Fundamentals of Engineering Physics 2019

Week 2.

Lecture 3 : Dynamics

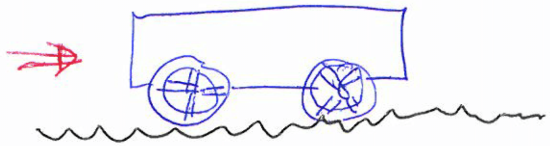
⊛ Aristotle's Law of Motion :

In a world dominated by friction ;

- ancient Greek communities
- even in some back streets of old cities

⋮

* Dynamics of a heavy cart with wooden wheels on an unpaved road :



Eg., cart, rickshaw, 牛馬車, ...

* From (empirical) experience, we know that ;

- To make it move, we need to push it (i.e., exert a force on it)
- If we stop pushing, a cart will come to rest.

Aristotle's Law of Motion

Empirical Law:

- "velocity of any object" $\propto$ "total applied force"
- moreover, more force is needed to move heavier object.

$$\Rightarrow \boxed{\vec{F} = \mu \vec{v}}$$

where

μ: the resistance of the body to being moved.

Aristotle could have guessed that $\mu \propto m$

* There's no (internal) inconsistency with this law.

* It can be actually useful in some situations even now.

~ Langevin's equation for Brownian motion.

* This is a good approximated law when the friction dominates.

Friction is a force, too.

* $\boxed{\dfrac{d\vec{x}}{dt} = \dfrac{\vec{F}}{\mu}}$ is not a "fundamental law" of physics.

* Friction is a consequence of (wheels of) a body (eg., cart) interacting with a huge number of other tiny bodies (molecules and atoms).

* In general, the macroscopic motion of bodies surrounded by an external medium is accompanied by frictional ~~forces~~ processes which ultimately bring the motion to a stop. In this process, the kinetic energy of the bodies is converted into heat and dissipated.

"Kinetic energy of a cart"

Solve fundamental microscopic equations of motion for all molecules involved (~ "inefficient")

macroscopic, phenomenological description in terms of a viscous drag, "μ" ~ "efficient!"

"Heat"

The Law of Inertia

* Aristotle missed a point that an applied force should ~~o~~ overcome the force of friction.

* In fact, the Aristotle's law is no longer a good approximation for ice skating or a modern car on a highway.

(cf., in "curling" game, μ can be controlled by sweeping the floor).

* "An isolated object moving in free space, with no forces acting on it, requires nothing to keep it moving."

* $\boxed{\text{the Law of Inertia}}$

* If the body is initially at rest → it takes a force to start it moving.
* If it is moving in a particular direction → " to change the direction of motion.

Mass, Acceleration, and Force

* All of examples in the previous page involve a change in the velocity of an object, therefore an acceleration.

* Some objects have more inertia than others; it requires a larger force to change their velocities. (eg., trains vs Ping-Pong balls).

* The quantitative measure of an object's inertia: its Mass.

- Mass is a new concept that is defined in terms of "force" and "acceleration."
- But, so far we have not defined force.

"Force is defined by the ability to change the motion of a given mass."

"Mass is defined by the resistance to that change."

?

Are we in a logical circle?

⇒ "A closer look at how force is defined and measured in practice." on page 64-66. Read and convince yourself.

Newton's Second Law of motion

$$* \quad \boxed{\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}} \qquad (1),(2)$$

- Force equals mass times the rate of change of velocity

  (= acceleration)

  → no force leads to no change in velocity.

- Note that this is ~~an~~ a relation between vector quantities.

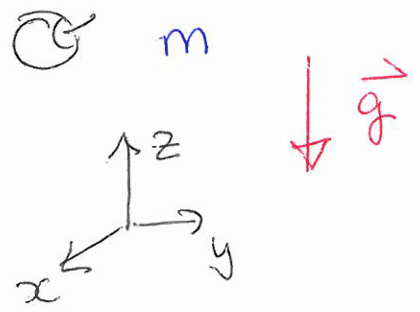
* A particular example when $\vec{F} = 0$, then from $m\frac{d\vec{v}}{dt} = 0$,

$\vec{v}$ = constant in time.

Newton's first Law of motion :

Every object in a state of uniform motion tends to remain in that state of motion, unless an external force is applied to it.

I. Fall of an object due to gravity:



- $\cancel{m}\dfrac{d\vec{v}}{dt} = \cancel{m}\vec{g} = -\cancel{m}g\,\hat{z}$

- Integrate once in time $\rightarrow$
$$V_x(t) = V_x(0)$$
$$V_y(t) = V_y(0)$$
$$V_z(t) = V_z(0) - gt$$

- Integrate once more to get:
$$X(t) = X(0) + V_x(0)t$$
$$y(t) = y(0) + V_y(0)t$$

and

$$z(t) = z(0) + V_z(0)t - \frac{1}{2}gt^2.$$

II. Simple Harmonic Oscillator (SHO) in 1d:

$$m\frac{dV_x}{dt} = m\frac{d^2}{dt^2}X = F_x = -kx \qquad (K: \text{spring constant})$$

$$\Rightarrow \boxed{\frac{d^2}{dt^2}X + \omega^2 \cdot X = 0 \quad , \text{ with } \omega^2 = k/m} \qquad (6)$$

* This equation appears frequently in many different context.

$$\boxed{\frac{d^2}{dt^2}X = -\omega^2 X} \qquad (6)$$

① We recall that both "sine" and "cosine" functions have these properties satisfied upon diffentiating twice.

