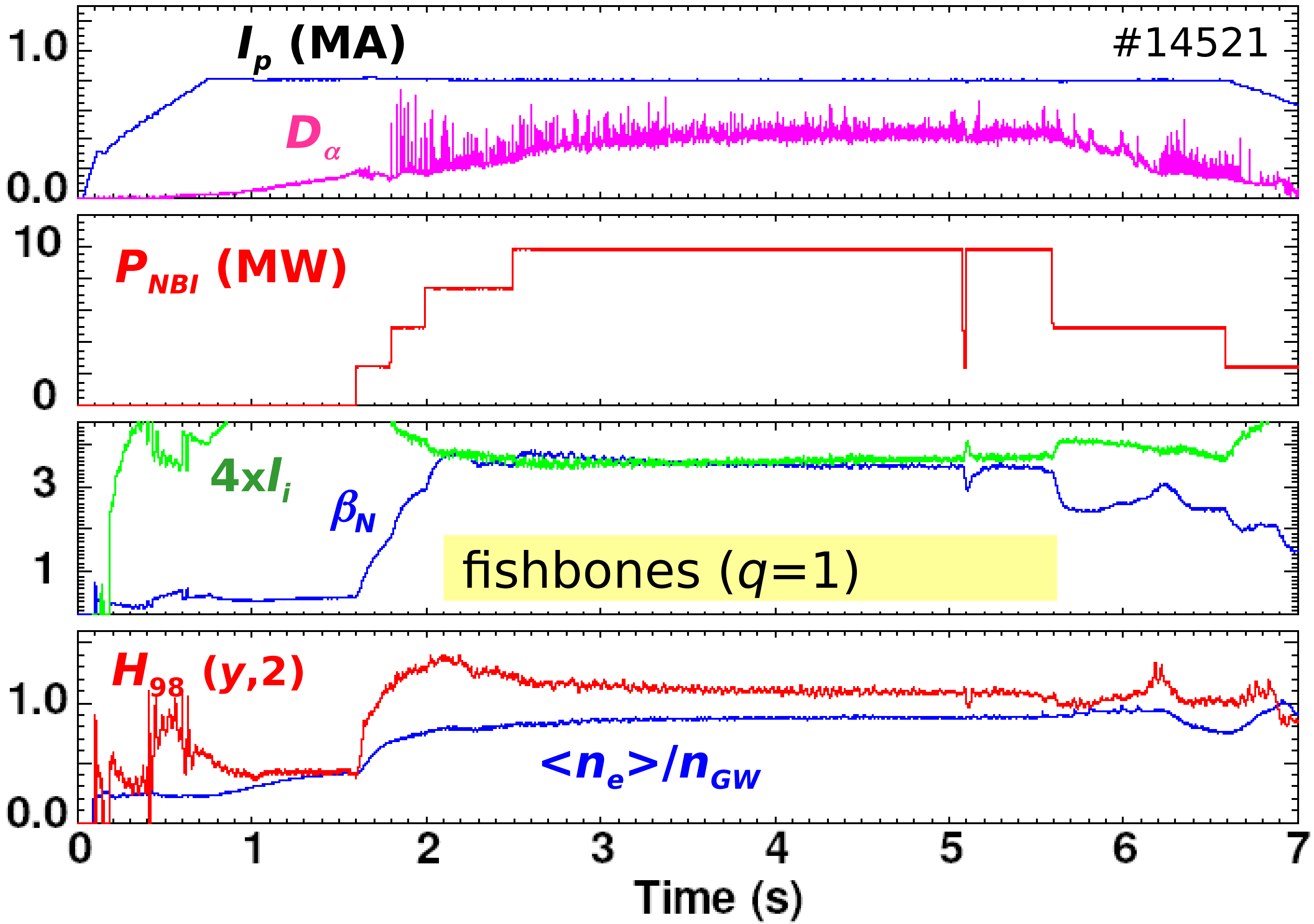
- High $<n_e>/n_{GW}$
- High β_N
- High $H_{98}(y,2)$
- Pulse length

→ Stability, confinement issue

Plasma Equilibrium, Stability and Transport

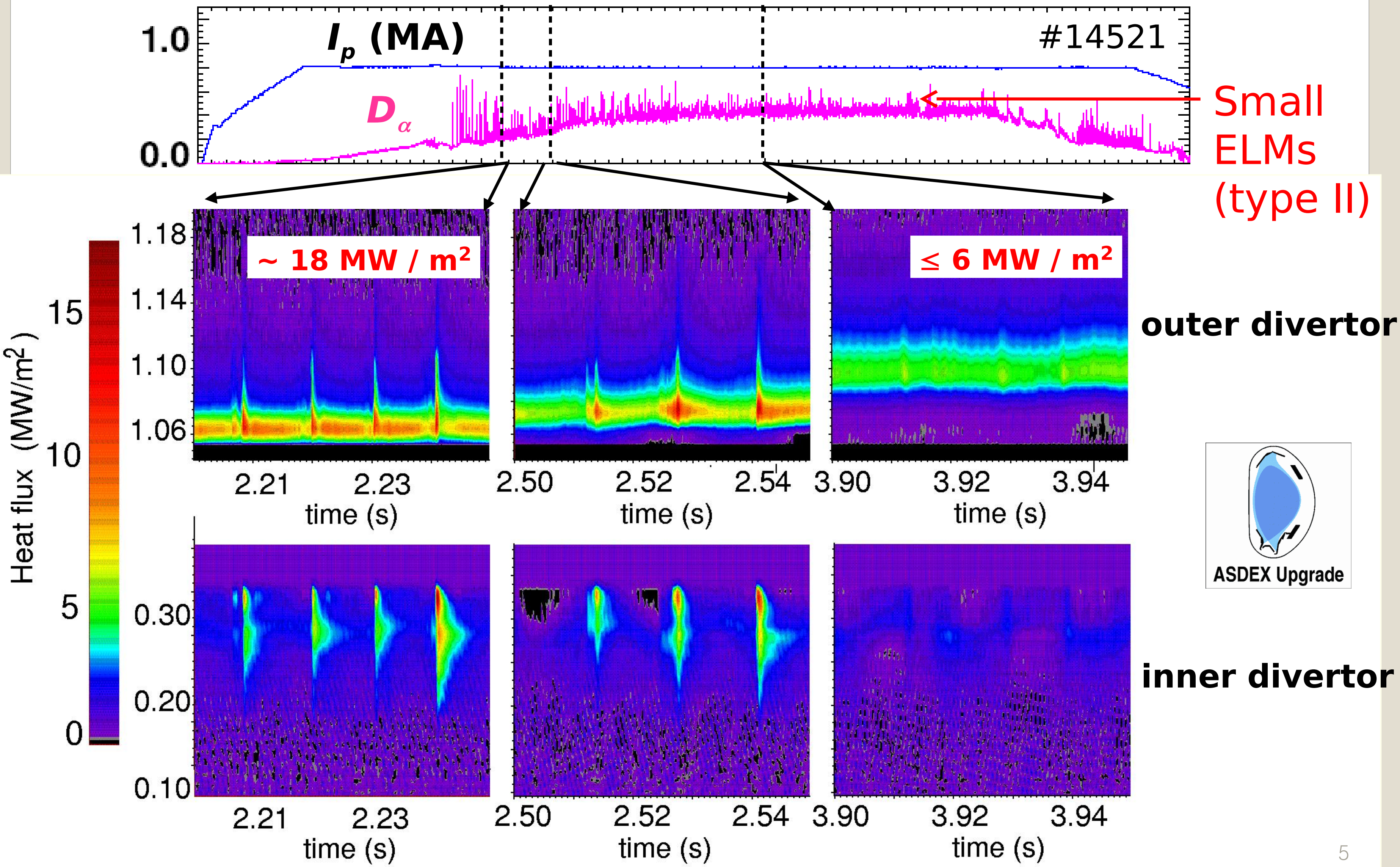


Objectives of the Tokamak Operation



- No sawteeth, good confinement, and $\beta_N \sim 3.5$, $T_i \sim T_e$, $\langle n_e \rangle / n_{GW} \sim 0.88$, averaged over 3.6 seconds ($\sim 50\ \tau_E$).

Objectives of the Tokamak Operation



Plasma as a Complex System

- **High-temperature plasma, confined by a magnetic field, is an exceptionally unusual physical object → complex physical system**

- Presence of macroscopic instabilities
- Local entropy production due to local plasma transport
- Rare Coulomb collisions (anomalous transport)
- Non-linear phenomena (noise source) in the edge plasma propagating inside the plasma core leading to transport enhancement
- Heating resulting in additional noise generation

cf) OH heating: drift current velocity of electrons ~ 10^5 m/s

≪ sound velocity (j ~ 1 MA/m^2, n_e ~ 10^{20} m^{-3})

$$V_{Alfven} = \sqrt{\frac{B_0^2}{\mu_0 \rho_0}}, \quad V_{adiabatic\ sound} = \sqrt{\frac{\gamma p_0}{\rho_0}}$$

Plasma as a Complex System

- A rational approach to study complex systems consists of a large number of experiments aimed at **understanding empirical laws**

  supported by development of a theoretical description and computer models.
- All this is actively used in modern tokamak studies.
- As experience with other complex systems shows, the general method of **scaling and dimensional approach** represents a powerful tool for their description.

Dimensional Analysis of Tokamaks

- **Dimensional approach**
  - All the laws of physics are based on mechanics.
  - Mechanics uses conventionally chosen units for mass, length, and time.
  - The objective laws of nature cannot depend on those units. These laws are invariant with respect to variations of measurement units chosen by man.
  - This invariance is seen more precisely when non-dimensional combinations of dimensional values are used.
  - The non-dimensional parameters define the internal physics of a complex system; indicators of the fundamental state of the system.

- Dimensional parameters look like some projection of a given system on the external world.

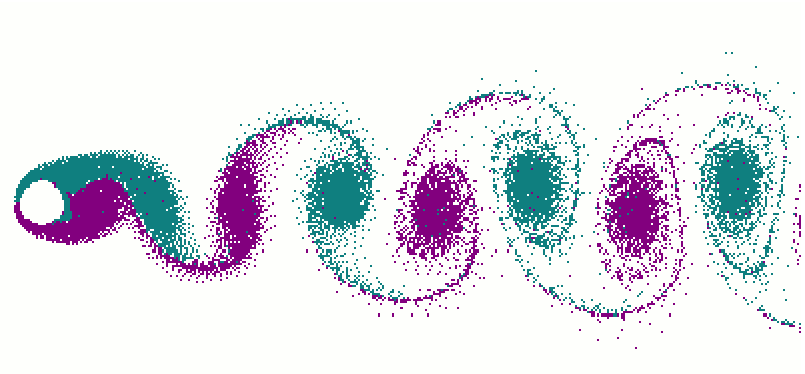
Dimensional Analysis of Tokamaks

- **Dimensional approach - Example**

  - Reynolds number

$$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho v L}{\mu} = \frac{vL}{\nu}$$

ρ: density of the fluid (kg/m³)
v: mean velocity of the object relative to the fluid (m/s)
L: a characteristic linear dimension, (travelled length of the fluid; hydraulic diameter when dealing with river systems) (m)
μ: dynamic viscosity of the fluid (Pa·s or N·s/m² or kg/(m·s))
ν: kinematic viscosity (*μ*/*ρ*) (m²/s)



A vortex street around a cylinder. This occurs around cylinders, for any fluid, cylinder size and fluid speed, provided that there is a Reynolds number of between ~40 and 103

Dimensional Analysis of Tokamaks

- **Dimensional approach**

  - Being immersed in the external physical world, each complex system can possess a non-unique set of dimensional parameters.

  - For a given set of dimensionless parameters the family of systems can exist with different sets of dimensional parameters.

  → Self-similarity

  - Therefore, all the objective laws of physics may be presented as relations between non-dimensional parameters.

  - Dimensional analysis should always be based on reasonable physical parameters which are specific for each particular case. Such an approach can allow us to pick out the most relevant parameters and to drop the unimportant ones.

Dimensionless Parameters

- The dimensional parameters

$$a,\ R,\ B_T,\ B_p,\ m_e,\ m_i,\ e,\ n,\ T,\ v$$

- Frequently used non-dimensional parameters for tokamak plasmas

$$A = R/a$$

$$q_a = aB_T / RB_p$$

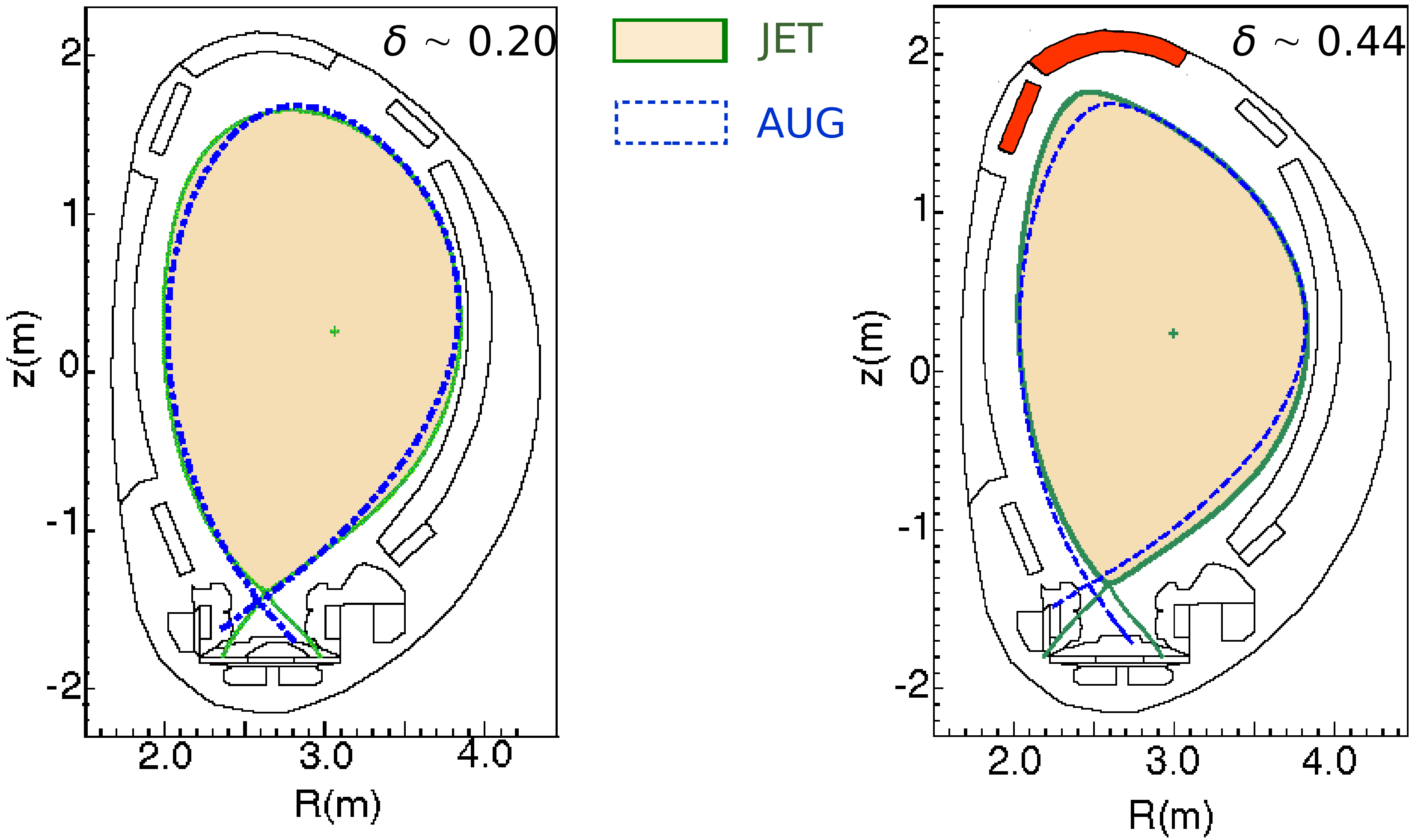
$$\beta = \frac{\text{plasma pressure}}{\text{magnetic pressure}} = \frac{2\mu_0 n(T_e + T_i)}{B^2}$$

$$\rho^* = \frac{\text{ion gyroradius}}{\text{minor radius}} = \frac{\rho_i}{a} = \left(\frac{2T_i}{m_i}\right)^{1/2} \frac{m_i}{eBa}$$

