

Fusion Plasma Theory I. 2019

Week 9

Magnetically Trapped Particles in Tokamak Plasmas

$$\textcircled{+} \text{ Equilibrium: } \vec{B} = \frac{B_0}{1 + \frac{r}{R_0} \cos\theta} \hat{\phi} + \frac{\epsilon}{q(r)} \frac{B_0}{1 + \frac{r}{R_0} \cos\theta} \hat{\theta}$$

" toroidal "

" poloidal "

- $\epsilon = r/R_0$: inverse aspect ratio

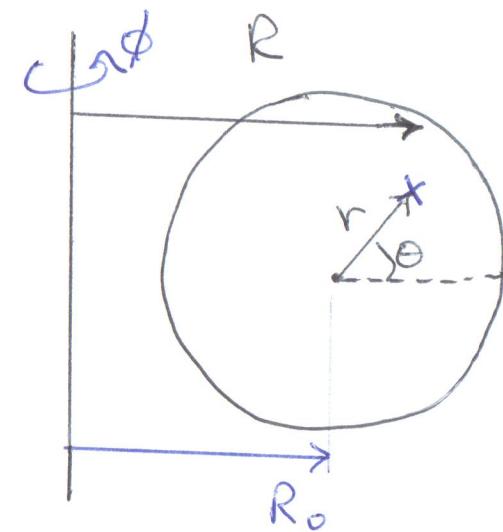
- $q(r) \approx r B_\phi / R B_0$: safety factor.

$$- B \equiv |\vec{B}| \approx B_\phi = \frac{B_0 R_0}{R} = \frac{R_0}{1 + \left(\frac{r}{R_0}\right) \cos\theta}$$

$\textcircled{+}$ (Axisymmetric)-Equilibrium:

No dependence on " ϕ ": $\frac{\partial}{\partial \phi}$ (equilibrium quantity) = 0,

" ϕ " is a cyclic (ignorable) coordinate.



- ⊕ ϕ is a cyclic variable \Rightarrow Canonical Angular momentum P_ϕ is conserved.

$$P_\phi = R(mv_\phi + \frac{ze}{c} A_\phi) \approx R(mv_{||} + \frac{ze}{c} A_\phi) = Rmv_{||} - \frac{ze}{c}\psi$$

—
for ion

where ψ is the poloidal flux function ($d\psi = RB_\theta dr$)

- ⊕ Unperturbed particle orbits in a tokamak stay on the surface of constant P_ϕ , constant μ , and constant E .

- ⊕ Let's consider a guiding-center motion along \vec{B} .
Let "l" be a distance along \vec{B} :

$$dl^2 = R^2 d\phi^2 + r^2 d\theta^2 = (gR_0)^2 d\theta^2 + r^2 d\theta^2 \approx (gR_0)^2 d\theta^2,$$

$$\text{where } g = \frac{d\phi}{dt} = rB\psi/RB_\theta, \Rightarrow v_{||} = \frac{dl}{dt} = gR_0 \frac{d\theta}{dt}$$

