Fundamentals of Engineering Physics 2019

Week 1.

Topics to be covered

1. Classical Mechanics

2. Electricity and Magnetism

3. Waves

4. (Prelude for) Quantum Mechanics

5. Statistical Mechanics.

I. Classical Mechanics

Lecture 1 : The Nature of Classical Physics

Classical Physics ⟺ Quantum Mechanics

⇕

Relativity — Relativistic Quantum Mechanics

" " : in a broad sense

: in a narrow sense.

* Classical Physics provides an accurate description of ~~dy~~ dynamical System at a scale greater than size of atoms and molecules.

* Classical physics (based on Newton's law etc) is valid for $v \ll c$.

  * Relativity is necessary for $v \lesssim c$.

The Classical Laws of Physics are "deterministic"

* If you know everything about a system at some instant of time, (initial condition) and also the equations that govern how the system changes, then you can predict the future.

(Governing Equation(s))

* The job of classical mechanics is to predict the future.
  Read a famous quote from "Pierre-Simon Laplace" on page I-2.

* The meaning of "reversible":

  - The laws are deterministic into the past as well as the future.

$$(t \rightarrow \text{"}-t\text{"})$$

  - "... for such an intellect, nothing would be uncertain and "the future" just like the past would be present before its eyes."

The Limits of Precision

* Laplace has been overly optimistic about how predictable the world is.

  1. "... equation for movements of atoms" → it would require Quantum Mechanics

  2. Even in classical physics, ability to know "the initial conditions with almost perfect precision"

     → extremely demanding and impractical for large enough system

  3. Furthermore, the space of initial states is, in many cases, ~~infinitely~~ continuously infinite.

     (uncountable)

* "Resolving Power": ability to distinguish the values of neighboring numbers.

* "Chaos"; very small differences in the initial conditions lead to large eventual differences in later time.

  It's a consequence of limited resolving power.

Lecture 2: Motion

* Particle Motion:

- Consider a point particle at a position $\vec{r}(t)$ specified by components x, y and z at time t.

- **Trajectory** : the path of the particle

- In classical mechanics, given the dynamical laws and initial conditions, one can figure out $\vec{r}(t)$.

- Velocity :
$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \equiv \lim_{\Delta t \to 0} \frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t} \equiv \dot{\vec{r}}$$

$v_i = \frac{dx_i}{dt}$ (for $i = 1, 2,$ and 3) is a short-hand notation for

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad \text{and} \quad v_z = \frac{dz}{dt}.$$

The velocity is a vector with a magnitude $|\vec{v}|$ (speed).

$|\vec{v}|^2 = v_x^2 + v_y^2 + v_z^2$. $|\vec{v}| = \left|\frac{d\vec{r}}{dt}\right|$, but $|\vec{v}| \neq \frac{d}{dt}|\vec{r}|$ in general.

Examples of Motion

* **Acceleration** : $a_i = \frac{dv_i}{dt} = \dot{v}_i$

- The time rate at which the velocity is changing.
- You feel acceleration when your velocity <u>vector</u> changes. So acceleration can happen at a constant speed.

Eg. 1, **Free Fall** : $x(t) = 0$, $y(t) = 0$, and $z(t) = z(0) + v_z(0)\,t - \frac{1}{2} g t^2$ for $t \geq 0$,

applying "$\frac{d}{dt}$", we obtain $v_x(t) = 0$, $v_y(t) = 0$, and $v_z(t) = v_z(0) - gt$.

" once more, " $a_x(t) = 0$, $a_y(t) = 0$, and $a_z(t) = -g$.

Eg 2. **Simple Harmonic Oscillator (SHO)**: $x(t) = \sin(\omega t)$

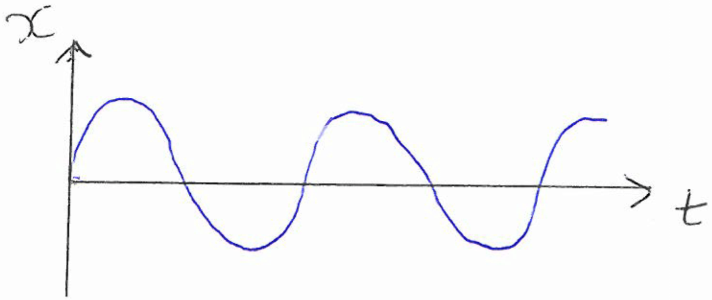


Fig. 1.

Simple Harmonic Oscillator

* $x(t) = \sin(\omega t)$ ⇒ applying "$\frac{d}{dt}$" to get $v_x(t) = \omega \cos(\omega t)$,

⇒ applying "$\frac{d}{dt}$" once more, we obtain $a_x(t) = $ "$-$" $\omega^2 \sin(\omega t)$.

* Note that; when the position "x" is at its — the velocity is

"maximum or minimum" → "zero"

"zero" → "maximum or minimum."

→ x and v_x are $90°$ (or $\frac{\pi}{2}$) out of phase.

* Significance of "minus" sign in $a_x(t) = $ "$-$" $\omega^2 \sin(\omega t) = $ "$-$" $\omega^2 x(t)$;

- Wherever the particle is, it is being accelerated back toward the orgin.