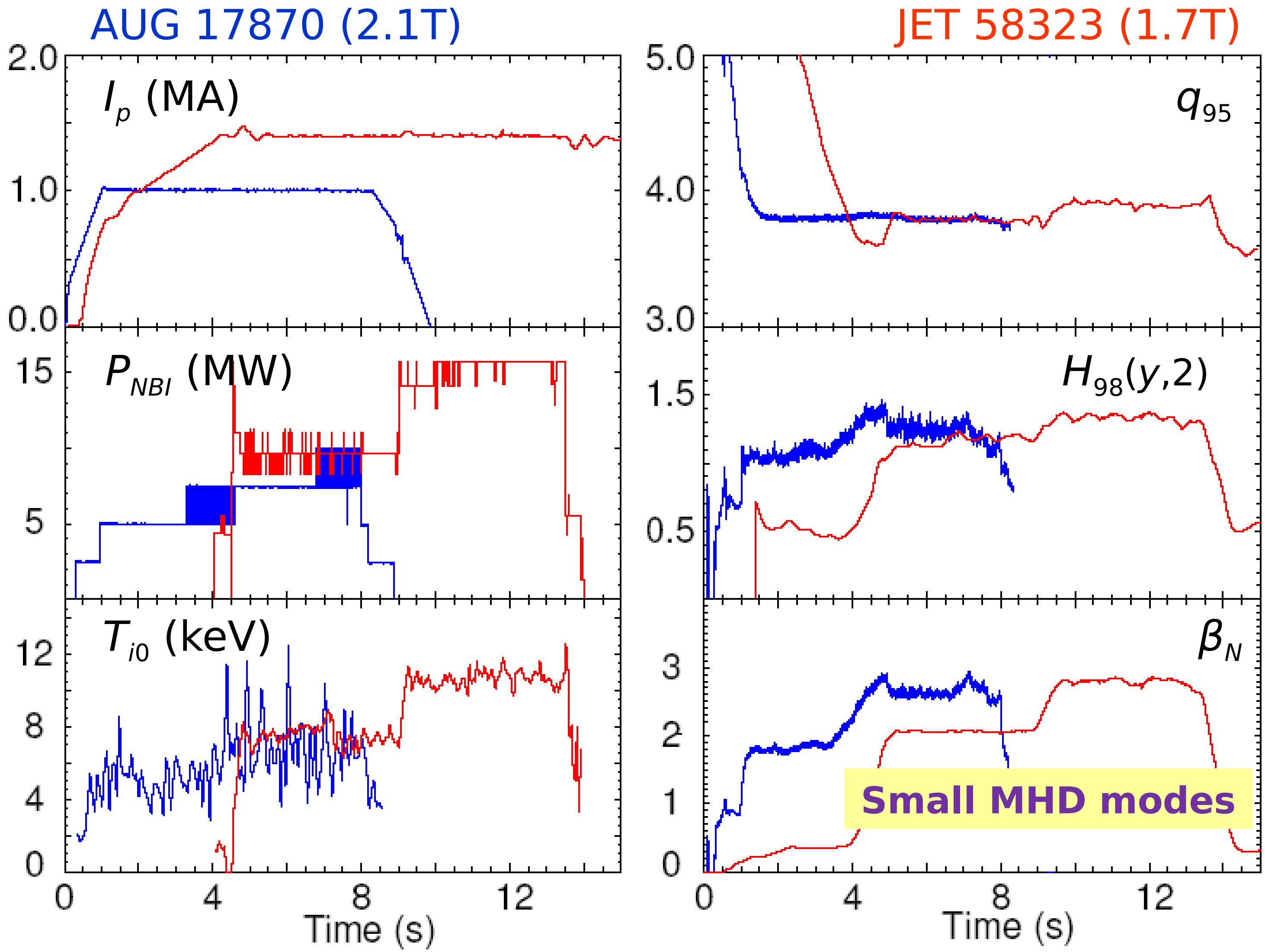
$$\nu^* = \frac{\text{connection length}}{\text{trapped particle mean-free-path}} = \nu_{ii}\left(\frac{m_i}{T_i}\right)^{1/2}\left(\frac{R}{a}\right)^{3/2} qR$$

Identity (Similarity) Experiments

- **Plasma shapes used in JET compared to ASDEX Upgrade**

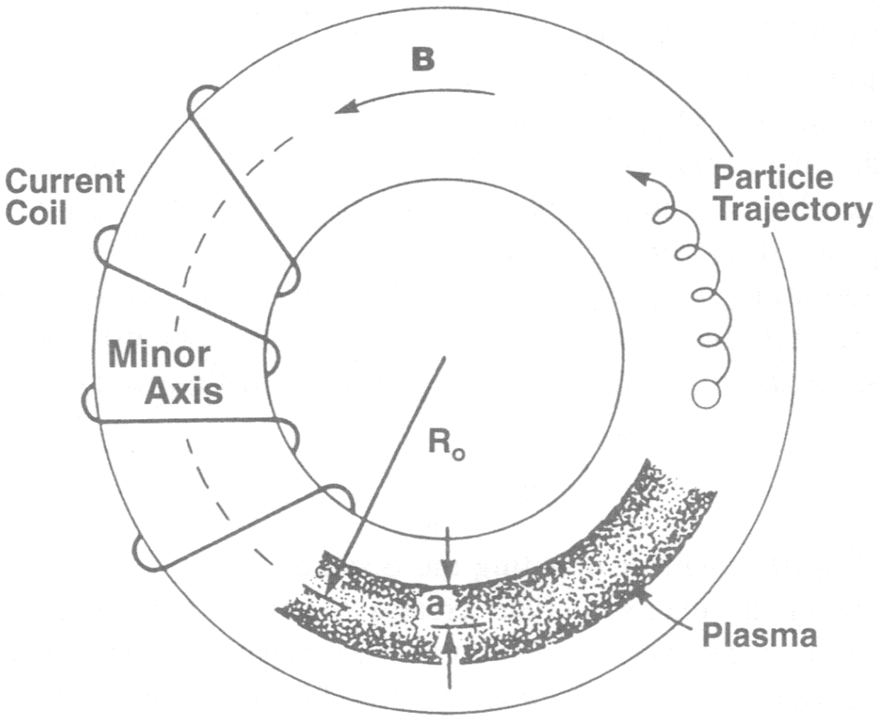


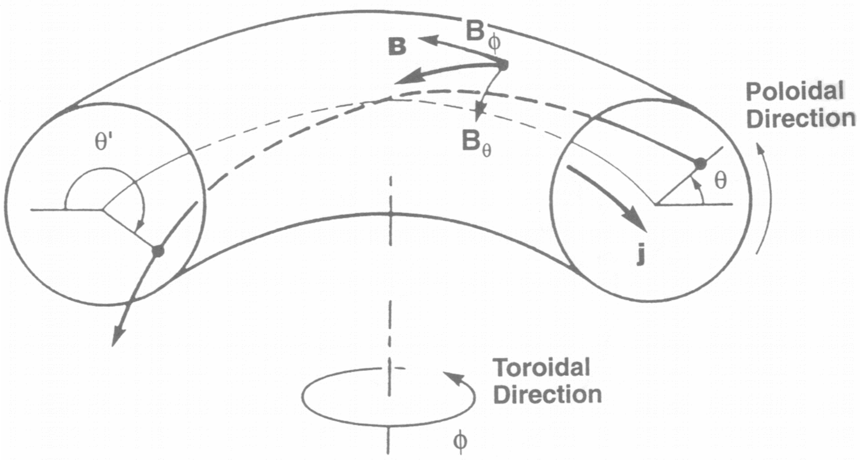
Identity (Similarity) Experiments



Basic Tokamak Variables

- **Cylindrical and local coordinates for a tokamak**





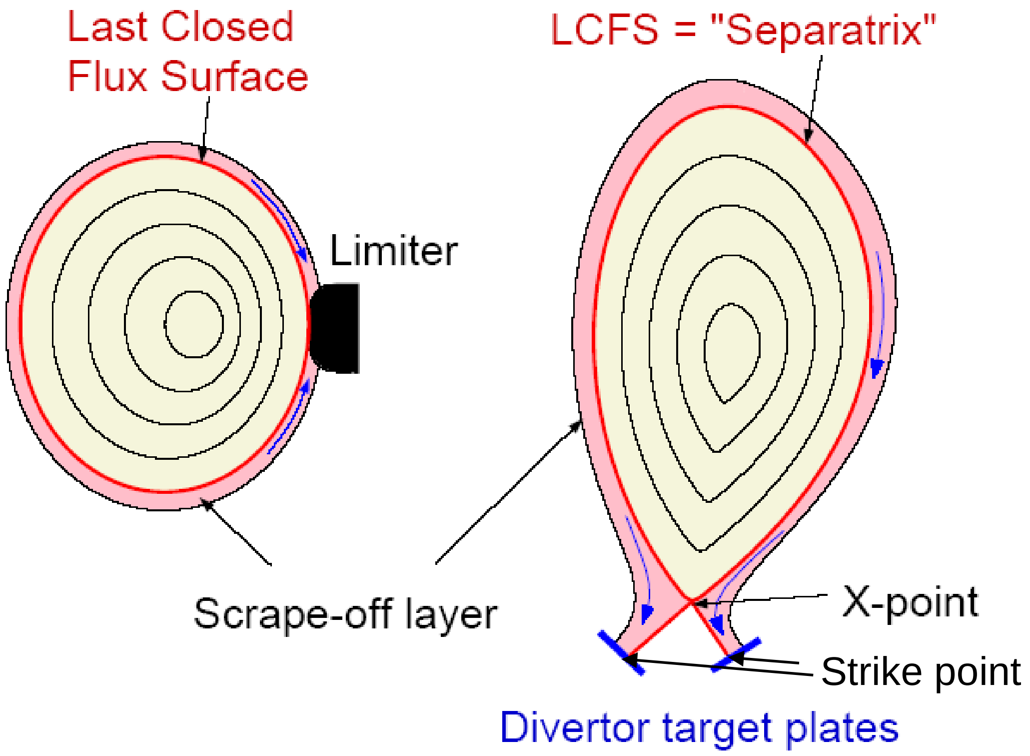
- Aspect ratio: $A = R_0/a \sim$ 3-5

  ex) KSTAR: 3.6, ITER: 3.1

- Inverse aspect ratio: $\varepsilon = a/R_0$

Basic Tokamak Variables

- **Plasma configuration**



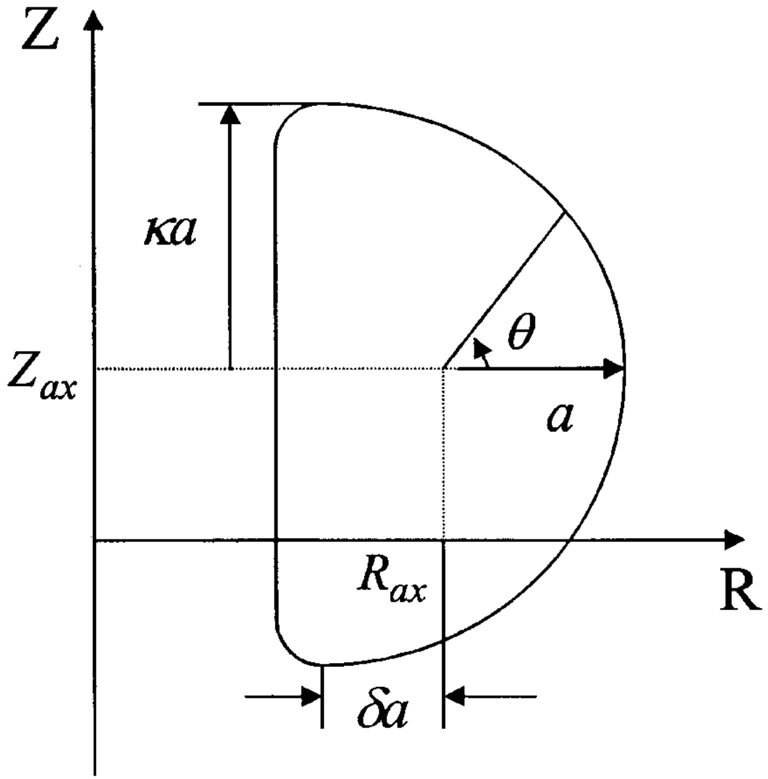
If no limiter and divertor?

Plasma diffusing into the whole vessel along the magnetic field → if touching the wall, impurities coming out

- Limiter configuration
- Divertor configuration

Basic Tokamak Variables

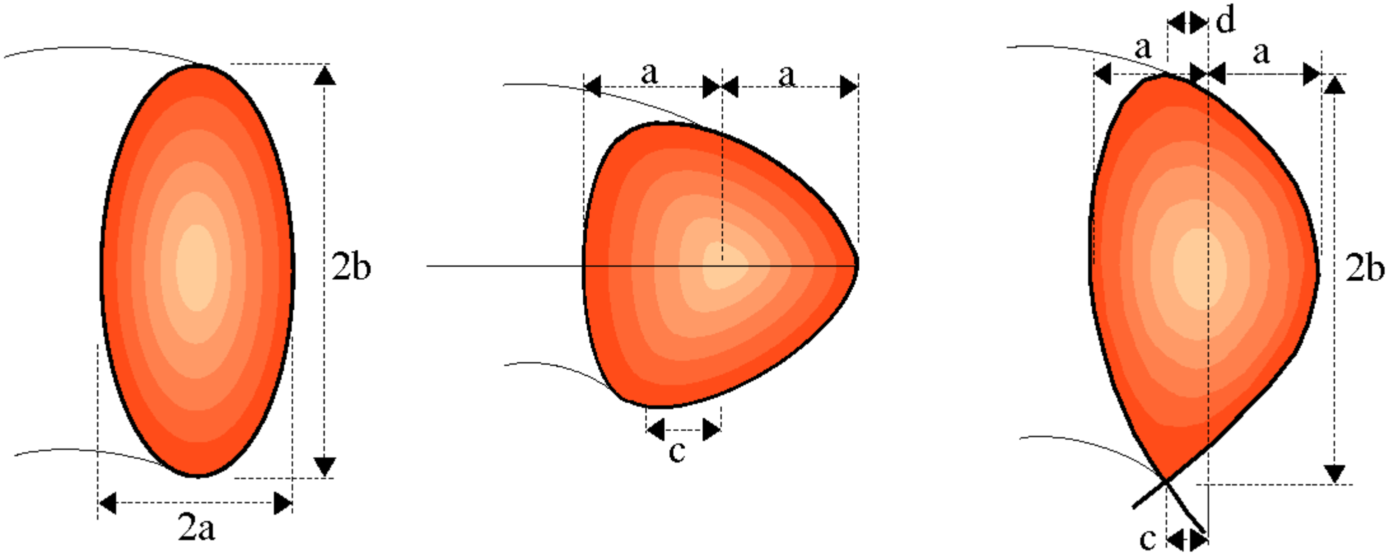
- **Plasma equilibrium parameters**



- Elongation: κ
- Triangularity: δ
- Squareness: ζ

Basic Tokamak Variables

- **Plasma equilibrium parameters**



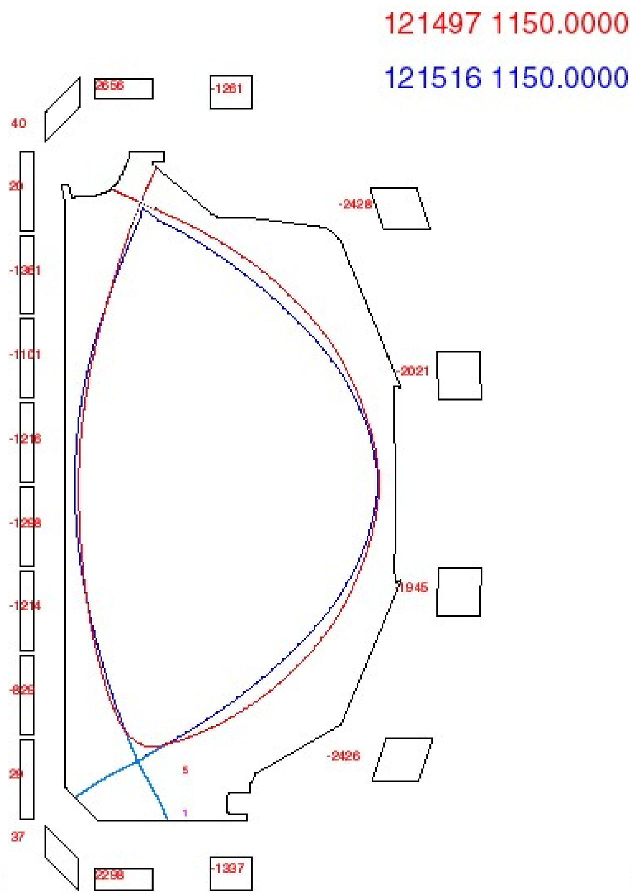
$$\kappa = \frac{b}{a} \qquad \delta = \frac{c}{a} \qquad \delta = \frac{c+d}{2a}$$