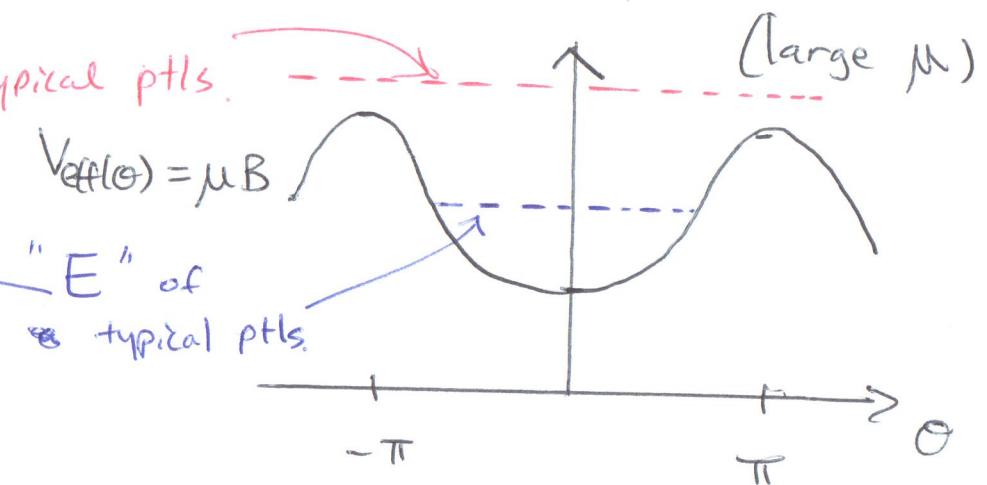
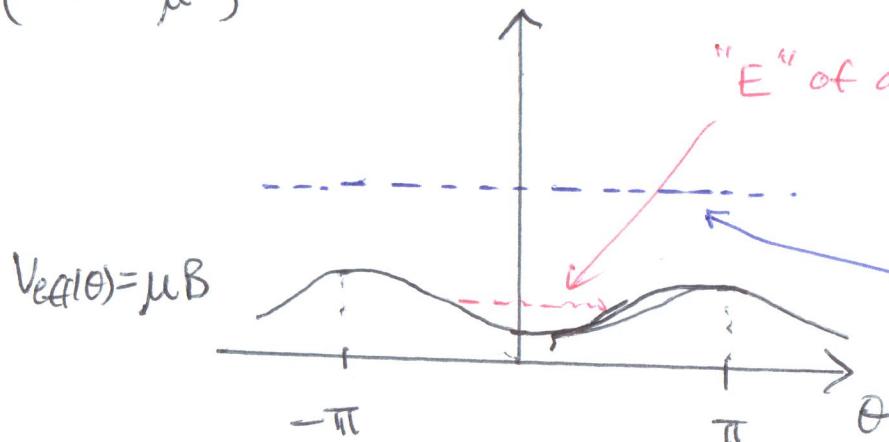
④ For $\frac{r}{R_0} \ll 1$, $\vec{\Phi}_0 = 0$, $\vec{E}_0 = 0$

$$E = \frac{1}{2}mv_{\parallel}^2 + \frac{1}{2}mv_{\perp}^2 = \frac{1}{2}mg^2R_0^2\left(\frac{d\theta}{dt}\right)^2 + \underline{\mu B_0(1 - \frac{r}{R_0}\cos\theta)}$$

- An effective potential for motion along \vec{B} projected to θ originated from \perp kinetic energy.

⑤ Note that the value of μ is a constant in time, but different for different particles.

(small μ)



⊕ For trapped particles, there is a θ_T such that

$$E = \mu B(\theta = \theta_T) = \frac{1}{2} m v_{\parallel}^2(\theta=0) + \mu B(\theta=0).$$

θ_T : is the turning point where $v_{\parallel}=0$.

⊕ For passing particles, $v_{\parallel} \neq 0$ even at $\theta = \pm\pi$ where B is maximum, (and $|v_{\parallel}|$ is minimum). as a function of θ .

→ trapped-passing boundary is determined from

$$\frac{1}{2} m v_{\parallel}^2(\theta=\pm\pi) + \mu B_0 \left(1 + \frac{r}{R_0}\right) = \frac{1}{2} v_{\parallel}^2(\theta=0) + \mu B_0 \left(1 - \frac{r}{R_0}\right).$$

By requiring $v_{\parallel}^2(\theta=\pm\pi) > 0$ for passing pts, we obtain

$$v_{\parallel}^2(\theta=0) > 4 \frac{r}{R_0} \frac{\mu B_0}{m},$$

④ Fraction of trapped pts = $\frac{\# \text{ of trapped pts}}{\# \text{ of total pts}} \propto \sqrt{\epsilon}$
 for isotropic distribution ~~of~~ in \vec{v} .