→ "$X(t) = A\cos(\omega t) + B\sin(\omega t)$"

- We can then determine A and B in terms of $V_x(0)$ and $X(0)$.
- Think about "the principle" allowing the linear sum of "cos" and "sine" as a solution.

② Note that (6) $\Rightarrow$ $\frac{d}{dt}V = -\omega^2 X$ and $\frac{dX}{dt} = V$ by definition.

Since
$$\left(\begin{array}{ll} \frac{d}{dt}V = (i\omega)(i\omega X) & \text{a)} \\ \frac{d}{dt}X = V & \text{b)} \end{array}\right) \xRightarrow[\text{a) + i}\omega\cdot\text{b)}]{\text{take}} \frac{d}{dt}(V + i\omega \cdot X) = i\omega(i\omega X + V) \; !$$

This is a first order differential equation for $V + i\omega X$.

Then, $V(t) + i\omega X(t) = (V(0) + i\omega X(0))e^{i\omega t} = (V(0) + i\omega X(0))[\cos(\omega t) + i\sin(\omega t)]$,

By equating the real and imaginary part of both sides respectively, we get the solution!

Interlude 3: Partial Differentiation

* Partial Derivatives

* Calulus of <u>multivariable functions</u> :

eg., $T(x, y, z)$: (local) temperature in 3d space

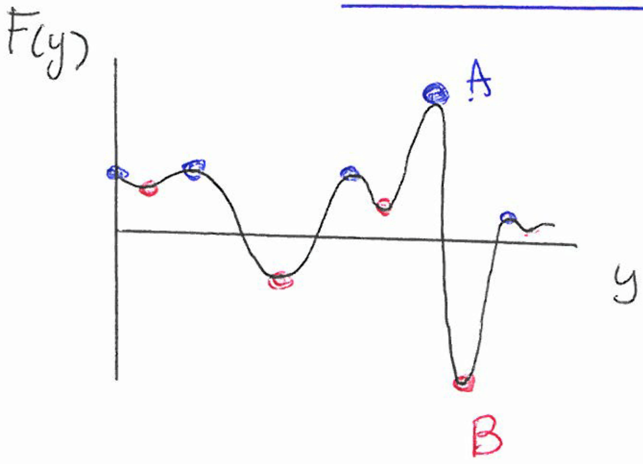
A partial derivative : $\dfrac{\partial T}{\partial x}$ implies that a derivative in x direction is taken at constant values of y and z.

$$\left(\equiv \left(\frac{\partial T}{\partial x} \right)_{y,z} \right)$$

– Obviously, $\dfrac{\partial^2 T}{\partial x \partial y} = \dfrac{\partial^2 T}{\partial y \partial x}$, i.e., the order of operations does not matter.

and $\dfrac{\partial}{\partial x}$ and $\dfrac{\partial}{\partial y}$ "commute" with each other.

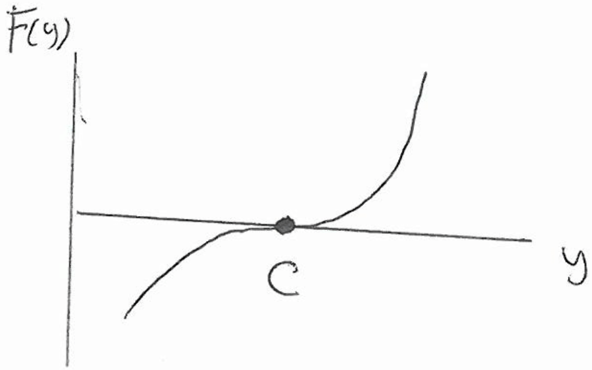
Stationary Points



- : local maxima , $\frac{d^2}{dy^2}F(y) < 0$
- : local minima , $\frac{d^2}{dy^2}F(y) > 0$

In particular, • A : global maximum

• B : global minimum



- Inflection point : $\frac{d^2}{dy^2}F(y) = 0$

All the points mentioned ~~on~~ this page is called the "Stationary Points",

at which $\boxed{\frac{d}{dy}F(y) = 0}$.

Stationary Points in Higher Dimensions

* Imagine a hilly terrain of Gwanak mountain.

$A(x,y)$: the altitude (height) as a function of latitude and longitude (x,y).

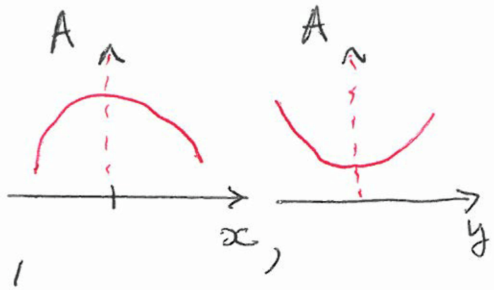
→ The "tops" of hills and the "bottoms" of valley are

↓ local maxima ↓ local minima of $A(x,y)$.

- From these places, you go down no matter which way you move. / you go up in all directions.

- The ground is locally level (horizontal) at these places.

- But there are other places where the ground is level; "Saddle Points."



where along one axis, the altitude increases in either direction,

but " another " (perpendicular to the former direction), " decreases " !