

Fusion Plasma Theory I. 2019

Week 13

14.3. Slowing-down of Beam Ions due to Collisions with background Ions

(+) Consider $M_b \gg M$ case first.

The situation is similar to Beam slowing down due to collisions with e^- 's up to some point, i.e.,

$$n_b M_b \frac{d\langle \vec{V} \rangle}{dt} = -M \int \frac{d\langle \vec{v} \rangle}{dt} f_i(v) d^3 v \quad (14.16)$$

where

$$\frac{d\langle \vec{v} \rangle}{dt} = -\nu_{ib} (\vec{v} - \vec{V}) \quad \text{and} \quad (14.17)$$

$$\nu_{ib} = \frac{n_b Z^2 Z_b^2 e^4 \ln \Lambda}{4\pi \epsilon_0^2 M^2 |\vec{v} - \vec{V}|^3} \quad (14.18)$$

For electron background, we had to perform integration in \vec{v} , but Now we can assume $|\vec{V}| \gg |\vec{v}|$ and integration in \vec{V} is unnecessary.

$$\Rightarrow \frac{d}{dt} \langle \vec{V} \rangle = - \underbrace{\frac{n_i Z^2 Z_b^2 e^4 \ln \Lambda}{4\pi \epsilon_0^2 M M_b V^3}}_{\sim \sim \sim} V \quad (14.19)$$

$$\textcircled{+} \quad \frac{d}{dt} W_b = - \frac{2^{1/2} n_i Z^2 Z_b^2 e^4 \ln N M_b^{1/2}}{8\pi \epsilon_0^2 M W_b^{3/2}} \cdot W_b \quad (14.20)$$

only depends on "W_b" not V_{T+} or T_i of backgnd ion.

Recall this case assumed M_b >> M.

\textcircled{*} Now, we assume M_b << M. This case is somewhat similar to electrons being pitch-angle scattered by heavier target ions.

While $(\Delta V_{\perp})^2 \gg (\Delta V_{\parallel})^2$ i.e., slowing-down is a subdominant process compared to the pitch-angle scattering,

A straigh fwd analysis (\sim the one for (11.11)) leads to

$$\frac{d}{dt} W_b = - \frac{M_b^2}{2M} \frac{d}{dt} (\Delta V_{\perp})^2 = - \frac{2^{1/2} n_i Z^2 Z_b^2 e^4 M_b^{1/2} \ln N}{8\pi \epsilon_0^2 M} \frac{W_b}{W_b^{3/2}} \quad (14.24)$$

i.e. the same with (14.20) for M_b >> M case.

→ We can deduce (14.20) and (14.24) apply to the case $M_b \approx M$ as well.

14.4. "Critical" Beam-Ion Energy.

Combining (14.13) and (14.20), we have

$$\frac{d}{dt} W_b = - \frac{2^{1/2} n_e Z_b^2 e^4 m_b^{1/2} \ln \Lambda}{6\pi^{3/2} \epsilon_0^2 M_b} \left(\frac{W_b}{T_e^{3/2}} + \frac{C}{W_b^{1/2}} \right) \quad (14.25)$$

where $C = \frac{3\pi^{1/2} Z M_b^{3/2}}{4m_b^{1/2} M} \approx 57$, for proton M_b and M, Z .

⊕ Two terms on RHS of (14.25) are equal for

$$\frac{W_{b,crit}}{T_e} = C^{2/3} \approx 15.$$

⊕ For $W_b > W_{b,crit}$, slowing-down on electrons (1st term) dominates,
and for $W_b < W_{b,crit}$ " ions (2nd term) dominates.

Homework

Problem 14.1 on page 239.