$$R = R_0 + a\cos\left(\theta + \sin^{-1}\delta\sin\theta\right)$$

$$Z = \kappa a \sin\theta$$

Basic Tokamak Variables

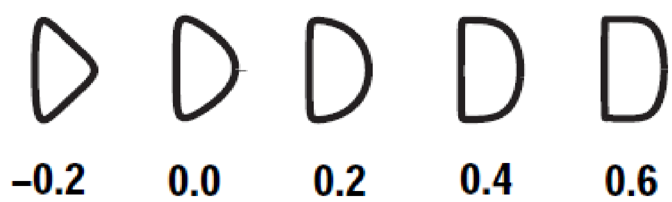
- **Plasma equilibrium parameters**



- Outer and inner squareness: $\zeta_{o,i}$

$$R = R_0 + a\cos\left(\theta + \sin^{-1}\delta\sin\theta\right)$$
$$Z = \kappa a\sin\left(\theta + \zeta_{o,i}\sin 2\theta\right)$$

HW: derive!

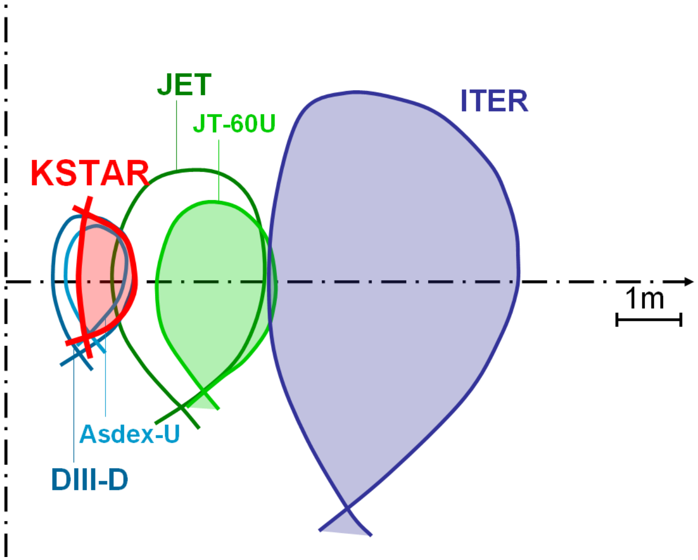


Basic Tokamak Variables

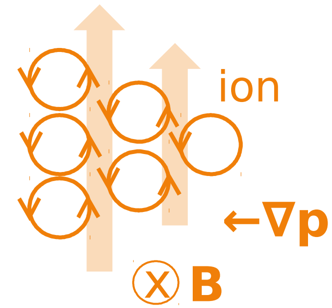
- **Plasma equilibrium parameters**

Parameters	KSTAR	ITER
Major Radius, R_0	1.8 m	6.2 m
Minor Radius, a	0.5 m	2.0 m
Plasma Current, I_P	2.0 MA	15 MA
Elongation, κ_x	2.0	1.85
Triangularity, δ_x	0.8	0.5
Toroidal Field, B_0	3.5 T	5.3 T
Pulse Length	300 s	500 s
Fuel	H, D	D, T

- Plasma shape



Basic Tokamak Variables



- **Plasma Equilibrium**

$$\boxed{\begin{gathered}\nabla p = J \times B \\ \nabla \times B = \mu_0 J \\ \nabla \cdot B = 0\end{gathered}}$$

→ Force balance

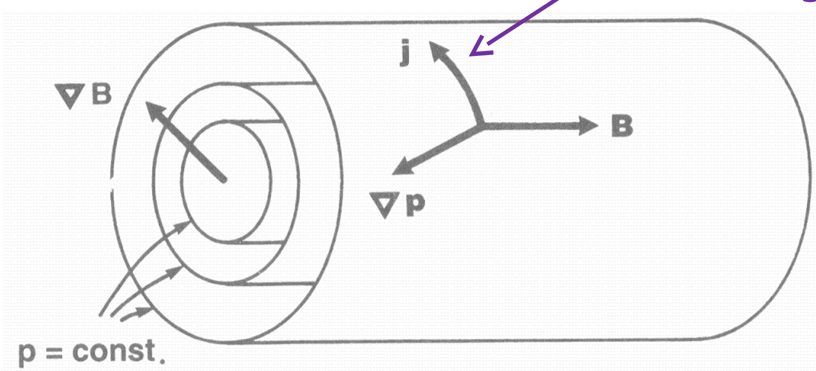
→ Ampere‘s law

→ Closed magnetic field lines

kinetic pressure
balanced by **J**x**B** (Lorentz) force

$$B \cdot \nabla p = 0 \qquad J \cdot \nabla p = 0$$

induced by the pressure gradient:
causing a decrease in **B** → diamagnetism



Diamagnetic current

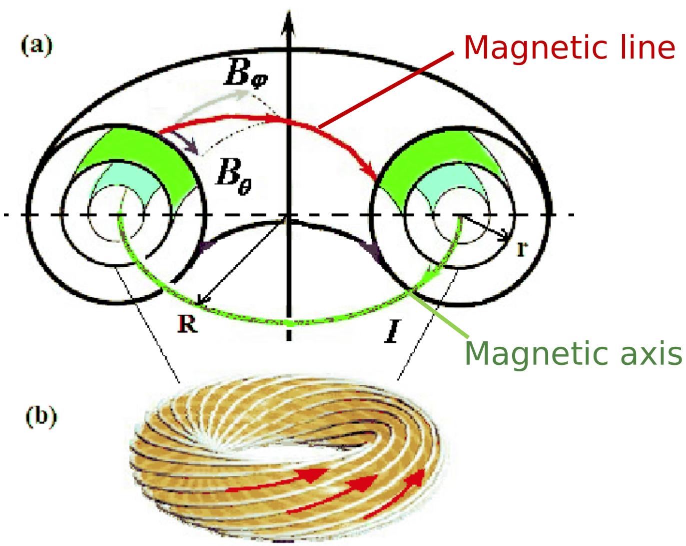
$$\vec{v}_{D,\nabla p} = -\frac{\nabla p \times B}{nqB^2}$$

$$\vec{J} = n_i q_i \vec{v}_{D,i} + n_e q_e \vec{v}_{D,e} = \frac{B \times \nabla p}{B^2}$$

- If B_z is applied, plasma equilibrium can be built by itself due to induction of diamagnetic current. $\nabla p = J \times B$

Basic Tokamak Variables

- **Magnetic Flux Surfaces**
  - In fusion configurations with confined plasmas the magnetic lines lie on a set of nested toroidal surfaces called flux surfaces.
  - Pressure is constant along a magnetic field line.
  - Magnetic lines lie in surfaces of constant pressure.
  - Flux surfaces are surfaces of constant pressure.
  - The current lines lie on surfaces of constant pressure.
  - The current flows between flux surfaces and not across them.
  - The angle between **J** and **B** is arbitrary.

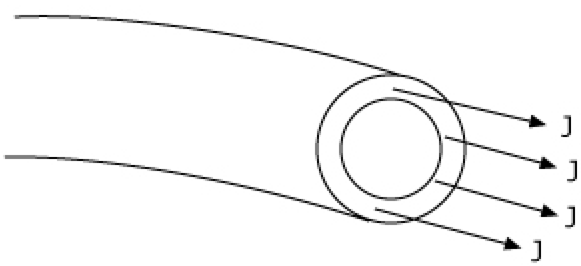


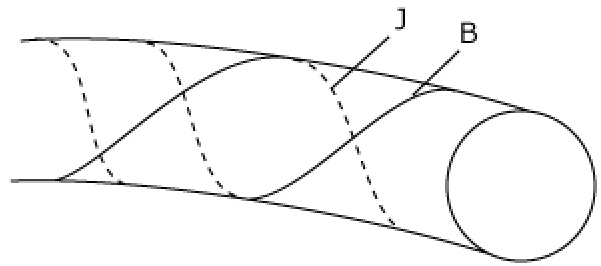
$$J \times B = \nabla p$$

$$B \cdot \nabla p = 0$$

$$J \cdot \nabla p = 0$$

If $J = J_{||}$,
$\mathbf{J} \times \mathbf{B} = \nabla p = 0$

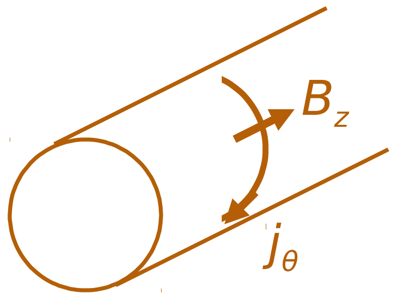




Basic Tokamak Variables

- **Magnetic Flux Surfaces**

  - Consider particle motion in a cylindrically symmetric configuration, i.e. $\partial/\partial\theta = 0$



$$B = \nabla \times A$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{j}'(\vec{r}')}{|\vec{r} - \vec{r}'|} d^3 r' \quad \text{vector potential}$$

$$\vec{B} = \nabla \times A_\theta \vec{e}_\theta = -\frac{\partial A_\theta}{\partial z}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\vec{e}_z$$

Equation of particle motion

$$m\frac{dv}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$= q\left\{\vec{E}_A + \left[\vec{v} \times \left(-\frac{\partial A_\theta}{\partial z}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\vec{e}_z\right)\right]\right\}$$

Basic Tokamak Variables

- **Magnetic Flux Surfaces**

$$\nabla \times \vec{E}_A = -\frac{\partial B}{\partial t} = -\nabla \times \frac{\partial \vec{A}}{\partial t}$$

$$\vec{E}_A = -\frac{\partial \vec{A}}{\partial t} = -\dot{A}_\theta \vec{e}_\theta$$

Equation of particle motion

$$m\frac{d\vec{v}}{dt} = q\left\{\vec{E}_A + \left[\vec{v} \times \left(-\frac{\partial A_\theta}{\partial z}\vec{e}_r + \frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\vec{e}_z\right)\right]\right\}$$

$$= q\left\{-\dot{A}_\theta \vec{e}_\theta + \left[\frac{v_\theta}{r}\frac{\partial}{\partial r}(rA_\theta)\vec{e}_r - \left(\frac{v_r}{r}\frac{\partial}{\partial r}(rA_\theta) + v_z\frac{\partial A_\theta}{\partial z}\right)\vec{e}_\theta + v_\theta\frac{\partial A_\theta}{\partial z}\vec{e}_z\right]\right\}$$

$$v_\theta = \dot{l}_\theta = \dot{r}\theta + r\dot{\theta}, \quad v_r = \dot{r}, \quad v_z = \dot{z}$$

$$\dot{v}_\theta = \ddot{r}\theta + 2\dot{r}\dot{\theta} + r\ddot{\theta}$$

Basic Tokamak Variables

- **Magnetic Flux Surfaces**

Equation of particle motion

$$m\frac{dv}{dt} = q\left\{ -\dot{A}_\theta \vec{e}_\theta + \left[\frac{v_\theta}{r}\frac{\partial}{\partial r}(rA_\theta)\vec{e}_r - \left(\frac{v_r}{r}\frac{\partial}{\partial r}(rA_\theta) + v_z\frac{\partial A_\theta}{\partial z} \right)\vec{e}_\theta + v_\theta\frac{\partial A_\theta}{\partial z}\vec{e}_z \right] \right\}$$

θ-component

$$m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right) = -\frac{q}{r}\left(\dot{r}A_\theta + r\dot{A}_\theta + r\dot{r}\frac{\partial A_\theta}{\partial r} + r\dot{z}\frac{\partial A_\theta}{\partial z} \right)$$

$$= -\frac{q}{r}\left(\dot{r}A_\theta + r\frac{\partial A_\theta}{\partial t} + r\frac{\partial r}{\partial t}\frac{\partial A_\theta}{\partial r} + r\frac{\partial z}{\partial t}\frac{\partial A_\theta}{\partial z} \right)$$

$$= -\frac{q}{r}\left(\dot{r}A_\theta + r\frac{dA_\theta}{dt} \right)$$

$$= -\frac{q}{r}\frac{d}{dt}\left[r(t)A_\theta(r,z,t) \right]$$

Basic Tokamak Variables

- **Magnetic Flux Surfaces**

Multiply by *r*

$$m\left(r^2\ddot{\theta}+2r\dot{r}\dot{\theta}\right)+r\frac{q}{r}\frac{d}{dt}(rA_\theta)=\frac{d}{dt}\left(mr^2\dot{\theta}+qrA_\theta\right)=\frac{d}{dt}(l)=0$$

→ Canonical momentum *l* due to the rotational motion about the z-axis conserved

$$rA_\theta\left(\frac{mr\dot{\theta}}{qA_\theta}+1\right)=\frac{l}{q}$$

$$rA_\theta\left(\frac{mr\dot{\theta}}{qA_\theta}+1\right)=rA_\theta\left(\frac{mr\dot{\theta}}{\frac{q}{2}rB_z(0)}+1\right)\longleftarrow$$

$$B_z=(\nabla\times\vec{A})\cdot e_z=\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)$$

$$A_\theta=\frac{1}{r}\int_0^r r'B_z(r')dr'\approx\frac{1}{2}rB_z(0)$$

$$=rA_\theta\left(\frac{2mv_\perp}{r|q|B_z(0)}+1\right)=rA_\theta\left(\frac{2r_L}{r}+1\right)=\frac{l}{q}=const.$$

Basic Tokamak Variables

- **Magnetic Flux Surfaces**

$$rA_\theta\left(\frac{2r_L}{r}+1\right)=\frac{l}{q}=const.$$

- $r_L/r << 1$ → The trajectories of the particles must lie on surfaces defined by $rA_\theta = const.$

→ Flux surface label:
The particle's guiding centers move on them in the absence of other forces (as a consequence of angular momentum conservation)

- Magnetic field lines lie within these surfaces which can be readily demonstrated by proving that the surface's normal is orthogonal to the field.