⑤ Radial Width of Orbits:

- Calculate the radial deviation from the reference flux surface ψ_0
 (as a function of θ):

Since $P_\phi = -\frac{Z|e|}{c} \psi + m R U_{||}(\theta) = -\frac{Z|e|}{c} \psi_0$ for each ptl,
 trapped

$$\Delta_b = \frac{2 \Delta \psi}{RB_\theta} = \frac{2}{RB_\theta} \frac{mc}{Z|e|} R U_{||}(\theta=0) = \frac{2mc}{Z|e|B_\theta} U_{||}(\theta=0),$$

for atypical trapped ptl, $U_{||}(\theta=0) \approx \sqrt{\epsilon} v$

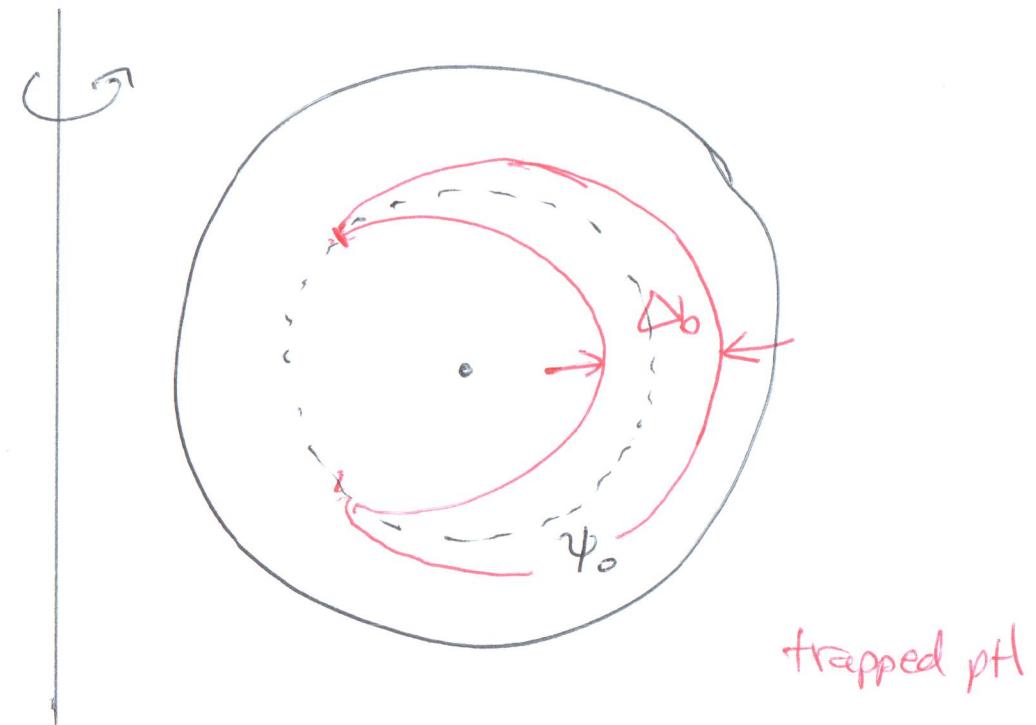
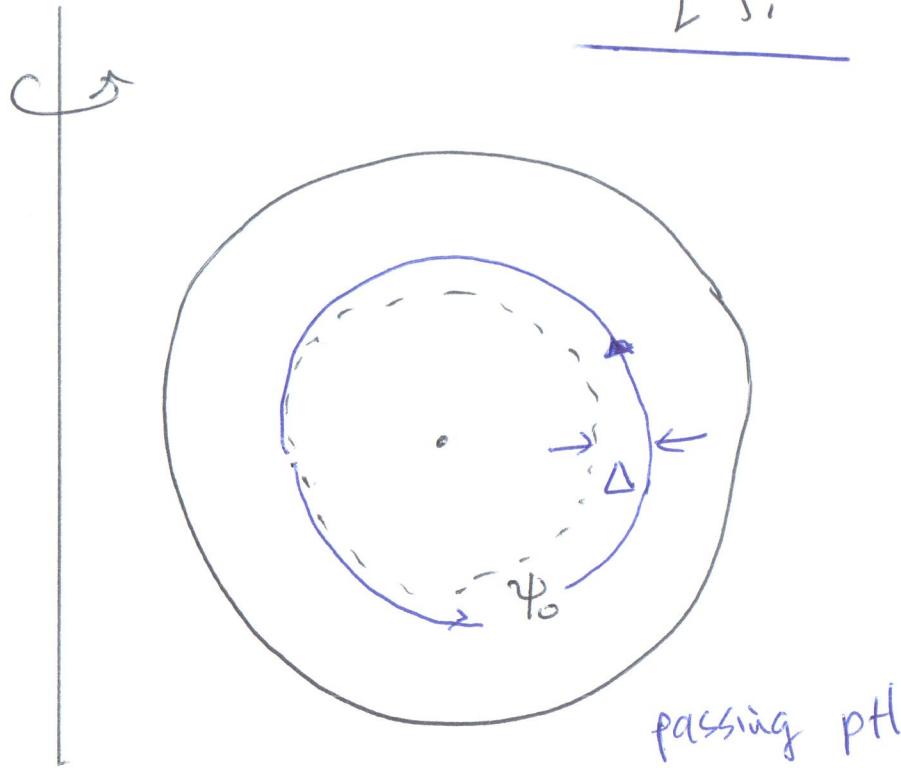
* $\Delta_b \approx \sqrt{\epsilon} \rho_i \frac{q}{E} \approx \frac{q}{\sqrt{\epsilon}} \rho_i$ banana width.