14.5. FOKKER-PLANCK EQN for Energetic Ions

- (*) For isotropic source of energetic ions, we can ignore the effects of pitch angle scattering and consider only slowing down due to collisions with background electrons and ions as described by Eqs (14.12) and (14.19). Then FP eqn without velocity space diffusion is:

$$\left(\frac{\partial f}{\partial t} \right)_{\text{coll.}} = - \frac{\partial}{\partial \vec{V}} \cdot \left(\frac{d \langle \Delta \vec{V} \rangle}{dt} f \right) \quad (14.28)$$

From (14.12), (14.19) and (14.28),

$$\frac{\partial f_b}{\partial t} = \frac{n_e Z Z_b^2 e^4 k u}{4 \pi \epsilon_0 M_b M} \frac{\partial}{\partial \vec{V}} \cdot \left[\frac{\vec{V}}{\sqrt{3}} \left(1 + \frac{V^3}{V_{\text{crit}}^3} \right) f_b \right] \quad (14.29)$$

where

$$\bar{V}_{\text{crit}} = \left(\frac{2 W_{b,\text{crit}}}{M_b} \right)^{1/2} = 3^{1/3} Z^{1/3} \left(\frac{\pi}{2} \right)^{1/6} \left(\frac{T_e}{m^{1/3} M^{2/3}} \right)^{1/2}$$

(*) In spherical coordinates in velocity space, the divergence operator for this situation with isotropic $f_b(v)$ simplifies:

$$\frac{\partial}{\partial \vec{v}} \cdot \vec{A} = \frac{1}{v^2} \frac{\partial}{\partial v} (v^2 A_r) + \frac{1}{V \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{V \sin \theta} \frac{\partial}{\partial \phi} A_\phi,$$

\Rightarrow Eq (14.29) becomes,

$$\frac{\partial f_b}{\partial t} = \frac{n_e Z^2 b^2 e^4 \ln \Lambda}{4\pi \epsilon_0^2 M_b M} \frac{1}{V^2} \frac{\partial}{\partial v} \left[\left(1 + \frac{V^3}{V_{\text{crit}}^3} \right) f_b \right] \quad (14.30).$$

(*) Consider the energetic ions created (or injected) at $V = V_0$ continuously in time at a source rate "S".

$$\left(\frac{\partial f_b}{\partial t} \right)_{\text{Source}} = \frac{S' S (V - V_0)}{4\pi V^2} \quad (14.31)$$

By definition, $\left(\frac{\partial n}{\partial t} \right)_{\text{Source}} = \int dV 4\pi V^2 \left(\frac{\partial f_b}{\partial t} \right)_{\text{Source}} = S$.

(*) For $V \neq V_0$, $\frac{\partial f_b}{\partial t} = 0$ at steady state ($(14.30) = 0$),

$$\Rightarrow \begin{cases} f_b = 0 & \text{for } V > V_0 \\ (1 + \frac{V^3}{V_{\text{crit}}^3}) f_b = C & \text{for } V < V_0 \end{cases}$$

* A constant "C" can be determined by applying

$$\lim_{\epsilon \rightarrow 0} \int_{V_0 - \epsilon}^{V_0 + \epsilon} dv \quad V^2 \cdot [\text{RHS of (14.30)} + \text{RHS of (14.31)}] = 0,$$

This leads to

$$-C \frac{n_e Z e_b^2 e^{4 \ln \lambda}}{4 \pi \epsilon_0^2 M_b M} + \frac{S}{4 \pi} = 0 \quad (14.33).$$

Finally, the steady state beam distribution function is given by

$$f_b(v) = \frac{S e^2 M M_b}{n_e Z Z_b^2 e^4 \ln \Lambda} \left(\frac{1}{1 + v^3 / V_{\text{crit}}} \right), \quad v \leq v_0$$

$$= 0 \quad v > v_0$$

"Slowing Down Distribution Function"