$$\vec{B}\cdot\nabla(rA_\theta)=B_r\frac{\partial(rA_\theta)}{\partial r}+B_z\frac{\partial(rA_\theta)}{\partial z}=0 \longleftarrow$$

$$B_r=\vec{e}_r\cdot\left(\nabla\times\vec{A}\right)=-\frac{\partial A_\theta}{\partial z}$$

$$B_z=\vec{e}_z\cdot\left(\nabla\times\vec{A}\right)=\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)$$

Basic Tokamak Variables

- **Plasma Equilibrium**

$$\nabla p = J \times B$$

$$\nabla \times B = \mu_0 J$$

$$\nabla \cdot B = 0$$

→ Force balance

→ Ampere's law

→ Closed magnetic field lines

kinetic pressure
balanced by **J**x**B** (Lorentz) force

$$\nabla p = (\nabla \times B) \times B / \mu_0$$

$$= [(B \cdot \nabla) B - \nabla (B^2 / 2)] / \mu_0$$

$$\nabla (p + B^2 / 2\mu_0) = (B \cdot \nabla) B / \mu_0$$

Assuming the field lines are straight and parallel

$$\frac{E^*_{mag}}{V} = \frac{BH}{2} = \frac{B^2}{2\mu_0}$$

$$p + \frac{B^2}{2\mu_0} = \text{constant}$$

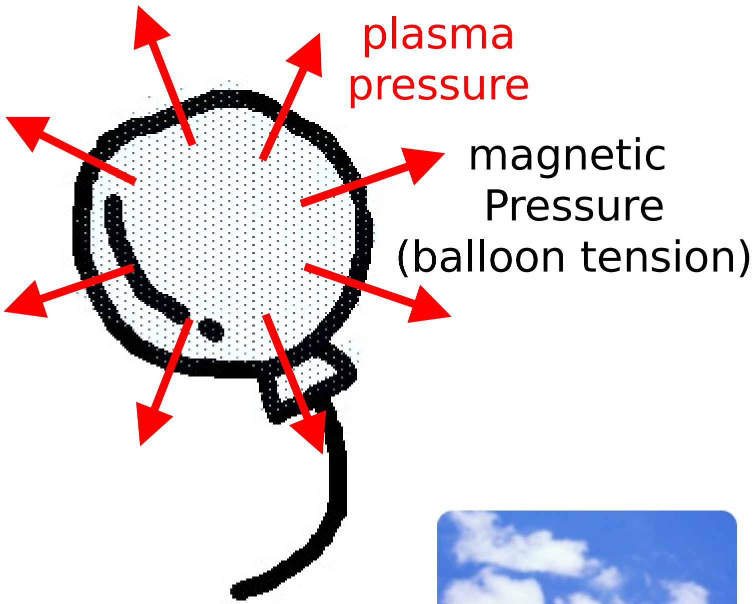
Total sum of kinetic pressure
and magnetic field energy density
will be a constant

The surfaces of constant B must also be surfaces of constant pressure.

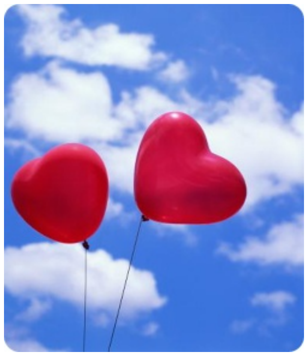
Basic Tokamak Variables

- **Concept of Beta**

$$\beta = \frac{p}{B^2 / 2\mu_0} = \frac{\sum_{i,e} nkT}{B^2 / 2\mu_0}$$



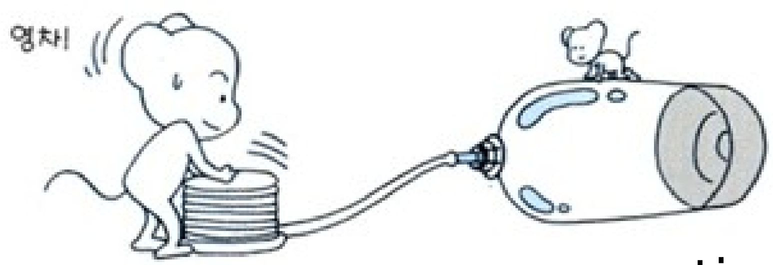
- The ratio of the plasma pressure to the magnetic field pressure
- A measure of the degree to which the magnetic field is holding a non-uniform plasma in equilibrium.
- In most magnetic configurations, fusion plasma confinement requires an imposed magnetic pressure significantly exceeding the particle kinetic pressure.



Basic Tokamak Variables

- **Concept of Beta**

Instability
(bad curvature region)
when with high *p*



plasma pressure

magnetic pressure

www.waterrocket.com/goachaik-28.htm
the43sunsets.tistory.com/tag/코카콜라

$$\beta = 2\mu_0 p / B^2$$

- β is related with fusion reactor economics and technology.
- Maximum allowable value is set by MHD equilibrium requirements and instabilities driven by the pressure gradient.

Basic Tokamak Variables

- **Concept of Beta**

  - Assuming that the magnetic surfaces have concentric, circular CXs and that conditions are independent of φ.

$$\beta_t = \frac{2\mu_0\langle p\rangle}{B_\varphi^2}, \quad \beta_p = \frac{2\mu_0\langle p\rangle}{B_{\theta a}^2} = \frac{8\pi^2 a^2\langle p\rangle}{\mu_0 I_p^2} \qquad \bar{\beta} \equiv \frac{2\mu_0\langle p\rangle}{B_\varphi^2 + B_{\theta a}^2} \qquad \frac{1}{\bar{\beta}} = \frac{1}{\beta_t} + \frac{1}{\beta_p}$$

$$\langle p\rangle = \int p dS \,/\, \int dS = \frac{2\pi}{\pi a^2}\int_0^a p(r) r dr$$

$$\nabla \times B = \mu_0 j$$ Ampère’s law

$$\frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = \mu_0 j_\varphi, \quad B_\theta = \frac{\mu_0}{r}\int_0^r j_\varphi(r')r'dr'$$

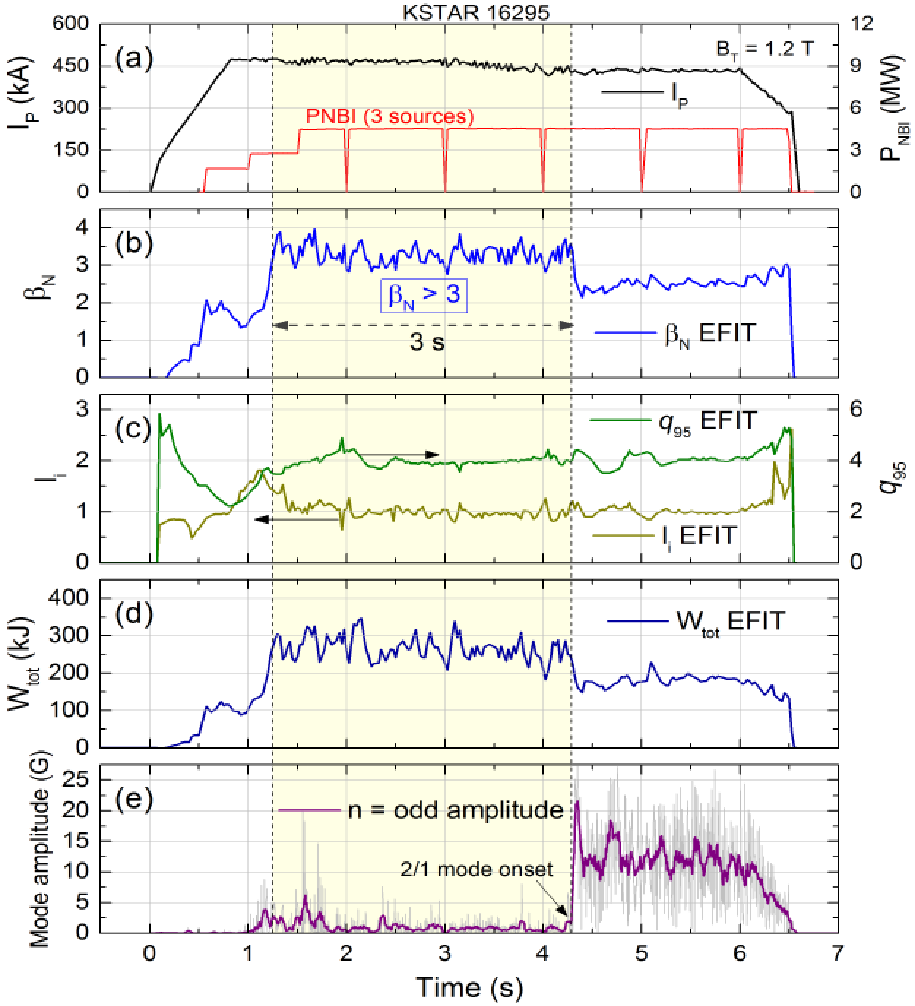
$$I_p = 2\pi\int_0^a j_\varphi r dr = 2\pi a B_{\theta a} / \mu_0$$

- The power output for a given magnetic field and plasma assembly is proportional to the square of beta.
- In a reactor it should exceed 0.1: economic constraint

Basic Tokamak Variables

- **Normalized beta – stability limit**

$$\beta_N = \beta_t \frac{aB_t}{I_p}$$



Basic Tokamak Variables

- **Normalized beta – stability limit**

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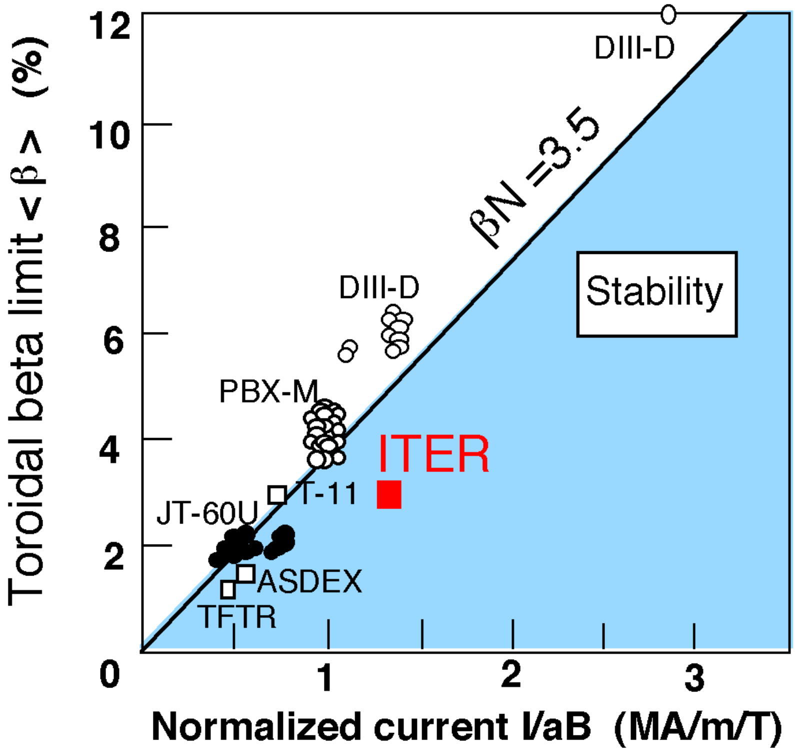


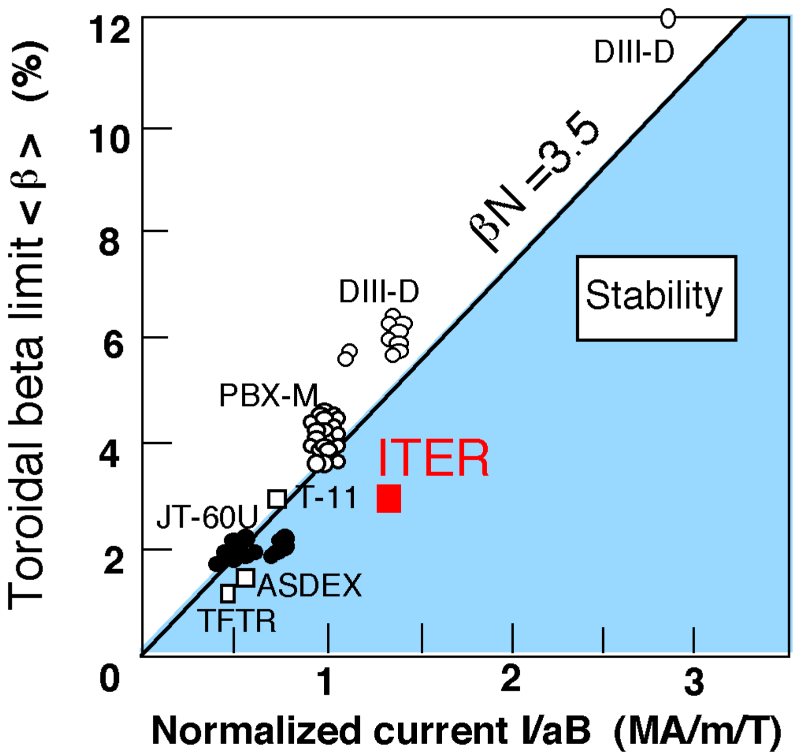
- Fundamental elements affecting the β_N-limit

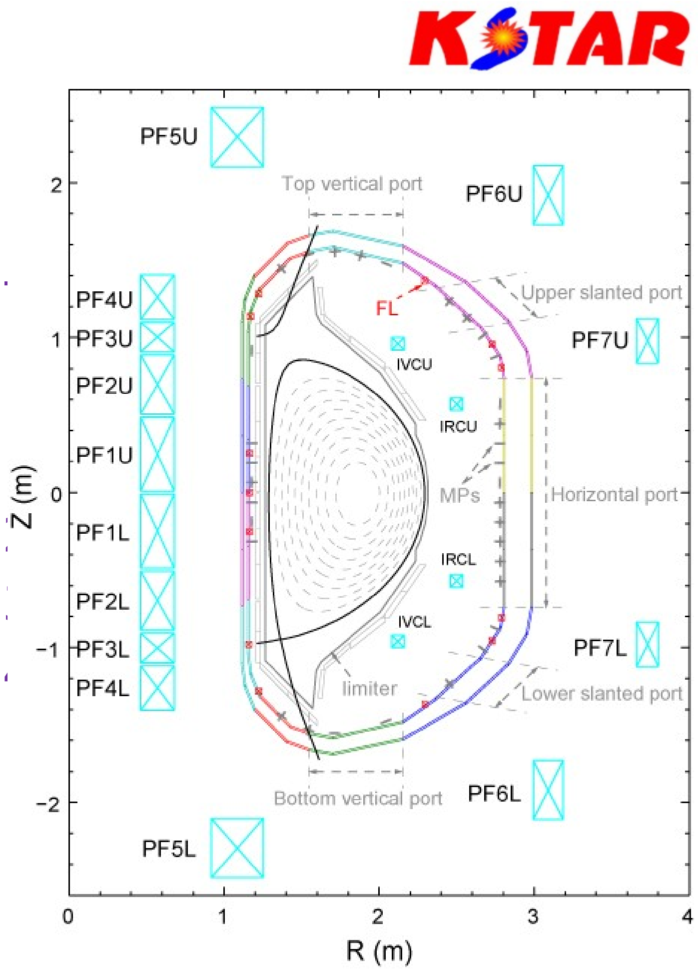
1. Current profile
2. Pressure profile
3. Plasma shape
4. Stabilising wall

Basic Tokamak Variables

- **Normalized beta – stability limit**

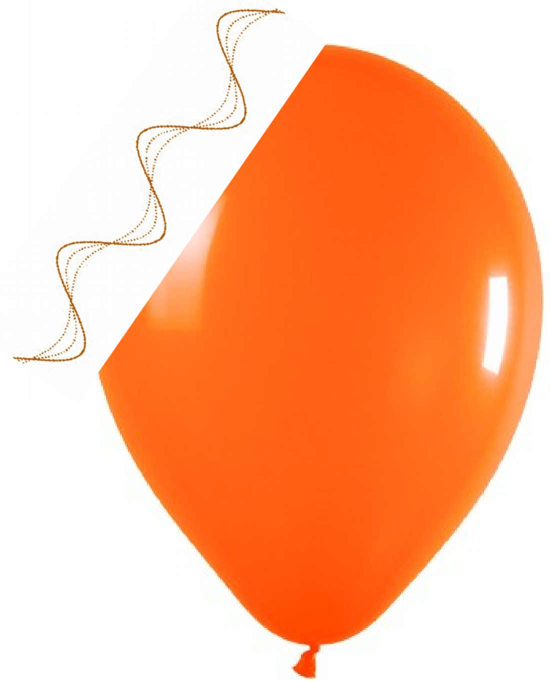
$$\beta_N = \beta_t \frac{aB_t}{I_p}$$





Basic Tokamak Variables

- **How to achieve high beta?**



- Providing high heating power by avoiding instabilities and reducing transport (sealing, insulation)
- Even without transport loss reduction, only transient high beta achievable with heating due to instabilities.