⊗ Meanwhile, for strongly passing pts, variation of v_{\parallel} along \vec{B} is not significant.

$$\rightarrow \underline{\Delta} \simeq \frac{2\Delta\psi}{RB_0} \simeq \frac{2}{RB_0} \frac{mc}{2|e|} v_{\parallel}(0=0) \Delta R \approx \frac{2mc}{2|e|B_0} \frac{r}{R} v_{\parallel}$$

$$\simeq \frac{2q\beta_i}{r}$$

$$(v_{\parallel} \gg v_{\perp} \rightarrow v_{\parallel} \simeq v).$$



* Neoclassical Transport

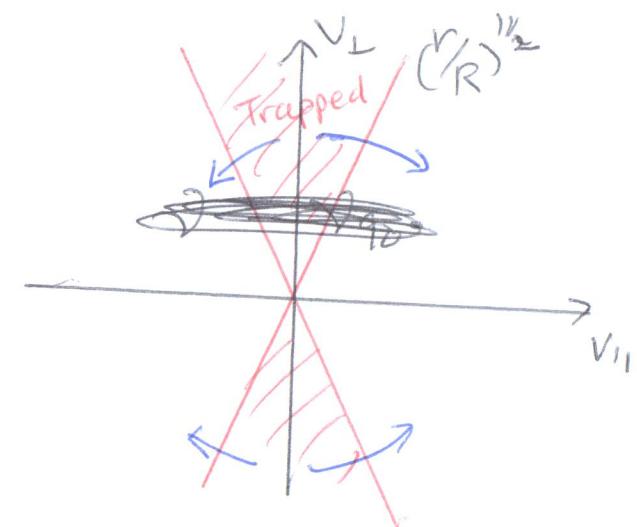
II-4.0

~ Coulomb collisional transport of charged particles influenced by toroidal geometry.

* Banana Orbits:

- Radial Width: $\Delta b \approx \left(\frac{r}{R}\right)^{1/2} \rho_\theta$, $\rho_\theta = \frac{v_{th}}{\Omega} \frac{B_\phi}{B_\theta} = \frac{B_\phi}{B_\theta} \rho$,
- Effective Collisional Frequency for scattering through pitch-angle $\Delta(v_{\parallel}/v)$:

$$v_{eff} \equiv v / [\Delta(v_{\parallel})]^2 \approx \left(\frac{R}{r}\right) v,$$

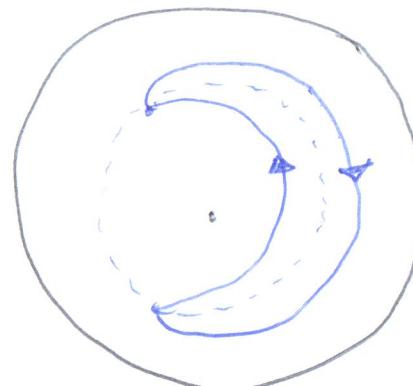


* Collisionality Regimes

I. Banana(Collisionality) Regime :

"Charged ptl can execute several banana orbits ~~without~~ before being scattered into loss cone and become passing ptl."

$$\rightarrow "V_{eff} < \omega_b"; \text{bounce frequency} \approx \left(\frac{r}{R}\right)^{1/2} \frac{v_{Th}}{qR}.$$



$$\omega_t = \frac{v_{Th}}{qR} : \text{transit frequency.}$$

$$\omega_b \sim \frac{1}{T_{bounce}}$$

of banana orbit
execution.