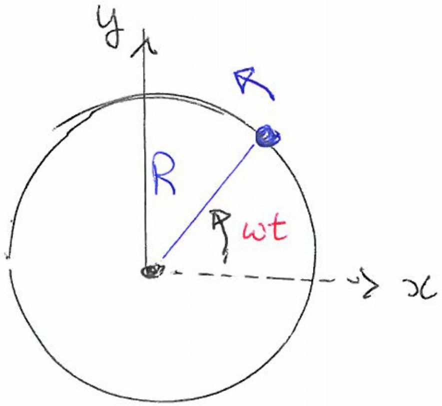
Uniform Circular Motion about the Origin

* $$\begin{bmatrix} x(t) = R\cos(\omega t) \\ y(t) = R\sin(\omega t) \end{bmatrix}$$

"ω" is called the angular frequency.

"T" $= \frac{2\pi}{\omega}$ is the period.

i.e., time taking for one full cycle of motion.



We can easily show that

$$a_x(t) = -R\omega^2\cos(\omega t)$$
$$a_y(t) = -R\omega^2\sin(\omega t)$$

* The acceleration of a circular orbit is parallel to the position vector, but it is oppositely directed. In other words, the acceleration vector points radially inward toward the origin.

* Note that a projection of the uniform circular motion to x-axis (or y-axis) leads to a SHO motion.

Homework

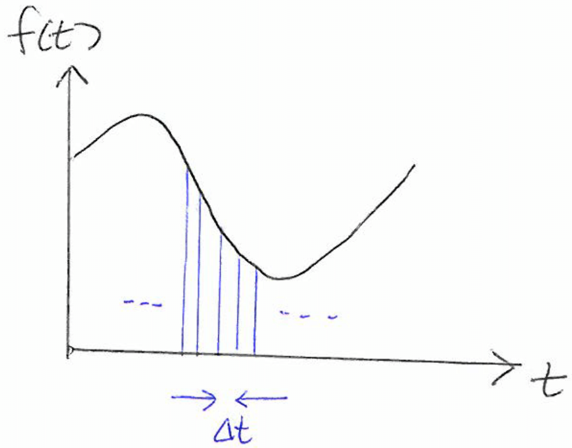
* Submit one week from the day of announcement.
* " after the class.

HW Set # 1 :

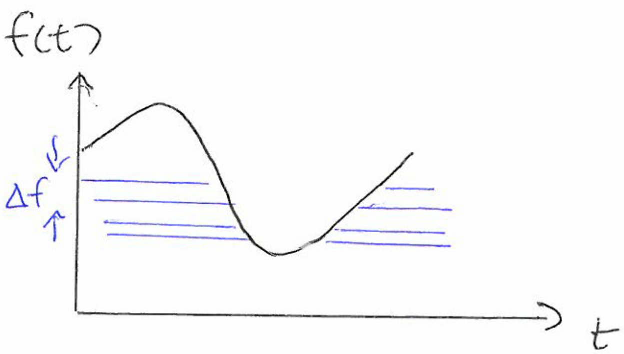
Exercise 7 and 8 on page 46.

Interlude 2: Integral Calculus

* Reverse operation of "differentiation".

* 

"Divide the area under the curve $y = f(t)$ into vertically oriented rectangles. Sum over those, taking $\Delta t \to 0$."

* 

"Why not horizontally oriented rectangles?"

→ "Lebesgue" integral.

* Integration by Parts (IBP)

* Suppose we encounter an integral of the form

$$\text{“}\ \underline{\int_a^b g(x)\left(\frac{df(x)}{dx}\right)}\ \text{”}.$$

and don't know how to do it directly.

* If you recognize that "$f(x)$" is not much more complex than $\dfrac{df(x)}{dx}$, but $\underline{\dfrac{dg(x)}{dx} \text{ is much simpler than } g(x)}$,

from $\frac{d}{dx}(f\,g) = f\,\frac{dg}{dx} + (\frac{df}{dx})\,g$, we obtain

$$\int_a^b g(x)\left(\frac{df(x)}{dx}\right) = \Big[f(x)\,g(x)\Big]_a^b - \underline{\underline{\int_a^b f(x)\left(\frac{dg(x)}{dx}\right)}}\ .$$

* If ═══ is an easier integral than ─── , it can work.
* It requires some experience and trial and error sometimes.

* Example of IBP ;

$$\int_0^{\pi/2} x\cos x\,dx = \int_0^{\pi/2} x\,\frac{d}{dx}(\sin x)\,dx = \Big[x\sin x\Big]_0^{\pi/2} - \int_0^{\pi/2}\frac{d}{dx}(x)\,\sin x\,dx$$

$$= \frac{\pi}{2}\sin\frac{\pi}{2} - \underline{\int_0^{\pi/2}\sin x\,dx}$$: easy to do !

Poor boy's attempt :

$$\int_0^{\pi/2} x\cos x\,dx = \int_0^{\pi/2}\frac{d}{dx}\left(\frac{x^2}{2}\right)\cos x\,dx = \left[\frac{x^2}{2}\cos x\right]_0^{\pi/2} - \underline{\int_0^{\pi/2}\frac{x^2}{2}\cdot(-\sin x)\,dx}$$

—— = ??? ; more difficult integration ! , backward progress.

Substitution of Variables

* Suppose you encounter an integration of the form "$\underline{\int f(h(x))\,g(x)\,dx}$" where $g(x) = \dfrac{d\,h(x)}{dx}$.

→ Then by letting $h(x) = u$, we get "$\int f(u)\,du$."

* Eg.. $$\int_0^{\pi/2} e^{-\sin x}\cos x\,dx = \int_0^1 e^{-u}\,du = \cdots\rightarrow$$

(recognizing, $u = \sin x$)