Basic Tokamak Variables

- **Plasma internal inductance**

Normalised internal inductance per unit length associated with the toroidal current flowing in the plasma

$$l_i = \frac{L_i / 2\pi R_0}{\mu_0 / 4\pi} = \frac{2}{\mu_0^2 I_p^2 R_0} \int B_\theta^2(r) d^3V \qquad L_i = \frac{1}{\mu_0 I_p^2} \int B_\theta^2(r) d^3V$$

- For flat current density profile, circular cx

$$J = J_0 \quad (r \le a)$$

$$J = 0 \quad (a < r \le b)$$

$$B_\theta = \frac{\mu_0}{2\pi r} \int J \, 2\pi r dr \qquad B_{\theta a} = \frac{\mu_0 I_p}{2\pi a}$$

$$B_\theta = \frac{\mu_0 I_p r}{2\pi a^2} \quad (r \le a)$$

$$L_i = \frac{1}{\mu_0 I_p^2} \int B_\theta^2(r) 2\pi R_0 2\pi r dr$$

$$B_\theta = \frac{\mu_0 I_p}{2\pi r} \quad (a < r \le b)$$

$$l_i = \frac{1}{2} - 2 \ln \frac{a}{b}$$

Basic Tokamak Variables

- **Plasma internal inductance**

  - For Bennett current density profile, circular cx

$$J = \frac{I_p a^2}{\pi \left(r^2 + a^2\right)^2} \quad (r \le a)$$

$$J = 0 \quad (a < r \le b)$$

$$B_\theta = \frac{\mu_0 I_p}{2\pi} \left(\frac{r}{r^2 + a^2} \right) \quad (r \le a)$$

$$B_{\theta a} = \frac{\mu_0 I_p}{4\pi a}$$

$$B_\theta = \frac{\mu_0 I_p}{4\pi r} \quad (a < r \le b)$$

$$l_i = \frac{1}{2} \left(\ln \frac{4b}{a} - 1 \right)$$

Basic Tokamak Variables

- **Plasma internal inductance**

  - For more general current density profile, circular cx

$$J = J(0)\left(1 - \frac{r^2}{a^2}\right)^{\nu} \quad (r \le a)$$

$$J(0) = \frac{I_p(\nu + 1)}{\pi a^2}$$

$$J = 0 \quad (a < r \le b)$$

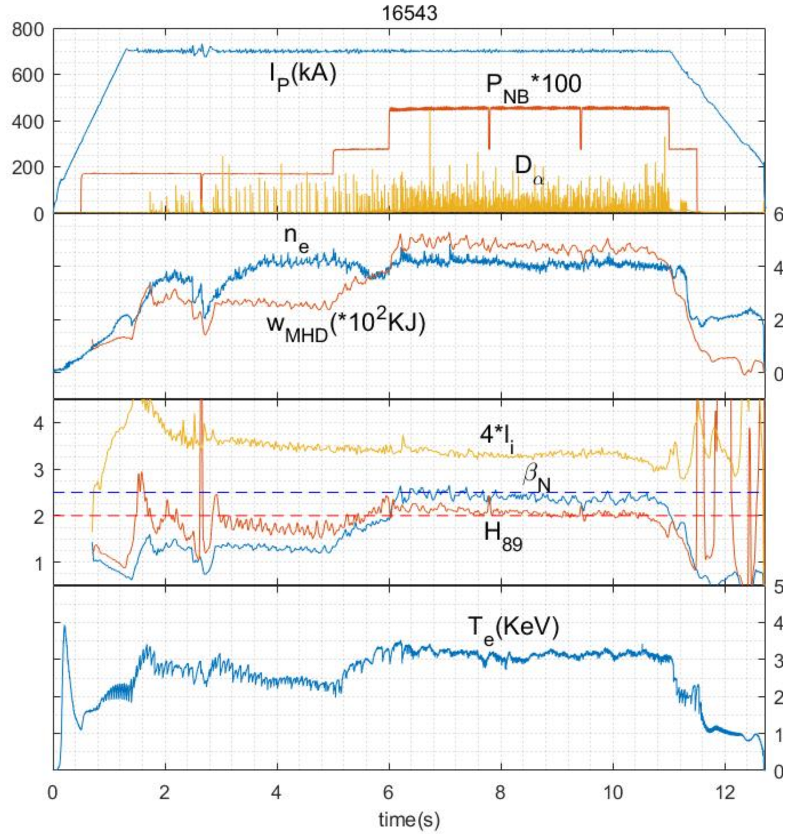
$$B_\theta = \frac{\mu_0 J(0) a^2}{2(\nu + 1) r}\left(1 - \left(1 - \frac{r^2}{a^2}\right)^{\nu + 1}\right) \quad (r \le a)$$

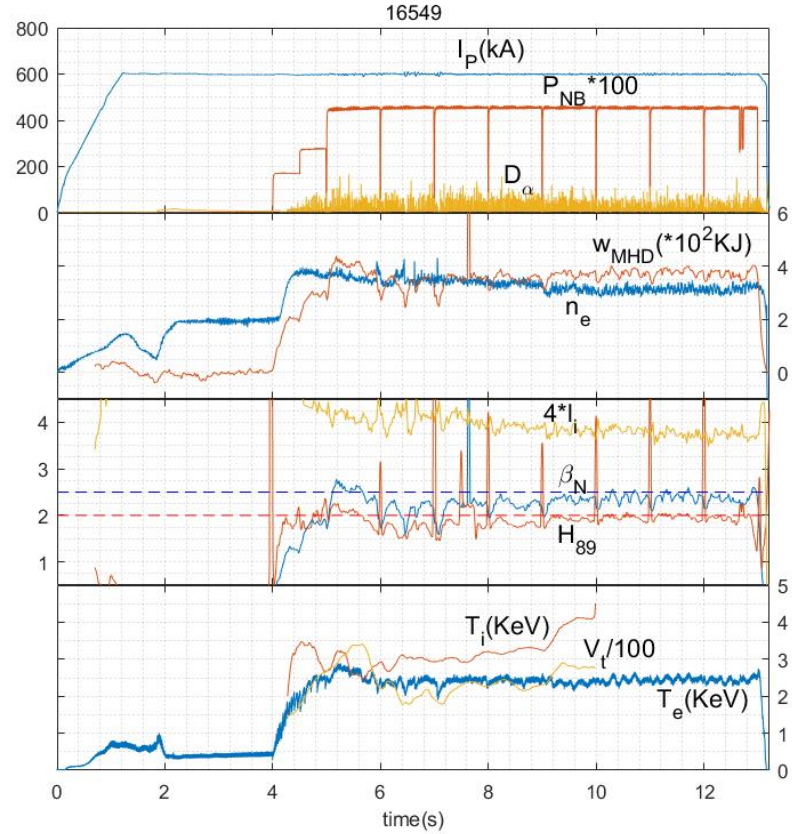
$$B_\theta = \frac{\mu_0 J(0) a^2}{2(\nu + 1) r} \quad (a < r \le b)$$

$$l_i = ?$$

Basic Tokamak Variables

- **Plasma internal inductance**





HW: What is li(3)?

Fusion Reactor Technology 2
(459.761, 3 Credits)

Prof. Dr. Yong-Su Na

(32-206, Tel. 880-7204)

Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**

- Rotational transform: *Δθ*? when *ΔΦ*=2π

$$\iota = \frac{\Delta\theta}{\frac{\Delta\phi}{2\pi}} = \frac{\frac{B_\theta}{r}}{\frac{B_\phi}{2\pi R}}$$

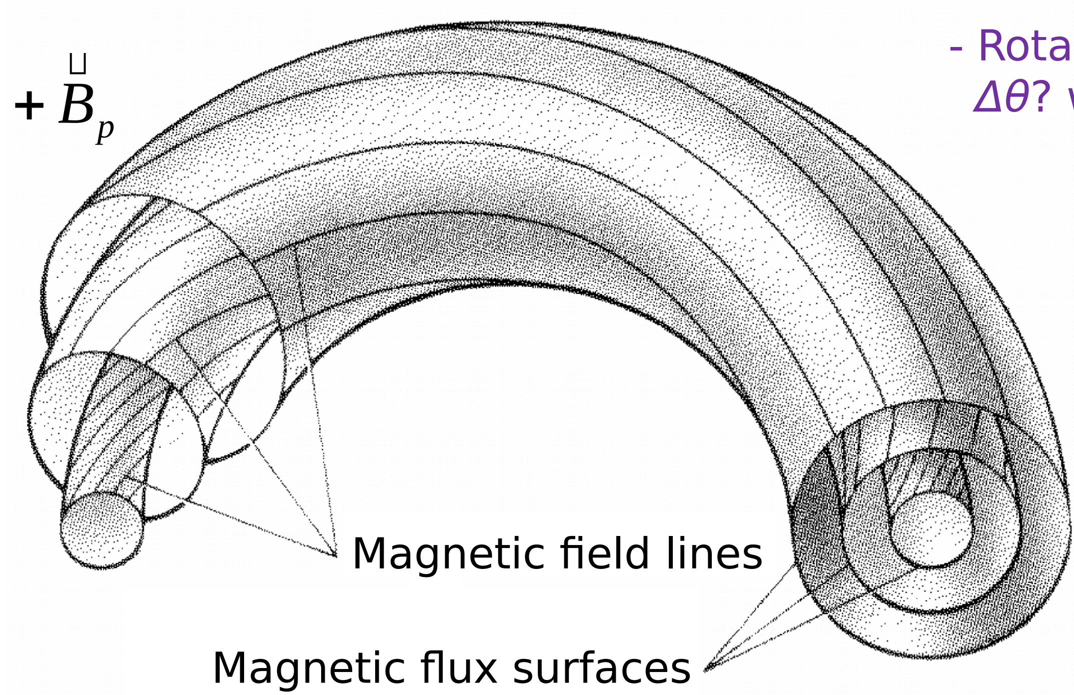
$$\frac{R d\phi}{B_\phi} = \frac{r d\theta}{B_\theta}$$

$$q = \frac{\text{number of toroidal windings}}{\text{number of poloidal windings}} = \frac{2\pi}{\iota} = \frac{r}{R}\frac{B_\phi}{B_\theta}$$

Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**

$$B = B_\phi + \vec{B}_p$$



- Rotational transform:
  $\Delta\theta$? when $\Delta\Phi = 2\pi$

$$\iota = \frac{\Delta\theta}{\frac{\Delta\phi}{2\pi}} = \frac{\frac{B_\theta}{r}}{\frac{B_\phi}{2\pi R}}$$

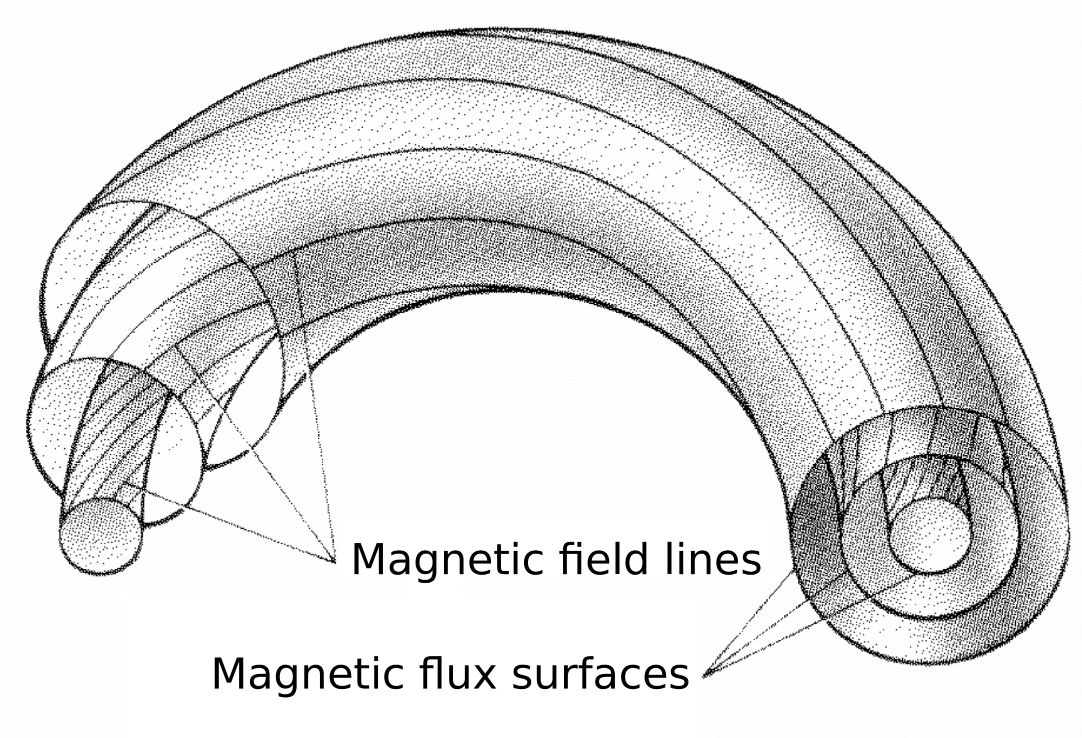
$$\frac{R d\phi}{B_\phi} = \frac{r d\theta}{B_\theta}$$

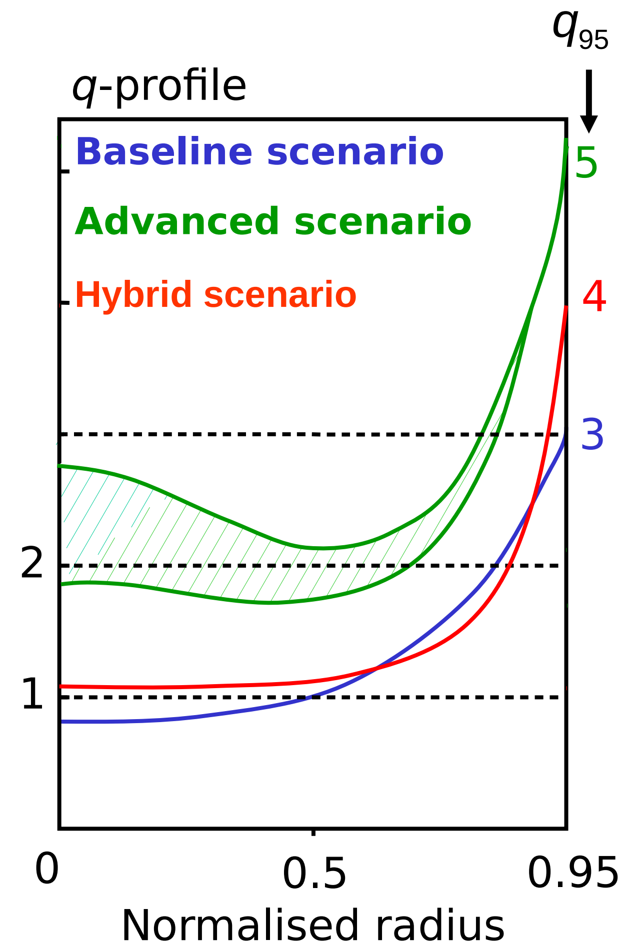
- The effect of the twisted magnetic field lines–each of which completely traces out a magnetic flux surface by its revolutions around the toroidal and poloidal axes–is to create a system of nested toroidal flux surfaces which guide ion motion.

$$q = \frac{\text{number of toroidal windings}}{\text{number of poloidal windings}} = \frac{2\pi}{\iota} = \frac{r}{R}\frac{B_\phi}{B_\theta}$$

Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**





Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**

Large aspect ratio tokamak with a circular CX

$$q(r) = \frac{rB_\varphi}{R_0 B_\theta}$$

$$q_a = \frac{aB_\varphi}{R_0 B_{\theta a}} = \frac{2\pi a^2 B_\varphi}{\mu_0 I_p R_0}, \quad \langle j_\varphi \rangle = \frac{I_p}{\pi a^2}$$

$$\mu_0 \langle j_\varphi \rangle = \frac{2B_\varphi}{R_0 q_a}, \quad q_0 = \frac{2B_\varphi}{\mu_0 j_{\varphi 0} R_0}$$

$$B_\theta = \frac{\mu_0}{r} \int_0^r j_\varphi(r') r' dr'$$

HW. Derive this!