II. "Plateau" Regime;

for

$$\left(\frac{r}{R}\right)^{3/2} < \gamma / \left(\frac{v_{th}}{qR}\right) < 1$$

- Somewhere between "banana" and "P-S" regime
- Diffusion rate is almost independent of collisionality " γ "
 → "plateau".

III. Pfirsch - Schlüter Regime;

for $\gamma \cdot (qR/v_{th}) > 1$, i.e., mean free path of untrapped pts
 is shorter than the "connection length" $\sim qR$ between
 top and bottom of torus.

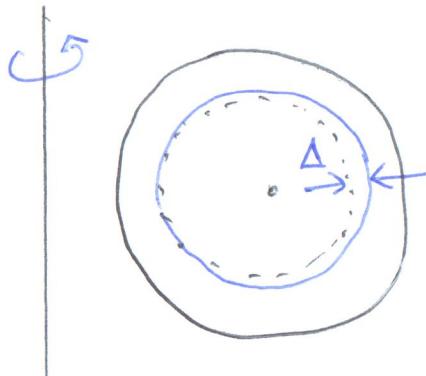
- "Banana orbits cannot maintain their identities even during one excursion."

④ Radial Diffusion Coefficient

$$\textcircled{4} \quad D \approx \left(\frac{\text{fraction of participating ptls}}{\text{effective collision freq.}} \right) \times \left(\frac{\text{radial stepsize}}{\Delta t} \right)^2$$

$\sim \frac{\Delta x^2}{\Delta t}$ from random walk.

III. Pfirsich-Schlüter Regime:



Δ : radial deviation from reference flux surface of passing ptls. $\approx qS$

$$D_{ps} \approx \nu (qS)^2 \approx q^2 (\nu S^2)$$

$$(\quad) \approx \left(1 - \left(\frac{r}{R} \right)^2 \right) \approx 1$$

"classical" diffusion coeff.

(in cylinder).

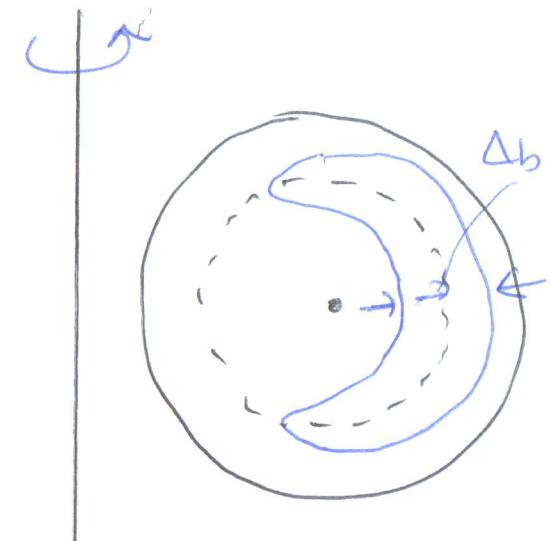
① Banana Regime:

$$D_{\text{Banana}} \approx \left(\frac{r}{R}\right)^{1/2} \cdot D_{\text{eff}} \cdot \Delta b^2$$

$$\approx \left(\frac{r}{R}\right)^{1/2} \cdot \frac{\gamma}{(r/R)} \cdot \left(\frac{\gamma}{\left(\frac{r}{R}\right)^{1/2}} \rho\right)^2$$