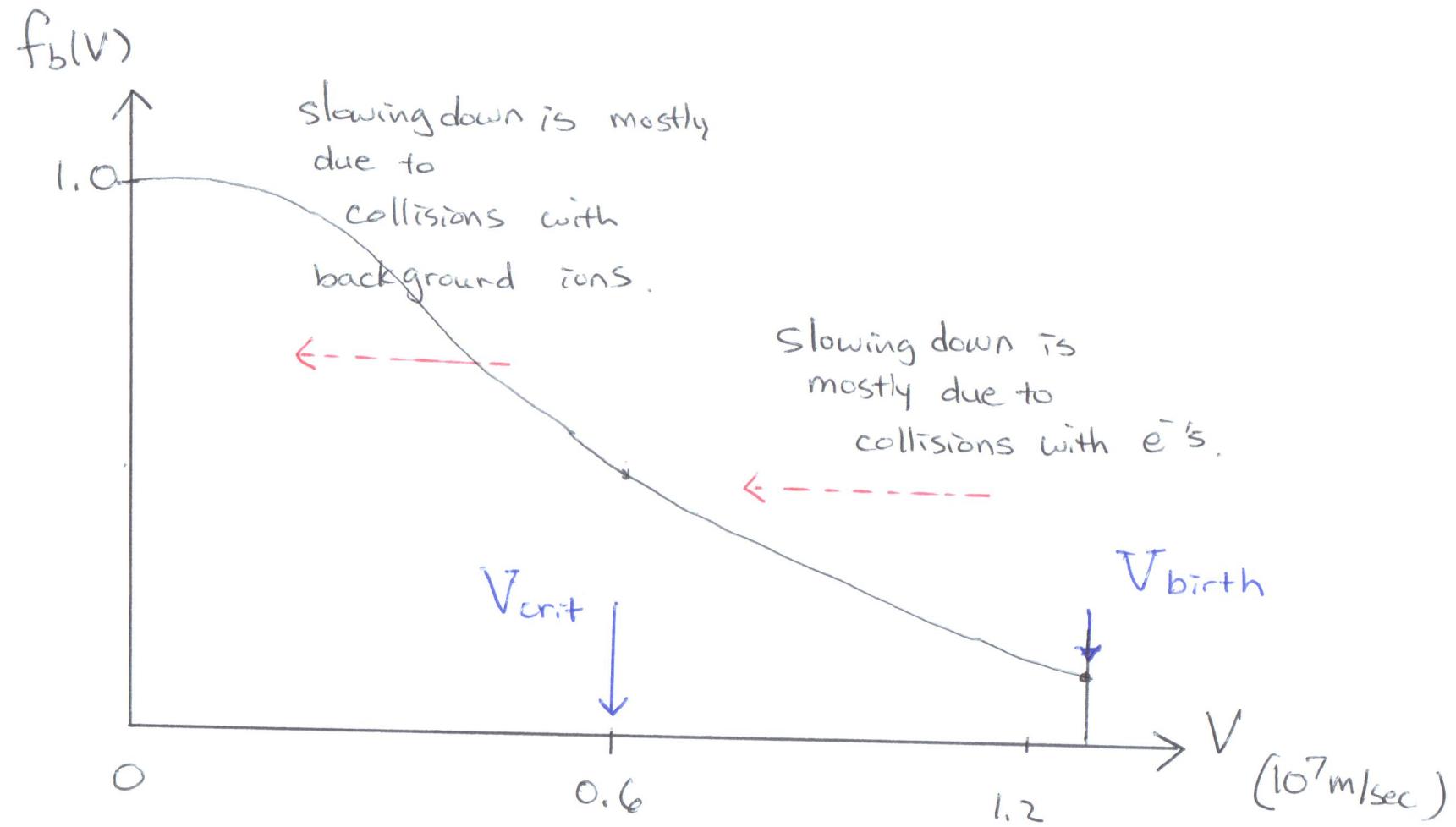
- (*) Application to Fusion product Alpha particles from D-T fusion reaction:

$$S = n_D n_T \underbrace{\langle \sigma v \rangle_{DT}}_{}, \quad \text{strong ftn of temperature.}$$

$$v_0 = 1.3 \times 10^7 \text{ m/sec} \quad \text{for} \quad E_\alpha = 3.5 \text{ MeV}$$

$$\langle \sigma v \rangle_{DT} = 4.2 \times 10^{-22} \text{ m}^3/\text{sec} \quad \text{at} \quad T_i = 20 \text{ keV.}$$

For this, $V_{b\text{-crit}} \approx 30 T_e$ (i.e., 600 keV for $T_e = 20$ keV),



For $T_e = 20$ keV, $E_b = 3.5$ MeV

Fig 14.2.