Why do stellarators not use q but the rotational transform?

$$\frac{q_a}{q_0} = \frac{j_{\varphi 0}}{\langle j_\varphi \rangle}$$ Current profile peakedness

Basic Tokamak Variables

- **Safety factor *q* = number of toroidal orbits per poloidal orbit**
  - General definition

$$q = \oint \frac{B_\varphi}{R_0 B_\theta} ds$$

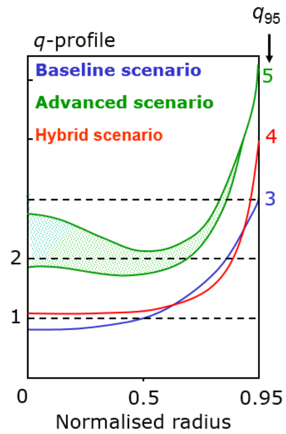
Integral is along a closed path enclosing the minor axis and lying on a specific magnetic surface; thus *q* is a surface quantity.

$$q_{95} = \frac{5a^2 B_T}{R I_{MA}} f$$

f : describing the role of plasma shape

$$f = \frac{1+\kappa^2\left(1+2\delta_{95}^2 - 1.2\delta_{95}^3\right)}{2} \frac{\left(1.17 - 0.65A^{-1}\right)}{\left(1 - A^{-2}\right)^2}$$

A: aspect ratio



ITER Physics Basis, Nucl. Fusion **39** 2169 (1999)

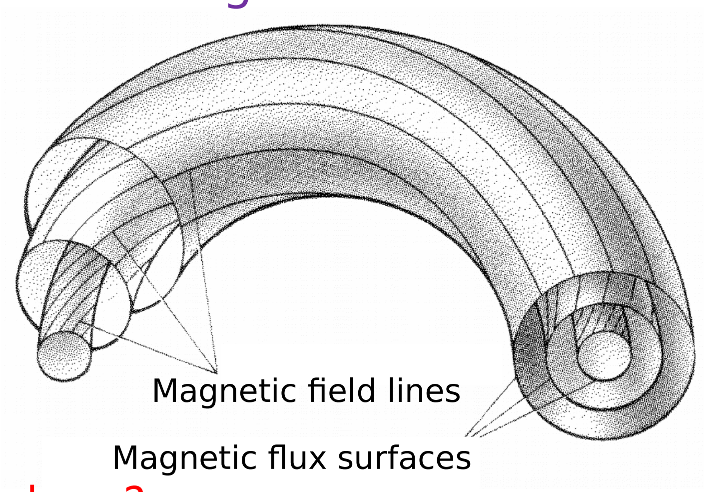
Basic Tokamak Variables

- **Magnetic Shear**

  - Measuring the change in pitch angle of a magnetic field line from one flux surface to the next

  - Playing an important role in stabilizing MHD instabilities, particularly those driven by the pressure gradient:
    A perturbation aligned with **B**(r) will, at a point with increased minor radial distance $r+dr$, encounter field lines at a different angle which again will vary as the perturbation grows to another distance $r+dr'$.
    Any helically resonant instabilities are thus radially localised.

$$s(r) \equiv \frac{r}{q}\frac{dq}{dr}$$



What are the shortcomings of large magnetic shear?

Its impact on tearing modes, internal kink modes?

Basic Tokamak Variables

- ***Z*-effective**

$$Z_{eff} = \frac{\sum_s n_s Z_s^2}{n_e}, \quad n_e = \sum_s n_s Z_s$$

Z_s: charge number for the *s*-type ion

- Tokamaks usually have several types of ion in their plasmas, due mainly to impurities entering from the walls. *Z*-effective defined as a convenient measure of the extent to which the plasma contaminated.
- Z_{eff} = 1 in a pure hydrogen plasma

Basic Tokamak Variables

- ***Z*-effective**
  - Method to determine Z_{eff}
  - Impurity concentration determined by analyzing resonance line intensities in the vacuum UV, supplemented by measurements of soft X-ray spectra; this data, coupled with a theory for ionisation rates
  - Visible Bremsstrahlung radiation
  - Spitzer’s formula for the parallel resistivity (particularly for Ohmic plasmas)

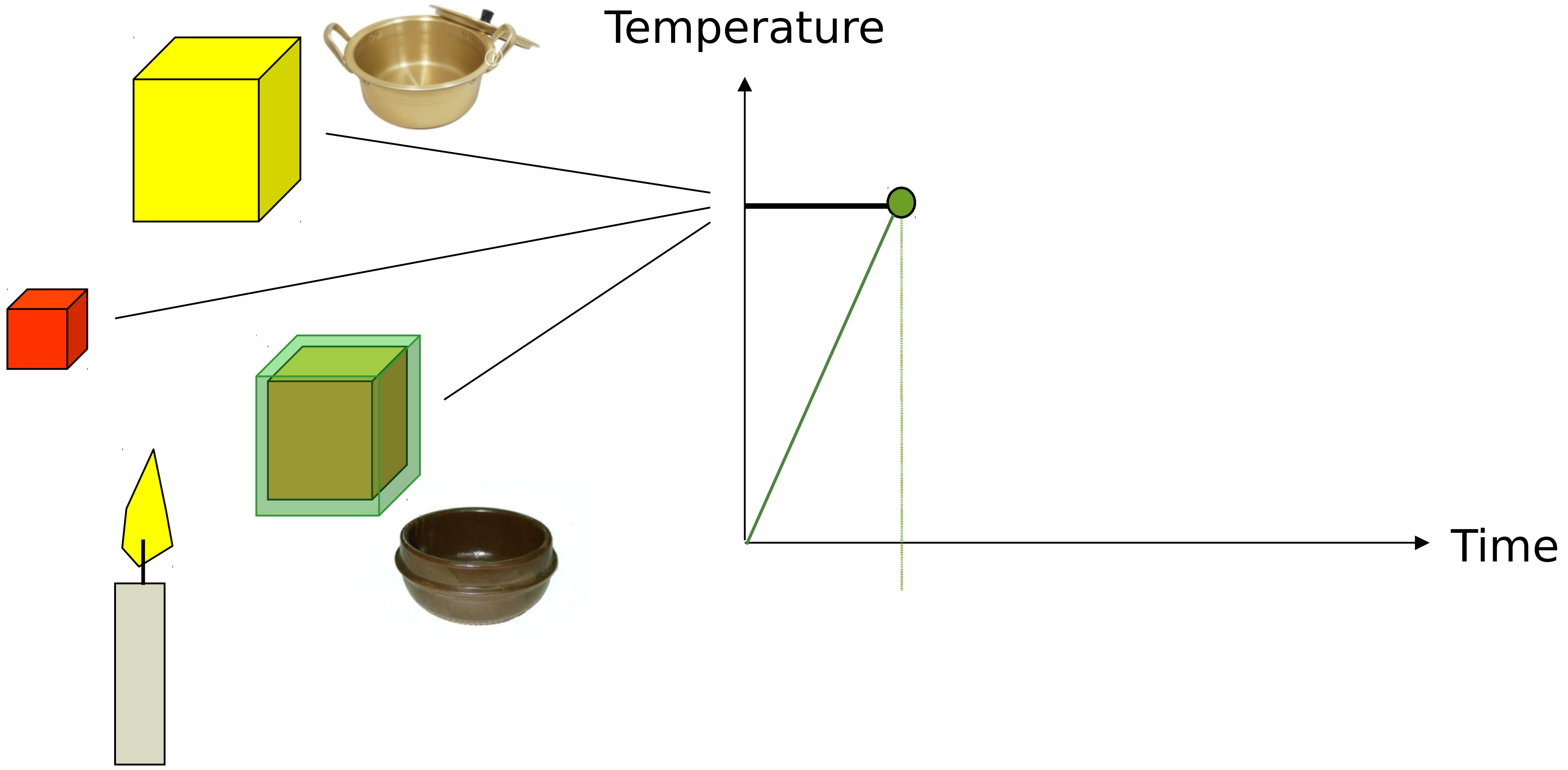
$$P_{br} = A_{br} n_i n_e Z^2 \sqrt{kT_e}$$

$$\eta = 3.80 \times 10^3 \frac{Z \ln \Lambda}{\gamma_E T^{3/2}} ohm \cdot cm \approx \frac{\pi Z e^2 m^{1/2}}{(4\pi\varepsilon_0)^2 (kT_e)^{3/2}} \ln \Lambda$$

L. Spitzer, Jr., *Physics of Fully Ionized Gases*, Interscience Publishers, p.84 (1956)

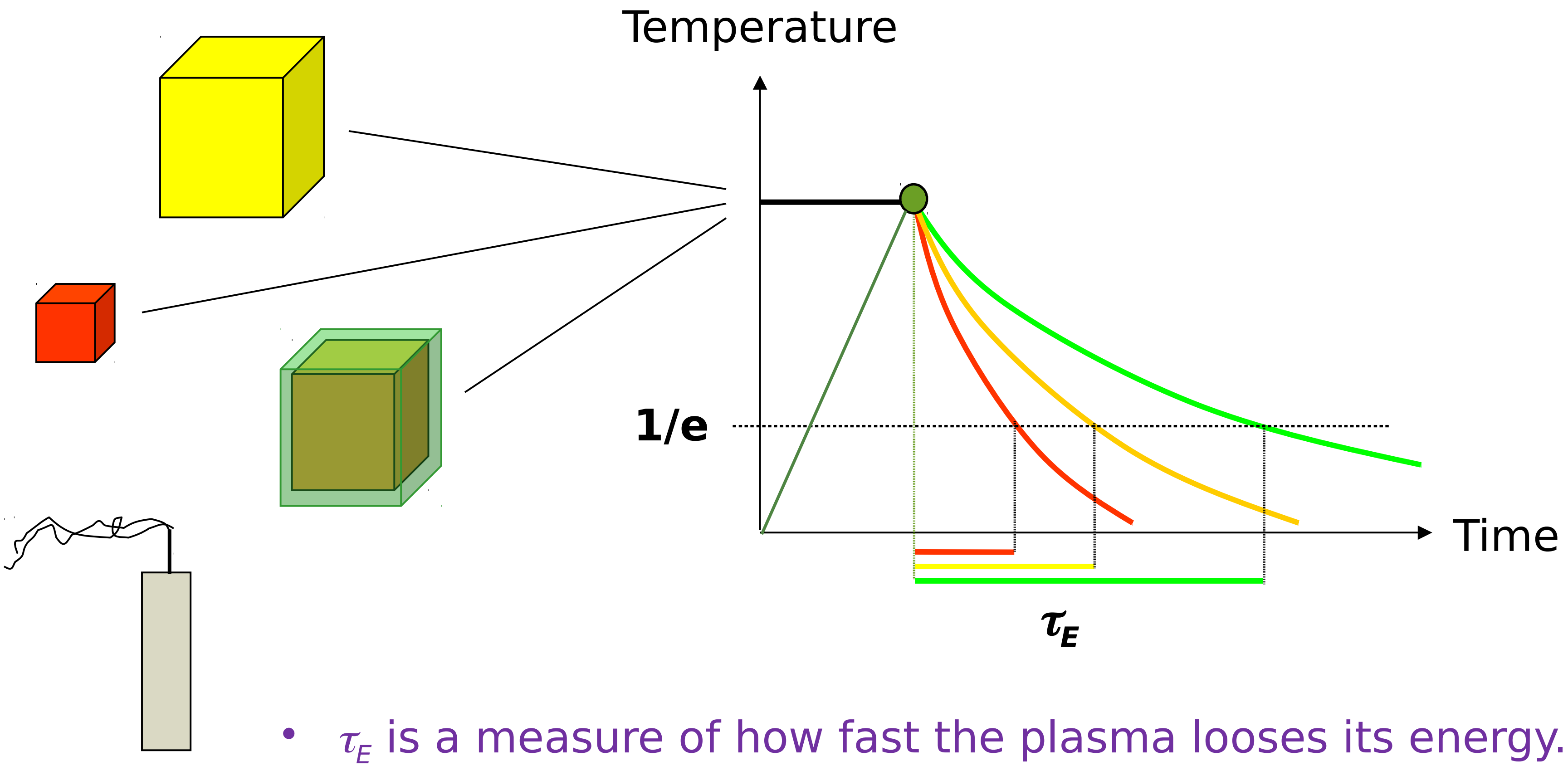
Basic Tokamak Variables

- **Energy confinement time**



Basic Tokamak Variables

- **Energy confinement time**



- τ_E is a measure of how fast the plasma looses its energy.
- The loss rate is smallest, τ_E largest

  if the fusion plasma is big and well insulated.