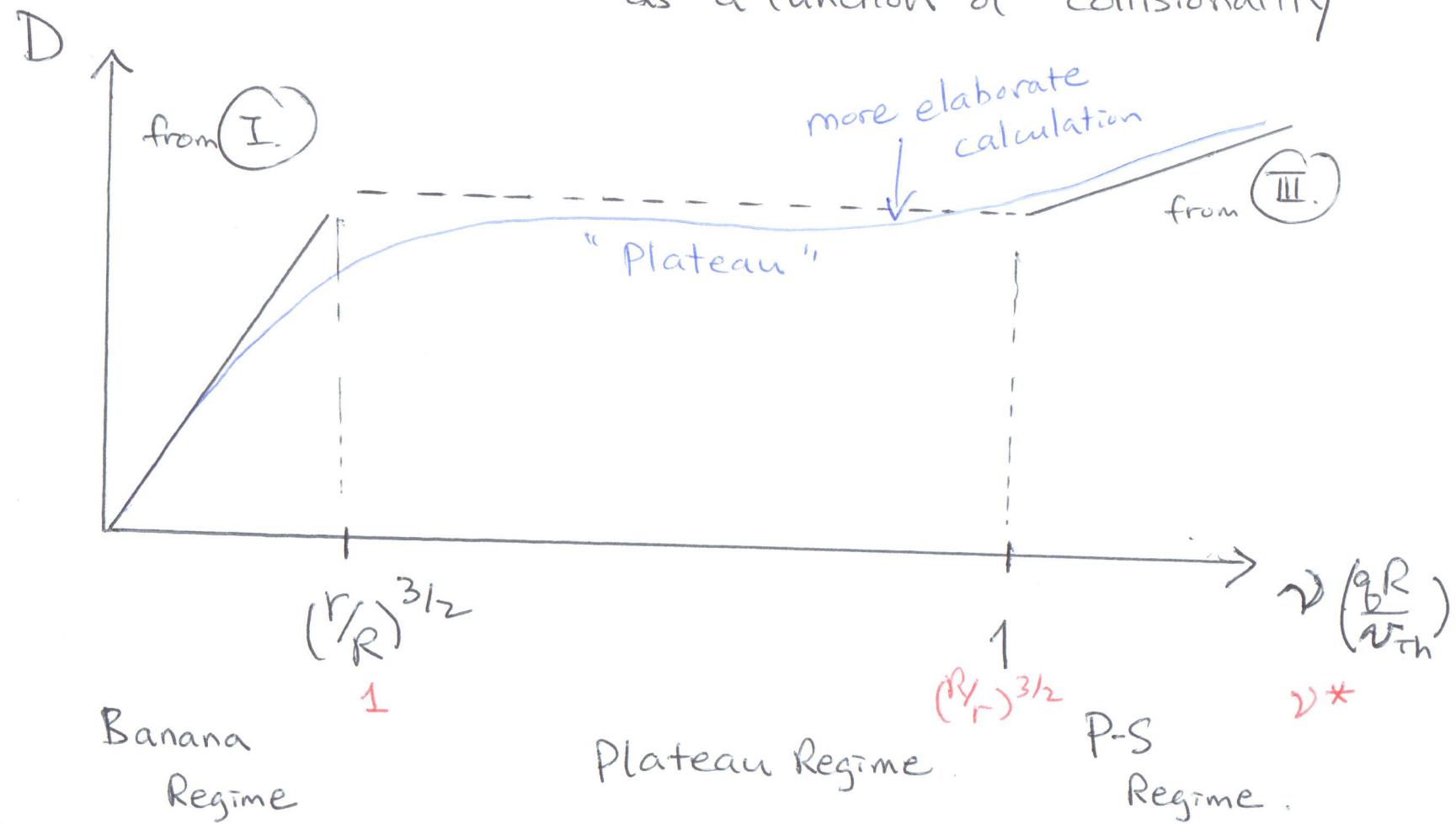
$$\approx \frac{\gamma^2}{(r/R)^{3/2}} (\rho^2) \approx \left(\frac{r}{R}\right)^{1/2} \gamma \rho_\theta^2.$$

banana enhancement over
factor classical diffusion



* Neo classical Diffusion

as a function of collisionality



(*)

$$\nu^* = \nu_{eff}/\omega_b = \nu \left(\frac{qR}{v_{th}} \right) \left(\frac{R}{r} \right)^{3/2}$$

Standard Definition !

④ Remarks on NeoClassical Transport :

- * Disparities among γ_{ee} , $\gamma_{ei} \leftrightarrow \gamma_{ii} \leftrightarrow \gamma_{ie}$ still exist.
- * Ambipolar constraints for particle diffusion still exist for axi-symmetric tokamaks,
- * Predicted rate of transport is much lower than experimental estimations unless turbulence is sufficiently suppressed.
- * It can be relevant for heavy metal impurities due to $Z_I \gg 1$.