Basic Tokamak Variables

- **Energy confinement time**

$$W = \int_0^a \frac{3}{2} k(n_e T_e + n_i T_i) r dr$$ ~ total thermal energy in the torus

total heat flux   radiation energy loss rate

$$\frac{\partial(\rho u)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left[r(\rho h v_D + Q_r)\right] = j_\varphi E_\varphi - L \quad (\rho u = \frac{3}{2}p, \quad \rho h = \frac{5}{2}p)$$

internal energy   enthalpy density

Basic Tokamak Variables

- **Boltzmann Equation**

$$\frac{\partial f_\alpha}{\partial t} + \vec{u} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(\vec{E} + \vec{u} \times \vec{B}) \cdot \nabla_u f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_c$$

- **Fluid Equations**

$$\int Q_i \left[\frac{df_\alpha}{dt} - \left(\frac{\partial f_\alpha}{\partial t}\right)_c \right] d\vec{u} = 0$$

$Q_1 = 1$ mass

$Q_2 = m_\alpha \vec{u}$ momentum

$Q_3 = m_\alpha u^2 / 2$ energy

$$\frac{\partial n_j}{\partial t} + n_j \nabla \cdot \vec{u}_j = S_{nj}$$

$$m_j n_j \frac{du_j}{dt} + \nabla \cdot \overset{\leftrightarrow}{P}_j - q_j n_j (\vec{E} + \vec{u}_j \times \vec{B}) = \sum_k^l \vec{R}_{jk} - m_j \vec{u}_j S_{nj}$$

$$\frac{3}{2} n_j \frac{dT_j}{dt} + \overset{\leftrightarrow}{P}_j : \nabla \vec{u}_j + \nabla \cdot \vec{h}_j = \sum_k^l Q_{jk} + S_{Ej} + \left(\frac{m_j u_j^2}{2} - \frac{3}{2} T_j\right) S_{nj}$$

Basic Tokamak Variables

- **Energy confinement time**

$$W = \int_0^a \frac{3}{2} k(n_e T_e + n_i T_i) r dr$$ ~ total thermal energy in the torus

total heat flux radiation energy loss rate

$$\frac{\partial(\rho u)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left[r(\rho h v_D + Q_r)\right] = j_\varphi E_\varphi - L \quad (\rho u = \frac{3}{2}p, \quad \rho h = \frac{5}{2}p)$$

internal energy enthalpy density

$$\frac{\partial}{\partial t}(\ln W) + \frac{1}{\tau_E} = \frac{1}{\tau_E^*} - \frac{1}{\tau_E^R}$$

$$\tau_E \equiv \frac{W}{\left[r\left(\frac{5}{2}pv_D + Q_r\right)\right]_{r=a}}, \quad \tau_E^* \equiv \frac{W}{\int_0^a j_\varphi E_\varphi r dr}, \quad \tau_E^R \equiv \frac{W}{\int_0^a L r dr}$$

Energy confinement time Energy replacement time by OH heating Radiation loss time

Basic Tokamak Variables

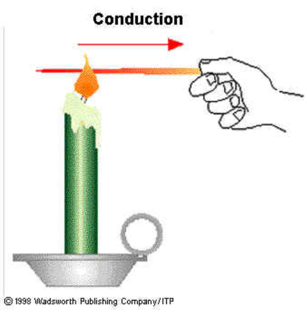
- **Energy confinement time**

$$\tau_E = \frac{W}{\frac{W}{\tau_E^*} - \frac{\partial W}{\partial t}} = \frac{W}{P_{in} - \frac{\partial W}{\partial t}} \approx \frac{W}{P_{in}} = \frac{\text{stored energy}}{\text{applied heating power}}$$

In steady conditions, neglecting radiation loss, replacing Ohmic heating by total input power

- To predict the performance of future devices, the energy confinement time is one of the most important parameter.
- Since tokamak transport is anomalous, empirical scaling laws for energy confinement are necessary.

$$\tau_{th,E} \sim a^2 / \chi$$



- Empirical scaling laws: regression analysis from available experimental database.

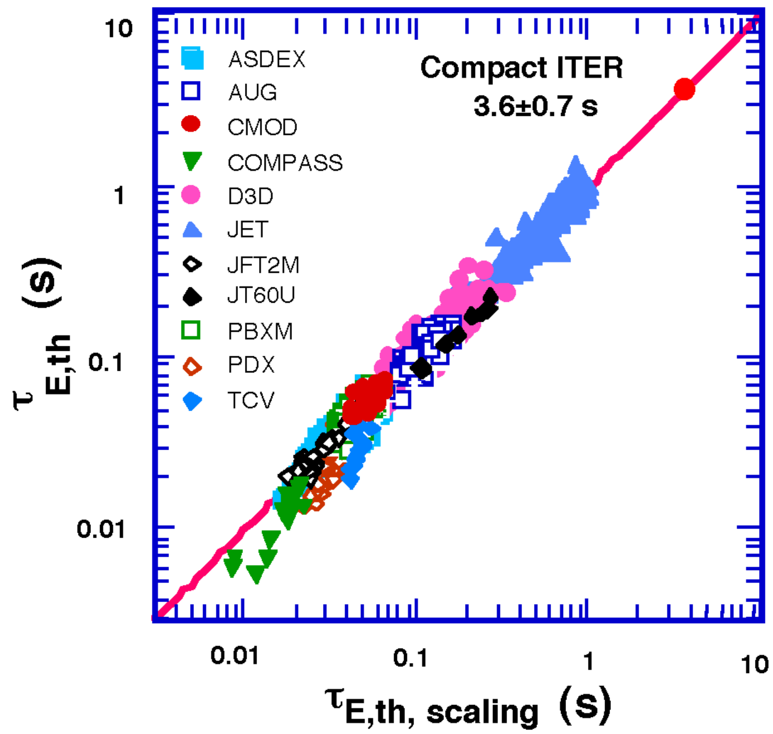
$$\tau_{th,E}^{fit} = C I^{\alpha I} B^{\alpha B} P^{\alpha P} n^{\alpha n} M^{\alpha M} R^{\alpha R} \varepsilon^{\alpha \varepsilon} \kappa^{\alpha \kappa}$$

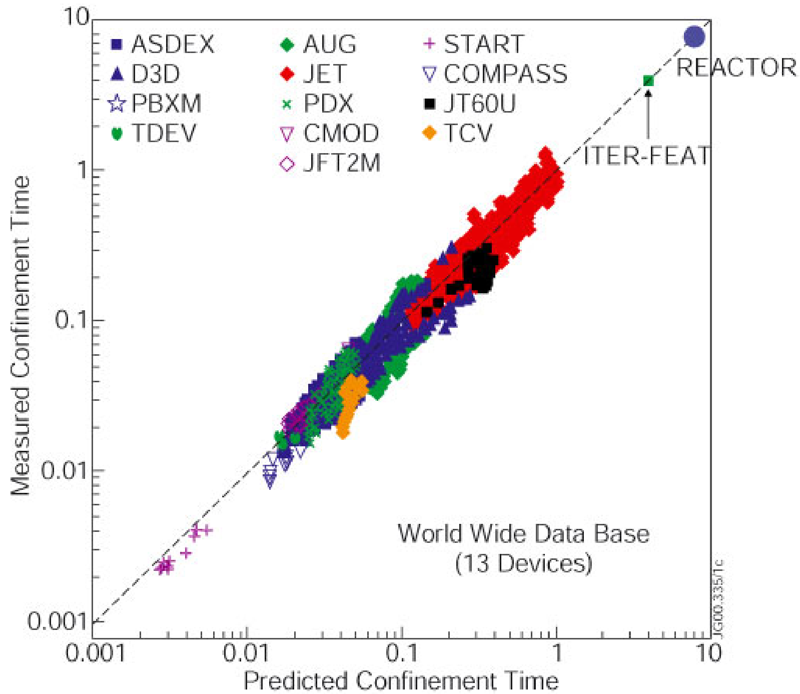
in engineering variables

Basic Tokamak Variables

- **Energy confinement time**

$$\tau_{th,E}^{IPB98(y,2)} = 0.0562 I^{0.93} B^{0.15} P^{-0.69} n^{0.41} M^{0.19} R^{1.97} \varepsilon^{0.58} \kappa_a^{0.78}$$





Homework: τ_E in KSTAR and ITER? Why should ITER be large?

Express IPB98(y,2) scaling with dimensionless physical parameters

Basic Tokamak Variables

- **Energy confinement time**
  - Power balance in the ignition condition

$$\int_o^{\tau_b} \frac{dE^*_{th}}{dt} dt = \cancel{E^*_{aux}} + E^*_{fu} - E^*_n - E^*_{rad} - \int_o^{\tau_b} \frac{E^*_{th}}{\tau^*_E} dt = 0$$

$$\frac{E_{th}}{\tau_E} = \frac{1}{5} P_f - P_b$$

For D-T reaction, bremsstrahlung radiation only

$$E_{th} = \frac{3k}{2}\left(\langle n_e \rangle \langle T_e \rangle + \sum_j \langle n_j \rangle \langle T_j \rangle \right) V_P$$

$$\tau_E \approx \frac{4.81 \times 10^{-2} \langle n_e \rangle \left[10^{20} m^{-3}\right] \langle T_e \rangle [keV] V_P \left[m^3\right]}{\frac{1}{5} P_f - P_b}$$

Basic Tokamak Variables

- **Energy confinement enhancement factor**

$$H_{98(y,2)} = \frac{\tau_E}{\tau_{th,E}^{IPB98(y,2)}}$$

$$H_{89} = \frac{\tau_E}{\tau_E^{89}}$$

Basic Tokamak Variables

- **Particle confinement time**

$$\frac{\partial n_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r n_e v_D) = S_e(r)$$ electron number density source

$$\tau_p = \tau_p^*$$ In steady state

$$\tau_p \equiv \frac{\int_0^a n_e r dr}{\left[r n_e v_D \right]_{r=a}}, \quad \tau_p^* \equiv \frac{\int_0^a n_e r dr}{\int_0^a S_e r dr}$$

particle confinement time

particle replacement time

Basic Tokamak Variables

- **Momentum confinement time**

$$\frac{\partial}{\partial t}(\rho v_{\varphi}) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r v_{\varphi}) + \nabla\cdot\Pi\cdot\hat{\varphi} = F_b\cdot\hat{\varphi}$$

Momentum equation having the toroidal component

$$\tau_{\varphi} = \tau_{\varphi}^{*}$$

In steady state

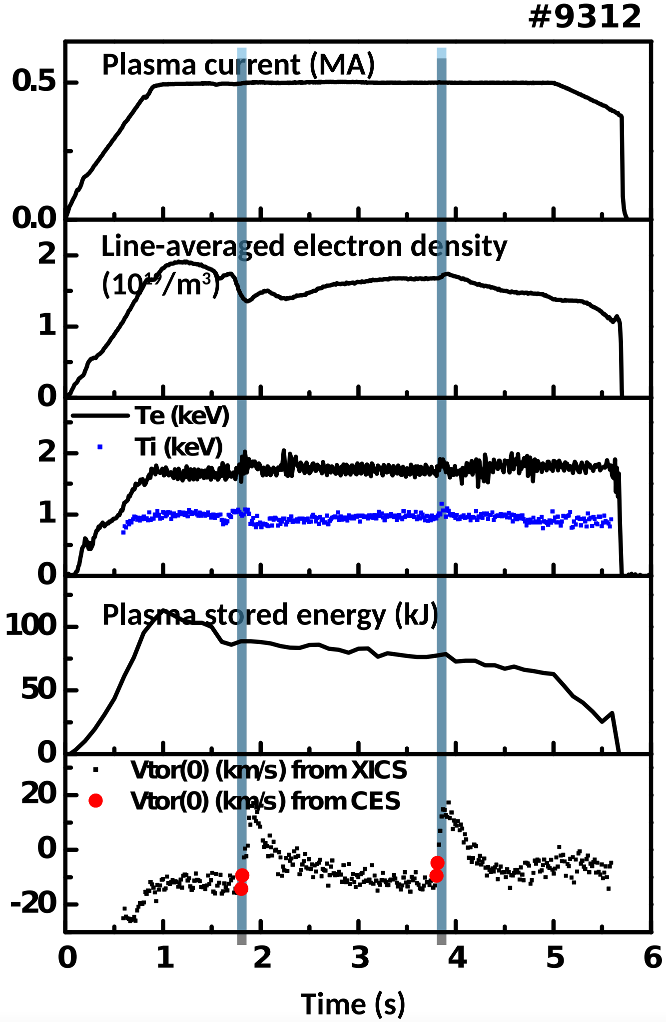
$$\tau_{\varphi} \equiv \frac{H_{\varphi}}{\int_0^a \nabla\cdot\Pi\cdot\hat{\varphi}\, r\,dr + \left[r\rho v_r v_{\varphi}\right]_{r=a}},\quad \tau_{\varphi}^{*} \equiv \frac{H_{\varphi}}{\int_0^a F_b\cdot\hat{\varphi}\, r\,dr} = \frac{2\pi^2 R_0^2 a^2 H_{\varphi}}{\text{Beam torque}},\quad H_{\varphi} \equiv \int_0^a \rho v_{\varphi} r\,dr$$

Toroidal momentum confinement time

Toroidal momentum replacement time

Basic Tokamak Variables

- **Momentum confinement time**



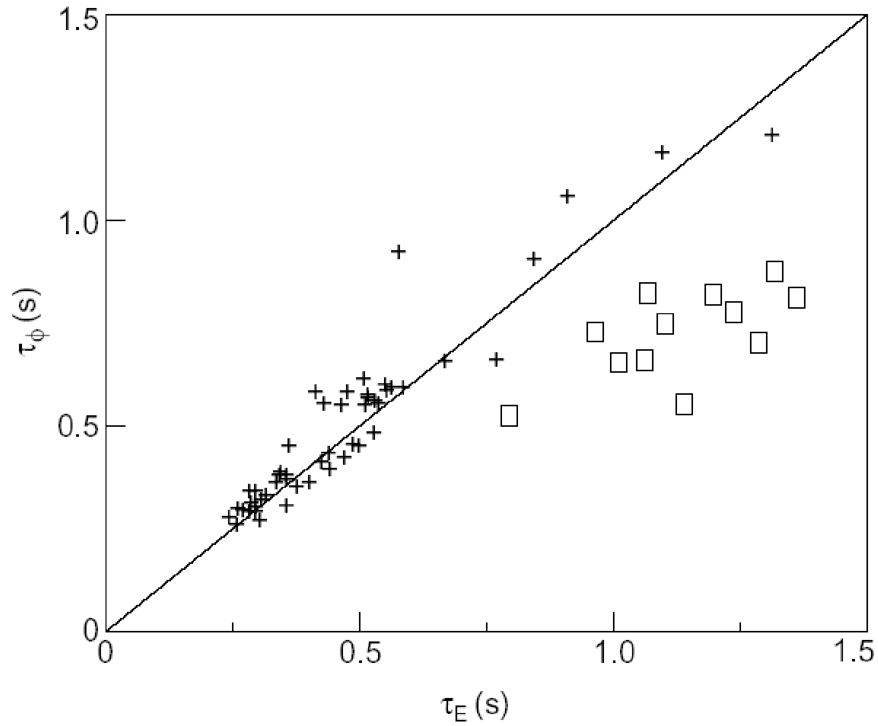
Effects of NBI blips in Ohmic discharges

- An approximate value of the momentum confinement time can be obtained directly by switching off the beam and determining the e-folding time for the toroidal rotation to decay to Ohmic levels.

- What is the impact of the remaining torque after turning off NBI?
- What is the effect of the density variation?

Basic Tokamak Variables

- **Momentum confinement time**

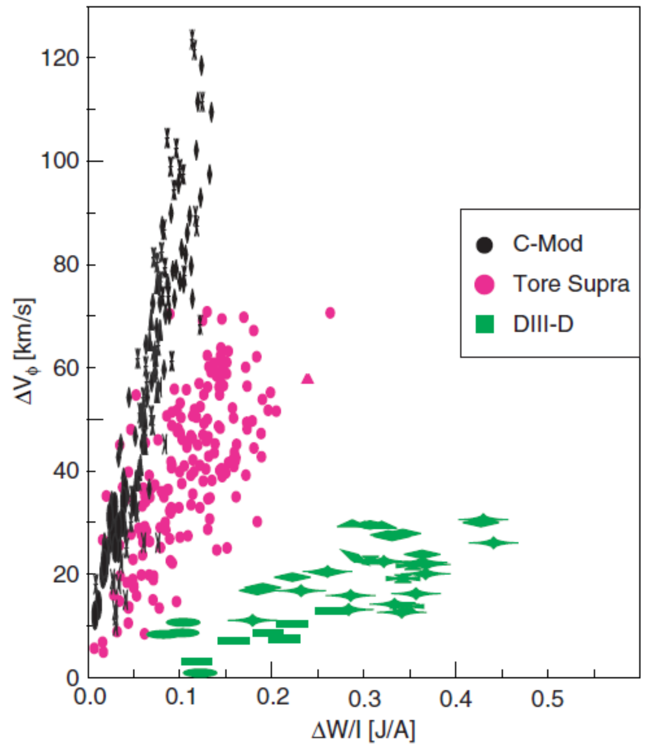


- Toroidal angular momentum confinement time of thermal particles during NBI versus simultaneously measured energy confinement time for steady state L-mode and ELMy H-mode discharges (crosses), and for transient ELM free phase of hot ion H-mode discharges (squares) in JET

Basic Tokamak Variables

- **Intrinsic rotation**

$$\frac{\partial}{\partial t}(\rho v_\varphi) + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r v_\varphi) + \nabla \cdot \Pi \cdot \hat{\varphi} = F_b \cdot \hat{\varphi}$$

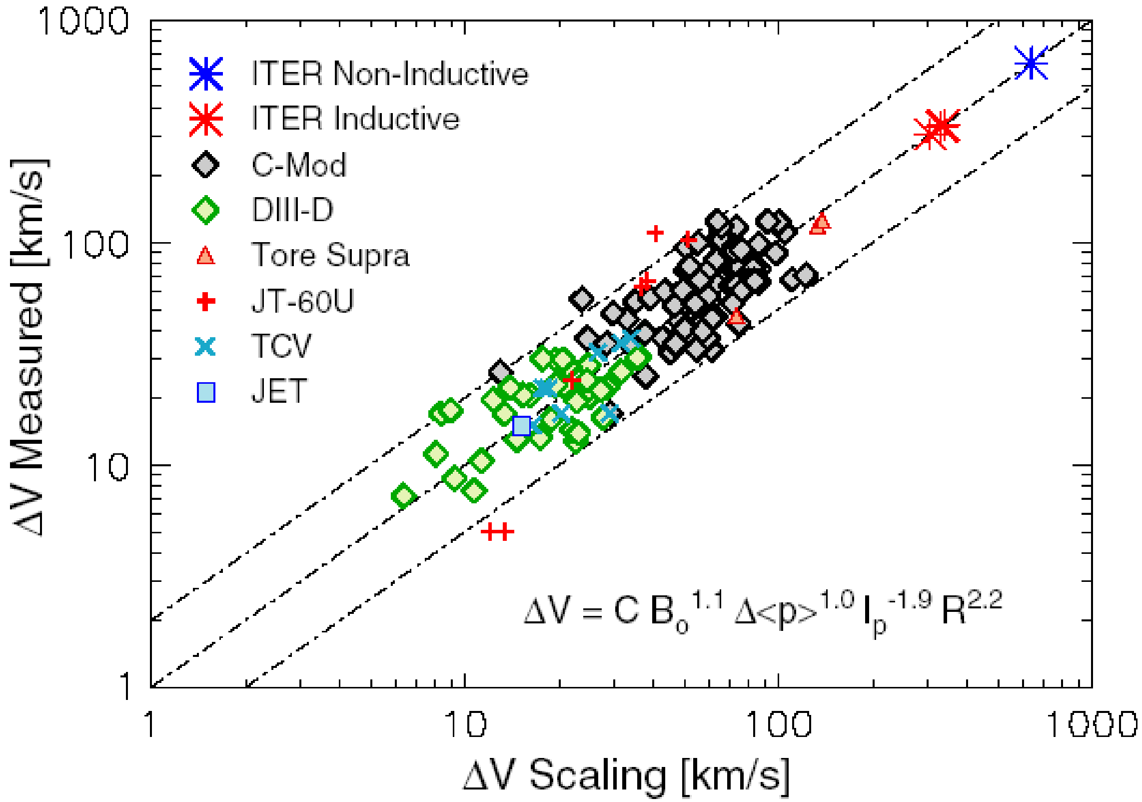


- The intrinsic rotation velocity (the difference between the L-mode velocity and the enhanced confinement value) as a function of the change in the stored energy normalized to the plasma current

J. E. Rice et al, Nucl. Fusion ***47*** *1618 (2007)*

Basic Tokamak Variables

- **Intrinsic rotation**



J. E. Rice et al, Nucl. Fusion ***47*** *1618 (2007)*

Basic Tokamak Variables

- **Greenwald density**

$$\bar{n} = \kappa \bar{J} \quad (1)$$

measured in 10^{20} m^{-3}, where κ is the plasma elongation and $\bar{J}$ is the average plasma current density, with the I_p area measured in MA·m^{-2}. Figures 4a to 4d are modified Hugill plots for several machines, showing the results of this scaling. They should be compared with Fig. 3. For elliptical machines this scaling for the density limit can be written as $\bar{n}_{max} = I_p/\pi a^2$, and for high aspect ratio, low beta, circular machines it can be written as $(5/\pi) \times B/qR$. A few comments on

A NEW LOOK AT DENSITY LIMITS IN TOKAMAKS

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ABSTRACT. While the results of early work on the density limit in tokamaks from the ORMAK and DITE groups have been useful over the years, results from recent experiments and the requirements for extrapolation to future experiments have prompted a new look at this subject. There are many physical processes which limit the attainable densities in tokamak plasmas. These processes include: (1) radiation from low Z impurities, convection, charge exchange and other losses at the plasma edge; (2) radiation from low or high Z impurities in the plasma core; (3) deterioration of particle confinement in the plasma core; and (4) inadequate fuelling, often exacerbated by strong pumping by walls, limiters or divertors. Depending upon the circumstances, any of these processes may dominate and determine a density limit. In general, these mechanisms do not show the same dependence on plasma parameters. The multiplicity of processes leading to density limits with a variety of scaling has led to some confusion when comparing density limits for different machines. The authors attempt to sort out the various limits and to extend the scaling law for one of them to include the important effects of plasma shaping, i.e. $\bar{n}_e = \kappa \bar{J}$, where n_e is the line average electron density (10^{20} m^{-3}), κ is the plasma elongation and $\bar{J}$ (MA·m^{-2}) is the average plasma current density, defined as the total current divided by the plasma cross-sectional area. In a sense, this is the most important density limit since, together with the q-limit, it yields the maximum operating density for a tokamak plasma. It is shown that this limit may be caused by a dramatic deterioration in core particle confinement occurring as the density limit boundary is approached. This mechanism can help explain the disruptions and Marfes that are associated with the density limit.

Basic Tokamak Variables

- **Greenwald density**

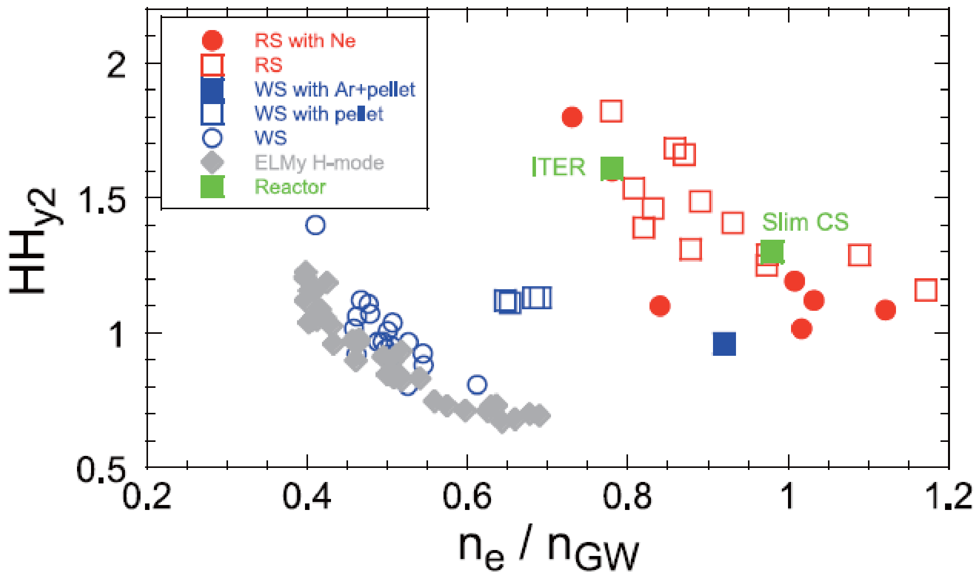
$$n_G = \frac{I_p}{\pi a^2}$$

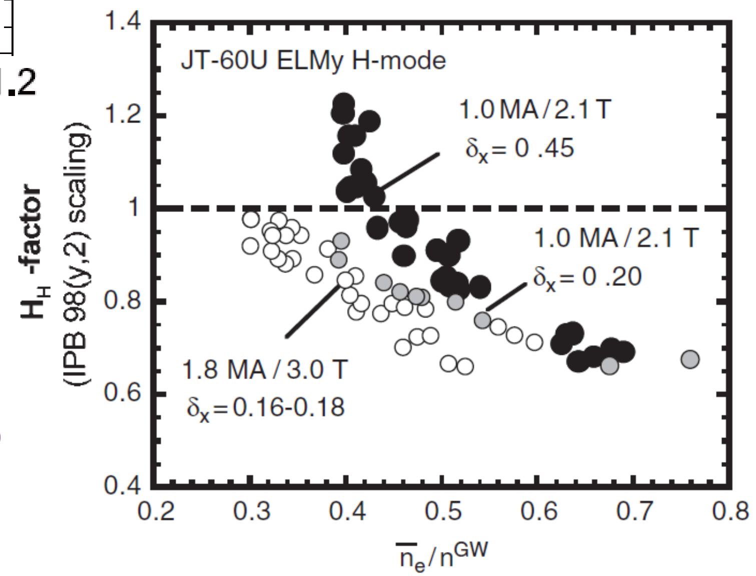
- As the limit is approached, the plasma becomes increasingly susceptible to disruption and data become sparser.

*M. Greenwald et al, NF **28** 199 (1988): one of the most cited paper in NF*
*Martin Greenwald, PPCF **44** R27 (2002)*

Basic Tokamak Variables

- **Greenwald density**





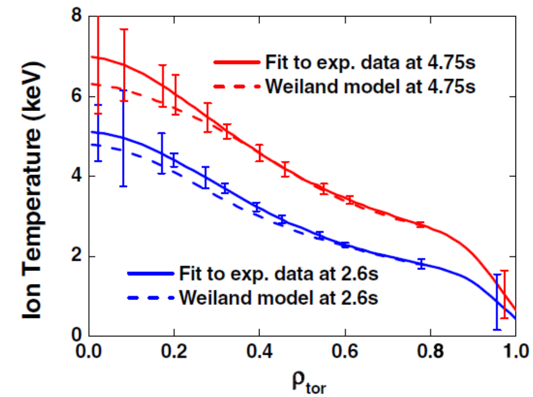
*Y. Sakamoto et al, PFR **5** S1008 (2010)*
*H. Urano et al, PPCF **44** 11 (2002)*

Basic Tokamak Variables

- **Flux coordinates**

\- Normalised toroidal magnetic flux coordinate

$$\phi = \int \frac{F}{R} dS_\phi \qquad F = B_\phi R$$



Yong-Su Na et al, NF ***46*** 232 (2006)

$$\rho_\phi^* = \sqrt{\frac{\phi}{\pi B_0}}$$

ϕ: the toroidal magnetic flux $\phi = \pi \rho_\phi^{*2} B_0$

S_ϕ: the toroidal magnetic flux surface

B_0: the magnetic field at the center of the vacuum vessel

$$\rho_\phi = \frac{\rho_\phi^*}{\rho_{\phi,b}^*} = \sqrt{\frac{\phi}{\phi_b}}$$

ρ_ϕ: the normalized toroidal magnetic flux coordinate

\- Normalised poloidal magnetic flux coordinate

$$\psi_N = \frac{\psi - \psi_a}{\psi_b - \psi_a}$$

ψ_a: the poloidal magnetic flux at magnetic axis

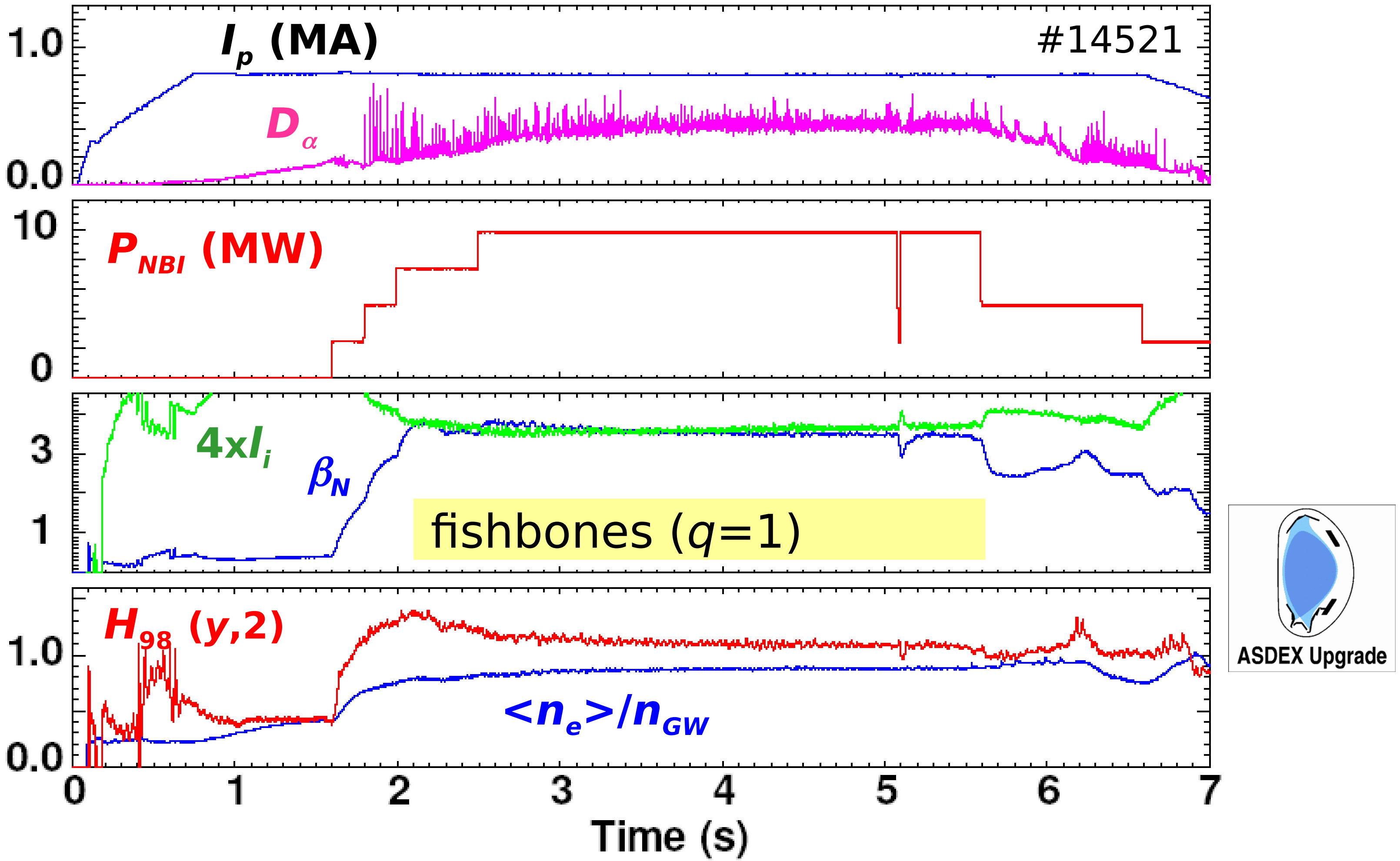
ψ_b: the poloidal magnetic flux at last closed magnetic flux surface

ψ: the poloidal magnetic flux at each magnetic surface

$$\rho_\psi = \sqrt{\psi_N}$$

ρ_ψ: the normalized poloidal magnetic flux coordinate

Objectives of the Tokamak Operation



- No sawteeth, good confinement, and $\beta_N \sim 3.5$, $T_i \sim T_e$, $\langle n_e \rangle / n_{GW} \sim 0.88$, averaged over 3.6 seconds ($\sim 50\ \tau_E$).